



CALIBRATION OF ANALYTICAL MODELS FOR LAYERED SYSTEMS USING VIBRATION RECORDS

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ABSTRACT

Layered systems are those whose vibrations can be formulated as a one dimensional wave propagation problem, such as multi-story buildings and layered soil media subjected to seismic waves. Typically, vibrations are recorded at certain floors in buildings, and certain depths in soil layers. This paper first introduces a methodology to estimate vibration time histories at non-instrumented layer interfaces from those at the instrumented interfaces by assuming that the mode shapes of the layered system can be approximated as a linear combination of the mode shapes of a shear beam and a bending beam. Knowing the vibrations at all interfaces, the paper then introduces a methodology to calibrate analytical models based on the observation that, when deconvolved by the uppermost layer's (e.g., roof or ground surface) record, the records in the layers become a simple downgoing wave. The key implication of this observation is that any structural change in a layer's characteristics would only change the deconvolved records at the layers below. In other words, the deconvolved record at a layer depends only on the properties of the layers above. In order to develop the analytical model for a multi-story building, for example, we use the roof-deconvolved records starting from the top story. A simple way for calibration would be to take the top-to-bottom spectral ratios of the deconvolved records for each story, starting from the top story. We can determine the stiffness of each story by matching the dominant frequency of the corresponding spectral ratios. Since any stiffness change in a lower story does not affect the deconvolved signals in upper stories, each story stiffness can be identified uniquely by using the spectral ratios of the deconvolved records, provided that we start from the top.

INTRODUCTION

Layered systems are usually instrumented at a limited number of locations (e.g., certain floors in buildings, and certain depths in layered soil), and the displacements are calculated by double integrating the acceleration records at these locations. The unknown displacements at non-instrumented layer interfaces, such as building floors, can be estimated by using those at the instrumented floors. Knowing the displacements at every floor is important for drift calculations and calibration of analytical models. The common approach to calculate the unknown displacements is to

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interpolate the known displacements over the height of the system by using linear, quadratic, or cubic interpolations. For buildings, these techniques require that a sensor must be installed both at the base and the roof of the building, as well as some intermediate floors, in order to have an acceptable estimate of the displacements at non-instrumented floors. Even though the previous studies states that a cubic polynomial interpolation provides a reasonably good estimates of motions at non-instrumented floors (Naeim, 1997; Naeim et al., 2004), a conclusion is made in Goel (2008) that the cubic polynomial interpolation does not work if the building is not instrumented at regular intervals over its height, and additional instruments are needed at locations where the stiffness changes significantly.

In this study, we present a new method, called Mode-Shape-Based-Estimation (MSBE), that eliminates the limitations of the interpolation techniques. We assume that the mode shapes of a layered system can be approximated as a linear combination of the mode shapes of a shear beam and a bending beam (i.e., Euler-Bernoulli beam).

Once the vibration time histories are known at every layer, a simple methodology, based on the concept of *Interferometric Imaging*, is introduced to develop and calibrate analytical models of layered systems. The methodology utilizes top-to-bottom spectral ratios of the top-deconvolved records at each layer, and shows that these spectral ratios are not influenced by any structural changes in the layers below. Thus, starting from the top layer, the properties of each layer can be determined uniquely by matching the predominant frequencies of the spectral ratios.

MODE SHAPES OF BENDING AND SHEAR BEAMS

Differential equations of the motion of a bending- and a shear-beam are given below, in Eq. 1a and 1b, respectively (Seon et al., 1999)

$$m \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} = f(x, t) \quad (1a)$$

$$\rho A \frac{\partial^2 u}{\partial t^2} - GA_s \frac{\partial^2 u}{\partial x^2} = f(x, t) \quad (1b)$$

where u , m , E , and I are the deflection, mass per unit length, Young modulus of elasticity, and second moment of area of the cross-section of the beam, GA_s is the elastic stiffness of the shear-beam (G is the modulus of rigidity and A_s is the effective shear area), ρA is the mass per linear length of the shear-beam where ρ is the mass density and A is the cross sectional area, and $f(x, t)$ is the loading per unit length of the beam.

The mode shapes of bending and shear beams $\phi_b(x)$ and $\phi_s(x)$ can be found by solving the differential equations above. The expressions for the mode shapes and the details of the derivations can be found in Seon et al. (1999) or in most textbooks in Structural Dynamics. The mode shapes are plotted in Fig. 1.

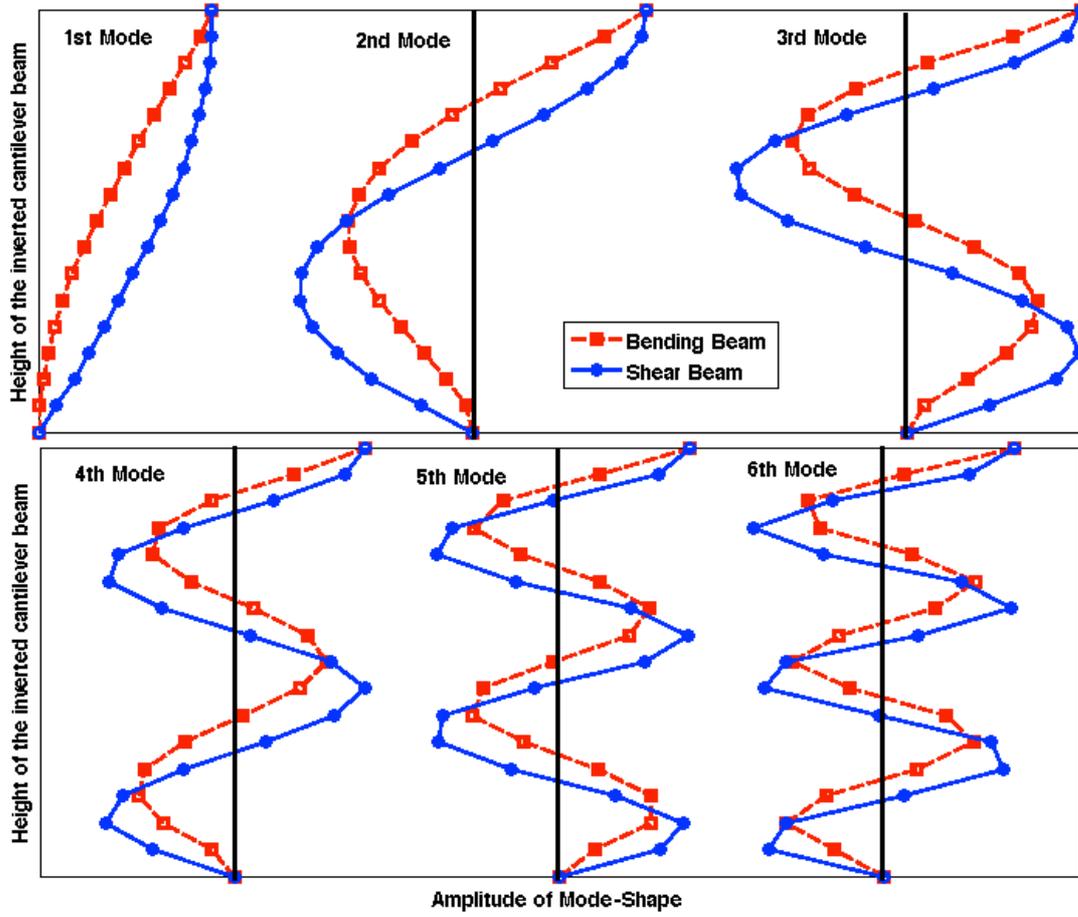


Figure 1. The first six normalized mode shapes of the inverted bending and the shear beams. Due to the differences in boundary conditions, the two beam models result in two different mode shapes: curvature, amplitude, local maximum along the length of the beam, and the shape of the mode.

ESTIMATION OF DISPLACEMENTS AT NON-INSTRUMENTED INTERFACES

Displacements of a N -story linear elastic building can be calculated by the superposition of modal displacements as (Chopra, 2007)

$$u_{j,k}(t) = \Gamma_j \cdot \phi_{j,k} \cdot q_j(t) \quad (2a)$$

$$u_k(t) = \sum_{j=1}^N \Gamma_j \cdot \phi_{j,k} \cdot q_j(t) \quad (2b)$$

where $u_{j,k}(t)$ is the time variation of the j^{th} mode's relative displacement at the k^{th} floor; $u_k(t)$ is the time variation of the k^{th} floor's relative displacement; Γ_j is the j^{th} modal participation factor; $\phi_{j,k}$ is the amplitude of the j^{th} mode shape at the k^{th} floor; and $q_j(t)$ is the time-variations of the displacement of the j^{th} mode of a single-degree-of-freedom system, as

$$\ddot{q}_j(t) + 2\zeta_j\omega_j\dot{q}_j(t) + \omega_j^2q_j(t) = -\ddot{u}_g(t) \quad (3)$$

where ζ_j and ω_j are the damping ratio and the frequency of the j^{th} mode, respectively, $\ddot{u}_g(t)$ is the time variation of the ground acceleration. Denoting modal displacement as $D_j(t) = \Gamma_j \cdot q_j(t)$, the Equation 2a and 2b will lead to

$$u_{j,k}(t) = \phi_{j,k} \cdot D_j(t) \quad (4a)$$

$$u_k(t) = \sum_{j=1}^N \phi_{j,k} \cdot D_k(t) \quad (4b)$$

The MSBE method assumes that any mode shape of a layered system can be estimated as a linear combination of the corresponding mode shapes of a shear beam and a bending beam as given in Eq. 2:

$$\phi_{j,k} = C_{s,j} \cdot \phi_{s,j,k} + C_{b,j} \cdot \phi_{b,j,k} \quad (5)$$

where $\phi_{s,j,k}$ and $\phi_{b,j,k}$ are the amplitudes of the j^{th} mode shapes of a shear beam and a bending beam, respectively, at the k^{th} interface; $\phi_{j,k}$ is the amplitude of the j^{th} mode shape of the building at the k^{th} floor; $C_{s,j}$ and $C_{b,j}$ are the unknown weighting coefficients for the j^{th} mode. The error in the estimation for the j^{th} mode can be expressed as the square sum of the differences over the instrumented floors between the recorded modal displacements, $y_{j,k}(t)$, and the calculated modal displacements, $u_{j,k}(t)$.

$$\varepsilon_j(t) = \sum_{i=1}^{NIF} [y_{j,k}(t) - u_{j,k}(t)]^2 \quad (6)$$

where $\varepsilon_j(t)$ is the error function for the j^{th} mode and NIF is the number of instrumented floors. In order to calculate the recorded modal displacements, $y_{j,k}(t)$, first the modal frequencies of the building are identified by using Fourier/spectral analysis. Next, the recorded accelerations at each instrumented floor is band-pass filtered around each modal frequency of the building, and then double integrated. The summation in the error function (6) is only over the instrumented floors; therefore, the coefficients of $C_{s,j}$ and $C_{b,j}$ can be estimated by making the partial derivatives equal to zero; that is:

$$\frac{\partial \varepsilon_j}{\partial C_{s,j}} = 0, \quad \frac{\partial \varepsilon_j}{\partial C_{b,j}} = 0 \quad (7)$$

which yields two linear equations for the two coefficients

$$\begin{bmatrix} \sum_{j=1}^{NIF} \phi_{s,j,k}^2 & \sum_{j=1}^{NIF} \phi_{s,j,k} \cdot \phi_{b,j,k} \\ \sum_{j=1}^{NIF} \phi_{s,j,k} \cdot \phi_{b,j,k} & \sum_{j=1}^{NIF} \phi_{b,j,k}^2 \end{bmatrix} \cdot \begin{pmatrix} C_{s,j} \cdot D_j(t) \\ C_{b,j} \cdot D_j(t) \end{pmatrix} = \begin{bmatrix} \sum_{j=1}^{NIF} \phi_{s,j,k} \cdot y_{j,k}(t) \\ \sum_{j=1}^{NIF} \phi_{b,j,k} \cdot y_{j,k}(t) \end{bmatrix} \quad (8)$$

Equation 8 has to be satisfied at every time step, t . Note that the matrix on the left-hand side is time-independent; therefore, it needs to be calculated only once. However, the vector on the right-hand side is time-dependent and must be calculated at every time step, t .

Once the coefficients $C_{s,j}$ and $C_{b,j}$ are determined, the time histories of modal displacements and total displacements are calculated from Eqs. 4 and 5. More detail on the MSBE method is given in Kaya et al. (2014).

EXAMPLE FOR THE MSBE METHOD

To present an example for the MSBE method, we will use the data from a densely instrumented building, the Factor Building at UCLA campus in Los Angeles. The building is a 17-story moment-resisting steel-frame structure with a 72-channel accelerometer network, composed of 4 horizontal accelerometers at every floor above the basement, plus 2 vertical and 2 horizontal accelerometers at two basement levels. More information about the building and the instrumentation can be found in Kohler et al. (2005).

Although the records are available from all the floors of the building, in the example, we will assume that the records are available only from three locations: the ground, roof, and an arbitrary mid-floor between the ground and the roof. Only the records from the west side of the building will be considered. First, the modal frequencies of the building are determined by spectral analysis of accelerations at the three instrumented floors (Safak et al., 2010). Next, the acceleration records are band-pass filtered around each modal frequency and then double integrated to determine the modal displacement time histories at the instrumented floors.

By using the MSBE procedure presented above, the modal and total displacements at the remaining non-instrumented floors are determined. The same are also calculated by using linear and cubic polynomial interpolation techniques. Assuming that the instrumented mid-story is the 6th story, the comparison of the calculated displacements at the non-instrumented floors with the recorded displacements are shown in Fig. 2 for the MSBE method, and linear and cubic interpolations.

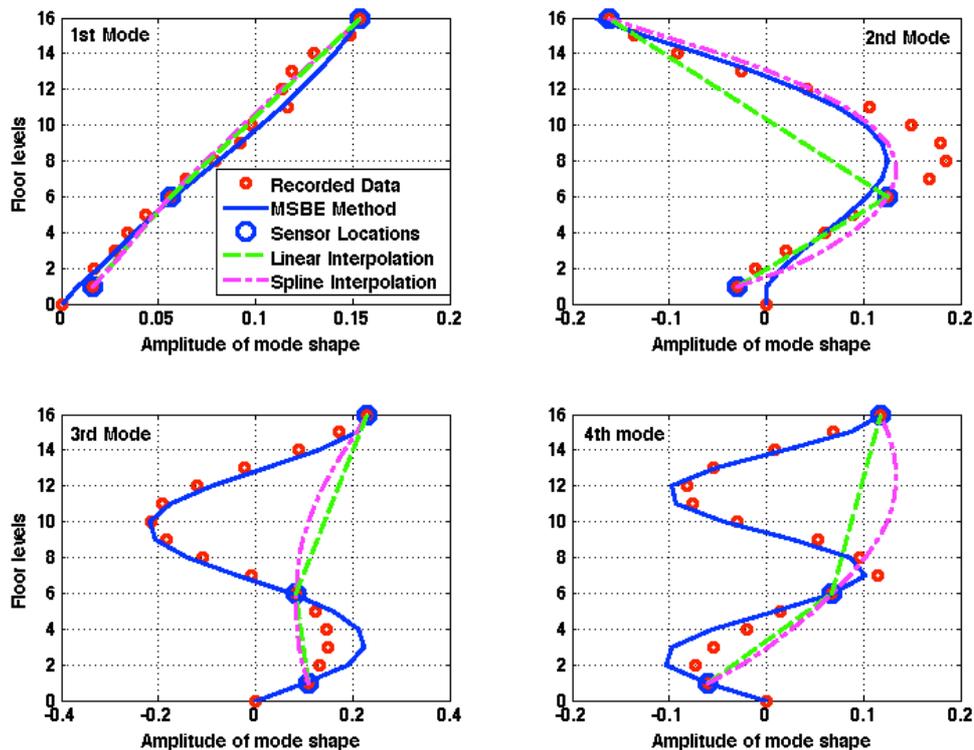


Figure 2. The first four-mode shapes of the Factor building; the red circles are the relative displacement amplitudes (calculated from the recorded accelerations) of the building at each floor level, the blue circles show the location of the sensors of configuration, and the dashed green line, the dashed magenta line, and the solid blue line show the calculated mode shapes (calculated displacement amplitudes at each floor level) using the linear interpolation, cubic spline interpolation, and the MSBE methods, respectively.

The figure shows that the MSBE method provides a much better approximation, especially at higher modes. Similar comparisons are also made using the records from the east, north, and south side sensors, and also assuming other instrumented mid-floors. The results were similar. More examples can be found in Kaya et al. (2014).

CALIBRATION OF ANALYTICAL MODELS

Knowing the displacement time histories at every interface, we present a simple methodology to develop analytical models for layered systems. The methodology is based on the concept of *Interferometric Imaging*. *Interferometry* utilizes the correlations among the synchronized records collected from different locations, and provides a new approach to analyze seismic data from instrumented structures (Snieder and Safak, 2006). It can be shown that when deconvolved by the top-layer record (e.g., roof record or the ground surface record), the accelerations at layer interfaces become a simple downgoing wave. The analytical proof for this is given in (Snieder and Safak, 2006).

We will show an example of this by using the same data set from the Factor Building. Figure 3 shows a 1-sec. portion of accelerations in the Factor building, after they are deconvolved by the roof accelerations. The horizontal axes denote the time and the floor level, and the vertical axis is the accelerations deconvolved by the recorded roof accelerations. The accelerations are color-coded based on their amplitudes. As clearly seen in the figure, when deconvolved by the roof accelerations, the accelerations in the building become a simple downgoing wave.

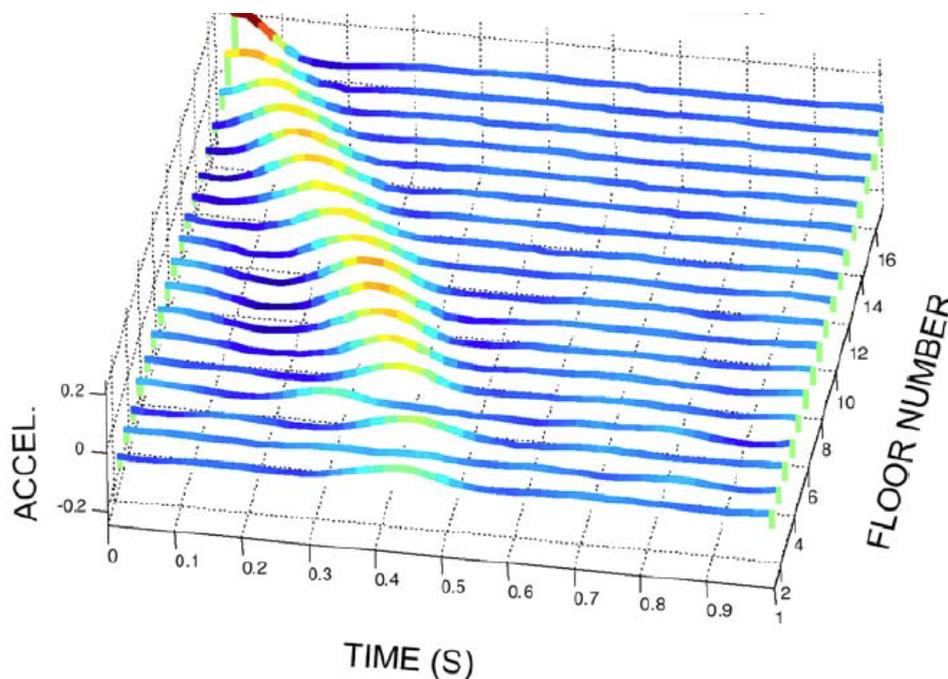


Figure 3. Accelerations of Factor Building after they are deconvolved by roof accelerations

The key implication of this observation is that any structural change in a story would only change the deconvolved records at the floors below that story. In other words, the deconvolved record at a floor depends only on the properties of the stories above. This is not valid for the original records (i.e., without the deconvolution) since they are formed by the combination of both upgoing and downgoing waves (Safak, 1999). In order to develop the analytical model for the building, we should use the roof-deconvolved records starting from the top story. A simple way would be to take the top-to-bottom spectral ratios of the deconvolved records for each story. Starting from the top story, we can determine the stiffness of each story by matching the dominant frequency of the corresponding spectral ratios. Since any stiffness change in a lower story does not affect the deconvolved signals in upper stories, each story stiffness can be identified uniquely by using the spectral ratios of the deconvolved records, provided that we start from the top. Similar approach can be used to detect and identify soil-structure interaction effects in buildings (Safak, 1995).

To illustrate the concept, we use a 12-story simulated building subjected to an earthquake excitation. The stiffness and the mass of each story are 0.05 N/m and 5.625e-4 ton, respectively, and the damping ratio is 1% for all modes. Damage is simulated at the 5th story of the building by reducing the story stiffness by 40%. The modal characteristics of the undamaged and the damaged buildings can be found in Kaya et al. (2014). The top-to-bottom spectral ratios of the accelerations at each story are calculated for the undamaged and damaged cases. The comparison of spectral ratios are plotted in Fig. 4. As the figure clearly shows, the dominant frequencies of the spectral ratios for the undamaged and damaged cases are identical for floors above the damaged floor, but different for the damaged floor and the floors below.

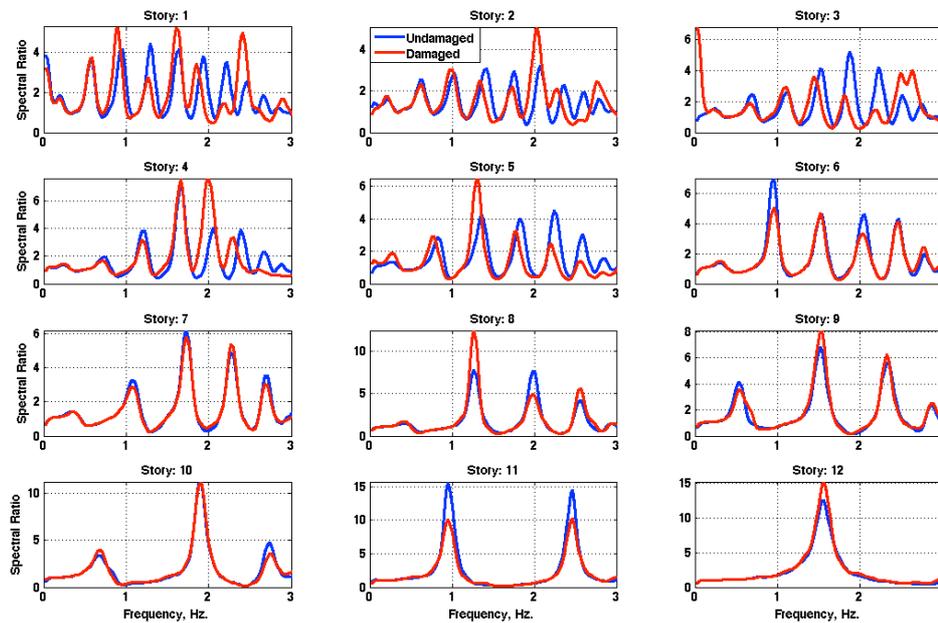


Figure 4. Spectral ratios of each story for a 12-story building with and without damage: the damage is simulated at the 5th story by reducing the story stiffness by 40%.

SUMMARY AND CONCLUSIONS

In this paper, a new method called Mode Shape Based Estimation (MSBE), is proposed to calculate the vibration time histories at the non-instrumented interfaces of layered systems by using the recorded vibrations at the instrumented interfaces. The formulation for the MSBE method is derived based on the assumption that the mode shapes of a layered system can be assumed as a linear combination of the mode shapes of a bending beam and a shear beam. Earthquake records from the densely instrumented Factor Building at UCLA campus in California are used to test the performance of the MSBE method, and the results are compared with both the linear and the cubic spline polynomial interpolation methods. It is shown that the MSBE method has advantages over both interpolation methods for all sensor placement combinations, especially at higher modes.

Once the vibration time histories are known at every interface, a simple methodology, based on the concept of *Interferometric Imaging*, is introduced to develop and calibrate an analytical model of the system. The methodology utilizes top-to-bottom spectral ratios of the records at each layer (e.g., at each story in a building), and shows that these spectral ratios are not influenced by any structural changes in the layers below. Thus, starting from the top layer, the properties of each layer can be determined uniquely by matching the predominant frequencies of the spectral ratios.

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