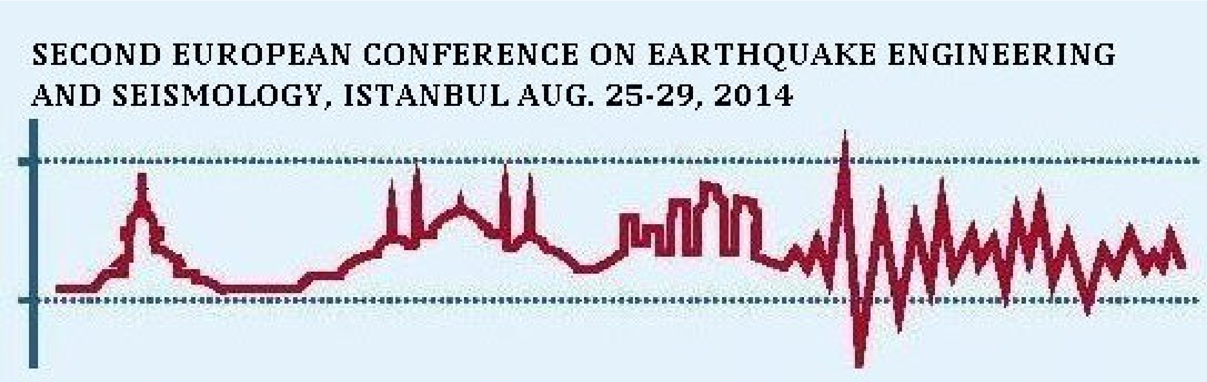


THE AVERAGE FUNCTION OF LOCAL EXCEEDANCE RATES AND AREA-EQUIVALENT GROUND MOTION RELATIONS IN SEISMIC HAZARD ANALYSIS

Mathias RASCHKE[1]

ABSTRACT

The exceedance function is the relation between a fixed value of the local ground motion intensity and its average annual exceedance rate. It is estimated by a seismic hazard analysis. Ground motion relations (also called ground motion prediction equations) are an important part of the seismic hazard analysis respectively the hazard model. Raschke (2013) has discovered that area-equivalent ground motion relations have an equivalent influence on the exceedance function of the sites surrounding the current source. We introduce the average function that is the average of the exceedance functions of a larger region. Furthermore we mathematically derive that area-equivalent ground motion relations result in equivalent average functions of a region under the condition that some other parameters of the seismic hazard analysis are equivalent. The average function characterizes a hazard model in a simple and manageable way and can also be used for comparing different hazard models. Additionally, we introduce the functions of the variation coefficient for the quantification of the spatial variability of the hazard.

INTRODUCTION

The level of local shaking intensity is estimated for building codes and the earthquake-resistant design of different facilities by probabilistic seismic hazard analysis (PSHA). Therein, the average annual exceedance rate of local earthquake ground motion intensity such as the peak ground acceleration (PGA) is estimated. An important element of PSHA is the ground motion relation (GMR). It describes the relation between the local ground motion intensity and different event parameters such as the magnitude (see, e.g., Bommer and Abrahamson 2006). It is also called ground motion prediction equation. We use the term GMR because we are searching for an appropriate model for the PSHA, and the entire relation is formed by more than one equation. Raschke (2013) has discovered that area-equivalent GMRs have equivalent influence on the hazard of sites surrounding the considered source point. We will extend this concept to all source points in a region and introduce the average function of the exceedance rates of all sites in a region in the following section. Therein we also explain briefly the area-equivalence of GMRs. Additionally, we discuss the issue of finite modelling of the geo-space in the PSHA in relation to the average function. Then we will demonstrate the utility of the average function for comparing hazard models and researching the influence of model parameters by a numerical experiment. Furthermore, we introduce a combination of parameters of the seismicity and the GMR with equivalent influence on the average function. Finally, we formulate the variance function for quantifying the spatial variability of the annual exceedance rate.

[1] Free researcher, Wiesbaden, Germany, mathiasraschke@t-online.de

PSHA, AREA-EQUIVALENT GMR AND THE AVERAGE FUNCTION

The average annual rate, also called frequency, that a random shaking intensity Y (e.g., PGA) exceeds a fixed level y at site $\mathbf{s}$ is described by the annual exceedance function (AEF) $\lambda(y,\mathbf{s})$. The AEF of a site with coordinate vector $\mathbf{s}$ is computed in PSHA by an integral of all possible source points with coordinate $\mathbf{t}$ in the region. A source point is not necessarily a point source but the source allocation. The random variable Y has the cumulative distribution function $F_y(y)$ for the condition of vector $\mathbf{X}_E$ of source parameters and the concrete coordinate vectors $\mathbf{s}$ and $\mathbf{t}$. $F_y(y)$ describes the probability of non-exceedance with $Y\leq y$ and is parameterized here by expectation $E(Y)$ and variance $V(Y)$. These parameters are determined by the GMR (see, e.g., Raschke 2013)

$$Y = \varepsilon\, g(\mathbf{X}), \quad E(Y) = g(\mathbf{X}), E(\varepsilon) = 1, V(Y) = V(\varepsilon) g^2(\mathbf{X}), \tag{1}$$

whereby $g(\mathbf{X})$ is a regression function with the vector $\mathbf{X}$ of predicting variables like source parameters and distance that is a function of the coordinates $(\mathbf{s},\mathbf{t})$. If the random component ε is log-normally distributed, then we can logarithm the GMR with

$$\ln(Y) = g^*(\mathbf{X}) + \xi, E(\ln(Y)) = g^*(\mathbf{X}), E(\xi) = 0, V(\ln(Y)) = V(\xi), \tag{2a}$$

$$E(Y) = g(\mathbf{X}) = \exp(g^*(\mathbf{X}) + V(\xi)/2), \tag{2b}$$

$$V(Y) = \exp(2g^*(\mathbf{X}))\exp(V(\xi))(\exp(V(\xi)) - 1) \text{ and} \tag{2c}$$

$$\varepsilon = \exp(\xi) - \exp(V(\xi)/2). \tag{2d}$$

Under certain conditions the logarithm can be also applied as an approximation in case of not exactly log-normally distributed random components.

Furthermore, we need to know the multivariate probability density function f_E of the source parameters $\mathbf{X}_E$ and the seismicity ν of each source point $\mathbf{t}$ for the computation of the AEF. The unit of the seismicity is the average occurrence density per year and per area (region source) or length (line source) for earthquakes with random magnitude $M\geq m_{\min}$. The entire integral for the AEF is

$$\lambda(y,\mathbf{s}) = \int_{\mathbf{t}} \nu(\mathbf{t}) \int_{\mathbf{X}_E} f_E(\mathbf{X}_E,\mathbf{t})\left(1 - F_y\left(y; E(Y(\mathbf{s})) = g(\mathbf{X}_E,\mathbf{s},\mathbf{t}), V(Y(\mathbf{s})) = g^2(\mathbf{X}_E,\mathbf{s},\mathbf{t})V(\varepsilon)\right)\right) d\mathbf{X}_E d\mathbf{t} \tag{3}$$

with $\mathbf{X}=(\mathbf{X}_E,\mathbf{s},\mathbf{t})$. We write $Y(\mathbf{s})$ instead of Y because the random variable corresponds to site $\mathbf{s}$. The same is valid for ν and source point $\mathbf{t}$. Our formulation differs from many other formulations, e.g., of McGuire (1995) or Habenberger (2006), because we consider each source point separately and do not compute the distribution of the random distance between site $\mathbf{s}$ and each possible source like a region source. The computation is easier this way, the spatial nature of the issue is better presented, and the formulation is more comparable to the formulations of the mathematical statistics especially of max-stable random fields (Schlather 2002, Kabluchko et al. 2009). However, we consider the exceedance probability similarly to McGuire (1995).

There is the possibility to formulate an average function of the AEFs of all sites of the entire geo-space that is an infinite plane in our mathematical model, and all points $\mathbf{s}$ and $\mathbf{t}$ are on this plane. We simply formulate the average function with normalization by area A

$$\gamma(y) = \frac{\int \lambda(y,\mathbf{s}) d\mathbf{s}}{A}, \; A = \int d\mathbf{s} = \int d\mathbf{t}. \tag{4}$$

Now we consider equation (3) in (4) and write

$$\gamma(y) = \frac{\int_{\mathbf{s}}\int_{\mathbf{t}} \nu(\mathbf{t}) \int_{\mathbf{X}_E} f_E(\mathbf{X}_E,\mathbf{t})\left(1 - F_y\left(y; E(Y(\mathbf{s})) = g(\mathbf{X}_E,\mathbf{s},\mathbf{t}), V(Y(\mathbf{s})) = g^2(\mathbf{X}_E,\mathbf{s},\mathbf{t})V(\varepsilon)\right)\right) d\mathbf{X}_E d\mathbf{t} d\mathbf{s}}{A}. \tag{5}$$

We can change the order of integration and write

$$\gamma(y)=\frac{\int_{\mathbf{t}}\int_{\mathbf{s}}\nu(\mathbf{t})\int_{\mathbf{X}_E} f_E(\mathbf{X}_E,\mathbf{t})\left(1-F_y\left(y;E(Y(\mathbf{s}))=g(\mathbf{X}_E,\mathbf{s},\mathbf{t}),V(Y(\mathbf{s}))=g^2(\mathbf{X}_E,\mathbf{s},\mathbf{t})V(\varepsilon)\right)\right)d\mathbf{X}_E d\mathbf{s} d\mathbf{t}}{A} \quad .(6)$$

The solution of the two inner integrals of equation (6) is nothing else than the influence function $\lambda^*(y,\mathbf{t})$ of Raschke (2013, equation (14)) that describes the entire influence of a source point **t** and the corresponding function f_E on the hazard of all surrounding sites s. We can write for the average function

$$\gamma(y)=\frac{\int\lambda(y,\mathbf{s})d\mathbf{s}}{A}=\frac{\int\lambda^*(y,\mathbf{t})d\mathbf{t}}{A} \qquad (7)$$

and state that the average function is the average of all AEFs and the average of all influence functions and thus a global characterization and quantification of a hazard model. We know by Raschke (2013) that area-equivalent GMRs result in equivalent influence functions if all other elements in the integral for the PSHA (e.g. the dependence of $V(Y)$ from $g(\mathbf{X})$) are equal. Equal influence functions lead to equal average functions. We state that area-equivalent GMRs result in equivalent average function if all other model elements are equal, including $\nu(\mathbf{t})$.

Area-equivalence means that the amount of points **s** with $g(\mathbf{X}_E,\mathbf{s},\mathbf{t})\geq y$ is the same for two GMRs. If the functions $g(\mathbf{X}_E,\mathbf{s},\mathbf{t})$ are monotone decreasing with increasing source distance, then area-equivalence also means that the isolines (counter lines) include equal area. We show an example of two area-equivalent GMRs in a one-dimensional geo-space in Fig. 1a. Both GMRs have the same area function describing the relation between y and the area with $g(\mathbf{X}_E,\mathbf{s},\mathbf{t})\geq y$ (Fig. 1b).

Area-equivalence is also the reason why the difference between the estimated GMR and observations may not simply be interpreted as a result of the random component. Otherwise, the variances $V(\xi)$ and $V(\varepsilon)$ are over-estimated, which leads to a global bias in the PSHA because of the well-known fact that a larger variance leads to a larger AEF (Ambraseys et al. 1996). The only exception is the one described by Raschke (2013).

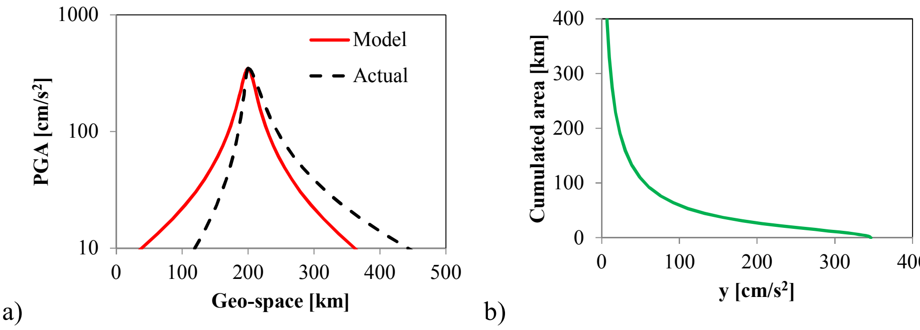


Figure 1. Example of an area-equivalent GMR: a) actual and modelled function g(X) for **t**=200, b) corresponding area-function

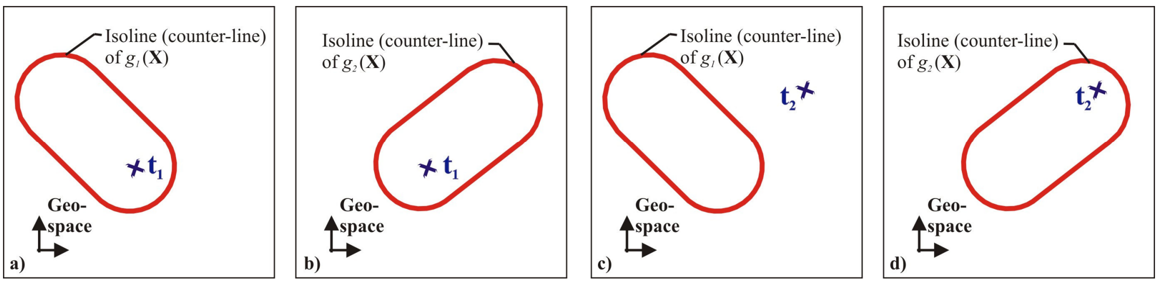


Figure 2. A schematic example of area-equivalent GMRs and different definitions of source points: a) initial situation, b) as a) but with area-equivalent GMR, c) other source point definition than in a) but with the same GMR, d) other source point definition and other area-equivalent GMR than in a)

Now we extend this principle of equivalence. For this purpose, we consider different combinations of definitions of source allocations with area-equivalent ground motion relations. In Fig. 2a, we show a source point $\mathbf{t_1}$ and the corresponding GMR is represented by an isoline of $g_1(\mathbf{X})$. In Fig. 2b, the same source point $\mathbf{t_1}$ is presented with an isoline of $g_2(\mathbf{X})$ of the corresponding GMR. Both GMRs are area-equivalent because the isolines include obviously equal area.

There is no absolute definition for the source point of a GMR. The source point is simply a kind of geographic allocation required for computation such as for equation (3). The source point can be the epicentre; '10 km north of the epicentre' would also be a possible definition and could be applied in the computation. The definition only has to be applied consistently. Therefore, we can change the source point definition of the GMR of Fig. 2a and get the situation of Fig. 2c; the source point-related functions and parameters are also displaced. Both variants result in exactly the same influence on each surrounding site **s**. Furthermore we can apply the GMR of Fig. 1b to the source point definition of Fig. 2c, which results in the situation of Fig. 2d. Obviously the situations of Fig. 2a and d result in equivalent average functions because we have for point $\mathbf{t}_1$ in Fig. 2a the point $\mathbf{t}_2$ in Fig. 2d with equal influence function. The seismicity $\nu(\mathbf{t})$ is not the same for both variants.

Every continuous hazard model includes a set of source points **t** with $\nu(\mathbf{t})$, $f_E(\mathbf{X}_E, \mathbf{t})$ and the other parameters. If two hazard models include a set of equivalent combinations as described above, then the average functions are also equal. The simplest possibility to provide equivalence for such combinations of source points is obviously the application of a function $f_E(\mathbf{X}_E)$ being independent of **t** for both models, and of seismicity models with $\nu_1(\mathbf{t}) \neq \nu_2(\mathbf{t})$ for the most points **t** and with

$$\int_{\mathbf{t}} \nu_1(\mathbf{t}) d\mathbf{t} = \int_{\mathbf{t}} \nu_2(\mathbf{t}) d\mathbf{t} \,. \qquad (8)$$

More complex equivalent combinations are possible.

In practice, we consider only finite site and source regions in the PSHA. For such cases, we can construct an infinite region according to Fig. 3. However, we can also consider finite regions for applying the average function according to equation (4). The appropriate definition of the bounds and the region size depends on the research interest. Do we want to know the average influence of a region of source on the hazard of the surrounding sites or do we want to study the average hazard of the sites? In the first case, the size of the region of sites should be much larger than the considered source region that includes line source, point source and/or region sources. If we want to know the average of the local hazard, then the considered source region should be much larger. The problem is a bit similar to the correct definition of the lower bound $m_{\min}$ for the magnitudes that is considered in the PSHA.

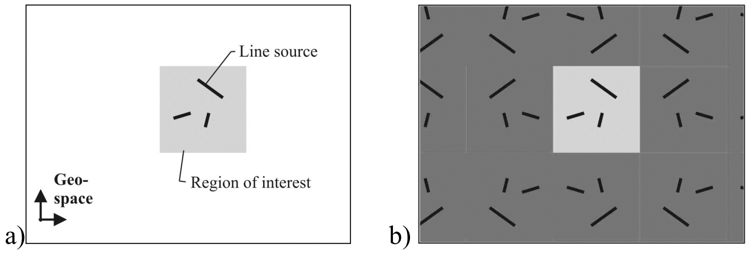


Figure 3. Generating infinity: a) finite region of interest, b) generated infinite region by copies of the finite one

We can also formulate a finite version for the average function with

$$\gamma_{finite}(y) = \int_{finite}^{finite} \lambda(y, \mathbf{s}) d\mathbf{s} = \int_{finite}^{finite} \lambda^*(y, \mathbf{t}) d\mathbf{t} \,. \qquad (9)$$

The average function is now a simple aggregation. It is relatively stable against changes of the site region in the range of large local shaking intensities y. If the site region is extended, then this should not lead to a significant change of the finite version of the average function, provided the site region is already large in relation to the considered source region.

A NUMERICAL EXPERIMENT

We demonstrate accuracy and potential of our approach in a simple numerical experiment. A quadratic region of sites is assumed according to Fig. 4. We compute the average function for all sites in this region according to equation (4) with the area of the site region. Therein we consider two variants of earthquake source: a line source of length 100 km and a small region source with a size 20 km by 25 km. The geography of these variants is shown in Fig. 4a and b.

The entire exceedance frequency $N(m)$ of magnitudes is the same for both regions, is pictured in Fig. 5a and is written with

$$N(m) = K(1 - F_m(m)), \quad K = \int \nu(\mathbf{t})d\mathbf{t}, \quad F(m) = \frac{1-\exp(-\beta(m-m_{\min}))}{1-\exp(-\beta(m-m_{\max}))} \,. \tag{10}$$

The entire exceedance frequency of magnitudes $M \geq m$ of the source is the product of aggregated seismicity K and the magnitude distribution F_m that is modelled by the truncated exponential distribution according to Cosentino et al. (1977). The aggregated seismicity K is the average annual number of events with $M \geq m_{min}$. We set $m_{\min}$=4, $m_{\max}$=7.5, β=2.3 and K=4.4 for the numerical experiment. The exceedance function for the random magnitudes is shown in Fig. 5a. The seismicity $\nu(\mathbf{t})$ is homogenous for both source models with $\nu(\mathbf{t})$ =0.044km^{-1} for the line source and $\lambda(\mathbf{t})$=0.0088km^{-2} for the region source.

Furthermore we assume a simple point source model for a GMR according to equation (2a), which is written with

$$g^*(\mathbf{X}) = \theta_0 + \theta_1 m + \theta_2 \ln(\sqrt{d^2+h^2}) + \theta_3(\sqrt{d^2+h^2}) + \theta_{site}, \mathbf{X} = (d,m) \,. \tag{11}$$

This formulation is already applied in different GMRs (Douglas 2011). The distance between site **s** and source point **t** is d with unit [km]; here **t** is the epicentre. The source depth is parameterized by h and the magnitude is the predicting variable m. The site effect is considered by parameter θ_{site} and is 0 for our standard research. The parameter θ_2 describes the geometrical and θ_2 the anelastic (material) attenuation according to Ambraseys and Bommer (1991). We set θ_0=0, θ_1=0.7, θ_2=-1.0, θ_3=-0.005 and h=10. The standard deviation of the random component is set $\sqrt{V(\xi)}$=0.2763, resulting in a value of 0.12 for log_{10}. This is relatively small (compared with Douglas 2011), but we only want to demonstrate the principals. For the experiment, we assume a normally-distributed random component ξ according to equation (2) although this cannot be for actual GMRs (Raschke 2013). This GMR is applied to the region source and is shown for different magnitudes in Fig. 5b. Additionally, we apply an anisotropic, elliptic variant of this GMR to the line source. It is area-equivalent to the circular variant according to Raschke (2013). The larger axis of the ellipse is in the direction of the source line; the ration of the radii of the main axis is 1.2.

The vector $\mathbf{X}_E$ of source parameters in equations (3,5,6) contains only magnitude m in our example. The probability density function f_E of equations (3,5,6) is the probability density function f_m, which is the first derivation of F_m of equation (10).

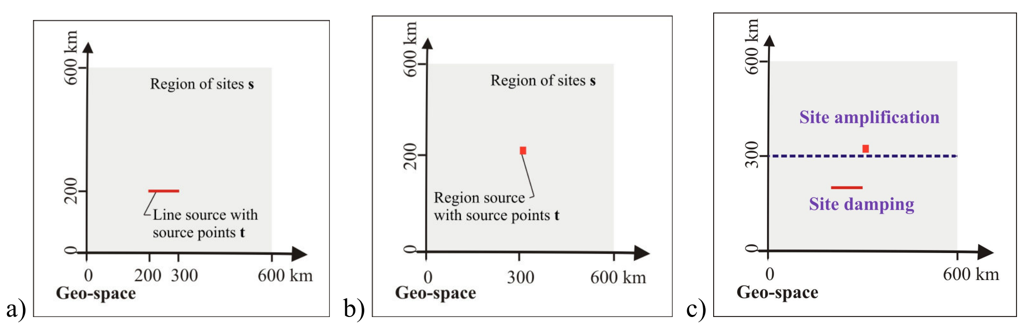
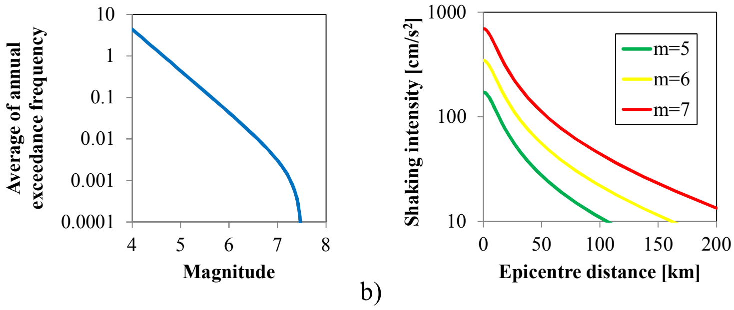


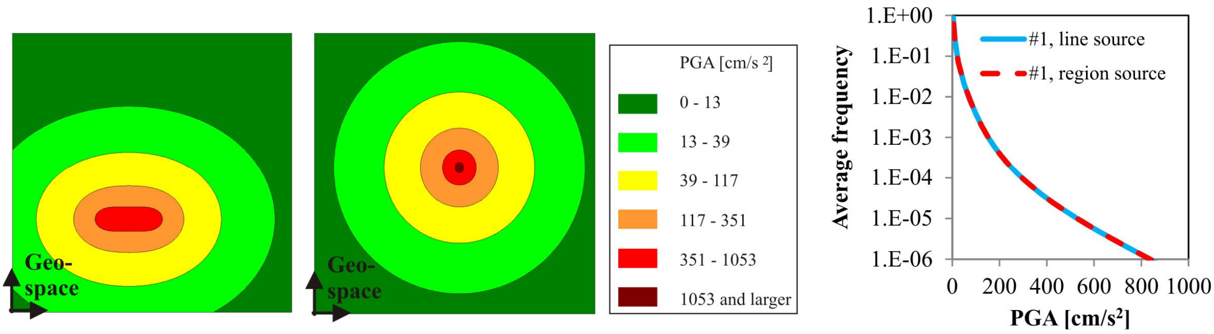
Figure 4. Variants of sources and non-random site effects: a) line source model in the geo-space, b) the region source model, c) non-random site effects in the geo-space (#3 in Tab. 1)



a) b)

Figure 5. Elements of the hazard model: a) applied function of the annual average of exceedance frequency of the earthquake magnitudes, b) applied GMR

We have computed the seismic hazard of each site for both variants of earthquake sources and shown the results in Fig. 6. The computation has been done by self-developed software on the basis of VB.net. The maps are for a local PGA with an exceedance frequency of 1/475, equal to the return period of 475 years. The differences between the maps are large but not surprising. This demonstrates clearly that two hazard models should not only be compared by the hazards maps because we know that basic elements of the hazard models are equal. Additionally, there are many possible hazard maps for different return periods. The only possibility of a comprehensive and global comparison is offered by the average functions. We present these in Fig. 6d. As expected, the average functions of the two models are equal because the aggregated seismicity is equal and the GMRs are area-equivalent.



a) b) c) d)

Figure 6. Results of the PSHA for the two variants of sources: a) local PGAs with a return period of 475 years for the line source, b) local PGAs with a return period of 475 years for the region source, c) legend of the maps, d) average functions for the source models

Table 1. Parameters of the considered hazard models

#	Variant	Site effects	Modification
1	Standard	No	No
2	Site effect A	Random	Random site effect
3	Random effect	No	$\sqrt{V(\xi)}=0.3454$, $\theta_0=3.9785$
4	Site effect B	For large regions	Site effects for large region
5	Upper magnitude	No	$m_{max}=8.0$
6	β-value	No	$\beta=2.1$
7	Seismicity	No	$K=8.8$
8	Geometric damping	No	$\theta_2=-0.9$
9	Source depth	No	$h=5$
10	Material damping	No	$\theta_3=-0.002$

The average function can also be applied to study the influence of different model components on the entire hazard of the considered region of sites. We demonstrate this and change the parameters for different variants according to Tab. 1. The standard variant with #1 means the line source and region source model with the parameters defined above.

At first we research the influence of random components (#2 in Tab. 1), which includes the site effects with large variability in a small area. The contribution of each site coordinate to the average function includes 50% amplification and 50% damping. This acts like an additional variance respectively standard derivation of the random component according to Joyner and Boore (1993). We set $\theta_{site}=ln(1.2)$ and $\theta_{site}=ln(0.8)$ for the site effects. Both effects together have no influence on the expectation $E(Y)$ in equation (1). The average function of this variant #2 is shown in Fig. 7a. The difference to the standard variant #1 is principally equal to the difference of an average function of variant #3 with changed standard derivation of the normally-distributed random component ξ (θ_0 is changed to fulfil the condition $E(\varepsilon)=1$ in equation (1)). The additional spreading of the random component results in a higher AEF and influence function and thus in larger average functions (larger exceedance frequency for the same y). This influence depends on other parameters, e.g. the upper bound magnitude m_{max} (see, e.g., Raschke 2013). If the site effects are constant for a larger region then the influence of the site effects can be more important. We have considered this example in variant #4. The geography of the site effects is shown in Fig. 4c and the results are presented in Fig. 7b. The average functions differ considerably. Therefore, the global comparison between hazard models for a region should be done either with random site effects or without site effects.

In variants #5 to 7, parameters of the seismicity are modified. The resulting average functions are presented in Fig. 7c. The influence is relatively small in comparison to the standard model. At last, we consider modified parameters of the GMR in variants #8 to 10. Their average functions are shown in Fig. 7d. The smaller parameters θ_3 for the geometric damping and the smaller source depth h result in a much larger hazard than the standard variant. The material damping parameter θ_4 has a smaller influence. Of course, the influences are relative and depend on the changes in the single parameter, which are not simply comparable.

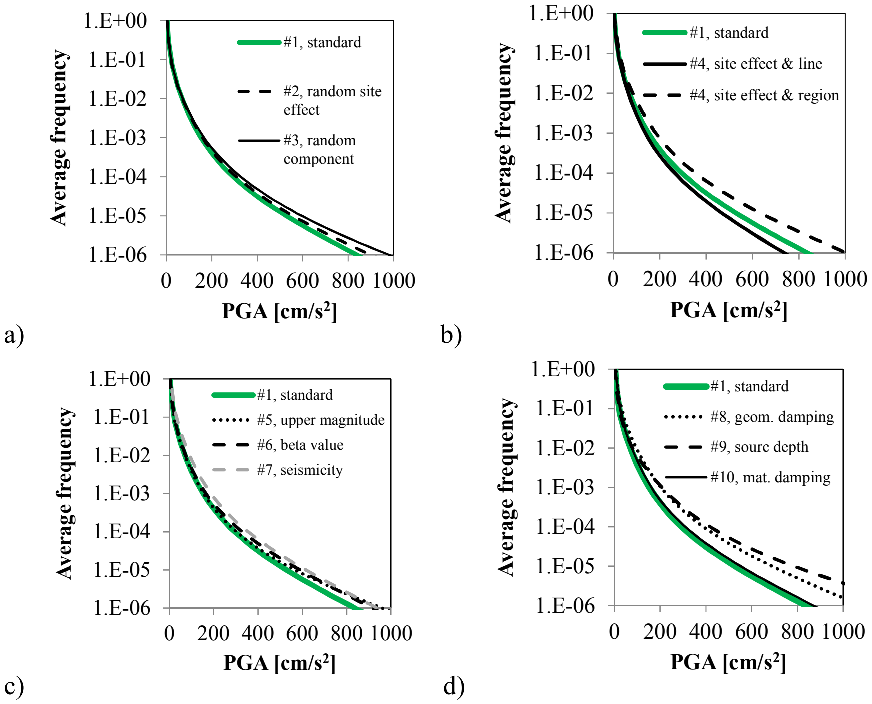


Figure 7. Influence of the model components on the average function, parameters according to Tab. 1

A FURTHER EQUIVALENCE

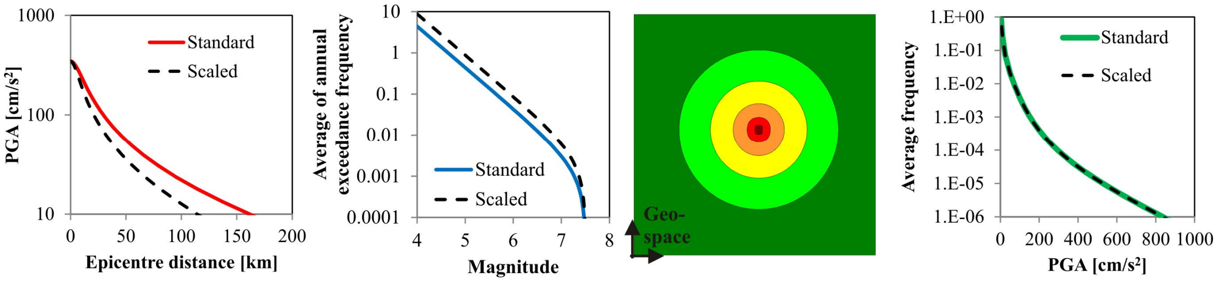
We can formulate a further equivalence for the combination of GMR and magnitude frequency. If the area in the isolines of the GMR is scaled by $1/\theta_{scale}$ with $\theta_{scale}>0$, and the magnitude frequency is scaled by factor θ_{scale}, then the resulting average function is the same as for the unscaled case with $\theta_{scale}=1$. For example, the GMR can simply be modified as follows

$$g^*_{scaled}(\mathbf{X})=\theta_0+\theta_1 m+\theta_2\ln(\sqrt{\theta_{scale}d^2+h^2})+\theta_3(\sqrt{\theta_{scale}d^2+h^2})+\theta_{site}, \quad (12)$$

and the modified magnitude frequency is

$$\mathrm{K}_{scale}=\theta_{scale}\mathrm{K}. \quad (13)$$

We apply a scaling θ_{scale}=2 for the model with region source and show the corresponding functions and resulting hazard map and the average function for the region source in Fig. 8. As expected, the latter is equivalent to the average function of the unscaled case but the hazard map differs significantly from the unscaled case of Fig. 6b. Obviously, a simple comparison of hazard maps is not an appropriate method for the global comparison of two models for the PSHA.



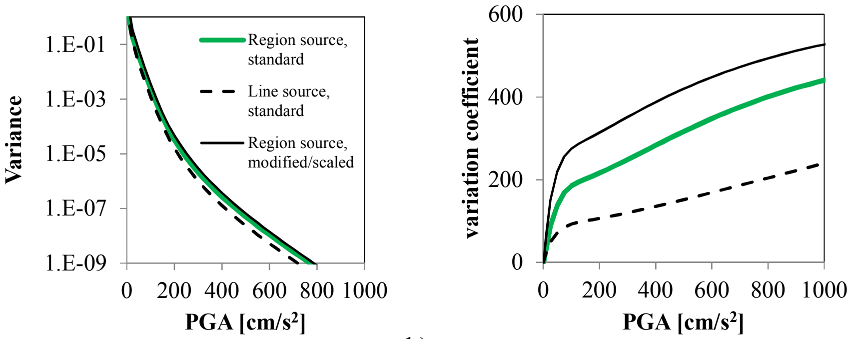
a) b) c) d)

Figure 8. Example of equivalent combination of GMR and magnitude frequency: a) GMR, b) magnitude frequencies, c) locale PGAs with a return period of 475 years for the scaled model (legend: Fig. 6c), d) average functions

THE SPACIAL VARIABILITY OF THE AEF

The average hazard can be well described by the average function. However, this function is not a good measure for the spatial variability. This can be quantified using the variance function. The average function is defined very similarly to the expectation of a random variable. The variance function is also similar defined as variance of a random variable. We apply the average function $\gamma(y)$ according to equation (4) to the variance function and write

$$\varphi(y) = \frac{\int (\lambda(y,\mathbf{s}) - \gamma(y))^2 d\mathbf{s}}{A} = \frac{\int \lambda(y,\mathbf{s})^2 d\mathbf{s}}{A} - \gamma^2(y) \, . \tag{14}$$



a) b)

Figure 9. Variation of the local exceedance frequencies: a) variance, b) variation coefficient

In Fig. 9a, the variance function $\varphi(y)$ is shown for the region source and the line source with standard parameters and the region source with the modification of the latter section. The functions are very similar. The differences are much larger when we apply the variation coefficient that is the ratio of variance function and squared average function. The variation coefficient is shown in Fig. 9b. The spatial variation of the line source is the smallest. The reason for this could be that the seismicity is less concentrated in the line source model. The highest spatial variation is for the scaled variant of the region source. The values of the variation coefficient for small PGAs could be influenced by the geographic bounds of the site region and by the lower bound m_{min} of the magnitudes.

CONCLUSIONS

We have derived the average function as the average of the influence functions of the source point **t** and the average of the AEFs of the sites **s**. We have discussed the aspects of finite and infinite geo-space and demonstrated the opportunities of the average function for characterizing a hazard model, comparing different hazard models and researching the influence of model elements. The average function is the only simple possibility for a global comparison of two models. Any more detailed comparison would require different comparisons of further functions and/or maps. Additionally, we state that the variation coefficient is an appropriate measure to quantify the spatial variation of the AEFs.

Furthermore, we emphasise that formulating the AEF according to equation (3) is more practicable and considers the spatial nature of the PSHA in a better way. It is also more compatible to approaches of spatial statistics. The equivalences could not be discovered in the conventional formulations because they do not include an integral over space.

Finally, we want to draw attention to the consequences of area-equivalence of GMRs. The difference between the "truth" and the estimated, area-equivalent regression model $g(\mathbf{X})$ according to equation (1) must not be simply interpreted as part of the random component because this would lead to an overestimation of the variance $V(\varepsilon)$ (square of standard deviation) and a global bias in the hazard estimation. Therefore, regression analysis cannot simply be applied to estimate the variances of the random components. For details s. Raschke (2013).

REFERENCES

Ambraseys NN and Bommer J (1991) "The attenuation of ground accelerations in Europe", *Earthquake Engineering and Structural Dynamics* 20: 1179-1202

Ambraseys, NN, Simpson KA and Bommer JJ (1996) "Prediction of horizontal response spectra in Europe", *Earthquake Engineering and Structural Dynamics* 25, 371-400

Bommer JJ and Abrahamson A (2006) "Why do modern probabilistic seismic hazard analyses often lead to increased hazard estimates?", *Bulletin of the Seismological Society of America* 96: 1967-1977

Cosentino P, Ficarra V and Luzio D (1977) "Truncated exponential frequency-magnitude relationship in earthquake statistics", *Bulletin of the Seismological Society of America 67*: 1615-1623

Douglas J (2011) "Ground-motion prediction equations 1964–2010", *Final Report BRGM/RP-59356-FR* (http://peer.berkeley.edu/publications/peer_reports/reports_2011/reports_2011.html)

Habenberger J (2006) „Gefährdungskonsistente Erdbebenszenarien für das Gebiet von Deutschland", *D-A-CH Mitteilungsblatt*, *Band 81, Bauingenieur*. Springer VDI Verlag

Joyner WB and Boore DM (1993) "Methods for regression analysis of strong-motion data", *Bulletin of the Seismological Society of America* 83(2): 469-487

Kabluchko Z, Schlather M, de Haan L (2009) "Stationary max-stable random fields associated to negative definite functions", *The Annals of Probability* 37 (5): 2042-2065

McGuire RK (1995) "Probabilistic Seismic Hazard Analysis and Design Earthquakes: Closing the Loop", *Bull Seismol Soc Am* 85: 1275-1284

Raschke M (2013) "Statistical modeling of ground motion relations for seismic hazard analysis", *Journal of Seismology*, 17: 1157-1182

Schlather M (2002) "Models for Stationary Max-Stable Random Fields", *EXTREMES* 5(1): 33-44