THREE-DIMENSIONAL ANALYSIS OF AN IRREGULAR GROUND WITH EMBEDDED FOUNDATION
PART II. INCIDENT SURFACE WAVES

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ABSTRACT

In the first part of this study (Part I), an effect of an irregular ground with a rigid embedded foundation on the body wave propagation has been examined. In the second part of this study, the objective is set to investigate the effect of this three-dimensional configuration of soil-structure interaction from the viewpoint of surface wave propagation. The reason why surface waves are considered is because the microtremor, or ambient vibration, wave field is believed to consist of various kinds of surface waves that propagate in various directions. Based on this hypothesis, soil profiles are estimated by applying inversion techniques in which the ground is assumed as horizontally stratified. It is often the case, however, that the ground in an actual condition has irregularities such as slopes and inclined layer boundaries. In such a situation, a simplistic assumption of a horizontally stratified ground may not be applicable. From this context, the effect of an irregular ground, i.e. inclined layer boundary and an existence of an embedded foundation, on the microtremor wave field was investigated in this study. The analysis method is a combination of 2.5-dimensional and three-dimensional finite element methods. It was found from the study that the microtremor wave field is very much affected by the existence of irregularities.

INTRODUCTION

It is essential to know the condition of the ground when considering earthquake disaster estimation and mitigation. It is well known that the surface soil condition and micro topography, or landform, influence the seismic intensity of the ground and hence impact structural damage to the buildings during earthquakes. For example, it has been reported that damage due to liquefaction during the 2011 Tohoku earthquake showed an extensive non-uniform distribution (Sekiguchi and Nakai, 2012). It is also reported that piled foundations of a building were found to be damaged during this earthquake possibly due to a varying local soil condition in a very small area (Kaneko and Nakai, 2013). It is not, however, an easy task to obtain information on the ground condition, such as soil profiles, over a wide area from a practical perspective. One of the most popular approaches to estimate the ground condition is to conduct microtremor (ambient vibration) measurements on the ground surface from which dynamic properties including soil profiles can be obtained (Aki, 1957; Capon, 1969; Arai and Tokimatsu, 2004; Cho et al., 2006). All the approaches proposed so far, however, are based on a parallel layer assumption in that the ground consists of a number of horizontally stratified layers. A difficulty arises when the ground has an irregularity which is often the case in an actual situation. For example, Fig. 1 shows a soil profile of a ground and horizontal to vertical Fourier spectral ratios, or

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H/V spectra, at a number of locations on the ground. As can be seen in the figure, the soil profile changes by a great deal in a small distance and H/V spectra also change very much according to the locations.

Figure 1. H/V spectra at various locations on the surface of an irregular ground

A number of studies on wave propagation in an irregular ground have been reported so far (e.g., Hisada and Yamamoto, 1996; Kawase, 1996). However, a horizontal layering assumption is made for the far field ground in almost all three-dimensional studies (e.g., Bielak et al., 1998, 2003). In the previous studies, the authors have looked at surface wave propagation in a slope ground and pointed out that H/V spectra and phase velocity dispersion curves can be influenced to some extent by the existence of a slope even in a distant location (Nakai and Nakagawa, 2011).

In this paper, the effect of a three-dimensional ground irregularity on the surface wave propagation is studied. More specifically, a two-dimensional two-layered ground that has a horizontal ground surface but has an inclined boundary between the surface layer and the underlying bedrock. In addition, a case in which there exists a rigid foundation with embedment is considered. This soil-foundation system is subject to incident Rayleigh and Love waves of different modes that travel in a variety of directions, which constitutes the microtremor wave field. The analysis method used in the study is a combination of three-dimensional and 2.5-dimensional finite element methods in conjunction with a substructure technique (Nakagawa and Nakai, 2010).

METHOD OF ANALYSIS

Problem under Study

As one of the typical irregular grounds, a two layered ground with an inclined layer boundary is considered as shown in Fig. 2. The width of the inclined part of the boundary, hereafter called a slope, is limited and its inclination does not change along the longitudinal direction of the slope. The rest of the boundary is completely horizontal and so is the ground surface as well. Thus, the configuration is two-dimensional. Nevertheless, the problem under study is three-dimensional since incident surface
waves propagate in a variety of directions which are not necessarily parallel nor perpendicular to the longitudinal direction of the slope. Besides, there exists an embedded foundation.

According to the work done by Tokimatsu and Arai (1998), the microtremor wave field can be considered as the weighted sum of all possible modes of both Rayleigh and Love waves. Since it is not possible to determine the propagation direction of surface waves, a few number of directions are considered.

![Figure 2. Schematic illustration of an irregular ground under study](image)

**Methodology Used in the Analysis**

The methodology used in the analysis is basically the same as the one described in the first part (Part I.) of this study. The difference from Part I. of this study is that incident waves are surface waves instead of body waves traveling in a variety of directions. In the analysis, a semi-infinite medium is divided into two parts, i.e. a near field and a far field, where the near field is modeled as an assembly of a number of finite elements and the far field is represented by a combination of the impedance functions of the "excavated far field" and the driving forces due to an incident wave.

The detailed procedure of the method of analysis is described elsewhere (e.g. Nakagawa and Nakai, 2010; Nakai and Nakagawa, 2012; Nakagawa and Nakai, 2014).

![Figure 3. Analysis model (3-D finite element mesh layout)](image)
Analysis Model and Analysis Conditions

Fig. 3 shows the finite element mesh layout of the near field. As shown in the figure, the ground is basically two-layered but some part of the boundary between the surface layer and the underlying bedrock is inclined. In addition, a rigid foundation with embedment is placed in the central part of the slope. The thickness of the shallow part of the surface layer is 12 meters and that of the deep part is 24 meters. The angle of inclination of the slope is set to 45, 18.4 or 6.3 degrees which corresponds to the slope of 1/1, 1/3 or 1/9, respectively. The embedment depth of the foundation is 10 meters. Eight node linear elements are used in the three-dimensional analysis.

The microtremor wave field is considered as the sum of fundamental and higher modes of Rayleigh and Love waves with the incidence angles (propagating directions) of 0, 60, 120, 180, −120 and −60 degrees with respect to the x-axis. In the summation process, it is assumed that all the applied forces are the same among wave types and propagating directions. Corresponding mode participation coefficients, known as medium responses (Harkrider, 1964), can be computed from the two-dimensional thin layer element analysis (Nakagawa and Nakai, 2008). Fig. 4 shows the phase velocity dispersion curves and corresponding medium responses in the case of Rayleigh wave propagation for the shallow and deep surface layer grounds. Fig. 5 shows the synthesized phase velocity dispersion curves and H/V spectra which were obtained as the weighted sum of all type of waves and modes, medium responses being the weighting functions. As you can see, the fundamental mode of Rayleigh wave prevails but higher modes have some influence in the higher frequency region.

**Figure 4. Phase velocity and medium response of two-layered ground for Rayleigh wave propagation**

RESULTS AND DISCUSSIONS

Microtremor Wave Field of Two-Layered Ground with Irregularities

Fig. 6 shows the wave field due to an incident Rayleigh wave of fundamental mode. The color bar indicates the normalized amplitude of displacement vector with respect to the unit vertical motion at the control point which is located at the back left corner on the ground surface. Red color shows large and blue color shows small amplitude. As you can see from this figure, the displacement field shows a stripe pattern which may be resulted from the interference of the incident wave and the reflected and scattered waves due to the slope and the foundation. It is also noted that the amplitude is smaller in the downwind region of the foundation when compared to the upwind region. When the frequency is low, the displacement amplitude in the deep surface layer ground is larger compared to the shallow surface layer ground, while higher frequency leads to the opposite result. This can be understood from the characteristics of the medium response shown in Fig. 4.
H/V Spectra and Phase Velocity Dispersion Curves of Two-Layered Ground with Inclined Layer Boundary

Now we are investigating how the existence of irregularities such as an inclined layer boundary influences microtremor measurements from the viewpoint of H/V spectra and phase velocity dispersion curves. First, two-dimensional topography is considered although the problem is three-dimensional as mentioned earlier. Fig. 7 shows the variation of H/V spectra and phase velocity dispersion curves along the line perpendicular to the longitudinal direction of the slope. Evaluation
points denoted A through K are shown in Fig. 3. Here, the H/V spectrum at a selected location has been computed by summing up the displacements due to all the incoming waves. Summation was done in terms of the power of displacement amplitude as shown in the following expression (Nakagawa and Nakai, 2008):

\[
R_{(H/V)} = \frac{\sum (\beta \alpha s^2) (\beta v_x^s/\beta v_z^s)^2 \left\{ 1 + \gamma^2 (\beta v_x^s/\beta v_z^s)^2 \right\} / \sqrt{\sum (\beta \alpha s^2) (\beta v_x^s/\beta v_z^s)^2}}{\sum (\beta \alpha s^2) (\beta v_x^s/\beta v_z^s)^2}
\]

(1)

\[
\gamma^2 = 2 \sum (\beta \alpha s^2) (\beta v_x^s/\beta v_z^s)^2 / (R/L)^2 \sum (\beta \alpha s^2) - \sum (\beta \alpha s^2) (\beta v_x^s/\beta v_z^s)^2
\]

\[
(R/L)^2 = \frac{\sum (\beta \alpha s^2) (\beta v_x^s/\beta v_z^s)^2 + \gamma^2 (\beta v_x^s/\beta v_z^s)^2}{\sum (\beta \alpha s^2) (\beta v_x^s/\beta v_z^s)^2}
\]

in which \( W \) is the number of incident surface waves. In the above expression, \( v_i (i=x, y, z) \) represents a different kind of surface wave mode. The weighting factor \( \alpha \) was set to the medium response, shown in Fig. 4, for different modes and the constant value of 0.7 was assumed as the ratio \( R/L \) between Rayleigh and Love wave components (Arai and Tokimatsu, 2004). The phase velocity dispersion curve has been computed from the vertical component of the microtremor wave field based on the centerless circular array, or CCA, method (Cho et al., 2006) by assuming an array of hypothetical sensors that correspond to neighbouring nodes located at the ground surface of the finite element model shown in Fig. 3. One-dimensional results shown in Fig. 5 are also plotted in Fig. 7. From Fig. 7, it is possible to say the followings.

Figure 7. H/V spectra and phase velocity dispersion curves of two-layered ground with inclined layer boundary

- The difference between 1-D and 3-D results is fairly large especially for H/V spectra.
- The difference between them is not that small even if the inclination of the slope is relatively small.
- Overall tendency of phase velocity is that locations on the shallow surface soil give similar values to those of shallow part 1-D results, while locations on the deep surface soil give similar values to those of deep part 1-D results.
- However, the situation for H/V spectra is a little different in that the spectra show fairly large fluctuation when compared to the 1-D results and that values at locations on the shallow surface soil are smaller in the low frequency range and larger in the high frequency range.
- The mechanism that explains this tendency needs to be examined in detail by looking at the contribution of each wave component, but it can be said at this time that this large fluctuation may be resulted from higher mode contributions.

H/V Spectra and Phase Velocity Dispersion Curves of Two-Layered Ground with Inclined Layer Boundary and an Embedded Foundation

Fig. 8 shows the variation of H/V spectra and phase velocities in the case of two-layered ground with an embedded rigid foundation in addition to inclined layer boundary. The results are similar to Fig. 7 for the case of no foundation except that fluctuation is much stronger especially in the high frequency range. This strong fluctuation may be due to the interference of the incident wave, waves reflected from the inclined layer boundary and waves scattered by the foundation. This fluctuation may also be resulted from insufficient number of incident waves and insufficient capability of dashpots assumed as impedance function of the far field ground.

Figure 8. H/V spectra and phase velocity dispersion curves of two-layered ground with inclined layer boundary and an embedded foundation

CONCLUSIONS

In order to examine the effect of irregularity of a ground, i.e. an inclined soil layer boundary and a foundation in this study, on the microtremor wave field, a three-dimensional finite element analysis in
conjunction with a 2.5-dimensional thin layer and finite element analyses has been conducted based on
the widely accepted hypothesis that microtremors are a synthesis of various surface waves traveling
from a variety of directions. It was found from the study that:
- It is possible to conduct a three-dimensional analysis of a ground with basically a two-dimensional
topography by adopting an appropriate substructure technique.
- The microtremor wave field becomes very complex when there exists an inclined layer boundary. It
becomes even more complex when there exists an additional embedded foundation, causing a fairly
big difference between the results in 2.5 and three dimensions.
- There is a high possibility that H/V spectra and phase velocity dispersion curves at the locations in a
wide area not limited to the vicinity of the irregularity can be affected because of this.

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