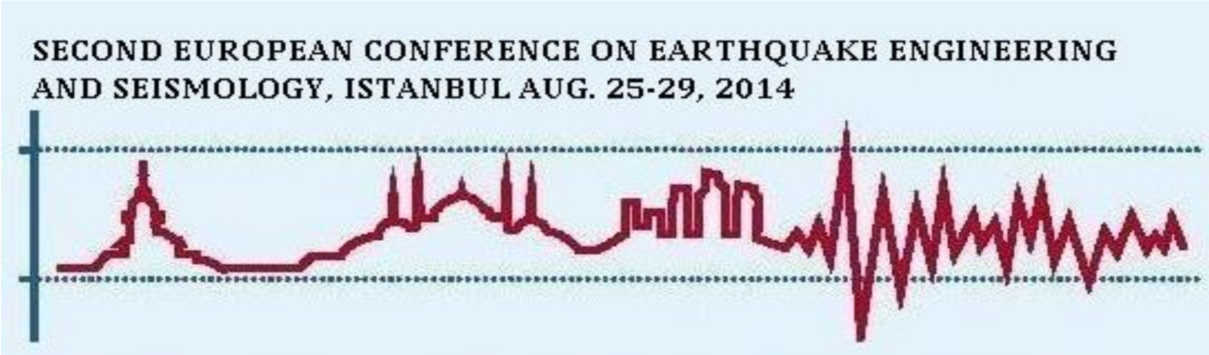

A STATISTICAL STUDY ON THE AMPLIFICATION FACTORS FOR AN UPDATED DISPLACEMENT DESIGN SPECTRUM

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The development of performance-based design methodologies and displacement-based approaches requires a special attention to the correct definition of the seismic input to be used for the analysis and in particular of reliable displacement response spectra. Past research works pointed out that the actual formulations of the displacement design spectrum adopted by the current seismic codes do not provide a realistic seismic displacement demand (Bommer and Elnashai 1999).

In this paper, the characterization of the schematized response spectra has been deeply investigated with the purpose of obtaining a more accurate displacement design spectrum and a simple but efficient criterion for the selection of recorded ground motions. The amplification factors are been modelled as random processes in order to identify their fundamental properties in terms of mean values and variability. It is noted that the magnitude of the amplification factor is quite correlated to the corresponding peak parameter (especially the pseudo-velocity and displacement).

Based on the obtained results, a formulation for the design displacement spectrum is proposed and compared with that suggested by the Eurocode 8.

INTRODUCTION

Over the last decades, response spectra and their design schematizations (Newmark and Hall 1982, Chopra 2007) have assumed a fundamental role for the definition of the input for seismic analyses and have been progressively adopted by seismic codes as the basic tools for response spectrum analysis. The conventional response spectrum analyses (Chopra 1995, 2007) is grounded on the traditional Force Based Design (FBD) and is usually conducted using the acceleration design spectrum, which is constructed starting from the knowledge of the peak ground acceleration (PGA) at the site as result of hazard analyses. Modern design codes also introduces displacement design spectrum which are simply derived from the corresponding acceleration spectrum employing the relationship between the displacement and the pseudo-acceleration.

In more recent years with the developments of Performance-Based-Design (PBSD) and Displacement-Based-Design (DBD) approaches (Vision 2000 1995, Bertero and Bertero2002, Priesteley et al. 2007) the issue of defining displacement spectra leading to realistic displacement demand has received increasing attention among the scientific community. In 2000 Bommer et al. showed that displacement spectra obtained by simple conversion of the design acceleration spectra given by codes result in unrealistic spectral shapes and amplitudes.

Many scientific works approached the issue of developing more realistic displacement spectra. Bommer and Elnashai 1999 have carefully processed a dataset of European strong motions and derived new frequency-dependent ground motion prediction equation for horizontal displacement response spectral ordinates, in order to provide simplified displacement spectral shapes in a linearised

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form. Tolis and Faccioli 1999 studied high-quality digital recordings from the 1995 Hyogoken-Nanbu (Kobe) record in order to identify possible trends in long-period spectral displacements. Lam et al. (2001) developed a simple and rational procedure, termed the “frame analogy soil amplification” (FASA) model, which can be used to build response spectra accounting for the effects of soil amplifications. Faccioli et al. 2004 have related the displacement spectral shape in the long-period range to the magnitude (high influence on the spectra) and the source distance (small influence), identifying a “corner period” Td beyond which the spectral ordinates remain roughly constant or gently decrease towards the peak ground displacements. Akkar and Özen 2005 recognized the fundamental effect of the peak ground velocity on single-degree-of-freedom deformation demands, for decreasing the dispersion due to record-to-record variability.

The above cited works highlighted the need of including further ground motions parameters for a reliable evaluation of appropriate displacement spectra rather than the only peak ground acceleration. Nonetheless, despite the number of research works devoted to the study of correlations between input parameters such as spectral ordinates at selected periods and structural response parameters, few systematic studies on the correlation between parameters characterizing the input (such as the peak ground parameters or other intensity measures) can be found in the scientific literature. Akkar and Özen (2005) provided some correlations analyses between PGV and PGA and between PGV and the effective duration. Akkar et al. 2006, investigating on the influence of long-period filter cut-off on the spectral displacement, studied the correlation between corner period Tc and magnitude. Aochi and Dougles 2006 studied the correlation between the Arias Intensity AI (Arias, 1970) and significant duration RSD (Trifunac and Brady, 1975).

In this paper, a systematic statistical analyses on the amplification factors is conducted considering an ensemble ground motion of 177 historical records. Based on the obtained results, a formulation for the design displacement spectrum is proposed.

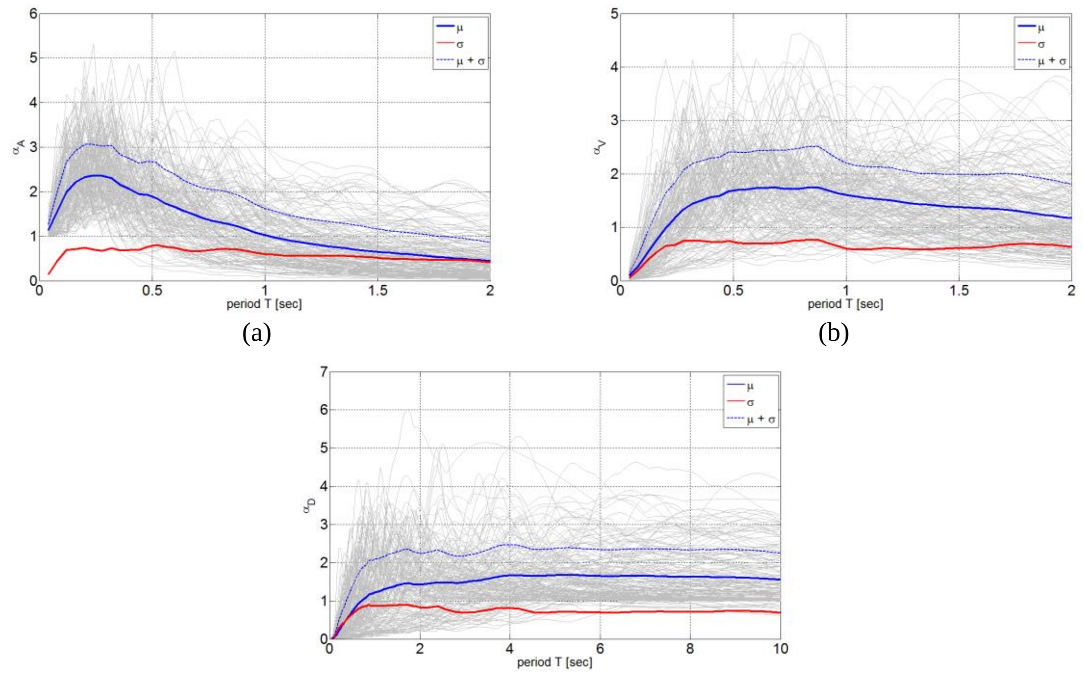
THE AMPLIFICATION FACTORS

Most of the studies on the tripartite spectrum are dated between the 60s and the 80s (Veletsos and Newmark 1964, Newmark and Hall 1982). Values of the corner periods as well as median values and median plus one standard deviation of the amplification factors α_A, α_V, and α_D, have been calculated by Newmark and Hall 1982 on the basis of the limited number of records available at the time. More recently, Malhotra 2006 evaluated amplification factors from a broader dataset of 64 records.

In this section, a statistical analysis is performed in order to evaluate mean values and variability of the amplification factors. The response spectrum is idealized as a stochastic process (Crandall and Mark 1963), with each single spectrum assumed as a realization of the process. In this study, 177 historic records characterized by a V_{S30} between 360 and 700 m/s have been selected within the PEER strong motion database (http://peer.berkeley.edu/products/strong_ground_motion_db.html). The amplification factors are evaluated through statistical analyses conducted both across-the-ensemble (to investigate on the variability due to the choice of the ensemble) and along-the-period (to investigate on the inherent variability). Then, the amplification factors are calculated over selected period assuming stationary processes within the identified period ranges.

Across the ensemble mean values (μ) and standard deviations (σ) of the amplification factors and their trends are presented in Figure1. Inspection of the graphs indicates that:

- As expected, each mean value appears approximately constant (for engineering purposes) within a specific period range, generally known as acceleration sensitive, velocity sensitive and displacement sensitive regions (Chopra 1995).
- The standard deviations of all the three processes are quite constant for wider ranges of periods and assume similar values (0.60 ÷ 0.90). The displacement amplification factor seems to show slightly larger standard deviations with respect to the other two amplification factors.



(c)

Figure 1. Across-the-ensemble variability: mean value, standard deviation and mean plus one standard deviation for (a) pseudo-acceleration amplification factor; (b) pseudo-velocity amplification factor; (c) displacement amplification factor.

Thus, for engineering purposes, the three processes may be assumed as mean-value stationary processes (Crandall and Mark 1963) within restricted domains. Table 1 provides the numerical ranges of the across-the-ensemble main descriptors (μ, σ, $\mu+\sigma$ and *COV*) of the three amplification factors, within the period ranges.

Table 1. Ranges of values of the across-the-ensemble main descriptors of the three amplification factors.

	Period range	Minimum value in the period range	Maximum value in the period range	Mean value in the period range
$\mu_{\alpha_D}(T_j)$	$T_d \le T_j \le T_e$	1.21	1.67	1.63
$\sigma_{\alpha_D}(T_j)$		0.69	0.89	-
$\mu_{\alpha_D}(T_j)+\sigma_{\alpha_D}(T_j)$		2.09	2.47	-
$COV_{\alpha_D}(T_j)$		**0.55**	**0.80**	**0.44**
$\mu_{\alpha_V}(T_j)$	$T_c \le T_j \le T_d$	1.60	1.75	1.71
$\sigma_{\alpha_V}(T_j)$		0.60	0.77	-
$\mu_{\alpha_V}(T_j)+\sigma_{\alpha_V}(T_j)$		2.20	2.52	-
$COV_{\alpha_V}(T_j)$		**0.39**	**0.43**	**0.42**
$\mu_{\alpha_A}(T_j)$	$T_b \le T_j \le T_c$	1.92	2.37	2.16
$\sigma_{\alpha_A}(T_j)$		0.66	0.76	-
$\mu_{\alpha_A}(T_j)+\sigma_{\alpha_A}(T_j)$		2.64	3.07	-
$COV_{\alpha_A}(T_j)$		**0.29**	**0.41**	**0.33**

Within each period range of approximately constant response, it is also meaningful to evaluate the along-the-period variability of each process. Figure 2 gives the graphical representation of mean and coefficient of variation COV for all considered ground motions (solid lines), together with their mean values over the *N* earthquakes (dotted lines). Table 2 provides the numerical ranges of the along-the-period main descriptors (μ, σ, $\mu+\sigma$ and *COV*) of the three amplification factors, within the corresponding period ranges.

Table 2. Ranges of values of the along-the-period main descriptors of the three amplification factors.

	Period range	Minimum value over all *N* earthquakes	Maximum value over all *N* earthquakes	Mean value over all *N* earthquakes
$\mu_{\alpha_{D,i}}$	$T_d \le T_j \le T_e$	0.85	3.8	1.61
$\sigma_{\alpha_{D,i}}$		0.0015	0.90	-
$\mu_{\alpha_{D,i}} + \sigma_{\alpha_{D,i}}$		4.55	0.88	-
$COV_{\alpha_{D,i}}$		0.0015	0.37	**0.16**
$\mu_{\alpha_{V,i}}$	$T_c \le T_j \le T_d$	0.39	3.7	1.70
$\sigma_{\alpha_{V,i}}$		0.07	1.11	-
$\mu_{\alpha_{V,i}} + \sigma_{\alpha_{V,i}}$		4.42	0.48	-
$COV_{\alpha_{V,i}}$		0.09	0.52	**0.23**
$\mu_{\alpha_{A,i}}$	$T_b \le T_j \le T_c$	1.45	3.21	2.16
$\sigma_{\alpha_{A,i}}$		0.18	1.55	-
$\mu_{\alpha_{A,i}} + \sigma_{\alpha_{A,i}}$		1.64	4.14	-
$COV_{\alpha_{A,i}}$		0.12	0.85	**0.28**

The results reported in Table 2 indicate that the along-the-period variability decreases going from α_A to α_V and from α_V to α_D, due to the fact that the displacement spectrum is smoother than the velocity spectrum, which, in turn, is smoother than the acceleration spectrum.

It is possible to recognise that the overall variability of the amplification factors is given by two contributions:

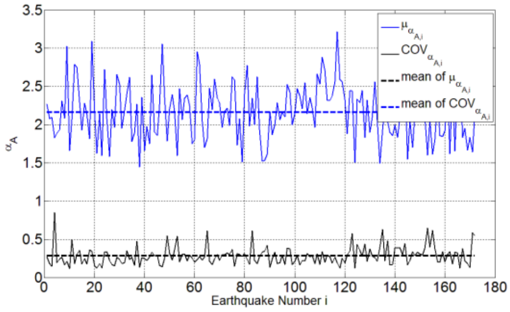
- the along-the-period variability, which is connected to the SDOF response to the single earthquake excitation;
- the across-the-ensemble variability, which is related to the ensemble of the selected ground motions.

Comparison of Tables 1 and 2 allows to note that, for each amplification factor, the mean value of the along-the-period coefficient of variation is sensibly lower than the mean value of across-the-ensemble coefficient of variation:

- for α_D: 0.16 < 0.44,
- for α_V: 0.23 < 0.42,
- for α_A: 0.28 < 0.33,

Note that, as per their definitions, the mean value of $COV_{\alpha_{D,i}}$ provides a measure of the along-the-period variability, whilst the mean value of the $COV_{\alpha_D}\left(T_j\right)$ provides a measure of the across-the-ensemble variability, which includes the along-the-period variability.

With the aim of reducing the across-the-ensemble variability to the actual intrinsic ground motion variability at a specific site (Al Atik et al. 2010), the correlation between the amplification factors and the peak ground parameters has been investigated. Correlograms and the linear regression lines (both mean and mean plus one standard error lines) are given in Figures 3. Also the cross-correlations (e.g. α_A vs *PGV* and α_A vs *PGD*, α_V vs *PGA* and α_V vs *PGD*, α_D vs *PGA* and α_D vs *PGV*) have been investigated, but are not reported herein given that no significant correlation has been found.



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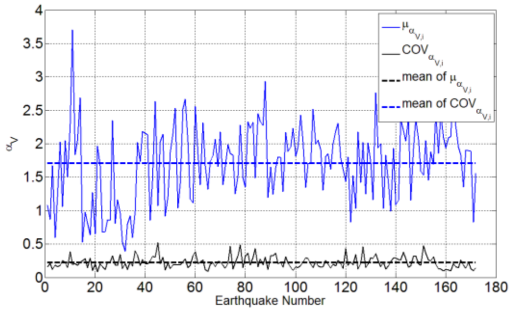

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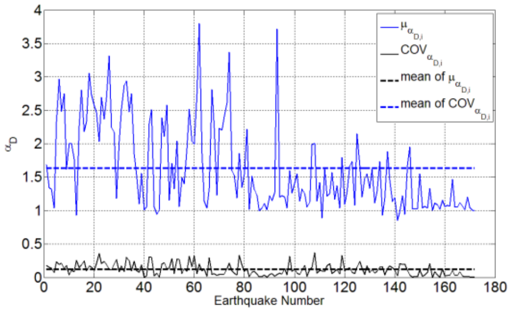

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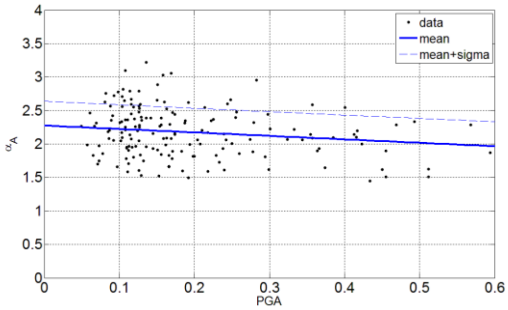
Figure 2. Along-the-period variability: (a) α_A; (b) α_V; (c) α_D.



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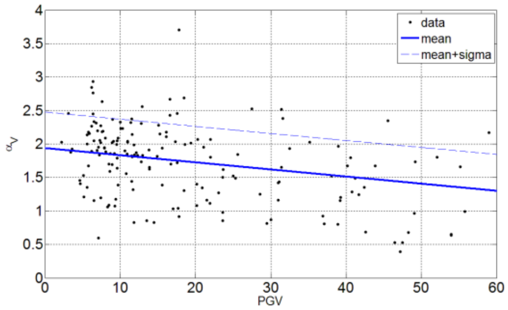

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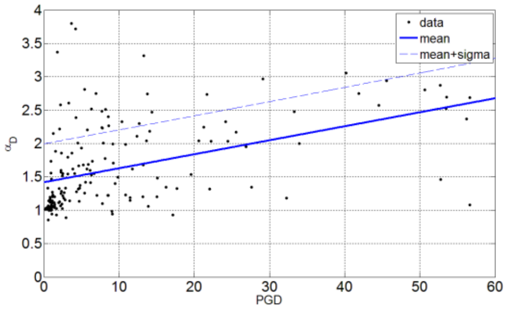


(c)

Figure 3. The correlograms and regression lines of (a) pseudo-acceleration amplification factor α_A; (b) pseudo-velocity amplification factor α_V; (c) displacement amplification factor α_D.

The graphs show that α_A is weakly correlated to the *PGA* and therefore, for design purposes, it can be assumed as constant, whilst α_V and α_D appear quite correlated to the *PGV* and *PGD*, respectively. The following linear relationships are proposed:

$$\mu_{\alpha_D} = 1.44 + 0.0206 \cdot PGD \text{ [cm]} \tag{1}$$

$$\mu_{\alpha_V} = 1.96 - 0.0139 \cdot PGV \text{ [cm/s]} \tag{2}$$

$$\mu_{\alpha_A} = 2.16 \tag{3}$$

If the mean values of the *PGD* and *PGV* reported in Table 2 are used in Eqs. 1 and 2 values of $\mu_{\alpha_D} = 1.65$ and $\mu_{\alpha_V} = 1.69$ are obtained.

The numerical values of the standard errors of estimate (Baten 1942) are equal to: SE_{α_D} = 0.78, SE_{α_V} = 0.38, and SE_{α_A} = 0.39 associated to Eqs. 1, 2 and 3 respectively. The standard error is here used as a measure of the variability of the across-the-ensemble mean value, by introducing the following coefficient of variation:

$$COV_{mean,\alpha_D} = \frac{SE_{\alpha_D}}{\mu_{\alpha_D}\left(\overline{PGD}\right)} \tag{4}$$

where $\overline{PGD}$ should be selected in order to be representative of a given site (by means of a specific probabilistic seismic hazard analysis carried out on the *PGD*). A similar definition applies to α_V, whilst:

$$COV_{mean,\alpha_A} = \frac{SE_{\alpha_A}}{\mu_{\alpha_A}} \tag{5}$$

provided that μ_{α_A} may be taken as constant.

An estimate of the overall intrinsic coefficient of variation of α_D may be obtained by using the SRSS rule (Al Atik *et al.* 2010) combining the along-the-period variability, $COV_{\alpha_{D,i}}$, with the variability of the across-the-ensemble mean value, COV_{mean,α_D}:

$$COV_{overall,\alpha_D} = \sqrt{COV_{\alpha_{D,i}}^{\ 2} + COV_{mean,\alpha_D}^{\ 2}} \tag{6}$$

The numerical values of $COV_{overall,\alpha_D}$, $COV_{overall,\alpha_V}$ and $COV_{overall,\alpha_A}$ are collected in Table 3. It can be seen that these estimates are consistent with the across-the-ensemble coefficients of variation shown in Table 1, thus indicating the applicability of Eq. 6. Moreover, as expected, the values of $COV_{overall,\alpha_D}$ and $COV_{overall,\alpha_V}$ are lower than the corresponding across-the-ensemble coefficients of variation provided that the discovered correlation between α_D and α_V with PGD and PGV respectively. On the contrary $COV_{overall,\alpha_A}$ is substantially coincident with the corresponding across-the-ensemble coefficients of variation , given that due to the weak correlation between α_A and PGA.

The quantification of the intrinsic variability of the amplification factors is useful in order to provide consistent percentiles of the amplification factors for design purposes. For sake of simplicity, the values reported in Table 3 are obtained assuming $\overline{PGD}$ = 30 cm, $\overline{PGV}$ = 30 cm/s, and $\mu_{\alpha_A} = 2.16$.

Table 3. Coefficients of variation of α_D, α_V and α_A.

	COV_{α_i}	$COV_{mean,\alpha}$	$COV_{overall,\alpha}$
α_D	0.16	0.39	**0.42**
α_V	0.23	0.24	**0.33**
α_A	0.28	0.18	**0.33**

The main results obtained from the study on the amplification factors may be summarized as follows:

- The pseudo-acceleration amplification factor does not appear to be correlated to any peak parameter and therefore a constant mean value can be adopted; the pseudo-velocity and displacement amplification factors appear to be correlated to the corresponding peak parameters.

- Linear relationship between displacement (or pseudo-velocity) amplification factor and peak ground displacement (or peak ground velocity) is proposed.
- The across-the-ensemble variability of the displacement and pseudo-velocity amplification factors can be reduced to their intrinsic values, if the dependency on the related peak parameters is accounted for.
- Amplification factors exhibit an intrinsic overall variability which can be decomposed into two contributions: (i) the along-the-period variability (coefficients of variation about 0.16 and 0.28); (ii) the variability of the across-the-ensemble mean (coefficients of variation about 0.18 and 0.39).

DISPLACEMENT SPECTRUM BASED ON THE THREE PEAK GROUND MOTION PARAMETERS

According to EC8, the displacement design spectrum is derived from the corresponding design acceleration spectrum, which only accounts for the *PGA* at the site (given by seismic hazard analysis) and soil amplification coefficient depending on the soil category based on the values of V_{S30}. The *PGV* and *PGD* are not directly taken into account. Previous studies demonstrated that such a way of constructing design spectra, even though reasonable for pseudo-acceleration spectrum (in a force-based-design approach), may lead to unrealistic displacement spectra both in shape and amplitude (Bommer and Elnashai 1999). Indeed, provided that the *PGV* and especially the *PGD* are weakly correlated to the *PGA*, the code displacement design spectra are not associated to a clear hazard in terms of probability of exceeding the spectral ordinate.

Based on the obtained results, the following expression of the displacement spectrum is here proposed:

$$
\begin{aligned}
S_D(T) &= \frac{\alpha_A \cdot PGA}{(2\pi)^2} \cdot T^2 && \text{for } T \le T_c \\
S_D(T) &= \varphi_D \cdot T && \text{for } T_c < T \le T_d \\
S_D(T) &= \varphi_D \cdot T_d && \text{for } T > T_d
\end{aligned}
\tag{7}
$$

with:

$$
\begin{aligned}
\alpha_A &= 2.16 \\
\varphi_D &= 0.31 \cdot PGV - 0.22 \cdot PGV^2 \\
T_c &= 2\pi \cdot (0.91 - 0.64 \cdot PGV) \cdot \frac{PGV}{PGA} \\
T_d &= 2\pi \cdot \left(\frac{1.44 + 2.06 \cdot PGD}{1.96 - 1.39 \cdot PGV} \right) \cdot \frac{PGD}{PGV}
\end{aligned}
\tag{8}
$$

The proposed displacement spectrum is compared with the EC8 spectrum together with the average spectrum as obtained from a group of 10 historical ground motions. All the selected ground motions are characterized by a similar *PGA* (mean values around 0.25 g) and have been selected from the PEER database. Three groups are considered:

- Group G1 is composed of 10 records with PGV between 15 and 31 cm/s (with an average of 23 cm/s) and PGD between 8 and 12 cm (with an average of 10 cm);
- Group G2 is composed of 10 records with PGV between 40 and 70 cm/s (with an average of 48 cm/s) and PGD between 30 and 40 cm (with an average of 34 cm);
- Group G3 is composed of 10 records with PGV between 40 and 77 cm/s (with an average of 51 cm/s) and PGD between 50 and 60 cm (with an average of 54 cm).

Figure 4 compares the proposed displacement spectrum with the EC8 displacement spectrum for the three groups of accelerograms. Inspection of Figure 4 allows the following observations:

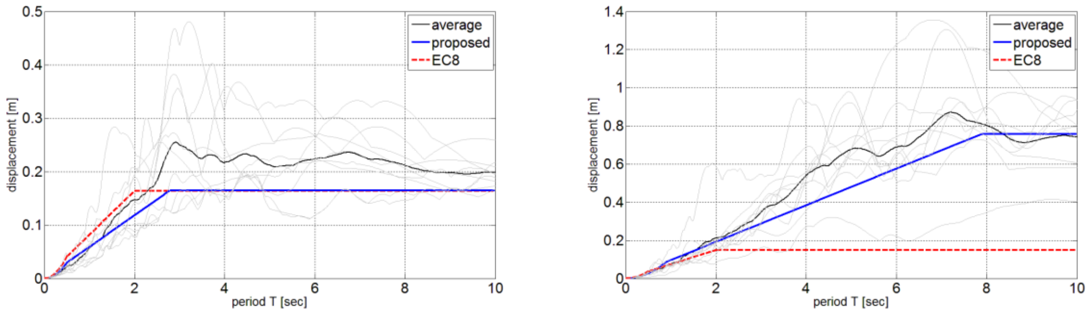
- Despite all the records exhibit a similar PGA, the average displacement response varies significantly moving from group G1 to group G3, due to the increase of the average PGD. Clearly, this behaviour cannot be captured by the EC8 spectrum which is based on the PGA only.
- The EC8 is only able to match the average spectrum of group G1 which is composed of ground motions selected in order to have values of peak ground parameters equal, on average, to the mean values of the entire ensemble. As far as groups G2 and G3 are concerned, it clearly appears that the EC8 spectrum is unconservative, especially at periods larger than approximately 1.0 s.
- The proposed spectrum is capable of reasonably capture both the shape and the amplitude of the mean spectrum for all the three groups.

CONCLUSIONS

In this paper, the amplification factors used for the characterization of the schematized response spectra has been deeply revised with the purpose of obtaining a more accurate displacement design spectrum through a statistical analysis performed on a large database composed of 177 historical seismic records for given soil conditions ($360 \leq V_{S30} \leq 750$ m/s), selected among the PEER strong motion database. The main findings can be summarized as follows:

- The pseudo-acceleration amplification factor does not appear to be correlated to any peak parameter; the pseudo-velocity and displacement amplification factors appear to be correlated to the corresponding peak parameters. Linear relationships are proposed between displacement and pseudo-velocity amplification factors and PGD and PGV, respectively.
- Amplification factors exhibit an intrinsic overall variability which can be decomposed into two contributions: (i) the along-the-period variability (coefficients of variation about 0.16 and 0.28); (ii) the variability of the across-the-ensemble mean (coefficients of variation about 0.18 and 0.39).

Then, a formulation for the displacement spectrum, which makes use of the three peak ground parameters (which should be available to the designer form specific seismic hazard studies) and of the revised amplification factors, has been proposed. Numerical examples showed the effectiveness of the proposed spectrum in matching the average response of different groups of ground motions selected on the basis of specific peak parameters.



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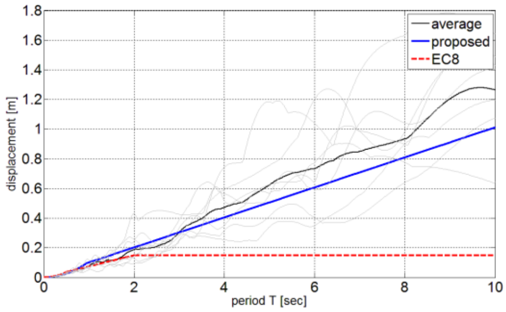

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Figure 4. Comparison between EC8 displacement spectrum and the proposed spectrum for: (a) Group G1; (b) Group G2 and (c) Group G3.

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REFERENCES

Al Atik L, Abrahamson N, Bommer JJ, Scherbaum F, Cotton F, Kuehn N (2010) The variability of groundmotion prediction models and its components. Seismol Res Lett 81:794–801. doi:10.1785/gssrl.81.5. 794.

Akkar S, Özen Ö (2005) Effect of peak ground velocity on deformation demands for SDOF systems. Earthq Eng Struct Dyn 34:1551-1571.

Akkar, S., Bommer, J. J. (2006). Influence of long-period filter cut-off on elastic spectral displacements. Earthquake engineering & structural dynamics, 35(9), 1145-1165.

Arias, A. (1970). MEASURE OF EARTHQUAKE INTENSITY. Massachusetts Inst. of Tech., Cambridge. Univ. of Chile, Santiago de Chile.

Baten, W. D. (1942). The standard error of the standard error of estimate. National Mathematics Magazine, 183-185.

Bertero RD, Bertero VV (2002) Performance-based seismic engineering: the need for a reliable conceptual comprehensive approach. Earthq Eng Struct Dyn 31(3): 627-652.

Bommer JJ, Elnashai AS (1999) Displacement spectra for seismic design. J Earthq Eng 3(1):1-32.

Chopra AK (1995) Dynamics of Structures. Theory and applications to earthquake engineering. Prentice Hall, Englewood Cliffs.

Chopra AK (2007) Elastic response spectrum: A historical note. Earthq Eng Struct Dyn 36:3-12.

Crandall SH, Mark WD (1963) Random vibration in mechanical systems. Academic Press, New York.

Eurocode 8 (2002) Design of structures for earthquake resistance. prEN 1998-1.

Faccioli E, Paolucci R, Rey J (2004) Displacement spectra for long periods. Earthq Spectra 20(2):347-376.

Lam NTK, Wilson JL, Chandler AM (2001) Seismic displacement response spectrum estimated from the frame analogy soil amplification model. Eng Struct 23(11):1437-1452.

Malhotra, P. K. (2006). Smooth spectra of horizontal and vertical ground motions. Bulletin of the Seismological Society of America, 96(2), 506-518.

Newmark NM, Hall WJ (1982) Earthquake Spectra and Design. Earthq Eng Res Inst, Berkeley.

PEER strong ground motion website: http://peer.berkeley.edu/products/strong_ground_motion_db.html

Priestley MJN, Calvi GM, Kowalsky MJ (2007) Direct Displacement-Based Seismic Design of Structures. IUSS Press, Pavia.

SEAOC Vision 2000 (1995) Framework for Performance Based Earthquake Engineering. Struct Eng, Sacramento.

Trifunac, M. D., Brady, A. G. (1975). A study on the duration of strong earthquake ground motion. Bulletin of the Seismological Society of America, 65(3), 581-626.

Tolis SV, Faccioli E (1999) Displacement design spectra. J Earthq Eng 3(1):107-125

Veletsos AS, Newmark NM (1960) Effect of Inelastic Behavior on the Response of Simple Systems to Earthquake Motions. Proc 2nd World Conf Earthq Eng 895-912.