



## **NON LINEAR SEISMIC RESPONSE OF PLANAR ASYMMETRIC SYSTEMS**

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### **ABSTRACT**

The seismic behavior of one-storey asymmetric structures has been extensively studied since 1970s by a number of researches studies which identified the coupled nature of the translational-to-torsional response of those class of systems leading to severe displacement magnifications at the perimeter frames and to significant increase of local peak seismic demand to the structural elements. These studies identified the fundamental parameters governing the torsional behavior of in-plan asymmetric structures and trends of behavior. It has been clearly recognized that asymmetric structures can be grouped into torsionally-stiff systems, and torsionally-flexible systems. The latter may be affected to significant torsional effects when subjected to earthquake excitations. Previous research works by some of the authors proposed a simple closed-form estimation of the maximum torsional response of one-storey torsionally stiff elastic systems leading to the so called “Alpha-method”.

The present paper provides an extension of the “Alpha Method” by removing the assumption of linear elastic response of the system. The main objective is to evaluate how the excursion of the structural elements into the inelastic field affects the seismic displacement demand of one-storey in-plan asymmetric structures. The simple system proposed by Goel and Chopra in 1991 is used to perform the non-linear analysis varying all the fundamental parameters of the system, including the inelastic demand by varying the force reduction factor from 2 to 5. Corner displacement magnification factors for different force reduction factors are evaluated and comparisons with the results obtained from linear analysis are provided.

### **INTRODUCTION**

Since the late 1970s it is well known that structures characterized by non coincident center of mass and center of stiffness, commonly defined as eccentric (or asymmetric) systems, when subjected to dynamic excitation develop a coupled lateral-torsional response that may considerably increase their local peak response, such as the corner displacements (Kan and Chopra 1977, Hejal and Chopra 1987, Rutenberg 1992).

In order to effectively apply the performance-based design approach to seismic design there is a growing need for code-oriented methodologies aimed at predicting deformation parameters. Thus,

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the estimation of the displacement demand at different locations, especially for eccentric structures, appears a fundamental issue. Furthermore, the ability to predict the torsional response of eccentric systems can be also useful to improve the capability of one of the most actually used seismic design approaches (i.e. push-over analysis, Perus and Fajfar 2005).

Since the early 1990s Nagarajaiah *et al.* 1993, investigating the torsional behavior of base isolated structures, observed that, for the specific class of torsionally-stiff asymmetric structures, the maximum center mass displacement can be well approximated by the maximum displacement of the equivalent not-eccentric system.

In previous research works (Trombetti and Conte 2005, Trombetti *et al.* 2008), some of the authors identified a structural parameter, called “alpha”, related to the attitude of one-storey asymmetric linear systems to develop a rotational response in free vibration and proposed a simplified procedure, called “Alpha-method”, for the estimation of the maximum torsional response. In its original formulation, the “Alpha-method” was based on the assumption of equal maximum displacement response between the eccentric system and the equivalent not-eccentric system. Recently (Palermo *et al.* 2013) the “Alpha-method” has been extended to all classes of one-storey asymmetric systems, thus including both torsionally stiff and torsionally flexible systems. All those results were obtained by performing linear analyses (either in free vibration or under earthquake excitation) with a fixed damping ratio equal to 0.05.

In this paper a further generalization of the “Alpha Method” is proposed by removing the assumption of linear elastic response of the system. First linear-elastic analyses (free vibration and seismic analyses) are performed varying the damping ratio from 0.02 to 0.30. Then, nonlinear seismic analyses are performed by assuming an elastic-perfectly plastic response of the structural elements through the introduction of the force reduction factor  $R$ . The main objective is to evaluate how the excursion of the structural elements in the inelastic field affects the displacement demand of one-storey in-plan asymmetric structures.

## PROBLEM FORMULATION

Let us consider the one-storey eccentric structure (i.e. a system characterized by non-coincident center of mass,  $C_M$ , and center of stiffness,  $C_K$  leading to a one-way eccentricity  $E_x=E$ ) displayed in Fig.1 (the origin of the reference system is located at  $C_M$ ). It is assumed that the diaphragm is infinitely rigid in its own plane, and that the lateral-resisting elements are massless and axially inextensible. The self torsional stiffness ( $k_\theta$ ) of each lateral-resisting element is neglected. Under these assumptions, the system may modelled as a 3-dof system: longitudinal center mass displacement,  $u_{y,CM}$ ; transversal center mass displacement,  $u_{x,CM}$ ; center mass rotation,  $u\theta,CM$  (coincident with the floor rotation,  $u\theta$ ). One-way dynamic excitation (e.g. free vibrations or seismic input) along the longitudinal direction (namely, the  $y$ -direction) is considered.

From simple trigonometric relationships, with reference to the plan view of the system given in Fig.1, the longitudinal corner side displacements, i.e. the displacement of the so called stiff side  $u_{y,s}$  (e.g. point B or B', the closer to CK) and flexible side of the system  $u_{y,f}$ , (e.g. point A or A', the farther from CK), at any instant of time  $t$ , are given by:

$$\begin{aligned} u_{y,f}(t) &= u_{y,CM}(t) - u_\theta(t) \cdot \frac{L}{2} \\ u_{y,s}(t) &= u_{y,CM}(t) + u_\theta(t) \cdot \frac{L}{2} \end{aligned} \quad (1)$$

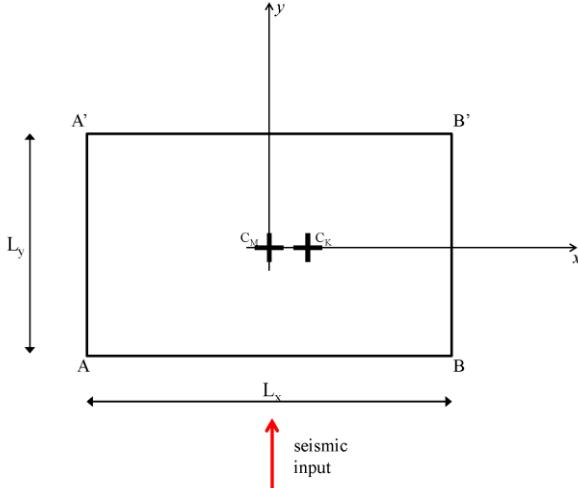


Figure 1. Plan view of the system.

Estimating corner displacements according to Eq. (1) need time-history analyses to be performed. Nonetheless, the practical engineer is only interested in the peak response, which, generally speaking, may be expressed as an appropriate combination of the longitudinal and rotational maximum responses. From the results of a previous research work (Palermo *et al.* 2013) the following expression of the maximum flexible side displacement of a linear system has been obtained:

$$u_{y,f,\max} = \delta \cdot u_{y,CM,\max,N-E} \cdot (1 + B \cdot A \cdot \alpha_u \cdot \phi) \quad (2)$$

Three contributions may be recognized:

- the translational contribution due to  $\delta = \frac{u_{y,CM,\max}}{u_{y,CM,\max,N-E}}$ , providing the center mass displacement amplification with respect to that of the equivalent not-eccentric system (N-E);
- the rotational contribution due to  $A \cdot \alpha_u = \rho \frac{u_{\theta,\max}}{u_{y,CM,\max}}$  ( $\alpha_u$  is a parameter given in closed-form by Trombetti *et al.* 2005 for the case of undamped free vibration response;  $\rho$  is the mass radius of gyration of the system,  $A$  is a parameter which has to be calibrated through numerical simulations);
- the correlation coefficient  $B$  between the maximum displacement response and the maximum rotational response;

$\phi = \sqrt{\frac{3(L_x/L_y)^2}{1+(L_x/L_y)^2}}$  is the shape factor for the simple rectangular plan geometry of Figure 1.

An estimation of  $\delta$  has been obtained in a closed form expression, while the other parameters have been calibrated through numerical simulations assuming linear response of the system (Palermo *et al.* 2013). The main objective of the present work is to obtain an estimation of the maximum corner side displacement (qualitatively similar to that of Eq. 3) by assuming a non-linear system response through the introduction of the force reduction factor  $R$  accounting for the possible excursion into the inelastic field.

## FREE VIBRATION RESPONSE

Under the following assumptions: (i) equal total lateral stiffness  $k$  along the  $x$ - and the  $y$ -direction (i.e.  $k=k_x=k_y$ , where  $k_x$  and  $k_y$  are the translational stiffness the  $x$ - and the  $y$ -direction, respectively);(ii) the rotational response  $u_\theta$  developed under dynamic excitation is small enough to allow the approximation  $u_\theta \approx \sin(u_\theta) \approx \tan(u_\theta)$ ; the dynamic coupled lateral-torsional response of the

dynamic system of Fig. 1 is governed by the following set of coupled differential equations of motion (Trombetti and Conte 2005), written in a reference system with the origin located at CM:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{u}_x(t) \\ \ddot{u}_y(t) \\ \rho \ddot{u}_\theta(t) \end{bmatrix} + [C] \begin{bmatrix} \dot{u}_x(t) \\ \dot{u}_y(t) \\ \rho \dot{u}_\theta(t) \end{bmatrix} + m\omega_L^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & (e\sqrt{12}) \\ 0 & (e\sqrt{12}) & \Omega_\theta^2 + 12e^2 \end{bmatrix} \begin{bmatrix} u_x(t) \\ u_y(t) \\ \rho u_\theta(t) \end{bmatrix} = \begin{bmatrix} p_x(t) \\ p_y(t) \\ \frac{p_\theta(t)}{\rho} \end{bmatrix} \quad (3)$$

where:  $m$  is the mass of the system;  $e=E/D_e$  is the relative eccentricity;  $D_e$  is the equivalent diameter equal to  $\rho_m \sqrt{12}$ ;  $\Omega_\theta=\omega_\theta/\omega_L$  is the torsional-to-lateral frequency ratio ( $\omega_L$  and  $\omega_\theta$  are the uncoupled translational natural frequency of vibration and the uncoupled torsional natural frequency of vibration, defined in a reference system with origin at C<sub>K</sub>, respectively); [C] is the damping matrix (classical damping is assumed). Values of parameter  $\Omega_\theta$  separate planar asymmetric systems in two categories:

- torsionally-stiff systems characterized by  $\Omega_\theta \geq 1.0$ ;
- torsionally-flexible systems characterized by  $\Omega_\theta < 1.0$ .

The undamped free vibration response (from an initial displacement  $a$  along the  $y$ -direction) of the studied system is given by (Trombetti and Conte 2005):

$$\begin{aligned} u_y(t) &= a \{ A_1 \cos(\omega_1 t) + A_3 \cos(\omega_3 t) \} \\ u_x(t) &= 0 \\ u_\theta(t) &= \frac{a}{\rho} A_4 \{ \cos(\omega_1 t) - \cos(\omega_3 t) \} \end{aligned} \quad (4)$$

Where:

$$\begin{aligned} A_1 &= \frac{1 - \Omega_3}{\Omega_1 - \Omega_3} \\ A_3 &= \frac{\Omega_1 - 1}{\Omega_1 - \Omega_3} \end{aligned} \quad (5)$$

$$\Omega_1 = (\omega_1 / \omega_L)^2 = 1/2 \left( 1 + \Omega_\theta^2 + 12e^2 - \sqrt{(\Omega_\theta^2 + 12e^2 - 1)^2 + 48e^2} \right) \quad (6)$$

$$\Omega_2 = (\omega_2 / \omega_L)^2 = 1 \quad (6)$$

$$\Omega_3 = (\omega_3 / \omega_L)^2 = 1/2 \left( 1 + \Omega_\theta^2 + 12e^2 + \sqrt{(\Omega_\theta^2 + 12e^2 - 1)^2 + 48e^2} \right)$$

In undamped free vibration the parameter “alpha” related to the ratio between the maximum rotational response and the maximum longitudinal displacement response has been derived in closed-form as a function of  $e$  and  $\Omega_\theta$  (Trombetti and Conte 2005):

$$\alpha_u = \frac{\rho u_{\theta,\max}}{u_{y,\max}} = \frac{4e\sqrt{3}}{\sqrt{(\Omega_\theta^2 + 12e^2 - 1)^2 + 48e^2}} \quad (7)$$

The correlation coefficient  $B_u$  has been formulated such as (Palermo et al. 2013):

$$u_{y,flex} = u_{y,\max} + B_u \cdot u_{\theta,\max} \cdot \frac{L}{2} \quad (8)$$

Looking to the “Argand diagram” representation of the solution of the equation of motion (see Figure 10 of Trombetti and Conte 2005) it has been noted that at the time instant which maximize the rotational response, the corresponding longitudinal displacement is equal to  $a \cdot (A_1 - A_3)$  which, in general, is quite close to the maximum longitudinal response  $a$ . Based on these considerations, a closed-form approximation (lower bound) of the correlation  $B_u$  may be obtained:

$$|A_1 - A_3| \cdot u_{y,\max} + u_{\theta,\max} \cdot \frac{L}{2} = u_{y,\max} + B_u \cdot u_{\theta,\max} \cdot \frac{L}{2} \quad (9)$$

leading to:

$$B_u = 1 - \frac{4A_3}{\alpha_u L_x} \rho_m \quad \text{for } A_l \geq A_3$$

$$B_u = 1 - \frac{4A_l}{\alpha_u L_x} \rho_m \quad \text{for } A_l < A_3$$
(10)

It can be noted that  $1 \geq B \geq 1 - \frac{2\rho}{\alpha_u L_x}$ .

The damped free vibration response of the system represented in Figure 1 is given by (Trombetti and Conte 2005):

$$u_y(t) = a \Lambda \text{Exp}(-\xi \omega_l t) \{ A_l \cos(\omega_{Dl} t + \theta) + A_3 \text{Exp}(-\xi(\omega_3 - \omega_l)t) \cos(\omega_{D3} t + \theta) \}$$

$$u_x(t) = 0$$

$$u_\theta(t) = \frac{a}{\rho} A_4 \Lambda \text{Exp}(-\xi \omega_l t) \{ \cos(\omega_{Dl} t + \theta) - \text{Exp}(-\xi(\omega_3 - \omega_l)t) \cos(\omega_{D3} t + \theta) \}$$
(11)

Where  $\xi$  is the damping ratio (constant for all modes);  $\Lambda = \sqrt{1 + \frac{\xi}{\sqrt{1 - \xi^2}}}$  and  $\theta = -\arctan\left(\frac{\xi}{\sqrt{1 - \xi^2}}\right)$ ;  $\omega_{Di} = \omega_i \sqrt{1 - \xi^2}$ ;  $i=1,2,3$ .

Similarly to the case of undamped free vibration it is possible to introduce the following “alpha” parameter:

$$\alpha_{d,\text{free}} = \rho \frac{u_{\theta,\text{max}}}{u_{y,\text{max}}} \quad (12)$$

Figure 2 displays  $\alpha_{d,\text{free}}$  as a function of  $e$  and  $\Omega_\theta$  for selected values of damping ratio. Values of  $\alpha_{d,\text{free}}$  has been obtained by numerically solving Eq. 15 for 8 different values of damping ratios (between 0.02 and 0.3). The graph of Figure 2 clearly shows that  $\alpha_{d,\text{free}}$  decreases as the damping ratio increases and that  $\alpha_u$  may be taken as an upper bound estimation of  $\alpha_{d,\text{free}}$ .

## LINEAR SEISMIC RESPONSE FOR DIFFERENT DAMPING RATIOS

The seismic response of the system for different values of damping ratio  $\xi$  has been evaluated by a performing linear elastic seismic analyses by varying the main system parameter as follows:

- 13 values of eccentricity  $e$  from 0.0 to 0.35,
- 20 values of  $\Omega_\theta$  from 0.1 to 2.0
- 8 values of damping ratio  $\xi$  from 0.02 to 0.30
- 7 values of uncoupled longitudinal period  $T_L$  from 0.1 s to 3.0 s

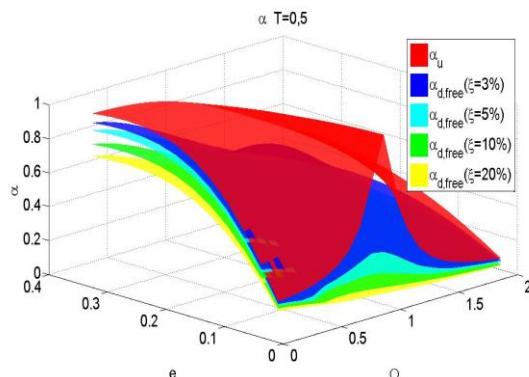


Figure 2. Variation of the damping ratio effect on the rotational parameter  $\alpha$  in free vibration response.

An ensemble ground motion composed of 50 ground motions selected from the PEER ground motion database has been used to perform the seismic analyses. The ground motions are selected based on the shear wave velocity  $V_{s,30}$  in the range of 360m/s to 800m/s (soil type B according to the Italian building code NTC08).

The following response parameters are computed from each analysis (Figures 3 to 5):

$$\alpha_{d,eq} = \rho \frac{u_{\theta,\max}}{u_{y,CM\max}} \quad (13)$$

$$B_{d,eq} = \frac{u_{y,\max} - u_{y,CM,\max}}{u_{\theta,\max} \cdot (L_x / 2)} \quad (14)$$

$$M_{CM,s} = \frac{u_{y,s,\max}}{u_{y,CM\max}} \quad (15)$$

$$M_{CM,f} = \frac{u_{y,s,\max}}{u_{y,CM\max}} \quad (16)$$

$M_{CM,s}$  and  $M_{CM,f}$  are the magnification factors (with respect to the center mass displacement) at the stiff side and flexible side, respectively.

Figures 3a and b compares the  $\alpha_{d,eq}$  curves for two different uncoupled lateral periods  $T_L$  (for a fixed damping ratio), while Figures 3a and b compares the  $\alpha_{d,eq}$  curves for different damping ratios and the same uncoupled lateral periods  $T_L$ . Inspection of the graphs allow the following observations:

- $\alpha_{d,eq}$  exhibits peak values larger than 1.0 for low  $T_L$  and  $\Omega_\theta$  and  $e$  around 0.2. The result indicates that the closed-form given in free vibration ( $\alpha_u$ ) is not always an upper bound. However, once the longitudinal period  $T_L$  increases the peak decreases and even vanish for  $T_L$  larger than 1.5sec.
- For fixed values of  $T_L$ , values of  $\alpha_{d,eq}$  decreases as the damping ratio  $\xi$  increases.
- Torsionally-stiff systems are more sensitive to the damping ratios with respect to torsionally-flexible systems.

Figures 4a and b compares the  $B_{d,eq}$  curves for two different uncoupled lateral periods  $T_L$  (for a fixed damping ratio), while Figures 4a and b compares the  $B_{d,eq}$  curves for different damping ratios and the same uncoupled lateral periods  $T_L$ . Inspection of the graphs allow the following observations:

- In general, values of  $B_{d,eq}$  increases as  $e$  and  $\Omega_\theta$  increase.
- For fixed values of damping ratio  $\xi$  the correlation coefficient  $B_{d,eq}$  decreases once the longitudinal period  $T_L$  increases,
- For fixed values of longitudinal period  $T_L$  the correlation  $B_{d,eq}$  increases once the damping ratio  $\xi$  increases.

Figures 5a and b compares the  $M_{CM,s}$  curves for two different damping ratios and same uncoupled lateral periods  $T_L$ , while Figures 5c and d compares the  $M_{CM,f}$  curves for two different uncoupled lateral periods  $T_L$  and same damping ratio. Inspection of the graphs allow the following observations:

- Values of  $M_{CM,s}$  are between 0.4 and 1.8.  $M_{CM,s}$  surface appears quite smooth for torsionally-stiff systems (values are less than 1.0), while it rapidly varies for torsionally-flexible systems (values are also larger than 1.0).
- For fixed values of  $T_L$  and  $\xi$  the magnification factor of the stiff side  $M_{CM,s}$  decreases once the eccentricity increases, except within an area characterized by small values of  $e$  and  $\Omega_\theta$  (high torsionally-flexible systems) where it exhibits peaks larger than 1.0 (maximum values are around 1.8).
- As the damping ratio increases, the range of  $\Omega_\theta$  within the magnification factor exhibit those peaks peak becomes smaller.

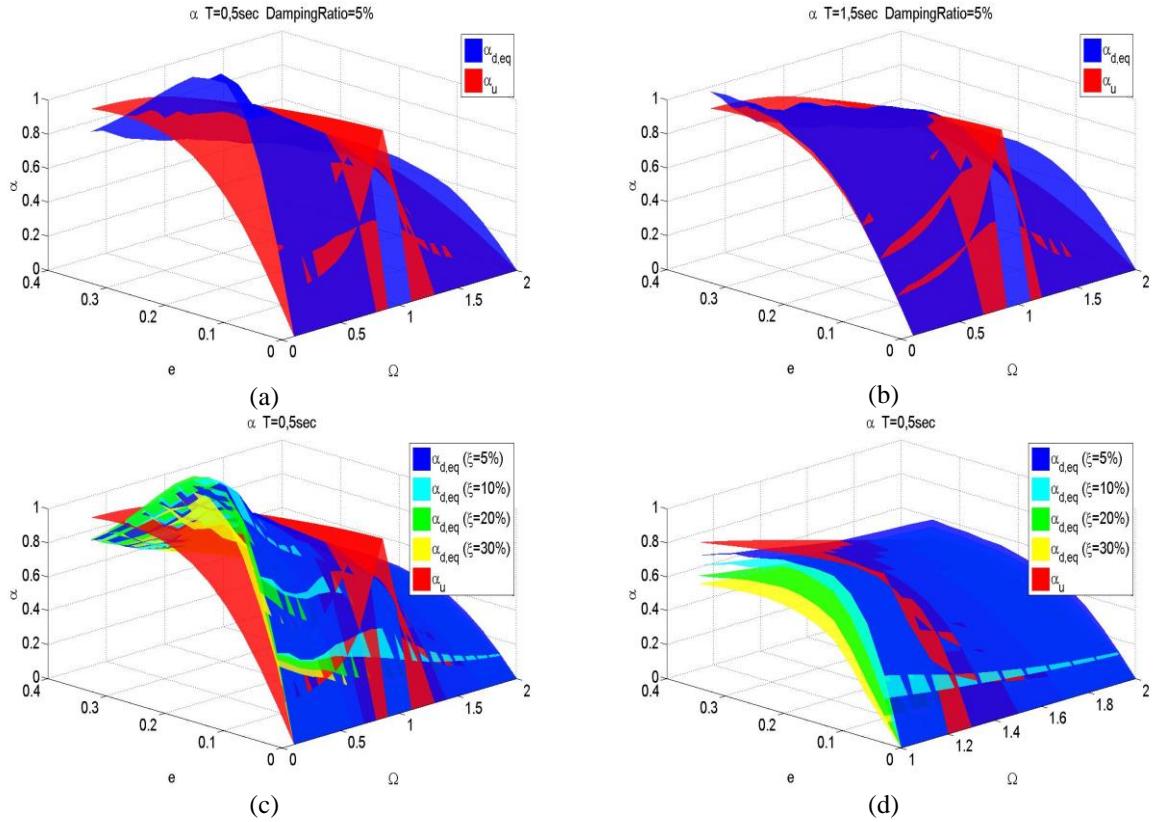


Figure 3. Rotational parameter as function of  $e$  and  $\Omega_0$  for  $T_L=0.5\text{sec}$  and  $\xi=5\%$  (a);  $T_L=1.5\text{sec}$  and  $\xi=5\%$  (b);  $T_L=0.5\text{sec}$  and different values of damping ratio  $\xi$  (c); focusing on torsionally-stiff structures (d)

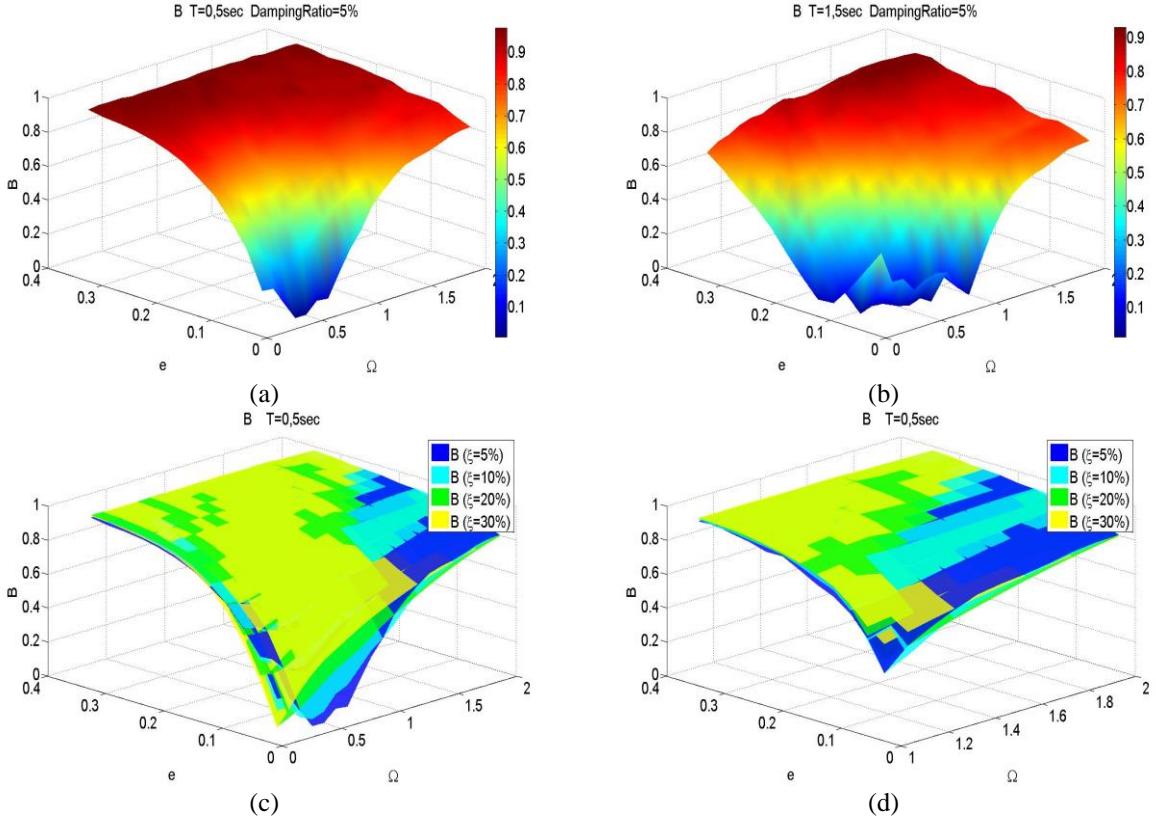


Figure 4. Correlation coefficient as function of  $e$  and  $\Omega_0$  for  $T_L=0.5\text{sec}$  and  $\xi=5\%$  (a);  $T_L=1.5\text{sec}$  and  $\xi=5\%$  (b);  $T_L=0.5\text{sec}$  and different values of damping ratio  $\xi$  (c); focusing on torsionally-stiff structures (d)

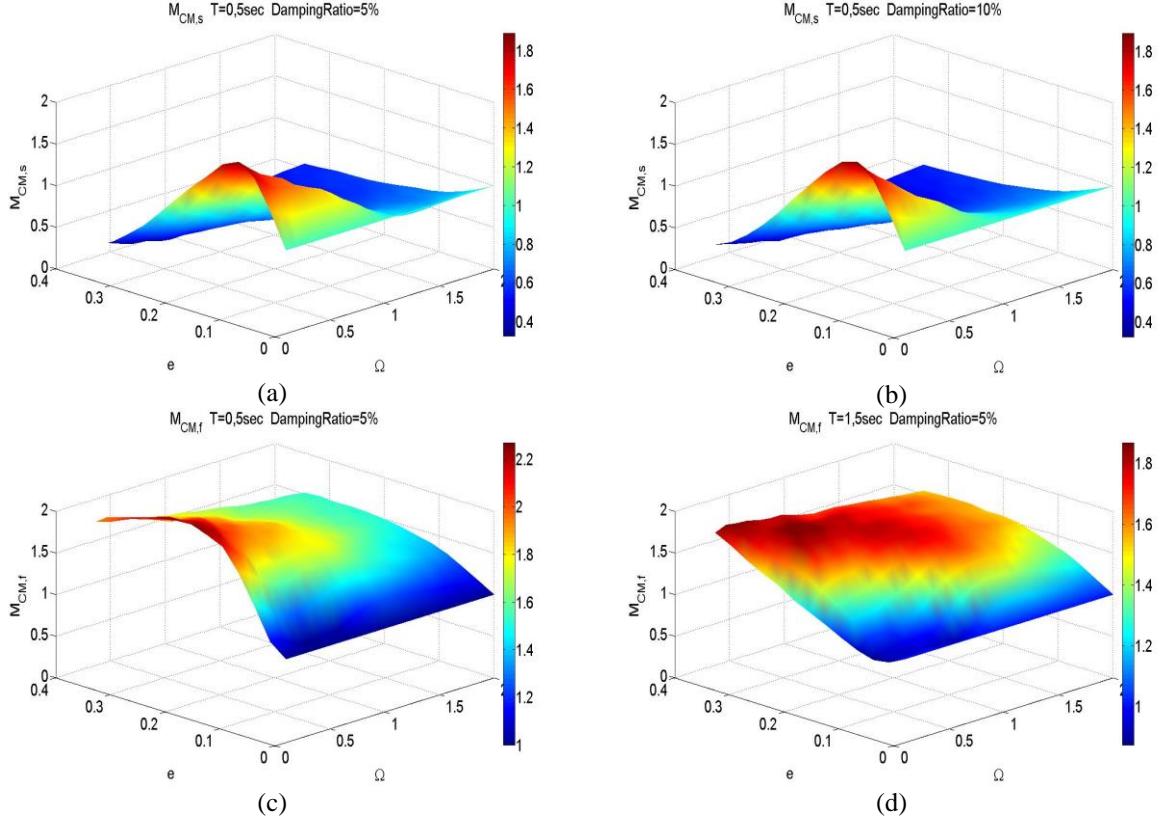


Figure 5. Magnification factor of the stiff side as function of  $e$  and  $\Omega_0$  for  $T_L=0.5\text{sec}$  and  $\xi=5\%$  (a);  $T_L=0.5\text{sec}$  and  $\xi=10\%$  (b); magnification factor of the flexible side as function of  $e$  and  $\Omega_0$  for  $T_L=0.5\text{sec}$  and  $\xi=5\%$  (c);  $T_L=1.5\text{sec}$  and  $\xi=5\%$  (d)

- Values of  $M_{CM,f}$  are between 0.9 and 2.2.  $M_{CM,f}$  surface appears quite smooth in the entire domain.

For fixed values of  $\xi$  and  $T_L$ , the magnification factor of the flexible side  $M_{CM,f}$  increases together with the eccentricity  $e$ , while its peak values decreases once the longitudinal period  $T_L$  increases.

## NON LINEAR SEISMIC RESPONSE FOR DIFFERENT FORCE REDUCTION FACTORS

The simple 3-dof system idealization used to the study the linear response cannot be used to study the non-linear response of planar asymmetric systems. For this reason, the still simple system idealization as proposed by Goel and Chopra 1991 is used to perform the non-linear analyses. The system consists of a roof diaphragm, assumed to be rigid in its own plane, supported by three frames, namely A, B and C (see Figure 6). Frame A is oriented along the  $y$ -direction, at a distance  $E$  from the center of mass (CM). Frames B and C are oriented along the  $x$ -direction, located at the same distance  $D/2$  from the CM; Frame B and C are assumed to have the same lateral stiffness ( $k/2$ ) so that the system is not eccentric along the  $y$ -direction. Frame A is assumed to have a lateral stiffness equal to  $k$ . Along the  $x$ -direction the eccentricity is equal to  $E$ . The rigid motion of the roof can be described by the same three DOFs defined at the CM of the slab: displacements  $u_x$ , in the  $x$ -direction and  $u_y$  in the  $y$ -direction, and torsional rotation  $u_\theta$  about the vertical axis. The equation of motion of the system is given by:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{u}_x(t) \\ \ddot{u}_y(t) \\ \rho \ddot{u}_\theta(t) \end{bmatrix} + [C] \begin{bmatrix} \dot{u}_x(t) \\ \dot{u}_y(t) \\ \rho \dot{u}_\theta(t) \end{bmatrix} + m \omega_L^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & (e\sqrt{12}) \\ 0 & (e\sqrt{12}) & 12e^2 + d^2/4 \end{bmatrix} \begin{bmatrix} u_x(t) \\ u_y(t) \\ \rho u_\theta(t) \end{bmatrix} = \begin{bmatrix} p_x(t) \\ p_y(t) \\ \frac{p_\theta(t)}{\rho} \end{bmatrix} \quad (17)$$

Where  $d=D/\rho$ .

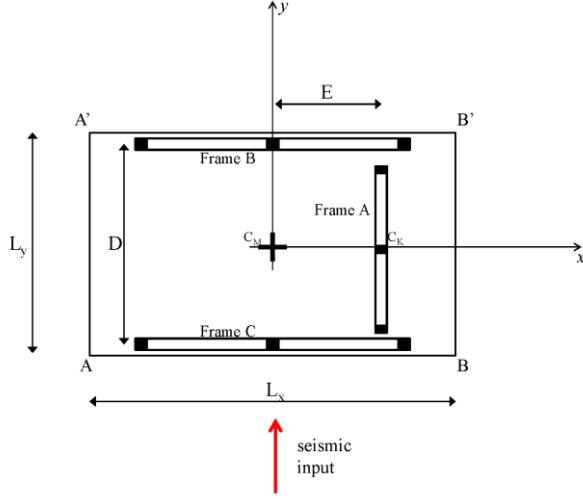


Figure 6. Idealized non linear eccentric system proposed by Chopra and Goel in 2007.

Each frame is characterized by an elastic-perfectly plastic response. The yield strength  $F_y$  is obtained by applying a force reduction factor  $R$  (Figure 7) between 2 to 5. Non linear time-history analyses have been developed using the same ground motion ensemble of the linear analyses. The same responses of the linear case have been computed (Figures 8 to 10):

$$\alpha_{d,NL,eq} = \rho \frac{u_{\theta,\max}}{u_{y,CM\ max}} \quad (18)$$

$$B_{d,NL,eq} = \frac{u_{y,\max} - u_{y,CM,\max}}{u_{\theta,\max} \cdot (L_x / 2)} \quad (19)$$

$$M_{CM,NL,s} = \frac{u_{y,s\max}}{u_{y,CM\ max}} \quad (20)$$

$$M_{CM,NL,s} = \frac{u_{y,s\max}}{u_{y,CM\ max}} \quad (21)$$

Figures 8a and b compares the  $\alpha_{d,NL,eq}$  curves for two different uncoupled lateral periods  $T_L$  (for a fixed damping ratio), while Figures 8c and d compares the  $\alpha_{d,NL,eq}$  curves for different damping ratios and the same uncoupled lateral periods  $T_L$ . Inspection of the graphs allow the following observations:

- The surface trends are qualitatively similar to those observed in the linear cases. The largest values of  $\alpha_{d,NL,eq}$  are observed for small values of  $T_L$  and  $\Omega_\theta$  and  $e$  around 0.2. Again peaks exceed 1.0 ( $\alpha_u$  is not always an upper bound estimation).
- Once the longitudinal period  $T_L$  increases, the peaks decreases and they even vanishes for  $T_L=1.5\text{sec}$ .

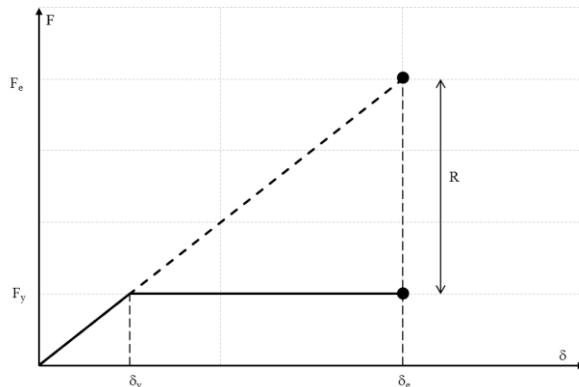


Figure 7. Force displacement response for the non-linear model.

- For a fixed longitudinal period  $T_L$ , the maximum rotational response tends to decrease as  $R$  increases. The dependence of  $\alpha_{d,NL,eq}$  on  $R$  is higher for the class of torsionally-stiff structures.

Figures 9a and b compares the  $B_{d,NL,eq}$  curves for two different uncoupled lateral periods  $T_L$  (for a fixed damping ratio), while Figures 9c and d compares the  $B_{d,NL,eq}$  curves for different damping ratios and the same uncoupled lateral periods  $T_L$ . Inspection of the graphs allow the following observations:

- The surface trends are qualitatively similar to those observed in the linear cases. In general values of  $B_{d,NL,eq}$  increases once  $e$  and  $\Omega_\theta$  increase.
- For a fixed force reduction factor the correlation decreases once the longitudinal period  $T_L$  increases.
- For a fixed longitudinal period  $T_L$ ,  $B_{d,NL,eq}$  increases once the force reduction factor decreases. The dependence of  $B_{d,NL,eq}$  on  $R$  is higher for torsionally-stiff structures.

Figures 10a and b compares the  $M_{CM,NL,s}$  curves for two different damping ratios and same uncoupled lateral periods  $T_L$ , while Figures 11c and d compares the  $M_{CM,NL,f}$  curves for two different uncoupled lateral periods  $T_L$  and same damping ratio. Inspection of the graphs allow the following observations:

- The surface trend of both  $M_{CM,NL,s}$  and  $M_{CM,NL,f}$  are qualitatively similar to those observed in the linear cases.
- $M_{CM,NL,s}$  appears quite smooth for torsionally-stiff systems (values are less than 1.0), while it rapidly varies for torsionally flexible systems (values are also larger than 1.0).
- For fixed values of  $T_L$ , the peaks of  $M_{CM,NL,s}$  gradually decreases as  $R$  increases. On the contrary in the area of torsionally stiff system  $M_{CM,NL,s}$  tends to slightly increase (the surface becomes almost flat for  $R=5.0$  with values between 0.8 to 1.0)
- The magnification factor  $M_{CM,NL,f}$  increases as the eccentricity increases. The peaks are observed for high torsionally flexible systems and eccentricities  $e$  around 0.2.
- For fixed values of  $R$  the peaks of  $M_{CM,NL,f}$  tends to decrease as  $T_L$  increases and the surface tends to flatten out.

## CONCLUSIONS

This paper provides a further insight into the dynamic response of one-storey non-linear asymmetric systems through the development of a systematic parametric analyses aimed at exploiting the influence of the fundamental parameters governing the response of non-linear planar asymmetric systems ( $e$ ,  $\Omega_\theta$ ,  $T_L$ ,  $\xi$ ,  $R$ ) magnification displacement at the corner side. In detail for linear undamped systems in free vibrations, closed-form expressions of the correlation coefficient between the maximum longitudinal and the maximum rotational response and estimations of the maximum rotational response for different damping ratios have been derived. Finally, the effect of the excursion into the inelastic field has been analysed by performing non-linear seismic analyses for different value of force reduction factor  $R$  and estimations of the corner displacement magnifications have been provided.

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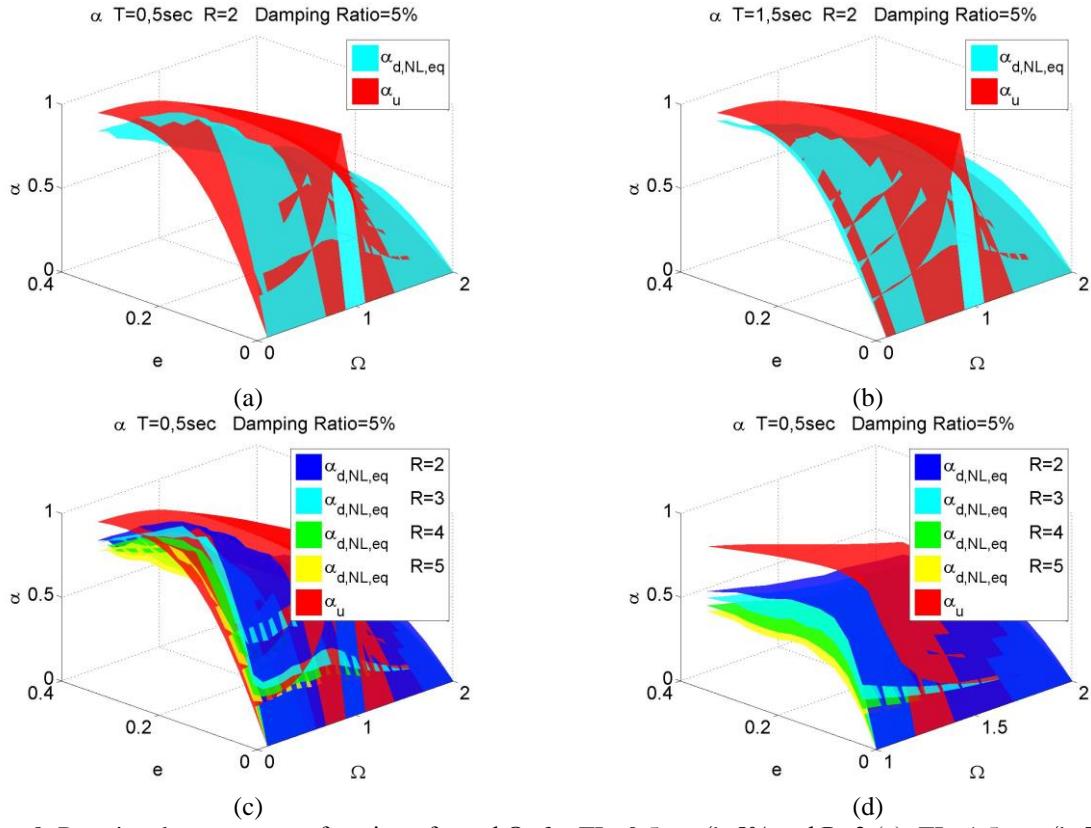


Figure 8. Rotational parameter as function of  $e$  and  $\Omega_0$  for  $T_L=0.5\text{sec}$ ,  $\xi=5\%$  and  $R=2$  (a);  $T_L=1.5\text{sec}$ ,  $\xi=5\%$  and  $R=2$ (b);  $T_L=0.5\text{sec}$ ,  $\xi=5\%$  and different values of force reduction factors  $R$  (c), focusing on torsionally-stiff structures (d).

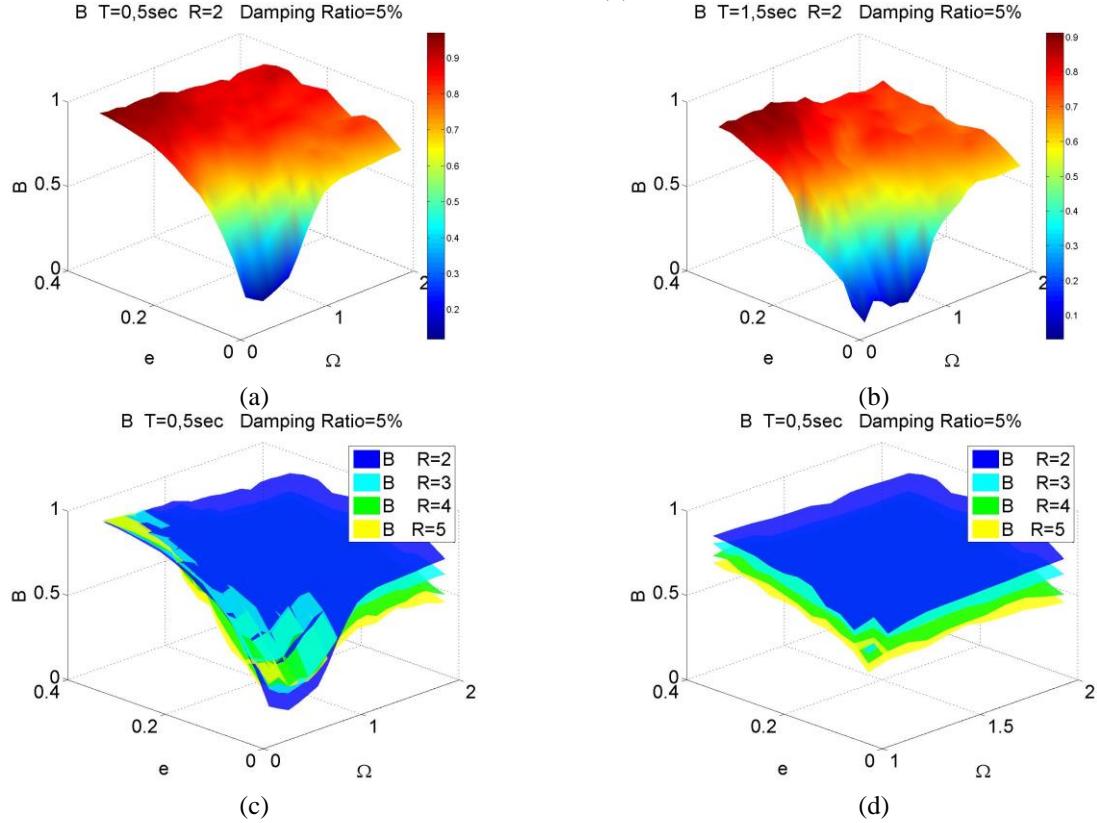


Figure 9. Correlation coefficient as function of  $e$  and  $\Omega_0$  for  $T_L=0.5\text{sec}$ ,  $\xi=5\%$  and  $R=2$  (a);  $T_L=1.5\text{sec}$ ,  $\xi=5\%$  and  $R=2$ (b);  $T_L=0.5\text{sec}$ ,  $\xi=5\%$  and different values of force reduction factors  $R$  (c), focusing on torsionally-stiff structures (d)

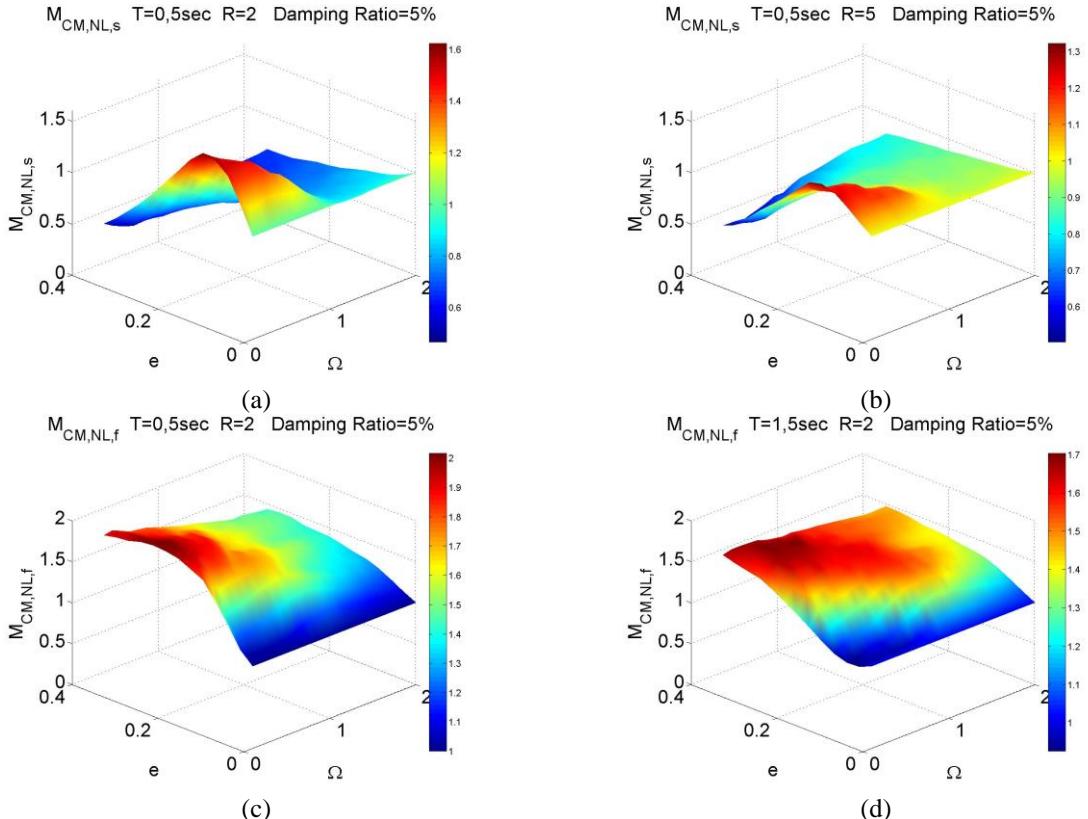


Figure 10. Magnification factor of the stiff side as function of  $e$  and  $\Omega_0$  for  $T_L=0,5\text{sec}$ ,  $\xi=5\%$  and  $R=2$  (a);  $T_L=0,5\text{sec}$ ,  $\xi=5\%$  and  $R=5$ (b); magnification factor of the flexible side as function of  $e$  and  $\Omega_0$  for  $T_L=0,5\text{sec}$ ,  $\xi=5\%$  and  $R=2$  (c);  $T_L=1,5\text{sec}$ ,  $\xi=5\%$  and  $R=2$ (d)

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