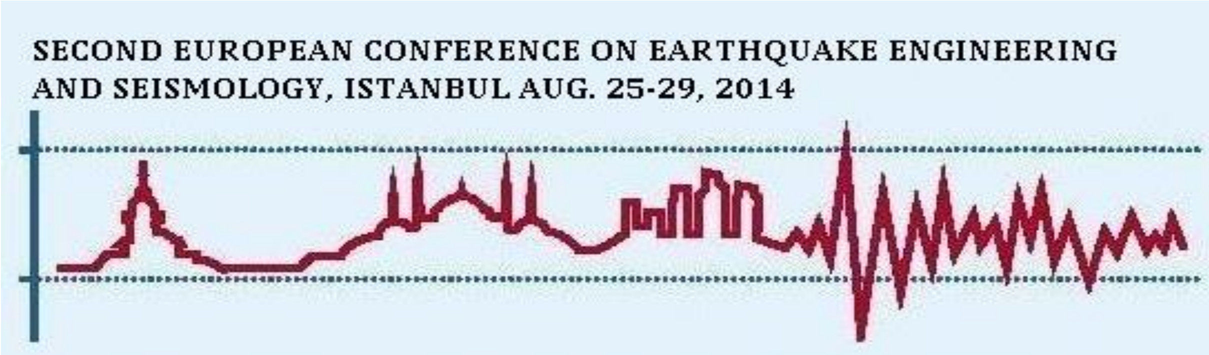

MEASUREMENTS OF THE FREE VIBRATION RESPONSE OF THE ASINELLI TOWER IN BOLOGNA AND ITS INTERPRETATION

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ABSTRACT

Since the early 20th century the two main towers of Bologna (Northern Italy), Asinelli and Garisenda, perhaps the main monumental landmark of the city, have been studied by researchers of the University of Bologna, in order to assess their structural safety (Cavani 1912).

Following the two mainshocks of the seismic sequence of Emilia Romagna (Northern Italy) (May 20th and 29th, 2013, respectively M 5.9 and 5.8, about 50 km far from Bologna), six seismic stations have been installed by the Istituto Nazionale di Geofisica e Vulcanologia (INGV) inside the two Towers in order to evaluate their seismic response, with special attention to the identification of the fundamental frequencies. Continuous measurements were carried out from June to September 2012. During the experiment the maximum magnitude earthquake recorded in the surrounding areas of Bologna was not greater than M 3.2. Despite this magnitude value, the level of seismic noise background in the centre of the city is too high to record seismograms useful for the analyisiys, therefore addressed only to the use of ambient noise.

The present paper provides the first results of a joint study developed by INGV and the University of Bologna aimed at providing an interpretation of the experimental data as obtained from the continuous monitoring (Azzara et al. 2013, 2014). In detail, analysing the free vibration response of the Asinelli Tower, the typical response of a certain class of in-plan asymmetric buildings has been clearly recognized. Starting from the dynamics of in-plan asymmetric buildings, a procedure to estimate the "equivalent eccentricity" of masonry towers is presented. The evaluation of the equivalent eccentricity may be of importance for the seismic assessment of masonry towers.

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INTRODUCTION

The Italian monumental heritage is worldwide recognized as one of the richest and various, thanks to the old history of the country. Among the many symbols of the Italian monumental heritage, the two medieval towers of Bologna, the Garisenda and the Asinelli Towers, located in the hearth of the town, are among the most known and attractive ones [Roversi 1989].

The Asinelli Tower, 97 m tall, was built between the XII and XIII century, an historical period characterized by strong political debates, especially between the Church and the Empire. It was commissioned by the one of the most influential family at the time, the Asinelli Family, as a symbol of their power. During its almost millennial life, the Tower was subjected to various accidents such as fires (the most destructive is dated 1398 and damaged most of the internal wood structures and the selenitic basement), lightnings (in 1754 a lightning caused damages to all the upper part of the tower, for about 30 m of length) and earthquakes. The 1399 earthquake caused the failure of the upper part of the Tower (the little bell tower) which was reconstructed. It fell down again during the 1505 Appennino Modenese earthquake. Historical documents report that also other portions of masonry fell down during that earthquake. Beside natural accidents, in 1943, during World War II, the Tower was hit by a bomb explosion occurred not far from it. In more recent years (end of 1990s) the Tower has been subjected to strengthening interventions as a joint project between the Municipality and the University of Bologna. Before the strengthening, in situ and laboratory tests were performed to characterize the mechanical properties of the masonry. From 2011, TECNO IN monitoring system S.p.A. was commissioned by the Municipality of Bologna to perform real-time measurements to evaluate the static response of the towers related to environmental agents. (A full description of the monitoring system is available on www.tecnoinmonitoraggi.it). Measurements include deformations across the main cracks through short-base deformometers, deformations of critical portion of masonry through short-base deformometers, tilt measurements, strains along the steel ties through strain gages, temperatures and wind speed through a meteorological station placed at the top of the Tower. In 2012, following the seismic sequence of Emilia Romagna, started on 20th May with an M 6.2 earthquake, the Istituto Nazionale di Geofisica e Vulcanologia (INGV) was called by the local authorities to design an experiment for the dynamical monitoring of the Towers. At the moment of preparation of this paper, thea report containing a description of the first results of the dynamical monitoring is nearing publication on the INGV website.

The objective of the present paper is to investigate on a particular dynamic phenomenon that has been recognized by analyzing the data of the dynamic monitoring. The phenomenon deals with beat waves and it was already recognized by some of the authors studying the behaviour of planar asymmetric systems.

THE ASINELLI TOWER

The Asinelli Tower is a 97 m high masonry tower with an inclination of 1.7° (corresponding to a overhanging of 2.5 m) along the West direction (Figure 1). The tower cross section dimension (almost square along the entire height) gradually decrease (almost linearly) from 8.5m side length at the base to 6.0m at the top, excepting a sudden discontinuity at a height of 34 m. The external walls are realized according to the “a sacco” technique (Figure 1): two skins of brick masonry with an internal rubble and mortar fill. The fill is composed of irregular materials including brick fragments and irregular stones bound by aerial mortar. Common solid bricks are used for the outer skins, while the basement is realized with selenitic bricks. The wall thickness (the two skins plus the internal fill) also gradually decreases from 3.15 m at the base to 0.45 m at the top, excepting for three main discontinuities at 11.5 m, 34.0 m and 56.0 m. The masonry assemblies are not regular, with variations in both the width of the bricks and the thickness of the mortar (from 1.0 cm to 3.0 cm).

Limited in-tests and laboratory tests were performed in order to characterize the material properties of the tower. In-situ tests included flat jack deformability tests (one compression test and two shear tests) according to ASTM 1197C standard and pointing hardness tests (six tests: one on the internal wall and 5 on the external walls) with the hammer pendulum (RILEM TC127MS D.2, . Laboratory tests were performed on masonry cores and included compression tests and measurements of density. The brick showed a compression strength around 10 MPa and a Young's modulus of approximately 8000 MPa. On average, the results of the analyses of the cores reveal that the internal portion of the masonry ("sacco") is characterized by a compression strength of 4 MPa and a Young's modulus of 2500 MPa. It has to be noted that the basement seems to have higher mechanical properties (a compressive strength of 6 MPa and a Young's moduls of 4000 MPa). Density measurements provided the following values: 22-24 kN/m^3 for the selenitic bricks at the base; 17-18 kN/m^3 for the masonry bricks and 16-18 m^3 for the internal fill. No measurements of tensile strengths and Poisson's coefficients are available and no experimental data are available for the selenitic basement at all. Values obtained from experimental tests (unpublished results of tests commissioned by the University of Bologna) on the selenitic basement of the near Garisenda tower (compressive strength equal to 11.9 MPa) may be considered as reasonable.

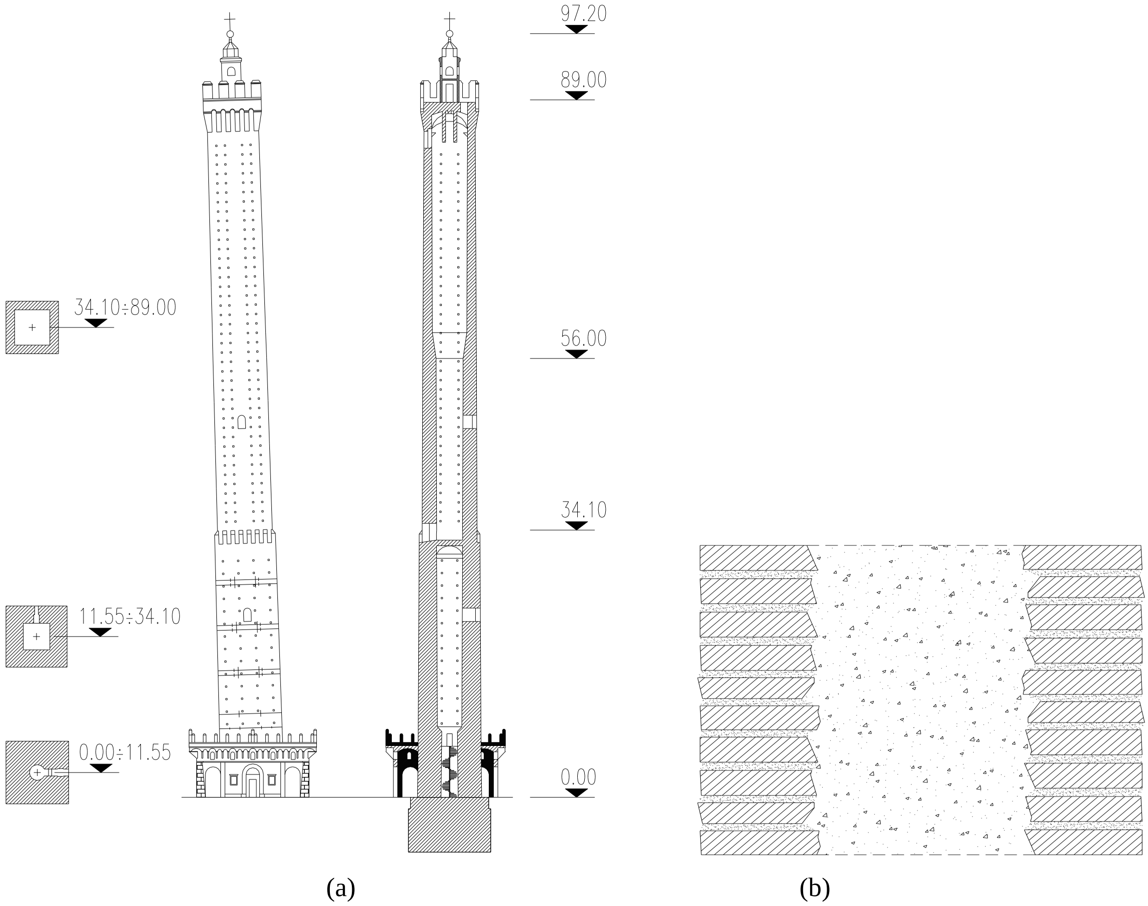


Figure 1. (a) The Asinelli Tower elevation with the indication of the main discontinuities. (b) A schematic view (vertical cross-section) of the "a sacco" masonry.

THE EXPERIMENT CONDUCTED BY THE INGV

Four seismic stations were installed inside the Asinelli Tower. All the stations were equipped with triaxial seismometers (Lennartz Le3d5s, 0.2 Hz eigenfrequency) coupled to AD 24-bit converters (Reftek 72A/07). Data, sampled at 200 sps, were continuously recorded from 22th June 2012 to 17h September 2012. The location of the three sites was driven by considerations about the observed structural changes along the Tower and on the results of studies conducted by the University of Modena about the slope of the Tower (Bertacchini et al. 2010). The seismic stations were located at 35 m, 70 m and ~~on~~ at the top. The fourth station was installed at the basement. Although the seismic sequence in Emilia-Romagna was active during the monitoring interval but no earthquake was effectively recorded due to the strong seismic noise background of the city center. Therefore the used dataset consists essentially of seismic ambient noise: the ground vibration induced by natural or artificial sources that propagate along the towers.

Average hourly Fast Fourier Transform (FFT) has been computed in order to estimate the Tower frequencies of vibration (Figure 2a). The experimental frequencies show a good agreement with those estimated through FE models (Ceccoli et al. 2011). The first three flexural frequencies fall within the range of 0.32-0.33 Hz, 1.3-1.5 Hz and 3.0-3.3 Hz, respectively. Looking at the spectra and focusing in the frequency range between 2 and 3 Hz, it is possible to identify a small frequency peak that, in agreement with the results of FE models (Ceccoli et al. 2011), could be associated to the first torsional mode.

In order to better discriminate the rotational component from the translational components two other seismic stations have been installed at the top of Asinelli. The two stations, geographically oriented, were placed at opposite corners of the summit terrace.

Figure 2b shows the deconvolution of the vertical component (that should not be influenced by torsional motion) from the anti-correlated portion of the signal recorded by the two seismic stations. The torsional frequency falls in the range 2.30 – 2.33 Hz fitting the results obtained by the theoretical models (Ceccoli et al. 2011).

THE TOWER DYNAMIC RESPONSE IN FREE VIBRATION

Seismic continuous monitoring has allowed to recognize the occurrence of frequent transients propagating along the towers that can be probably ascribed to the passage of heavy vehicles along the streets surrounding the tower. The oscillation produces a clear effect of damped beating (Figure 3) whose duration and amplitude can be related to the amplitude of the triggering signal. The frequency of the beat and the damping ratio can be related to the physical and geometric properties of the structure, understanding the mechanism and modelling the phenomenon can help to better investigate the dynamical behaviour of the Tower.

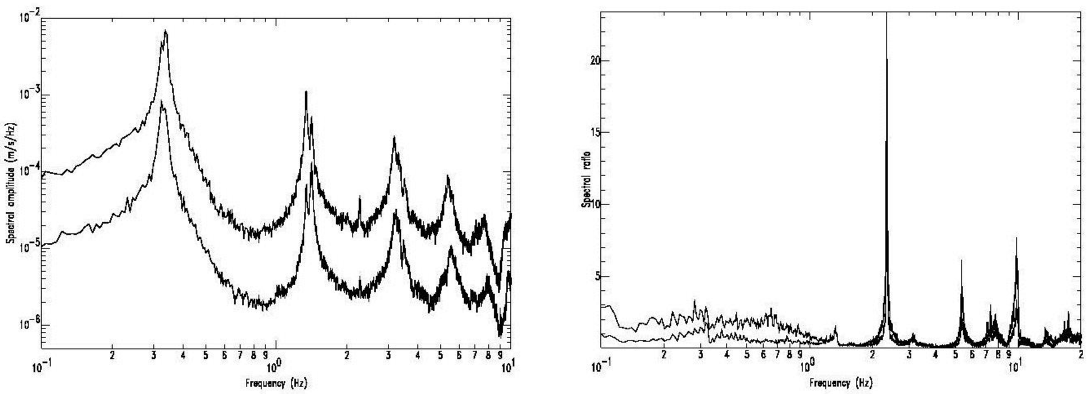


Figure 2. (a) Average hourly FFT computed on the two horizontal components; (b) Spectral ratio between the anti-correlated horizontal signal recorded by two seismic station installed at the top of the Asinelli Tower.

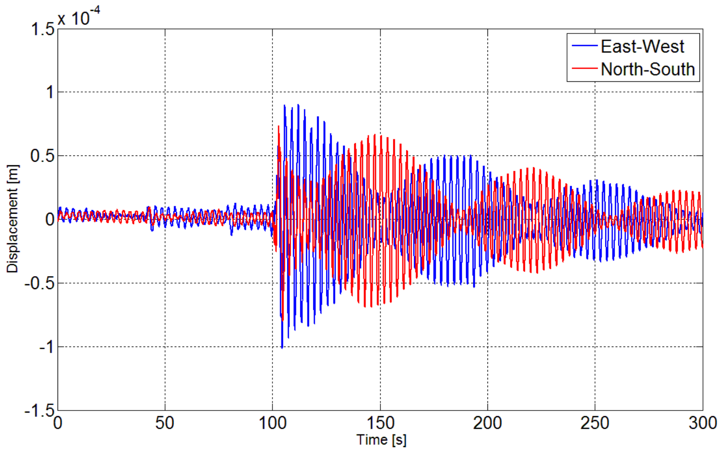


Figure 3. Beatings phenomenon induced by a transient signal as recorded at the top of the tower.

It is worth to note that, in general, the ratio between the fundamental torsional and lateral periods (referred to as torsional-to-lateral frequency ratio, Ω_θ, see Chopra 1995) of a uniform (i.e. constant geometrical and mechanical properties along the height) tower can be estimated using the following relationship (Den Hartog 1947):

$$\Omega_\theta = \frac{\omega_{1,t}}{\omega_{1,f}} = 2\pi \frac{l}{\rho} \cdot \sqrt{\frac{J_t}{2(1+\nu)J}} \qquad (1)$$

where $\rho = \sqrt{J/A}$ is the radius of inertia of the cross section of the tower. Typically, ρ/l ratios vary between 0.01 (for slender towers) and 0.06 (for squat tower) leading to torsional-to-lateral frequency ratios between 3.0 to 20.0 (assuming a Poisson's coefficient of 0.2). These values are common for high torsionally-stiff structures.

From a recorded displacement response (as the one displayed in Figure 3) the following information can be easily measured/obtained (Figure 4):

- Number of fast oscillations included in a slow oscillation:

$$N = \frac{\omega_{fast}}{\omega_{slow}} \qquad (2)$$

  where ω_{fast} and ω_{slow} are the frequency of the fast and slow oscillations, respectively.
- The ratio between the minimum and the maximum amplitudes of the ocsillation within an entire slow oscillation j:

$$R = \Delta u_{\min,j} / \Delta u_{\max,j} \qquad (3)$$

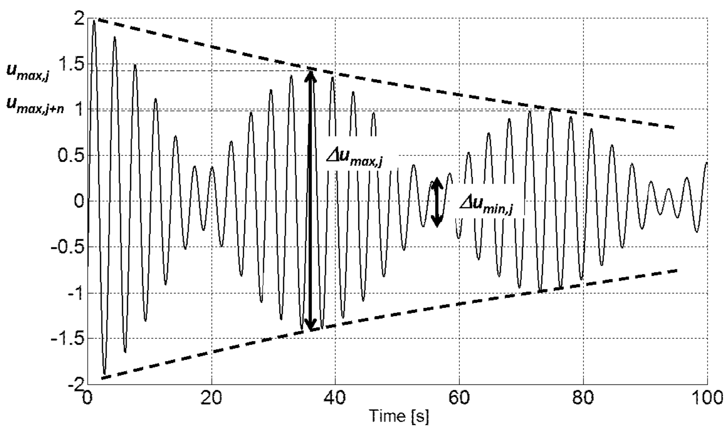


Figure 4. the graphical representation of the information which can obtained from the recorded displacement response.

R is clearly between 0.0 and 1.0

- the equivalent damping ratio (logarithmic decrement method Chopra 1995):

$$\xi = \frac{1}{2\pi n} \cdot \ln\left(\frac{u_{\max,j}}{u_{\max,j+n}}\right) \tag{4}$$

THE DYNAMIC BEHAVIOUR OF IN-PLAN ASYMMETRIC SYSTEMS

The dynamic behaviour of eccentric structures has been the objective of extensive research since the late 1970s (Kan and Chopra 1977, Hejal and Chopra 1987, Rutenberg 1992, Nagarajaiah *et al.* 1993, Perus and Fajfar 2005). In previous research works (Trombetti and Conte 2005, Trombetti *et al.* 2008), some of the authors identified a structural parameter, called "alpha", related to the attitude of one-storey torsionally-stiff systems to develop a rotational response in free vibration and proposed a simplified procedure, called "Alpha-method", for the estimation of the maximum torsional response. The method has been extended to all classes of one-storey asymmetric systems, including both torsionally stiff and torsionally flexible systems (Palermo *et al.* 2013).

Under the following assumptions:

- equal total lateral stiffness k along the x- and the y-direction (i.e. $k=k_x=k_y$, where k_x and k_y are the translational stiffness the x- and the y-direction, respectively);
- the rotational response u_θ developed under dynamic excitation is small enough to allow the approximation $u_\theta \cong \sin(u_\theta) \cong \tan(u_\theta)$;

the equations of motion for a 3D one-storey system idealization with rigid diaphragm (assumed as a 3-dof systems) can be written as follows (with reference to a system with the origin located at the center mass CM, see Figure 5):

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}\begin{bmatrix} \ddot{u}_x(t) \\ \ddot{u}_y(t) \\ \rho\ddot{u}_\theta(t) \end{bmatrix} + [C]\begin{bmatrix} \dot{u}_x(t) \\ \dot{u}_y(t) \\ \rho\dot{u}_\theta(t) \end{bmatrix} + m\omega_L^2 \begin{bmatrix} 1 & 0 & -e_y\sqrt{12} \\ 0 & 1 & e_x\sqrt{12} \\ -e_y\sqrt{12} & e_x\sqrt{12} & \Omega_\theta^2+12e^2 \end{bmatrix}\begin{bmatrix} u_x(t) \\ u_y(t) \\ \rho u_\theta(t) \end{bmatrix} = \begin{bmatrix} p_x(t) \\ p_y(t) \\ \frac{p_\theta(t)}{\rho} \end{bmatrix} \tag{5}$$

where: m is the mass of the system; $e_x=E_x/D_e$ and $e_y=E_y/D_e$ are the relative eccentricities in the x and y-direction, respectively; $e=\sqrt{e_x^{\ 2}+e_y^{\ 2}}$ is the global eccentricity ,; D_e is the equivalent diagonal equal to $\sqrt{12}\rho$ (for a rectangular plan D_e is equal to the diagonal); E_x and E_y are the eccentricities in the x and y-direction ; [C] is the damping matrix (classical damping is assumed). Values of parameter Ω_θ separate planar asymmetric systems in two categories:

- torsionally-stiff systems characterized by $\Omega_\theta \geq 1.0$;
- torsionally-flexible systems characterized by $\Omega_\theta < 1.0$.

The undamped free vibration response (from an initial displacement a along the y-direction) of the studied system is given by:

$$\begin{aligned} u_y(t) &= a\frac{e_x^{\ 2}}{e^2}\left\{A_1\cos(\omega_1 t) + \frac{e_y^{\ 2}}{e_x^{\ 2}}\cos(\omega_2 t) + A_3\cos(\omega_3 t)\right\} \\ u_x(t) &= a\frac{e_x e_y}{e^2}\left\{-A_1\cos(\omega_1 t) + \cos(\omega_2 t) - A_3\cos(\omega_3 t)\right\} \\ u_\theta(t) &= \frac{a}{\rho}\frac{e_x}{e}A_4\left\{\cos(\omega_1 t) - \cos(\omega_3 t)\right\} \end{aligned} \tag{6}$$

where A_1 , A_3 and A_4 are defined as follows (Trombetti and Conte 2005):

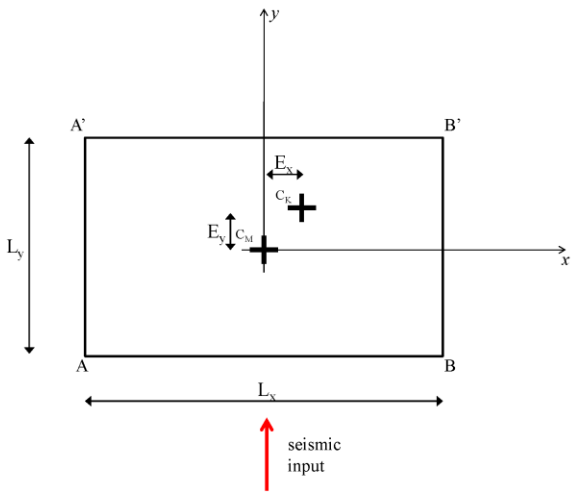


Figure 5. Plan view of a one-storey eccentric system.

$$A_1 = \frac{1-\Omega_3}{\Omega_1-\Omega_3}$$
$$A_3 = \frac{\Omega_1-1}{\Omega_1-\Omega_3} \tag{7}$$
$$A_4 = \frac{1}{e\sqrt{12}}\frac{(\Omega_1-1)(\Omega_3-1)}{\Omega_3-\Omega_1}$$

and:

$$\Omega_1 = (\omega_1/\omega_L)^2 = 1/2\left(1+\Omega_\theta{}^2+12e^2 - \sqrt{(\Omega_\theta{}^2+12e^2-1)^2+48e^2}\right)$$
$$\Omega_2 = (\omega_2/\omega_L)^2 = 1 \tag{8}$$
$$\Omega_3 = (\omega_3/\omega_L)^2 = 1/2\left(1+\Omega_\theta{}^2+12e^2 + \sqrt{(\Omega_\theta{}^2+12e^2-1)^2+48e^2}\right)$$

are the normalized (squared) natural frequencies of the system.

In the case of undamped free vibrations, the parameter "alpha", related to the ratio between the maximum rotational and the maximum longitudinal displacement response, has been derived in a closed-form as a function of e and Ω_θ (Trombetti and Conte 2005):

$$\alpha_u = \rho\frac{u_{\theta,\max}}{u_{y,\max}} = \frac{4e\sqrt{3}}{\sqrt{(\Omega_\theta^2+12e^2-1)^2+48e^2}} \tag{9}$$

The damped free vibration response of the system represented in Figure 5 is given by (Trombetti and Conte 2005):

$$\begin{aligned}
u_y(t) &= a\frac{e_x{}^2}{e^2}\Lambda Exp(-\xi\omega_1 t)\left\{A_1\cos(\omega_{D1}t+\theta)+Exp(-\xi(\omega_2-\omega_1)t)\frac{e_y{}^2}{e_x{}^2}\cos(\omega_{D2}t+\theta)\right.\\
&\left.+A_3 Exp(-\xi(\omega_3-\omega_1)t)\cos(\omega_{D3}t+\theta)\right\}\\
u_x(t) &= a\frac{e_x e_y}{e^2}\Lambda Exp(-\xi\omega_1 t)\left\{-A_1\cos(\omega_{D1}t+\theta)+Exp(-\xi(\omega_2-\omega_1)t)\cos(\omega_{D2}t+\theta)\right.\\
&\left.-A_3 Exp(-\xi(\omega_3-\omega_1)t)\cos(\omega_{D3}t+\theta)\right\}\\
u_\theta(t) &= \frac{a}{\rho}\frac{e_x}{e}A_4\Lambda Exp(-\xi\omega_1 t)\left\{\cos(\omega_{D1}t+\theta)-Exp(-\xi(\omega_3-\omega_1)t)\cos(\omega_{D3}t+\theta)\right\}
\end{aligned} \tag{10}$$

where ξ is the damping ratio (constant for all modes); $\Lambda = \sqrt{1+\frac{\xi}{\sqrt{1-\xi^2}}}$ and $\theta = -\arctan\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$;

$\omega_{Di} = \omega_i\sqrt{1-\xi^2}$; i=1,2,3.

It can be noted that for high torsionally-stiff structures A_3 is close to zero (Trombetti and Conte 2005) and the displacement responses $u_y(t)$ and $u_x(t)$ given by Eqs. 6 and 10 can be simplified as follows:

$$u_y(t) \cong a\frac{e_x^{\,2}}{e^2}\left\{A_1\cos(\omega_1 t)+\frac{e_y^{\,2}}{e_x^{\,2}}\cos(\omega_2 t)\right\}$$
$$u_x(t) \cong a\frac{e_x e_y}{e^2}\left\{-A_1\cos(\omega_1 t)+\cos(\omega_2 t)\right\} \quad (11)$$

$$u_y(t) \cong a\frac{e_x^{\,2}}{e^2}\Lambda Exp(-\xi\omega_1 t)\left\{A_1\cos(\omega_{D1}t+\theta)+Exp(-\xi(\omega_2-\omega_1)t)\frac{e_y^{\,2}}{e_x^{\,2}}\cos(\omega_{D2}t+\theta)\right\}$$
$$u_x(t) \cong a\frac{e_x e_y}{e^2}\Lambda Exp(-\xi\omega_1 t)\left\{-A_1\cos(\omega_{D1}t+\theta)+Exp(-\xi(\omega_2-\omega_1)t)\cos(\omega_{D2}t+\theta)\right\} \quad (12)$$

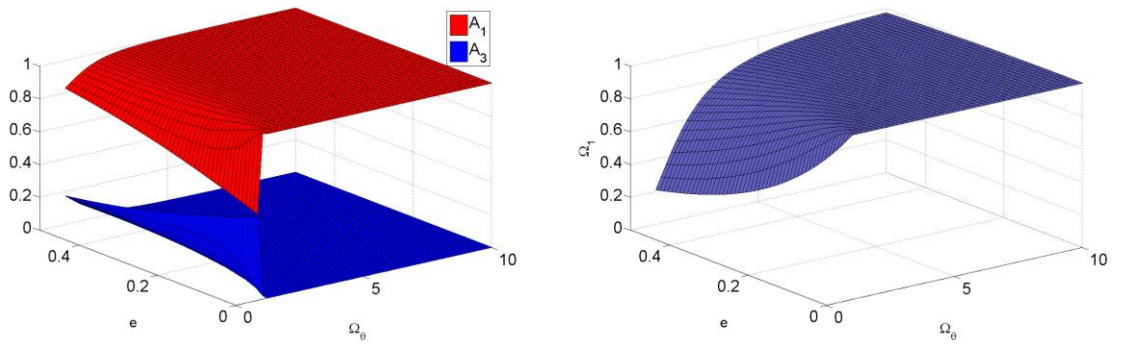
Eq. 11 is the sum of two undamped SDOF harmonic response having frequencies equal to ω_1 and ω_2 while Eq. 12 is the sum of two damped SDOF harmonic response having (undamped) frequencies equal to ω_1 and ω_2.

Furthermore for high torsionally-stiff systems A_1 is close to one, while ω_1 is close ω_2 (Figure 6). Under these two conditions (closely spaced frequencies and similar amplitudes): (i) the longitudinal responses $u_x(t)$ and $u_y(t)$ assume the shape of slow modulated waves containing fast modulated waves; (ii) when the envelope of the longitudinal displacement reaches its maximum value, that of the transversal displacement is at its minimum and vice versa. The phenomenon is known as beating phenomenon and induces a mutual energy/motion transfer which is maximized when e_x=e_y (Trombetti and Conte 2005). An illustrative example of undamped and damped beats is shown in Figure 7. A clear similarity between the Tower free vibration response and the free vibration response of a 3-dof eccentric system may be noted by comparing Figure 2 with Figure 7.

THE EQUIVALENT ECCENTRICTY FROM THE TOWER RESPONSE IN FREE VIBRATION

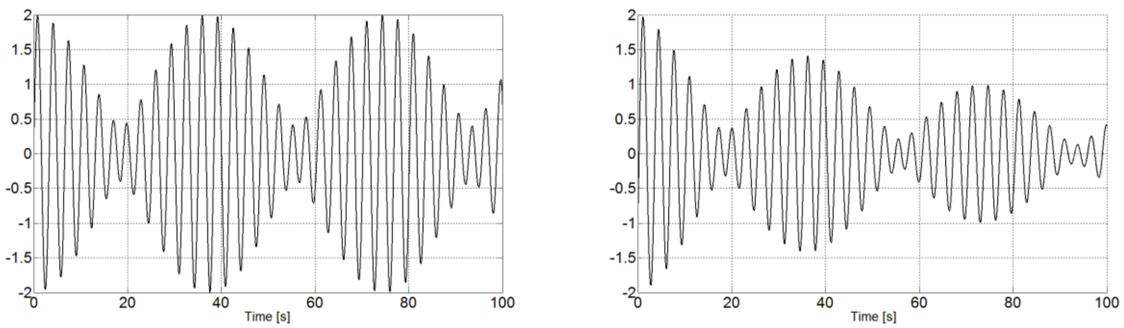
From the analogy between the observed dynamic response of the Tower and the theoretical dynamic response of a 3-dof eccentric system, a simple procedure for the evaluation of the equivalent eccentricity which make use of the recorded data can be developed.

From the relationships between *N*, fast and slow frequencies ω_{fast} and ω_{slow} and ω_1 and ω_2:



(a) (b)

Figure 6. (a) $A_1$and A_3 vs *e* and Ω_θ. (b) Ω_1 vs *e* and Ω_θ.



(a) (b)

Figure 7. Free vibration response for a system characterized by ω_1 =2.0 rad/s and ω_1 =2.1 rad/s. (a) undamped free vibration response; (b) damped free vibration response ($\xi = 0.003$).

$$N = \frac{\omega_{fast}}{\omega_{slow}}$$
$$\omega_{fast} = \frac{\omega_1 + \omega_2}{2} \tag{13}$$
$$\omega_{slow} = \frac{\omega_1 - \omega_2}{2}$$

and making use of Eq. 8 it is possible, after simple mathematical developments, to obtain the following close-form expression of *N* as a function of *e* and Ω_θ (Figure 8):

$$N = \frac{1 + \sqrt{1 + 1/2(\Omega_\theta^{\ 2} + 12e^2 - 1)\left(1 - \sqrt{1 + \frac{48e^2}{(\Omega_\theta^{\ 2} + 12e^2 - 1)^2}}\right)}}{1 - \sqrt{1 + 1/2(\Omega_\theta^{\ 2} + 12e^2 - 1)\left(1 - \sqrt{1 + \frac{48e^2}{(\Omega_\theta^{\ 2} + 12e^2 - 1)^2}}\right)}} \tag{14}$$

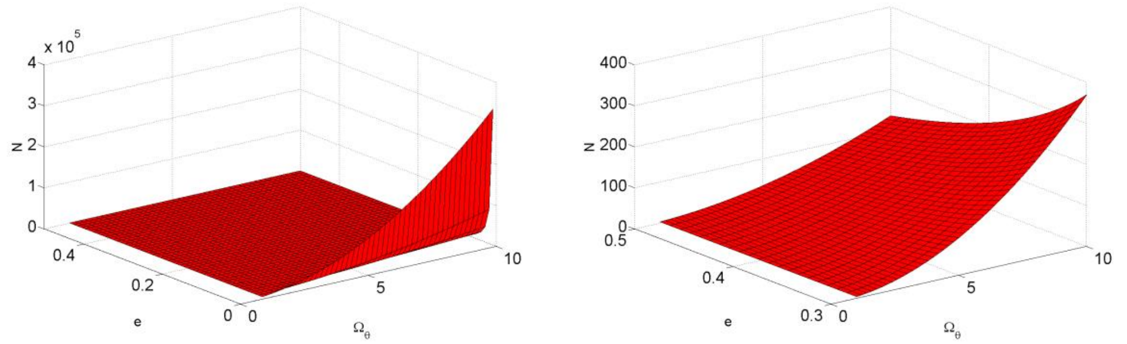
It can be noted that *N* remains quite flat (<10) for large values of *e* (>0.3) and limited torsionally-stiff systems ($1.0 < \Omega_\theta < 2.0$). On the contrary, as long as *e* decreases and/or Ω_θ increases, *N* grows very rapidly achieving values larger than 1000 (meaning really long slow oscillations). For the sake of clearness, 2-D curves representation of *N* versus *e* for selected Ω_θ are displayed in Figure 9. It can be noted that for *N* less than 100 and Ω_θ between 5 and 15 (typical of masonry towers) equivalent eccentricity may become quite large. Once *N* is measured and Ω_θ is estimated (Eq. 1 or similar) Eq. 14 allows to obtain the following close-form expression of the equivalent eccentricity (Figure 10):

$$e = \frac{\sqrt{N\left(\Omega_\theta^{\ 2}(N+1)^2 - (N-1)^2\right)}}{\sqrt{3}\left(N^2+1\right)} \tag{15}$$

It can be noted that, for a fixed *N*, the equivalent eccentricity *e* increases almost linearly with Ω_θ. On the contrary, for a fixed Ω_θ, eccentricity *e* decreases almost linearly with *N*. From a theoretical point of view the equivalent eccentricity *e* may become quite large and even fall out from the plan of the system (for a square plan an equivalent eccentricity e=0.35 leads to *E*=0.5*L*). For the Asinelli Tower, by assuming *N* around 55 (see Figure 4) and Ω_θ around 10-15 (Eq. 1), Eq. 15 leads to an equivalent eccentricity equal to 0.75-0.95.

Finally, once the equivalent eccentricity *e* is evaluated, it is possible to estimate its two components (e_x and e_y) by comparing the measured ratio *R* (Eq. 3) with that obtained from Eq. 11 (i.e from the two components of the longitudinal (or transversal) displacement):

$$\frac{\frac{e_x^{\ 2}}{e^2}A_1 + \frac{e_y^{\ 2}}{e^2}}{\frac{e_x^{\ 2}}{e^2}A_1 - \frac{e_y^{\ 2}}{e^2}} = R \tag{16}$$



(a) (b)

Figure 8. N vs e and Ω_θ. (a) entire domain; (b) zoom for e between 0.3 and 0.5 and Ω_θ between 1.0 and 2.0.

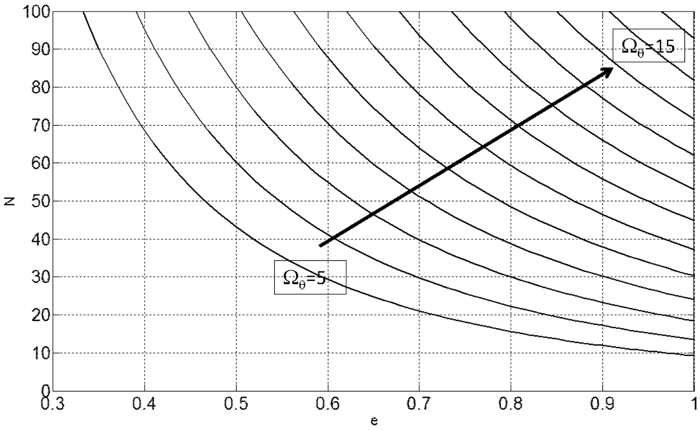
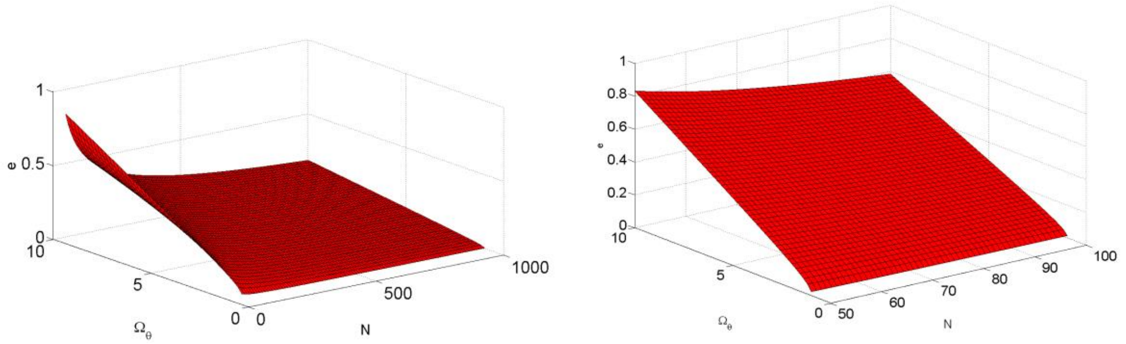


Figure 9. N vs e for selected Ω_θ from 5 to 15.



(a) (b)

Figure 10. e vs N and Ω_θ: (a) entire domain; (b) zoom for N between 50 and 100 and Ω_θ between 1.0 and 2.0.

leading to:

$$e_x = e\sqrt{\frac{R+1}{R(1+\mathrm{A}_1)+1-\mathrm{A}_1}} \tag{17}$$

As an illustrative example Figure 11 displays the interaction between e_x and R for A_1=1.0.

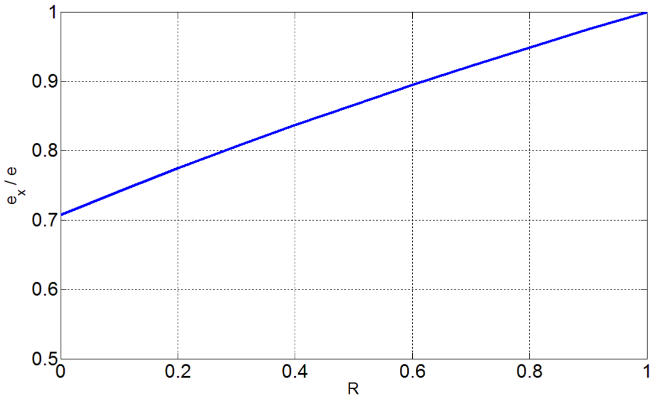


Figure 11. e_x vs r for the case of A_1=1.0.

CONCLUSIONS

In this paper the first results of a research work aimed at providing an interpretation of the data obtained from the dynamic monitoring of the Asinelli Tower are presented. The work is the result of a joint effort between the INGV which was in charge of carrying out the experiments and processing the data and the University of Bologna which was in charge of providing a structural interpretation of the recorded data. In detail the present work is mainly focused on the analysis of the free vibration response of the Tower triggered by the passage of heavy vehicles.

The analyses of the displacement response allowed to clearly recognize beat waves similar to those already observed by some of the authors studying the dynamic response of planar eccentric systems. From this analogy, a simple method for the estimation of the equivalent eccentricity of the Tower starting from the recorded response in free vibration has been developed.

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