SEISMIC STRENGTH REDUCTION FACTOR FOR SINGLE AND MULTI-STOREY SHEAR BUILDINGS CONSIDERING SOIL-STRUCTURE INTERACTION

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ABSTRACT

A parametric analysis has been performed to study the strength-ductility relationship of buildings with different fundamental periods considering Soil-Structure Interaction (SSI). Multi-storey shear buildings are combined with the cone model that represents a homogeneous soil half-space to provide a simplified SSI model. These buildings are designed with the IBC 2012 lateral force distribution and are then subjected to a group of 6 simulated spectrum compatible earthquakes. The effects of structure to soil stiffness ratio \(\alpha_s\), slenderness of the structure \(s\) and number of storeys \(N\) on the strength reduction factor \(R_\mu\) and the modification factor \(R_M\) have been investigated. It is concluded that using \(R_\mu\) calculated based on fixed-based systems in SSI design would underestimate the required strength for a given target ductility demand and, therefore, is not suitable for practical design purposes. It is also shown that the combined effect of \(\alpha_0\) and \(s\) invariantly results in an elongated period, while it can either increase or decrease the overall damping of SSI systems when compared to their fixed-base counterparts. The results indicate that the modification factor \(R_M\) for inelastic SSI systems is mainly affected by the structure to soil stiffness ratio, fundamental period and slenderness ratio of the building, but it is not sensitive to the variation of ductility demand and number of storeys.

INTRODUCTION

Modern seismic provisions allow structures to dissipate seismic energy by undergoing inelastic deformation in severe earthquake events. The expected level of inelasticity is usually achieved by reducing the elastic design response spectra. For example, the design lateral strength can be much lower than that required to maintain a structure in elastic state during strong ground motions. This reduction in strength is quantified by a reduction factor given by Equation 1 (Miranda, 1997)

\[
R = R_\mu \cdot R_M \cdot R_s
\]  

(1)

where \(R_\mu\) is associated with the reduction in strength for a single-degree-of-freedom (SDOF) structure due to its inelastic hysteretic behaviour; \(R_M\) is a modification factor that takes into account the multi-degree-of-freedom (MDOF) effects; and \(R_s\) accounts for the overstrength of structural components and other safety factors.

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In the past four decades, extensive studies have been dedicated to \( R_\mu \) (e.g. Riddell, et al., 1989; Fischinger et al., 1994) and \( R_M \) (Santa-Ana and Miranda, 2000; Moghaddam and Mohammadi, 2001) that are used in conventional design for fixed-base structures. In these studies, \( R_\mu \) and \( R_M \) were obtained over a wind range of structural period \( T \) for different values of the ductility ratio \( \mu \). A comprehensive review of previous studies on the \( R_\mu - \mu - T \) relationship was made by Miranda and Bertero (1994). On the other hand, the effects of soil flexibility have been seen to increase the natural period of the Soil-Structure Interaction (SSI) system. Moreover, the soil that is introduced to a SSI system dissipates some of the seismic energy absorbed by the structure through internal friction (hysteretic damping) and radiation damping. The period lengthening and damping effects have been regarded as beneficial and are coded into some of the design provisions (NERHP, 1997; ATC 3-06; FEMA 450). As a consequence, the above-mentioned reduction factors should be modified accordingly.

While a number of analyses have been performed to study the strength reduction factors of SDOF SSI systems (Avilés and Pérez-Rocha, 2005; Ghannad and Jahankhah, 2007; Jarernprasert, 2012), less attention has been paid to the inelastic force demands of multi degree-of-freedom (MDOF) SSI systems. In this study, the strength reduction factor \( R_\mu \) and the modification factor \( R_M \) for SSI systems are calculated and compared to those obtained from their fixed-base counterparts. Analysis is performed on a simplified SSI model subjected to a group of 6 simulated earthquakes compatible with IBC-2012 design spectra for Site E (soft soil profile). Effects of soil flexibility, fundamental period and slenderness ratio of the structure, target ductility demand and number of storeys are investigated.

**MODELLING AND ASSUMPTIONS**

The multi-storey buildings in this study are modelled as shear buildings, consistent with previous seismic analyses of building structures (e.g. Hajirasouliha and Pilakoutas, 2012). The reliability and accuracy of these models can be found in Lai et al. (1992). In shear-building models, the mass of each floor is assumed to be lumped at the centre of the beam and is connected by an elastic-perfectly-plastic spring that models the shear deformation. Each building is considered to have a storey height of \( h = 3m \) and a total mass \( m_{tot} \) which is uniformly distributed along the height. The strength and stiffness distribution is assumed to be proportional to the storey shear force distribution in accordance with the IBC 2012 lateral force pattern.

The homogeneous soil half-space is modelled by a discrete-element model based on the concept of the cone model (Ehlers, 1942). This model has been found to be adequate for practical applications (Meek and Wolf, 1992). A Sway-Rocking SSI model consisting of a translational and a rotational cone that supports a shear building is shown in Figure 1a. Both cones are constructed as an interconnection of a small number of masses, springs, and dampers whose properties are given by Wolf, (1994).

\[
k_h = \frac{8\rho V_s^2 r}{2-\nu}, c_h = \rho V_s \pi r^2
\]

\[
k_\theta = \frac{8\rho V_s^2 r^3}{3(1-\nu)}, c_\theta = \frac{\rho V_s \pi r^4}{4}
\]

\[
M_v = \frac{9}{128} (1-\nu) \pi^2 \rho r^5 \left( \frac{V_p}{V_s} \right)^2
\]

where \( k_h, k_\theta \) and \( c_h, c_\theta \) are the coefficients of springs (denoted by \( k \)) and radiation damping dashpots (denoted by \( c \)) for the horizontal (denoted with subscript \( h \)) and rocking (denoted with subscript \( \theta \)) motions, respectively, as illustrated in Figure 1a. The properties of the homogeneous soil half-space include its mass density \( \rho \), Poisson’s ratio \( \nu \), shear wave velocity \( V_s \) and dilatational wave
velocity $V_p$. The equivalent radius of the shallow foundation is assumed to be $r$. To account for the frequency dependence of the rocking components of the cone model in the time domain analysis, an additional rotational degree of freedom $\varphi$, with its own mass moment of inertia $M_\varphi$, is introduced as shown in Figure 1a.

![Figure 1. (a) Simplified Soil-Structure Interaction model; (b) Comparison of the IBC-2012 response spectrum for soil type E with the average spectrum of the synthetic earthquakes](image)

Six spectrum-compatible synthetic earthquakes are generated to represent the elastic design response spectra of IBC-2012 corresponding to soil types E (Figure 1b), which is described as a very soft soil profile with $V_s \leq 180m/s$. This design response spectrum is assumed to be an envelope of the possible ground motions that could occur at the site.

**MODELLING PARAMETERS**

The main properties of a SSI system can be well described by the following non-dimensional parameters:

1. The structure-to-soil stiffness ratio $a_0$, which is defined as

   $a_0 = \frac{w_n \bar{H}}{V_s}$  \hspace{1cm} (5)

   where $w_n = 2\pi/T_n$ is the circular frequency of the fixed-base structure corresponding to its first mode of vibration, with $T_n$ being the fundamental period, and the effective height of the structure $\bar{H}$ is approximated as 0.7 times the total height ($H_{tot}$). $a_0$ indicates the degree of influence of SSI and generally varies from zero, for fixed-base structures, to about two, where the soil becomes very flexible compared to the structure.

2. The slenderness ratio of the structure $s$, which is given by

   $s = \frac{\bar{H}}{r}$  \hspace{1cm} (6)

3. The structure-to-soil mass ratio $\bar{m}$ that is
\[ \bar{m} = \frac{m_{\text{tot}}}{\rho H_{\text{tot}} r^2} \]  

(7)

Where \( m_{\text{tot}} \) is the total mass of the structure. Additional parameters used in this study include the structural ductility ratio \( \mu_{\text{max}} \), which is the maximum inter-storey ductility at the critical storey; the foundation-to-structure mass ratio \( m_f/m_{\text{tot}} \), where \( m_f \) is the mass of the foundation that determines the mass moment of inertia of the foundation \( I_f (0.25m_f r^2) \); the damping ratio of the structure \( \zeta \); number of storeys \( N \); and Poisson’s ratio of the soil \( \nu \). The values of the parameters used in this study are presented in Table 1.

Table 1. Modelling parameters used in this study

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( a_0 )</th>
<th>( s )</th>
<th>( \bar{m} )</th>
<th>( \mu_{\text{max}} )</th>
<th>( m_f/m_{\text{tot}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>0,1,2,3</td>
<td>1,2,3,4,6,8</td>
<td>0.5</td>
<td>1,2,4,6</td>
<td>0.1</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>1,3,5,10,15</td>
<td>0.05</td>
<td>0.33</td>
<td>1800</td>
<td>3</td>
</tr>
<tr>
<td>( \nu )</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

RESULTS AND DISCUSSIONS

Strength Reduction Factor

For a SDOF system, \( R_\mu \) is a factor which reduces the elastic base shear to that required to avoid the maximum ductility larger than the design target ductility \( \mu_t \).

\[ R_\mu = \frac{V_{\text{SDOF}(\mu=1)}}{V_{\text{SDOF}(\mu=\mu_t)}} \]  

(8)

where \( V_{\text{SDOF}(\mu=1)} \) and \( V_{\text{SDOF}(\mu=\mu_t)} \) are the base shear demands of SDOF structures to remain elastic \( (\mu = 1) \) and to limit their maximum ductility ratios to the target value \( (\mu_t) \), respectively. In the present study, \( R_\mu \) factors are calculated from constant ductility spectra obtained for given \( a_0 \) and \( s \) values. The mean \( R_\mu \) factors, shown in Figure 2, are calculated by averaging the reduction factors obtained from the six spectrum compatible earthquakes.

As mentioned earlier, \( a_0 \) is a factor that controls the severity of the SSI phenomena, and \( s \) is the slenderness of the building. It is observed from Figure 2 that in general, regardless of \( a_0 \) and \( s \), \( R_\mu \) for nearly rigid structures \( (T_n \to 0) \) tends to one and increases with increasing the flexibility of the buildings. For fixed-base structures \( (a_0 = 0) \) having a long natural period \( (\text{i.e. very flexible structure}) \), \( R_\mu \) approaches the target ductility ratio. These observations are consistent with the evidence presented by Miranda and Bertero (1994). Figure 2 also shows that increasing \( a_0 \) (including SSI) reduces \( R_\mu \) of flexibly-supported structures when compared to their fixed-base counterparts. The reduction is more obvious when the \( a_0 \) value is smaller than 2, and is up to 75% for a structure with predominant SSI effects \( (a_0 = 3) \) as seen in Figure 2 (f). Unlike the rigidly-supported structures whose \( R_\mu \) factors are not affected by the slenderness ratio in SSI systems, \( R_\mu \) for slender structures is smaller than squatty structures. However, the strength reduction factor \( R_\mu \) is mainly influenced by the relative stiffness \( a_0 \), and to a lesser degree, by the slenderness of the structures. This indicates that using fixed-base \( R_\mu \) to design a slender building considering SSI can be unconservative.
Figure 2. Strength reduction factor $R_\mu$ for SDOF SSI systems, average of six spectrum compatible earthquakes

It should be noticed that the $R_\mu$ factor is not only a function of the dynamic properties of the SSI system, but also related to site condition and the ground input motion. However, the mean $R_\mu$ factor for a SSI system subjected to a given set of earthquake ground motions (ignoring any strength hardening or softening) can be expressed as a function of the effective system period $T$, damping ratio $\zeta$ and the target ductility demand $\mu_t$. The period and damping ratio of SSI systems can be calculated from their fixed-base counterparts $T_n$ and $\zeta_n$ using the modified expressions based on the work of Veletsos and Meek (1974)

$$\begin{align*}
T &= T_n \sqrt{1 + \left(\frac{2 - \nu}{5.6}\right) \tilde{m} a_0^2 s^{-1} + 1.093(1 - \nu)\tilde{m} a_0^2 s} \\
\zeta &= \left(\frac{T}{T_n}\right)^3 \left(\zeta_n + \frac{3}{256} \left(2 - \nu\right) \tilde{m} a_0^2 \left[\frac{2 - \nu}{s^2} + \frac{9(1 - \nu)^2}{2 - \nu}\right]\right)
\end{align*}$$

The effectiveness of these relationships is demonstrated in Figure 3, where the predicted curves from Equations (9) and (10) match very well with the data obtained from an Eigen analysis.
One of the controversial issues concerning the strength reduction factor is that $R_\mu$ obtained from a number of earthquakes may not be applicable to all cases given the variability of the ground motions and site conditions. It is worth mentioning that Ordaz and Pérez-Rocha (1998) proposed the following reduction rule, where $R_\mu$ is dependent on the elastic displacement spectra.

$$R_\mu = R_\mu \left(D(T_n, \zeta_s), D_{\text{max}} \right) = 1 + \left( \frac{D(T_n, \zeta_s)}{D_{\text{max}}} \right)^{\beta(\mu)} \left( \mu - 1 \right)$$

where $D(T_n, \zeta_s)$ is the elastic spectral displacement; $D_{\text{max}}$ is the peak ground displacement and $\beta$ is a curve fitting variable as a function of the ductility. The effects of damping, site conditions and ground motions are implicitly incorporated in the shape of the displacement spectra, which can account for the case-to-case variability. This rule was further extended to SSI systems by Avilés and Pérez-Rocha (2005).

**Modification Factor**

For multi-storey buildings, it is proposed that the $R_\mu$ factor in Equation (8) be multiplied by a modification factor to account for the possible concentration of the ductility in certain floors. This modification factor, denoted by $R_M$ is proposed as:

$$R_M = \frac{V_{MDOF(\mu=\mu_t)}}{F_{MDOF(\mu_{\text{max}}=\mu_t)}}$$

where $F_{MDOF(\mu_{\text{max}}=\mu_t)}$ is the base shear strength that is required to limit the maximum inter-storey ductility of a MDOF system to the target ductility $\mu_t$. The maximum ductility $\mu_{\text{max}}$ is defined as:

$$\mu_{\text{max}} = \text{MAX} \left( \frac{\Delta u_{1\text{max}}}{\Delta u_{1y}}, \frac{\Delta u_{2\text{max}}}{\Delta u_{2y}}, \ldots, \frac{\Delta u_{i\text{max}}}{\Delta u_{iy}}, \ldots, \frac{\Delta u_{N\text{max}}}{\Delta u_{Ny}} \right)$$

where $\Delta u_{\text{max}}$ and $\Delta u_{iy}$ are the maximum and yield lateral deformation exhibited by the $i^{th}$ floor during an earthquake, respectively. By substituting Equations (8) and (12) into Equation (1) and ignoring the overstrength of structural components (i.e. $R_5 = 1$), the base shear strength demand of a MDOF system ($F_{MDOF}$) to achieve the target ductility $\mu_t$ can be calculated by:
The modification factor $R_M$ was studied by Santa-Ana and Miranda (2000) where eight multi-storey steel moment-resisting frame buildings were subjected to 28 earthquake ground motions recorded on different sites. They found that to achieve a target ductility demand, elastic MDOF systems experienced less base shear when compared with their corresponding SDOF systems whereas inelastic MDOF structures attracted higher base shear than that of the equivalent SDOF structures. They also observed that the modification factor $R_M$ was mainly affected by the number of storeys, and to a small extent by the site conditions and fundamental period of the structure.

In this study, $R_M$ is obtained for SSI systems by using Equation (12), where the base shear strength of MDOF systems and their corresponding SDOF systems (having the same total mass) are calculated for the same $T_n, a_0, s$ and $\mu_t$ under a given excitation. The average results from the six spectrum compatible earthquakes are plotted against the number of storeys as presented in Figure 4. The data in Figure 4 (a) shows that $1/R_M$ (i.e. $F_{MDOF}/V_{SDOF}$) for $a_0 = 0$ (fixed-base) falls below $1/R_M = 1$, which agrees with the previously mentioned observation that elastic MDOF systems require lower base shear capacity than SDOF systems. Similar results are observed for structures under moderate SSI effects (i.e. $a_0 = 1$). On the contrary, for severe SSI effects (e.g. $a_0 = 2$ and 3), higher base shear strengths are required by MDOF systems than that of SDOF systems to remain elastic. Similar to $R_M$, the modification $R_M$ can also be expressed as a function of the system period $T(T_n, a_0, s)$, damping ratio $\zeta(\zeta_s, a_0, s)$ and the target ductility demand $\mu_t$. Besides, $R_M$ may be affected by the number of storeys $N$.

![Figure 4. Modification factor $R_M$ for MDOF SSI systems ($T_n = 1s$)](image-url)
The natural period and overall damping of SSI systems depend on the combined effects of $a_0$ and $s$. It is shown in Figure 3 that the period of a SSI system increases with an increase in $a_0$ and $s$. By increasing $a_0$, the damping ratio increases for squatty structures, while it decreases for slender structures. For example, the damping ratio of a SSI system with $a_0 = 3$ and $s = 1$ is five times larger than that of the fixed-base structure. On the other hand, if the slenderness ratio $s$ had been increased to four, the overall damping ratio would reduce by up to thirty percent. This can be explained by the fact that increasing $a_0$ increases the flexibility and damping of the system, while increasing $s$ decreases the radiation damping by reducing the contact area between the foundation and soil. This will hinder the geometric attenuation of wave energy arising from the inertial interaction.

Considering the results in Figure 4 (b) – (f), the modification factor $R_M$ for inelastic systems is mainly affected by $a_0$ and $s$, but it is not sensitive to the variation of target ductility demand and number of storeys. Generally, $1/R_M$ decreases with increasing $s$. The results show that $R_M$ does not change monotonically with increasing $a_0$, and in general, $a_0 = 0$ and 2 lead to a higher strength demand for MDOF systems.

The effect of fundamental period of structures on the modification factor $R_M$ is illustrated in Figure 5. The results indicate that $R_M$ is considerably more sensitive to the variations of $T_n$ for SSI systems compared to fixed-base systems. Generally, $1/R_M$ increases with increasing $T_n$ for SSI systems, while it is almost unaffected by $T_n$ for fixed-based systems. The results presented in Figures 4 and 5 indicate that, for the structures with similar fundamental period and slenderness ratio, the modification factor $R_M$ is not sensitive to the number of storeys.

**CONCLUSIONS**

This study aimed to investigate the effects of effects of Soil-Structure Interaction (SSI) on the strength-ductility relationship of SDOF and MDOF systems. Based on the results from this study, the following conclusions can be drawn:

- Soil structure interaction in general reduces the strength reduction factor $R_\mu$. This reduction can be up to 75% and decreases with increasing flexibility of the soil relative to the structure as well as the slenderness of the structure. Therefore, $R_\mu$ values which are calculated based on fixed-based systems in SSI design underestimate the required strength for a given target ductility demand and, therefore, are not suitable for practical design purposes.

- The combined effect of $a_0$ and $s$, though resulting in an elongated period, can either increase or decrease the overall damping of SSI systems when compared to their fixed-base counterparts. Generally, damping for flexibly-supported slender buildings is smaller than that of rigidly-supported structures, while SSI systems with squatty buildings have larger damping ratios than those of fixed-base buildings.
Elastic MDOF systems with moderate SSI effects require lower base shear capacity than SDOF systems. However, for severe SSI effects, higher base shear strengths are required by MDOF systems than that of SDOF systems to remain elastic.

The modification factor $R_M$ for inelastic SSI systems is mainly affected by the structure to soil stiffness ratio, fundamental period and slenderness of the building, while it is not sensitive to the variation of ductility demand and number of storeys.

REFERENCES


