



ESTIMATION OF MAXIMUM DAMPER FORCES IN SHEAR-TYPE BUILDINGS SUBJECTED TO SEISMIC INPUT

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ABSTRACT

In the last decades, the use of added viscous dampers for the mitigation of the effects due to the seismic action upon the structural elements has been worldwide spread. On the same time, a large research effort has been devoted to the study of the behaviour and performances of building structure equipped with added dampers with special attention to the development of procedures for the optimization of dampers performances through the use of complex algorithms and iterative procedures whose application requires, in most cases, a significant expertise beyond that of the average practitioner and its use is therefore essentially limited to researchers or to large firms specialized in the field of structural control. Nonetheless, less attention has been devoted to the development of practical design procedures. In this regard, on the knowledge of the authors, few practical procedures have been proposed.

Among others, the authors recently proposed a five-step procedure for the dimensioning of inter-storey viscous dampers to be added in frame structures (Silvestri et al., 2010). The procedure aims at guiding the practical design from the choice of a target reduction in the seismic response of the structural system (with respect to the response of a structure without any additional damping devices), to the identification of the corresponding damping ratio and the mechanical characteristics (i.e., damping coefficient values for chosen damping exponent, stiffness of the oil) of the commercially available viscous dampers. The application of the procedure requires the use of a computer program to perform linear and non-linear time-history analysis in order to evaluate the damping force in the added dampers.

In the present paper a simplified version of the 5-step procedure leading to a 3-step procedure which does not require any numerical analyses for the sizing of the viscous dampers is proposed. In detail, simple analytical expressions for the damping forces (leading to a specific target damping ratio) are derived starting from appropriate drift profiles representative of different stiffness distribution along the height.

INTRODUCTION

Manufactured viscous dampers are hydraulic devices which can be inserted in building structures in order to mitigate the seismic effects through the dissipation of part of the kinetic energy by the earthquake to the structure (Soong and Dargush, 1997; Constantinou et al., 1998; Hart and Wong, 2000; Chopra, 1995; Christopoulos and Filiatrault, 2006). The effectiveness of such devices in reducing the seismic demands on the structural elements have been demonstrated by a number of research works since the 1980s (Constantinou and Tadjbakhsh, 1983; Constantinou and Symans, 1993;

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Takewaki, 2000 and 2009; Singh and Moreschi, 2002; Trombetti and Silvestri, 2004, 2006 and 2007; Levy and Lavan, 2006; Silvestri and Trombetti, 2007; Silvestri et al., 2011; Diotallevi et al., 2012; Palermo et al., 2013b). The majority of the above-cited works basically proposed sophisticated algorithms for dampers optimization, i.e. damper size and location, sometimes leading to complex design procedures. Nevertheless, the application of such algorithms often requires computational expertise which is beyond the common capabilities of the practical engineers. Indeed, the issue of developing practical methods in order to size and locate the viscous dampers is still open.

In 1992, report NCEER-92-0032 (Constantinou and Symans, 1992) first investigated the problem of selecting the damping constant of linear viscous dampers in an elastic system to provide a specific damping ratio. In 2000, report MCEER-00-0010 (Ramirez et al., 2000) proposed an analytical relationship between the viscous damping ratio in a given mode of vibration and the damping coefficients on the basis of an energetic approach, assuming a given undamped mode shape. Later on other methods have been proposed. Among others, the most useful for the practitioners are likely to be the following ones: (i) Lopez-Garcia (2001) developed a simple algorithm for optimal damper configuration in MDOF structures, assuming a constant inter-storey height and a straight-line first modal shape; (ii) ASCE 7 (2005) absorbed the MCEER-00-0010 approach.

Nonetheless, other alternative approaches leading to practical design procedures for the sizing of the viscous dampers have been proposed in the last years: (i) Christopoulos and Filiatrault (2006) suggested a design approach for estimating the damping coefficients of added viscous dampers consisting in a trial and error procedure; (ii) Silvestri et al. (2010) proposed a direct design approach, called “the five-step procedure”.

The latter (five-step procedure) aims at guiding the professional engineer from the choice of the target objective performance to the identification of the mechanical characteristics (i.e. damping coefficient and oil stiffness, for given alpha exponent) of commercially available viscous dampers. The original procedure (Silvestri et al., 2010; Palermo et al., 2013a) requires the development of time-history analyses of FE models in order to estimate the maximum inter-storey velocity, necessary to size the damping force of the added dampers. On this regard, a recent work by Adachi et al. (2013) acknowledged that the distribution of the maximum inter-storey velocities is a key index in order to evaluate the along-height demand on viscous dampers and exhibit specific characteristics depending on the number of the stories of the building.

In the present paper, a study on the along-the-height distribution of the maximum inter-storey velocities for frame building structures equipped with inter-storey dampers has been conducted with the purpose of obtaining simplified analytical expressions for the evaluation of the design force for viscous dampers, thus not requiring the development of time-history analyses. The proposed formula may be useful in the case of preliminary design.

REVIEW OF THE 5-STEP PROCEDURE

In 2010 Silvestri et al. proposed a direct five-step procedure for the dimensioning of the damping coefficients (c) of viscous dampers to be added to frame building structures. The procedure allows the designer to estimate the damping coefficient c based on the knowledge of the floor masses and the fundamental period of vibration of the structure. It is composed of the following steps:

STEP 1. Identification of the target damping ratio $\bar{\xi}$ of the structure on the basis of a chosen target level $\bar{\eta}$ of structural performances; ($\bar{\eta}$ is usually known as the damping reduction factor).

STEP 2. Identification of the tentative characteristics of the linear viscous dampers for preliminary design ($c_L = \bar{c}_L$), i.e. first dimensioning of the linear damping coefficients.

STEP 3. Development of a series of preliminary time-history analyses of the building structure equipped with the viscous dampers identified in Step 2. This step allows to: (i) size the linear damping coefficients of the dampers to be added to the structure in order to achieve the desired level of actions (axial forces, shear forces, bending moments, etc.) on the structural members of the building; and (ii) identify the range of “working” velocities for the linear added viscous dampers.

STEP 4. Identification of the characteristics of the “equivalent” non-linear viscous dampers ($c_{NL} = \bar{c}_{NL}$, $\alpha = \bar{\alpha}$, where α is the damping exponent), i.e. identification of a real system of manufactured viscous dampers capable of providing the structure with actions (on the structural members) comparable to those obtained in Step 3 using the linear viscous dampers identified in Step 2.

STEP 5. Development of a series of final time-history analyses of the building structure equipped with the viscous dampers identified in Step 4. This last step is necessary in order to verify the effectiveness of Step 4 and obtain the forces through both the structural members and dampers which are to be used for the final design specifications.

The fundamental results of step 2 lies in the introduction of the following formula for the evaluation of the linear storey damping coefficient c_i and the storey damping coefficient c of a single inter-storey viscous damper (Eq. 28-29 of Silvestri et al., 2010):

$$c_i = \bar{\xi} \cdot \omega_1 \cdot m_{tot} \cdot (N+1) \quad (1)$$

$$c = \bar{\xi} \cdot \omega_1 \cdot m_{tot} \frac{(N+1)}{n} \quad (2)$$

where ω_1 is the fundamental frequency, m_{tot} is the building mass, N is the number of stories, n is the number of dampers placed at the generic storey.

The fundamental result of step 4 lies in the identification of a χ factor to be used to estimate the non linear damping coefficient c_{NL} based on the corresponding linear one (Eqs. (1) and (2)). It has been noted (Silvestri et al., 2010) that χ is fairly constant for a wide range of α and that for practical purposes it can be set equal to 0.8. Based on this assumption the non linear damping coefficient c_{NL} can be estimated according to (Eq. 32 of Silvestri et al., 2010):

$$c_{NL} = c \cdot (0.8 \cdot v_{max})^{1-\alpha} \quad (3)$$

where v_{max} is the maximum inter-storey velocity as obtained from linear time-history analyses.

Note that the above recalled results have been derived for a S-T structural schematization with uniform distribution of storey mass m , storey stiffness k and storey damping coefficient c along the height of the building. Nonetheless in a recent work (Palermo et al., 2013a) it has been showed that Eq. (1) keeps its validities also in the case of a generic moment resisting frame where the actual stiffness of the beams is considered.

AN ESTIMATION OF THE MAXIMUM INTER-STOREY VELOCITY

It is of common belief that the effectiveness of dampers allocation is closely related to the inter-storey drift demand. A recent work (Adachi et al., 2013) showed that, while this understanding is practically true in rather low- to medium-rise buildings, the key parameter for the estimation of the damping forces in high-rise buildings equipped with inter-storey dampers turns to be the distribution of the maximum inter-storey velocities which may substantially differs (from a qualitative point of view) with respect to that of the inter-storey drifts due to a significant higher modes contribution. In the same work the authors also introduced approximate predictions for the maximum horizontal force of linear oil dampers. In detail, amplification coefficients are introduced to account for the observed amplifications of the maximum inter-storey velocities at the bottom stories of high-rise buildings.

In this section, a study on the inter-storey velocity profiles is carried out with the purpose of obtaining an analytical estimation of the maximum inter-storey velocity starting from an assumed displacement profile along the height of the building. This profile is then corrected by an appropriate coefficient accounting for the influence of the higher modes. The study is aimed at obtaining an

analytical formulation to be used for an estimation of the damping force for a preliminary design of the added viscous dampers.

The following building schematization is considered:

- Shear-type building frames
- Uniform distribution of storey mass m along the building height ($m_{tot}=m \cdot N$ is the total mass of the building)
- Uniform distribution of storey stiffness k along the building height
- Constant storey damping coefficient c estimated according to Eq. (2).

The estimation of the maximum inter-storey velocity is based on the following displacement profile along the height of the building:

$$\{\phi\} = \beta \left\{ \begin{array}{c} \frac{N(N+1)}{2} \\ \dots \\ i \frac{N(N+1)}{2} - \sum_{j=0}^{i-1} \frac{j(j+1)}{2} \\ \dots \\ \frac{N^2(N+1)}{2} - \sum_{j=0}^{N-1} \frac{j(j+1)}{2} \end{array} \right\} \quad (4)$$

Eq. (4) represents the first approximation of the exact first eigenvector as obtained using the Rayleigh-Ritz method starting from a linear distribution of static forces along the building height. β is a constant provided that Eq. (4) represents just a shape. Figure 1 compares the drift profile of Eq. (4) with the exact profile of the first mode for a 30-storey building.

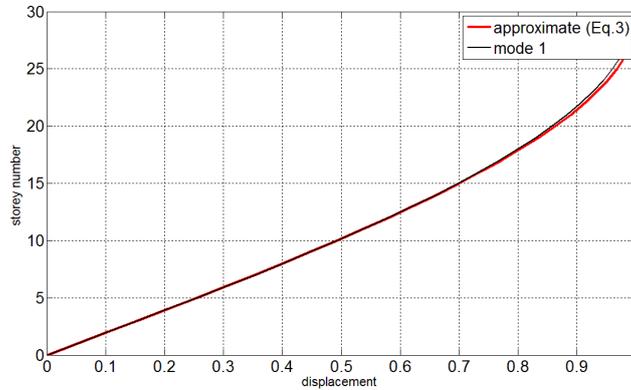


Figure 1. Comparison between the assumed displacement profile (Eq. (4)) and the 1st mode displacement profile.

Assuming Eq. (4) as the first mode shape, the pseudo-velocity and pseudo-acceleration profiles along the building height are given as follows:

$$\{\dot{\phi}\} = \omega_1 \beta \left\{ \begin{array}{c} \frac{N(N+1)}{2} \\ \dots \\ i \frac{N(N+1)}{2} - \sum_{j=0}^{i-1} \frac{j(j+1)}{2} \\ \dots \\ \frac{N^2(N+1)}{2} - \sum_{j=0}^{N-1} \frac{j(j+1)}{2} \end{array} \right\} \quad (5)$$

$$\{\ddot{\phi}\} = \omega_1^2 \beta \left\{ \begin{array}{c} \frac{N(N+1)}{2} \\ \dots \\ i \frac{N(N+1)}{2} - \sum_{j=0}^{i-1} \frac{j(j+1)}{2} \\ \dots \\ \frac{N^2(N+1)}{2} - \sum_{j=0}^{N-1} \frac{j(j+1)}{2} \end{array} \right\} \quad (6)$$

From structural dynamics (Chopra, 1995) the base shear, V_b , can be evaluated as follows:

$$V_b = m_{tot} \cdot \overline{S}_a \quad (7)$$

where \overline{S}_a is the pseudo-acceleration at the center of gravity.

By assuming that the base shear is given entirely by the 1st mode:

$$V_b = m \cdot \omega_1^2 \cdot \beta \cdot \sum_{i=1}^N \left[\frac{N(N+1)}{2} - \sum_{j=0}^{i-1} \frac{j(j+1)}{2} \right] = m \cdot \omega_1^2 \cdot \beta \cdot \frac{1}{24} N(1+N)(2+5N+5N^2) \quad (8)$$

The constant β is obtained by equating Eqs. (7) and (8):

$$\beta = \frac{\overline{S}_a}{\omega_1^2} \frac{24}{(N+1)(2+5N+5N^2)} \quad (9)$$

By substituting Eq. (9) into Eq. (4) the following expression of the displacement profile along the building height is obtained:

$$\{\phi\} = \frac{\overline{S}_a}{\omega_1^2} \frac{24}{(N+1)(2+5N+5N^2)} \left\{ \begin{array}{c} \frac{N(N+1)}{2} \\ \dots \\ i \frac{N(N+1)}{2} - \sum_{j=0}^{i-1} \frac{j(j+1)}{2} \\ \dots \\ \frac{N^2(N+1)}{2} - \sum_{j=0}^{N-1} \frac{j(j+1)}{2} \end{array} \right\} \quad (10)$$

The analytical expression of the inter-storey drift profile is then given by:

$$\{\delta\} = \frac{\overline{S}_a}{\omega_1^2} \frac{24}{(N+1)(2+5N+5N^2)} \left\{ \begin{array}{c} \frac{N(N+1)}{2} \\ \dots \\ \frac{N(N+1)}{2} - \frac{i(i-1)}{2} \\ \dots \\ \frac{N(N+1)}{2} - \frac{N(N-1)}{2} \end{array} \right\} \quad (11)$$

while the inter-storey pseudo-velocity profile is given by:

$$\{\dot{\delta}\} = \frac{\bar{S}_a}{\omega_1} \frac{24}{(N+1)(2+5N+5N^2)} \left\{ \begin{array}{c} \frac{N(N+1)}{2} \\ \dots \\ \frac{N(N+1)}{2} - \frac{i(i-1)}{2} \\ \dots \\ \frac{N(N+1)}{2} - \frac{N(N-1)}{2} \end{array} \right\} \quad (12)$$

Figure 2 represents the inter-storey pseudo-velocity profile as given by Eq. (12).

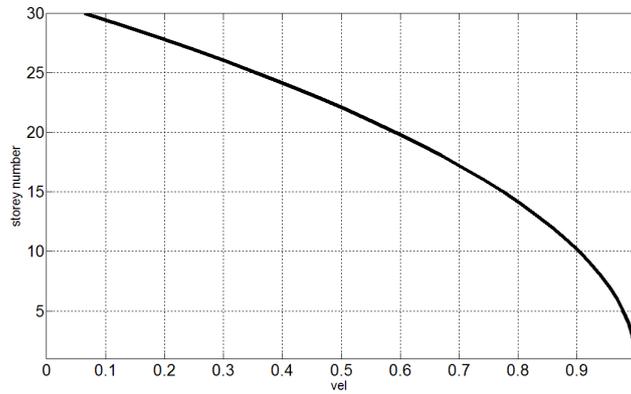


Figure 2. The inter-storey pseudo-velocity profile (Eq. (12)).

From Eq. (12) (or Figure 2) it can be noted that the maximum inter-storey velocity is achieved at the bottom storey:

$$\dot{\delta}_{\max} = \frac{\bar{S}_a}{\omega_1} \frac{12N}{(2+5N+5N^2)} \quad (13)$$

which can be approximated (for large N) by the following equation (see Figure 3):

$$\dot{\delta}_{\max} \approx \frac{12}{5} \frac{\bar{S}_a}{\omega_1} \frac{1}{(N+1)} \quad (14)$$

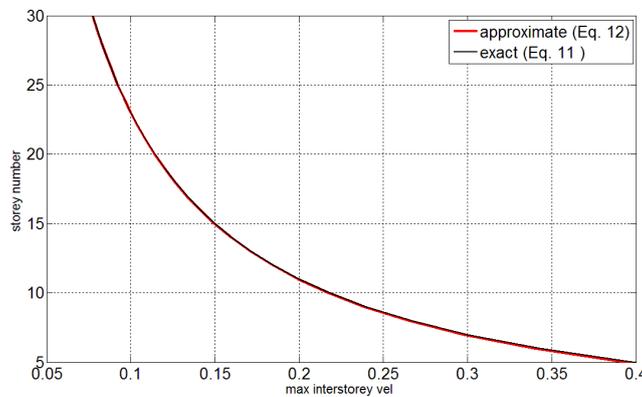


Figure 3. Comparison between the exact (Eq. (13)) and approximated maximum inter-storey velocity (Eq. (14)).

PARAMETRIC ANALYSIS TO VERIFY THE EFFECTIVENESS OF THE PROPOSED FORMULA FOR THE MAXIMUM INTER-STOREY VELOCITY

A parametric analysis has been carried out with the purpose of verifying the effectiveness of the estimations of the maximum inter-storey drift velocity (Eq. (13)). The three following systems are analyzed:

- shear-type building frame models with number of stories $N = 5, 15, 30$;
- uniform distribution of storey mass m ($m_{\text{tot}}=m \cdot N$ is the total mass of the building); $m=100\text{kN}$;
- uniform distribution of storey stiffness $k=1000000 \text{ kN/m}$;
- storey damping coefficient c estimated according to Eq. (1) in order to obtain selected damping ratios $\bar{\xi}$.

An ensemble of 100 recorded ground motions (Hatzigeorgiou and Beskos, 2009; Palermo et al. 2013b) is used to perform the time-history analyses. Time history analyses have been developed by direct integration of the equation of motion (using the Newmark method) and by also by modal integration (Chopra, 1995) in order to evaluate the contribution of the single modes. The accelerograms have been selected from the PEER strong motion database in order to be representative of ground motions recorded in a type B soil according to the Italian building code (NTC, 2008).

From the j -th time-analyses the ratio between the maximum recorded inter-storey velocity $v_{\text{max},j}$ and its estimation $\hat{\delta}_{\text{max},j}$ (Eq. (13)) is calculated:

$$\gamma_j = \frac{v_{\text{max},j}}{\hat{\delta}_{\text{max},j}} \quad (15)$$

The mean value of γ_j , $\mu_\gamma = \frac{1}{100} \sum_{j=1}^{100} \frac{v_{\text{max},j}}{\hat{\delta}_{\text{max},j}}$ and mean plus one standard deviation $(\mu + \sigma)_\gamma$ as

obtained from the time history analyses are represented in Figure 4 as a function of the storey number N . Also all γ_j values are displayed.

Inspection of Figure 4 clearly shows that coefficient γ increases as the storey number increases. For N equal to 5, μ_γ is close to 1.0; for $N=15$ μ_γ is around 2.0; for $N=15$ μ_γ is around 4.0. This means that the higher modes contribution increases as the storey number increases. Moreover γ values seems not to be significantly affected by damping coefficient. In general, as $\bar{\xi}$ increases, γ slightly decreases.

The following linear equations can be used to fit μ_γ and $(\mu + \sigma)_\gamma$:

$$\mu_\gamma = 0.14N + 0.38 \quad (16)$$

$$(\mu + \sigma)_\gamma = 0.30N + 0.17 \quad (17)$$

These equations are valid for N between 5 and 30. For N less than 5 μ_γ can be set equal to 1.0, while $(\mu + \sigma)_\gamma = 1.5$. For $\bar{\xi} > 0.05$ Eqs. (16) and (17) are slightly conservative.

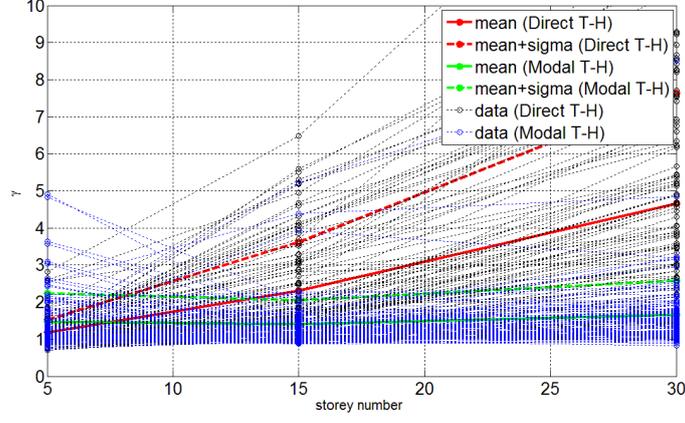
Based on the results of the parametric analyses, the following analytical expressions of the maximum inter-storey velocity, v_{max} (mean and mean plus one standard deviation) are proposed:

$$v_{\text{max,mean}} = \frac{\bar{S}_a}{\omega_1} \frac{12N}{(2 + 5N + 5N^2)} \quad \text{for } N < 5$$

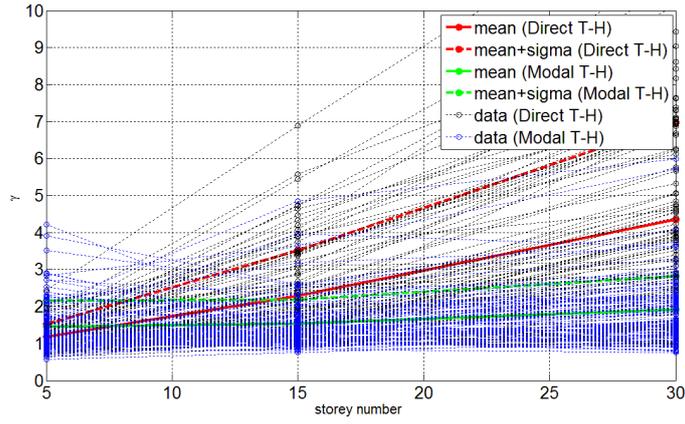
$$v_{\text{max,mean}} = \frac{\bar{S}_a}{\omega_1} \frac{12N \cdot (0.14N + 0.38)}{(2 + 5N + 5N^2)} \quad \text{for } 5 \leq N \leq 30 \quad (18)$$

$$v_{\max, \text{mean}+\sigma} = \frac{\bar{S}_a}{\omega_1} \frac{18N}{(2+5N+5N^2)} \quad \text{for } N < 5 \quad (19)$$

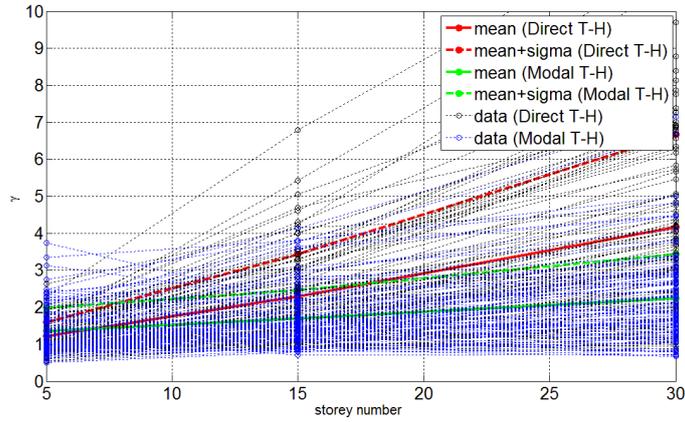
$$v_{\max, \text{mean}+\sigma} = \frac{\bar{S}_a}{\omega_1} \frac{12N \cdot (0.30N + 0.17)}{(2+5N+5N^2)} \quad \text{for } 5 \leq N \leq 30$$



(a)



(b)



(c)

Figure 4. coefficient γ as obtained from the time-history analyses. (a) $\bar{\xi} = 0.05$; (b) $\bar{\xi} = 0.15$; (c) $\bar{\xi} = 0.30$

AN ANALYTICAL ESTIMATION OF THE MAXIMUM DAMPER FORCE

The storey damping force profile can be obtained by combining Eq. (1) with Eq. (12):

$$\{F_d\}_i = c_i \cdot \{\dot{\delta}\} = 2 \cdot \bar{\xi} \cdot m_{tot} \cdot \bar{S}_a \frac{6}{(2+5N+5N^2)} \left\{ \begin{array}{c} \frac{N(N+1)}{2} \\ \dots \\ \frac{N(N+1)}{2} - \frac{i(i-1)}{2} \\ \dots \\ \frac{N(N+1)}{2} - \frac{N(N-1)}{2} \end{array} \right\} \quad (20)$$

The maximum storey damping force is achieved at the bottom storey:

$$F_{d,max,i} = 2 \cdot \bar{\xi} \cdot m_{tot} \cdot \bar{S}_a \cdot \frac{6(N+N^2)}{(2+5N+5N^2)} \quad (21)$$

The maximum damping force for the single damper is given by:

$$F_{d,max} = 2 \cdot \bar{\xi} \cdot m_{tot} \cdot \bar{S}_a \cdot \frac{6(N+N^2)}{n(2+5N+5N^2)} \quad (22)$$

which can be approximated (for large N) by the following simple expression:

$$F_{d,max} = 2 \cdot \bar{\xi} \cdot m_{tot} \cdot \bar{S}_a \cdot \frac{6}{5} \cdot \frac{1}{n} \quad (23)$$

On the basis of the results of the parametric analysis reported in the previous section, the following analytical expressions for the estimation of the maximum damping force of inter-storey dampers are proposed:

$$\begin{aligned} F_{d,max,mean} &= 2 \cdot \bar{\xi} \cdot m_{tot} \cdot \bar{S}_a \cdot \frac{12N}{(2+5N+5N^2)} && \text{for } N < 5 \\ F_{d,max,mean} &= 2 \cdot \bar{\xi} \cdot m_{tot} \cdot \bar{S}_a \cdot \frac{12N \cdot (0.14N + 0.38)}{(2+5N+5N^2)} && \text{for } 5 \leq N \leq 30 \end{aligned} \quad (24)$$

$$\begin{aligned} F_{d,max,mean} &= 2 \cdot \bar{\xi} \cdot m_{tot} \cdot \bar{S}_a \cdot \frac{18N}{(2+5N+5N^2)} && \text{for } N < 5 \\ F_{d,max,mean} &= 2 \cdot \bar{\xi} \cdot m_{tot} \cdot \bar{S}_a \cdot \frac{12N \cdot (0.30N + 0.17)}{(2+5N+5N^2)} && \text{for } 5 \leq N \leq 30 \end{aligned} \quad (25)$$

The formulas are slightly conservative for damping ratios $\bar{\xi} > 0.05$.

These formulas may be useful for preliminary design of shear-type structures equipped with linear viscous dampers and subjected to earthquake ground motions.

CONCLUSIONS

In this paper, simple analytical formulas for the estimation of the maximum damper force for a preliminary sizing of added viscous damper have been obtained. The estimations have been derived starting from a simple analytical expression of the displacement profile along the building height which is coincident with the first approximation of the Rayleigh-Ritz first eigenvector (starting from a linear displacement profile). After simple mathematical developments, a first simple formula for the estimation of the maximum inter-storey velocity has been obtained. Then, by mean of a parametric analysis conducted by performing linear time history analyses on shear type structures with added dampers, varying the storey number and the damping ratios of the added dampers, corrective coefficients to adjust the first formula have been obtained. The corrective coefficients account for the higher mode contributions. The analyses results showed that those contributions are not significant for low-rise buildings (storey number less than 5) and therefore they may be neglected, while they become predominant for the case of high-rise buildings. Finally, the estimated maximum inter-storey velocity is used to predict the maximum damper force in shear-type building subjected to seismic excitation.

REFERENCES

- Adachi F, Yoshitomi S, Tsuji M, Takewaki I (2013) "Nonlinear optimal oil damper design in seismically controlled multi-story building frame," *Soil Dyn Earthq Eng*, 44(1):1–13
- American Society of Civil Engineers, ASCE 7–05 (2005) Minimum design loads for buildings and other structures, Reston, VA
- Chopra AK (1995) Dynamics of structures. Theory and applications to earthquake engineering, Prentice-Hall, Upper Saddle River
- Christopoulos C, Filiatrault A (2006) Principles of passive supplemental damping and seismic isolation, IUSSPress, Pavia
- Constantinou MC, Symans MD (1992) Experimental and analytical investigation of seismic response of structures with supplemental fluid viscous dampers, NCEER-92-0032. National Center for Earthquake Engineering Research, Technical report, Buffalo
- Constantinou MC, Soong TT, Dargush GF (1998) Passive energy dissipation systems for structural design and retrofit, monograph No. 1, Multidisciplinary Center for Earthquake Engineering Research, Buffalo, New York
- Constantinou MC, Tadjbakhsh IG (1983) "Optimum design of a first story damping system," *Comput Struct*, 17(2):305–310
- Constantinou MC, Symans MD (1993) "Seismic response of structures with supplemental damping," *Struct Des Tall Build*, 2:77–92
- Diotallevi PP, Landi L, Dellavalle A (2012) "A methodology for the direct assessment of the damping ratio of structures equipped with nonlinear viscous dampers," *J Earthq Eng*, 16:350–373
- Hart GC, Wong K (2000) Structural dynamics for structural engineers. Wiley, New York
- Hatzigeorgiou GD, Beskos DE (2009) "Inelastic displacement ratios for SDOF structures subjected to repeated earthquakes," *Eng Struct* 31:2744–2755
- Levy R, Lavan O (2006) "Fully stressed design of passive controllers in framed structures for seismic loadings," *Struct Multidiscip Optimiz*, 32(6):485–498
- Lopez Garcia D (2001) "A simple method for the design of optimal damper configurations in MDOF structures," *Earthq Spectra*, 17(3):387–398
- NTC (2008) Norme Tecnica per le Costruzioni, Italian building code, adopted with D.M. 14/01/2008, published on S.O. n. 30 G.U. n. 29 04/02/2008
- Palermo M, Muscio S, Silvestri S, Landi L, Trombetti T (2013a) "On the dimensioning of viscous dampers for the mitigation of the earthquake-induced effects in moment-resisting frame structures," *Bulletin of Earthquake Engineering*, 11:2429–2446
- Palermo M, Silvestri S, Trombetti T, Landi L (2013b) "Force reduction factor for building structures equipped with added viscous dampers," *Bulletin of Earthquake Engineering*, 11:1661–1681
- Ramirez OC, Constantinou MC, Kircher CA, Whittaker AS, Johnson MW, Gomez JD, Chrysostomou CZ (2000) Development and evaluation of simplified procedures for analysis and design of buildings with passive energy dissipation systems. MCEER-00-0010. Technical report, Buffalo
- Silvestri S, Trombetti T (2007) "Physical and numerical approaches for the optimal insertion of seismic viscous dampers in shear-type structures," *J Earthq Eng*, 11(5):787–828

- Silvestri S, Gasparini G, Trombetti T (2011) "Seismic design of a precast r.c. structure equipped with viscous dampers," *Earthquake and Structures*, 2(3):297-321
- Silvestri S, Gasparini G, Trombetti T (2010) "A five-step procedure for the dimensioning of viscous dampers to be inserted in building structures," *J Earthq Eng*, 14(3):417-447
- Singh MP, Moreschi LM (2002) "Optimal placement of dampers for passive response control," *Earthq Eng Struct Dyn*, 31:955-976
- Soong TT, Dargush GF (1997) Passive energy dissipation systems in structural engineering. Wiley, Chichester
- Takewaki I (1997) "Optimal damper placement for minimum transfer functions," *Earthq Eng Struct Dyn*, 26:1113-1124
- Takewaki I (2000) "Optimal damper placement for critical excitation," *Prob Eng Mech*, 15:317-325
- Takewaki I (2009) Building control with passive dampers: optimal performance-based design for earthquakes. Wiley, Singapore
- Trombetti T, Silvestri S (2004) "Added viscous dampers in shear-type structures: the effectiveness of massproportional damping," *J Earthq Eng*, 8(2):275-313
- Trombetti T, Silvestri S (2006) "On the modal damping ratios of shear-type structures equipped with Rayleighdamping systems," *J Sound Vib*, 292(2):21-58
- Trombetti T, Silvestri S (2007) "Novel schemes for inserting seismic dampers in shear-type systems based upon the mass proportional component of the Rayleigh damping matrix," *J Sound Vib*, 302(3):486-526