



## EVALUATION OF ANALYTICAL FRAGILITY AND DAMAGE-TO-LOSS MODELS FOR REINFORCED CONCRETE BUILDINGS

Luís MARTINS<sup>1</sup>, Vítor SILVA<sup>2</sup>, Mário MARQUES<sup>3</sup>, Helen CROWLEY<sup>4</sup> and Raimundo DELGADO<sup>5</sup>

### ABSTRACT

This study is part of an ongoing research on the development of damage-to-loss functions for Portuguese reinforced concrete buildings. This paper presents a complete vulnerability analysis of a real concrete moment frame building. Only analytical methods for vulnerability analysis were considered and the structural performance was assessed through a 3D finite element model subjected to nonlinear incremental dynamic analysis using real ground motion records. Peak interstorey drift was considered as the engineering demand parameter for the development of global fragility functions, which represent the probability of exceeding a set of damage states, conditioned on the ground shaking intensity. Vulnerability functions (in terms of the ratio of cost of repair to cost of replacement, conditional on the level of ground shaking intensity) were derived through the estimation of member damage as a function of the chord rotation, and the assignment of different repair methods and costs as a function of the level of damage. Two different approaches for the estimation of losses have been applied, with one leading to more reasonable vulnerability functions for the Portuguese building stock.

### INTRODUCTION

Current seismic risk analysis requires loss estimation to be performed with the highest level of accuracy possible in order to provide decision makers with reliable information. Nevertheless, when one analyses each of the main steps in any seismic risk analysis it is clear that there is still room for improvement, namely in the definition of a damage-to-loss functions, arguably one of the major sources of uncertainty within an the analytical vulnerability assessment.

The first stage of an analytical vulnerability assessment is to use structural analysis to compute meaningful engineering demand parameters, EDPs (e.g. drift ratios, dissipated energy or floor accelerations) and convert them to structural damage in order to derive fragility functions. The issue of defining damage from structural performance parameters has been addressed in many previous studies, e.g. (Park *et al.* 1985; Calvi 1999; Borzi *et al.* 2008; Benavent-Climent 2011; Fardis *et al.* 2012), or technical guidelines and design recommendations, e.g. (FEMA 2003; CEN 2005). However, while EDP-to-damage estimation deals with concepts that are recognizable for most engineering practitioners and researchers and has been well documented, the same cannot be assumed about damage-to-loss assessment through so-called damage-to-loss or consequence models.

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<sup>1</sup> MSc and PhD student, Faculty of Engineering of the University of Porto, Porto, dec11007@fe.up.pt

<sup>2</sup> PhD, University of Aveiro, Aveiro, v.silva@ua.pt

<sup>3</sup> PhD, Faculty of Engineering of the University of Porto, Porto, mariom@fe.up.pt

<sup>4</sup> PhD, EUCENTRE - European Centre for Training and Research in Earthquake Engineering, Pavia, helen.crowley@eucentre.it

<sup>5</sup> Full Professor, Faculty of Engineering of the University of Porto, Porto, rdelgado@fe.up.pt

Despite efforts from a number of researchers to improve the field of loss modelling (e.g. Gunturi,(1993) Kappos *et al.*, (1998) Ramirez and Miranda, (2009) and Bal *et al.*, (2010), in general, the majority of available consequence models are still deterministic and/or based on limited empirical data. Furthermore, the majority of the existing damage-to-loss functions were developed and calibrated to represent the reality of a small number of countries and corresponding building stock, which may lead to a misleading vulnerability evaluation when applied to other regions of the world.

## CASE STUDY AND NUMERICAL ANALYSES

This study is based on a real residential dwelling building built in 1961 and located in Lisbon, Portugal (Figure 1). The structure is a five storey irregular reinforced concrete moment frame with a maximum height of 14.25m, a floor area of 151m<sup>2</sup> and a natural period of vibration of 0.665s. The building was designed following the general practice and regulations in force in Portugal at the date of construction (MOP 1958). In addition to the permanent loads a live load of 4 kN/m<sup>2</sup> was considered to act in all the floors except for the top one, for which it was reduced to 2.5 kN/m<sup>2</sup>.

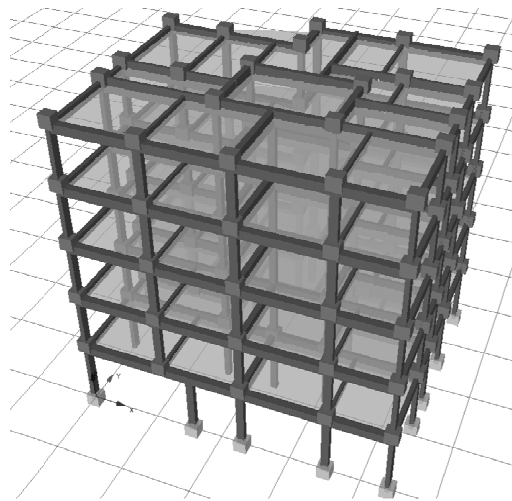


Figure 1. Case study building.

In order to assess the building's structural performance a 3D numerical model was built using the open source finite element (FE) software OpenSees (Mazzoni *et al.* 2005). The model was built using force based fibre elements, each with five Gauss-Lobatto integration points. Distributed plasticity models are widely acknowledged to be numerically unstable at high levels of ductility, especially if the element's section exhibits softening behaviour (Calabrese *et al.* 2010). Knowing that this may lead to numerical collapse for performance levels in which the structure is not expected to have collapsed, for every time step in which a numerical instability was detected, the implemented analysis code tested several solution algorithms (e.g. regular Newton-Raphson, modified Newton-Raphson, Newton-Raphson with line search and Broyden-Fletcher-Goldfarb-Shanno) before reducing the time step by a factor of 100 and increasing the tolerance in order to try to attain convergence. Finally, if convergence hasn't yet been achieved, the structure is reported as having collapsed.

A modified version of the incremental dynamic analysis (IDA) was applied for the structural assessment, in which different accelerograms were selected for each intensity level in order to better match the scenarios that contribute to the expected regional seismic hazard at different intensity levels; this approach is often referred to as multiple stripes analysis. The seismic hazard model and record selected framework described by Sousa *et al.* (2014) has been used wherein disaggregation of the hazard curve at a number of intensity measure levels is carried out, and based on the resulting magnitude, distance and epsilon triplets, the corresponding conditional spectra are estimated, and then records are selected which best-fit these spectra. The intensity measure (IM) chosen was the spectral acceleration on the fundamental vibration period ( $Sa(T_1)$ ).

The ground motion records were applied only on the direction of least stiffness. A total of 150 analyses were performed, divided by eight intensity levels ranging from 0.1g to 1.5g and with thirty ground motion records per intensity measure level (IML). A 2% tangent stiffness proportional damping was considered with the damping matrix being updated in all converged time steps. After applying the earthquake loading, the structure was allowed to vibrate freely until it finally stabilized in order to determine residual drifts and rotations.

## FRAGILITY ANALYSIS

After applying the seismic loads to the structure a damage evaluation analysis was undertaken for the development of fragility and vulnerability functions.

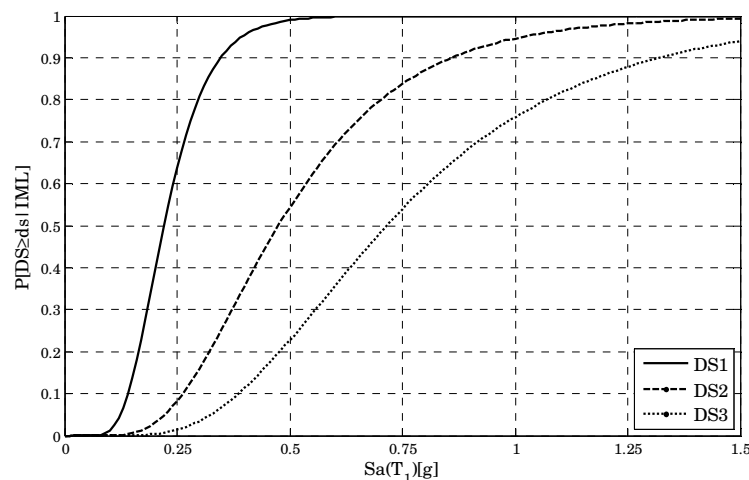
The structure's global performance was assessed using the maximum transient interstorey drift as the engineering demand parameter (EDP), and the damage criteria specified by FEMA 356 (FEMA 2000) for reinforced concrete moment frame mid rise structures was used for damage assessment (Table 1). Given that an irregular 3D FE model was used, which caused torsional effects, the maximum interstorey drift was computed using the SRSS of the horizontal displacements in the two orthogonal directions to define the total horizontal displacement at each structural node.

Table 1. Damage criteria.

Damage state	EDP	Limit value
Immediate occupancy (DS1)	Peak transient interstorey drift	0.01
Life safety (DS2)		0.02
Collapse prevention (DS3)		0.04

From a loss analysis perspective, the structural behaviour might not be completely characterized by the peak transient response if after the seismic loading the structure exhibits excessive deformation that might lead to the need for demolition. Therefore, in addition to the fragility curve given by the maximum transient interstorey drift, a collapse analysis is also provided. For this purpose the structure is deemed to have collapsed if at least one of the two following outcomes has occurred: i) the numerical analysis did not converge or ii) the residual interstorey drift exceeds 1.75% (which is the mean value suggested by Ramirez and Miranda (2012)).

The resulting global response fragility curves are plotted in Figure 2 and the respective mean ( $\theta$ ) and standard deviation ( $\beta$ ) of each curve are given in Table 2.



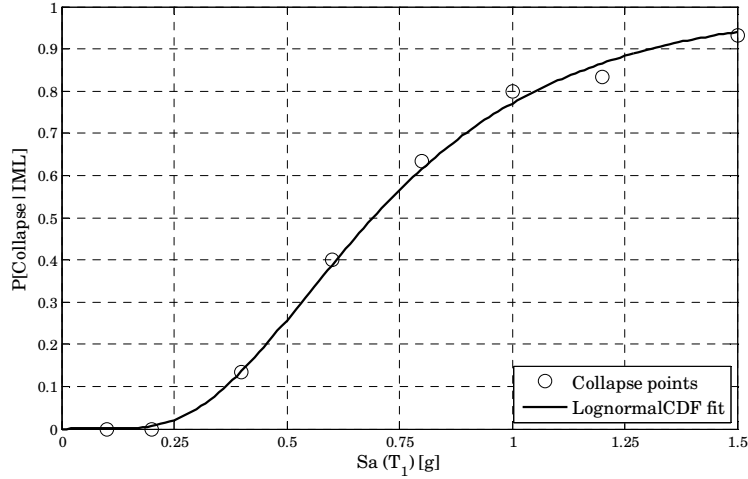


Figure 2. Top) Damage fragility functions based on interstorey drift (FEMA356); Bottom) Collapse fragility function

Table 2. Lognormal CDF fit parameters.

Damage state	Figure	Lognormal CDF fit parameters	
		$\theta$	$\beta$
Immediate occupancy (DS1)	Figure 2 (Top)	0.22	0.35
Life safety (DS2)	Figure 2 (Top)	0.48	0.46
Collapse prevention (DS3)	Figure 2 (Top)	0.71	0.48
Collapse curve	Figure 2 (Bottom)	0.69	0.50

When analysing the DS3 curve in the top plot and the Collapse curve in the bottom plot of Figure 2 it can be observed that the latter one exhibits a slightly lower mean value, meaning that following the criteria of numerical collapse and residual drift one would reach the collapse damage state before DS3. This evidence shows the importance of considering the residual drift in a loss analysis. Though the structure is not expected to have collapsed due to the seismic loading by FEMA's criteria, the excessive deformation would most likely lead to its demolition and therefore the need to consider its total loss in the analysis.

For the development of vulnerability functions, a member-based approach has been followed. In order to determine the damage level of each structural element, a modified version of the Park & Ang damage index proposed by Haselton *et al* (2008) (Eq. 1) was applied. The deformation damage index (DDI) is defined as the ratio of the maximum hinge rotation attained during seismic loading ( $\theta_{p\_transient}$ ) and the difference between the ultimate rotation capacity ( $\theta_u$ ) and the recoverable rotation at unloading ( $\theta_r$ ).

In monotonic loading the recoverable rotation may be approximately determined assuming the initial stiffness path as the unloading path; however for dynamic analysis the computation of plastic hinge rotations is not straightforward as the unloading path is a function of the loading history and varies from record to record (Chen and Lui 2006). Given that the structure was allowed to stabilize after the seismic loads were applied, one can estimate  $\theta_{pl}$  (Figure 3) from the residual rotations. Computing the recoverable rotation is then just a matter of subtracting the residual rotation from the maximum recorded rotation. The ultimate rotation capacity is a function of the section geometry, reinforcing steel pattern and the internal stresses and may be computed using the formulas provided in Annex A of Eurocode 8 Part 3 (CEN 2005).

$$DDI = \frac{\theta_{p\_transient}}{\theta_u - \theta_r} \quad (1)$$

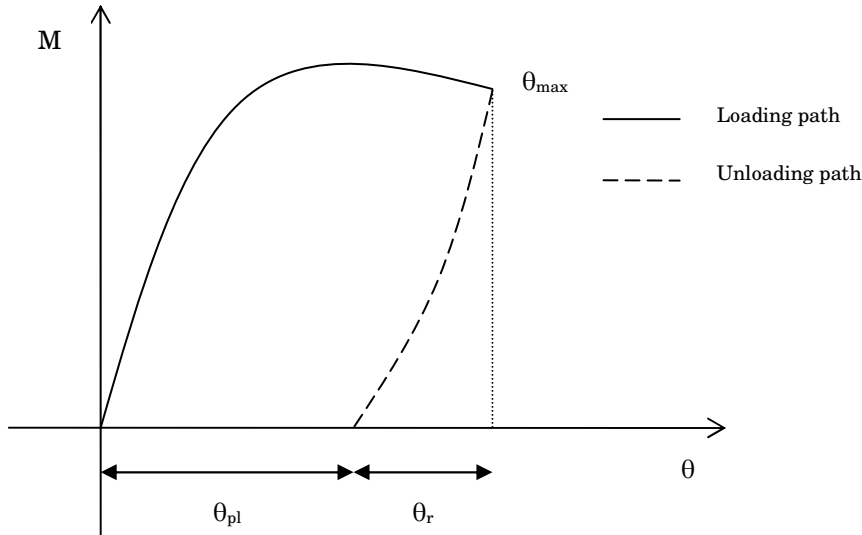


Figure 3. Defining rotations for seismic loading [adapted from (Chen and Lui 2006)].

The chord rotations at each end of the structural elements were determined using the geometrical method proposed by Romão *et al* (2010). Given that a 3D model was used, and therefore each node has six degrees of freedom, two values exist for the chord rotations at each element's end. To compute the damage index, the root mean square value of the chord rotations in all degrees of freedom was considered.

Using the fragility parameters presented in Table 3, it is possible to determine the probability of being in a given damage state, conditioned to a ground motion intensity level for every structural element, leading to Figure 4. This figure depicts the average probability of each damage state sorted by element type and floor.

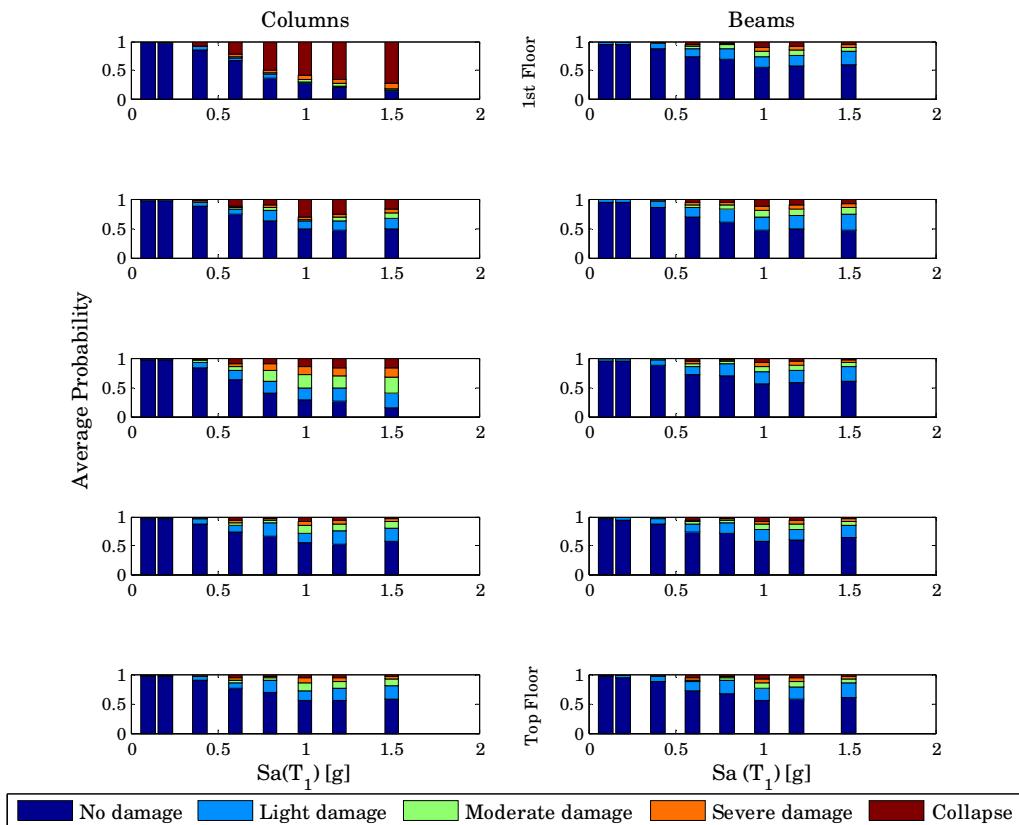


Figure 4. Distribution of the average probability of being in a given damage sorted by floor element type.

Table 3. Fragility parameters (Haselton *et al.* 2008)

Damage state	Fragility parameters	
	$X_m$ (DDI)	$\beta$
Light damage	0.08	1.36
Moderate damage	0.31	0.89
Severe damage	0.71	0.80
Collapse	1.28	0.74

Analysing Figure 4 one can observe that the main failure mechanism of the structure was a soft-storey at the first floor. The picture shows an increase in the probability of column collapse at the first floor level starting at 6.8% for 0.4g and going up to 73.7% for 1.5g. Given the fact that on average the beams have a much lower probability of collapse than the columns, it is possible to conclude that the building does not comply with the weak beam-strong column design rule, as expected from a building constructed during the 60's decade.

## VULNERABILITY ANALYSIS

Once the damage level of each element is known one can compute the expected loss and loss ratios by multiplying the damage state (DS) probability by the expected repair cost for that DS. To compute the repair cost of each damaged structural element this study followed the work of Haselton *et al.* (2008). The authors provide an estimate of the repair costs per damaged structural element for each damage state (Table 4) defined in terms of the DDI damage index (Eq.1).

Table 4. Repair cost parameters for each damage state (Haselton *et al.* 2008).

Damage state	Repair cost parameters	
	$X_m$ (US\$)	$\beta$
Light damage	8000	0.42
Moderate damage	22500	0.40
Severe damage	34300	0.37
Collapse	34300	0.37

The expected total repair cost of the building may be computed through Eq. (2) in which  $P[\text{Collapse} | IML]$  is the probability of collapse for a given intensity level,  $C_{\text{replacement}}$  is the replacement cost of the building and  $C_{\text{repair}}$  is the total repair cost of the structural elements. For research purposes the total building's replacement cost was simply defined as the product of the building's total area by the average construction cost predicted for 2014 by the Portuguese government (assumed to be 603€/m<sup>2</sup>).

$$E[C_{\text{total}} | IML] = P[\text{Collapse} | IML] \cdot C_{\text{replacement}} + (1 - P[\text{Collapse} | IML]) \cdot C_{\text{repair}} \quad (2)$$

To take into account the variability in the repair costs, 100 random numbers were sampled from the distributions in Table 4 that were later used to compute the repair cost of the elements. The total repair cost was taken as the average of the 100 values of the sum of the repair costs for all elements.

At any given intensity level and for any non-collapsed analysis one can estimate the probability of collapse associated with that ground motion record through the distribution proposed by Ramirez and Miranda (2012) that correlates the residual drift with the collapse probability through a lognormal distribution with median of 0.015 and logarithmic standard deviation of 0.30. Using these collapse probabilities and applying Eq. 2 one can estimate the global repair cost and the corresponding loss ratios for each IML and ground motion record (Figure 5).

From analysing Figure 5 one can observe that, as expected, the standard deviation of the mean loss ratio decreases with the intensity level. As the ground shaking intensity level increases the result of the cost-benefit analysis starts to lean towards replacing the building instead of repairing the damaged elements, thus the standard deviation of loss ratios should tend to zero.

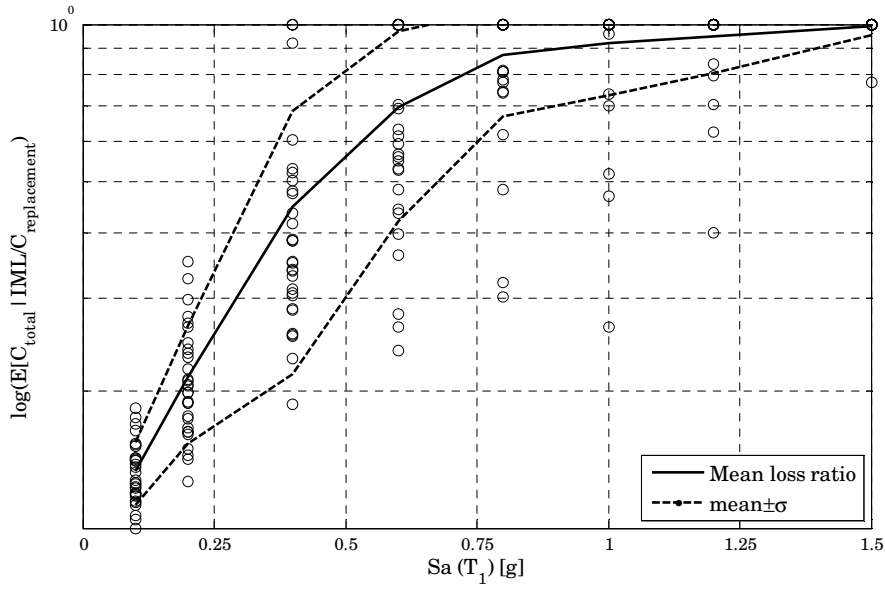


Figure 5. Expect loss ratios for case study.

The methodology proposed by Haselton *et al* (2008) assumes the same repair cost distributions for all structural elements and does not consider possible differences in the replacement cost between elements, however few would argue that a smaller and less important structural element should have a lower replacement cost.

In order to evaluate the influence of these assumptions in the overall loss estimation the distribution of the element replacement costs and the average loss ratio per IML were determined. To compute the replacement cost of each element ( $C_{rpl\_element}$ ) a percentage of the building's replacement cost ( $C_{rpl\_building}$ ) based on the ratio of the structural element's volume ( $V_{element\_i}$ ) and the total volume was considered (Eq. 3).

$$C_{rpl\_element} = C_{rpl\_building} \cdot \frac{V_{element\_i}}{\sum_{i=1}^{nElements} V_{element\_i}} \quad (3)$$

As depicted in Figure 6 (left) there is a significant variation on the replacement cost of each individual element which translates into excessive loss ratios for the smaller elements even for the lowest IML considered.

It is not expected to observe element loss ratios close to 1 for an intensity level of 0.1g for which most of the beams and columns did not suffered any damage. This can be explained by the fact that even though the probability of an element being damaged is very low, when it is used to compute repair costs using the functions provided in Table 4 the resulting value may be similar to the replacement cost of some of the smallest structural elements in the model. On the other end of the spectrum element loss ratios several orders of magnitude above 1 for the higher ground shaking intensities are also non acceptable.

It is acknowledged that repair operations are usually more expensive than building a new element due to extra tasks required prior to the actual work on the damaged element. However given the order of magnitude of the computed loss ratios one may say with a certain degree of confidence that the damage-to-loss functions provided by Haselton *et al* (2008) lead to an overestimation of the real losses when applied to the case study herein.

The insertion of this example in this study serves the purpose of giving a practical explanation of one of the major pitfalls of current vulnerability analysis methodologies. As stated in this paper's introduction the available damage-to-loss models were developed and calibrated to represent the

reality of a small number of countries and when applied to other regions of the world may lead to unreasonable loss estimation results.

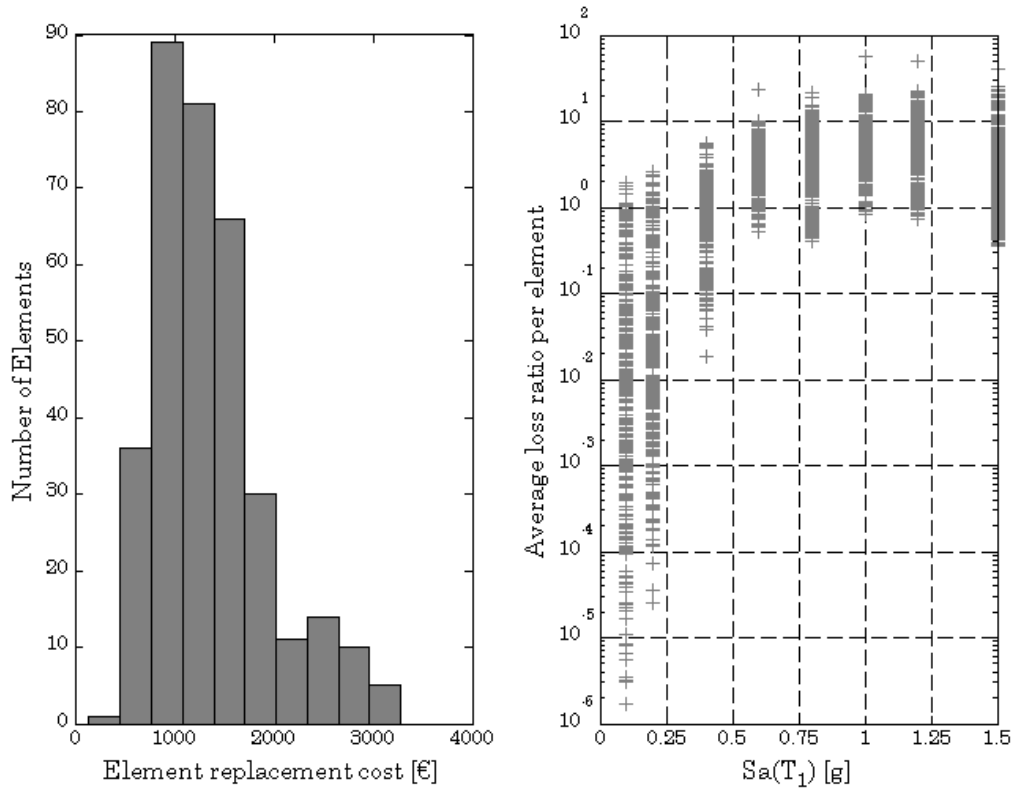


Figure 6. Left) Element replacement cost histogram; Right) Average loss ratio per element.

In order to provide a more suitable damage-to-loss model this study proposes a method that accounts for both the replacement cost of each element and the cost for the repair technique. The selected damage index used to assess the damage state of each element was the same modified version of Park & Ang previously presented in Eq.1.

In Haselton *et al* (2008) the authors state that if an element is deemed to be severely damaged or collapsed it cannot be repaired and has to be replaced; however for lower damage states the element can be repaired, and for lightly or moderate damaged elements the recommended repair techniques are epoxy injection and jacketing, respectively.

Therefore the methodology adopted in this study has a maximum value for the repair cost equal to the replacement cost given by Eq. 3. For moderate damaged elements that may need jacketing operations the value of 200€/m proposed by Calvi (2013) was adopted because it is deemed to be suitable for European constructions. At the current state of this ongoing research a reliable value for the cost of epoxy injection has not yet been found. For this study a percentage of the total repair cost for moderate damage limit state based on the ratio between the median DDI for light and moderate damage was used, assuming a linear variation of repair costs between the first and second DS. The resulting loss ratios are plotted in Figure 7.



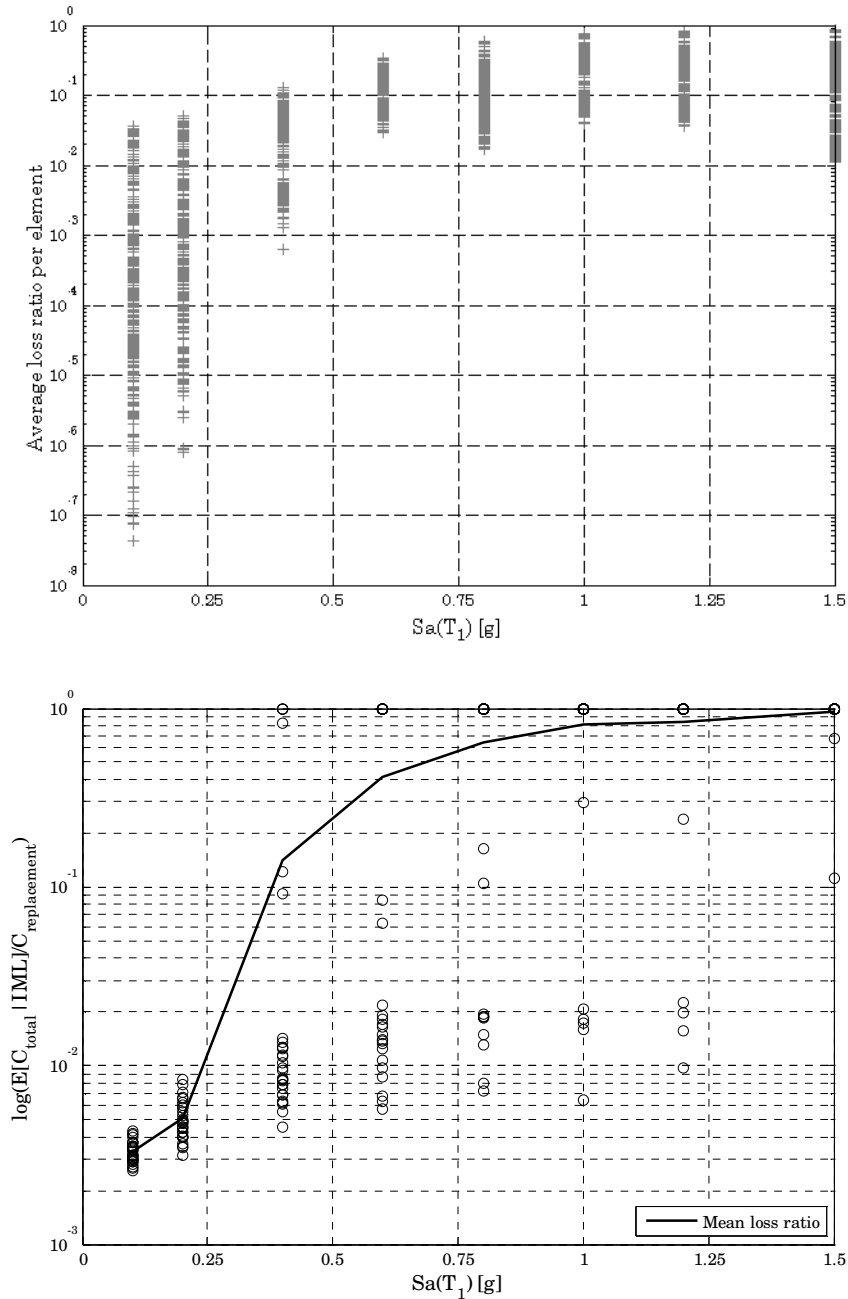


Figure 7. Top) Average loss ratio per element; Bottom) Expect loss ratios for case study.

As expected, this methodology yields an average loss ratio per element lower than those presented in Figure 6 and with a maximum of 1 being reached only by columns since on average the beams of the building have much lower damage.

The differences between the expected loss ratios for the whole building are only significant for the lower intensity levels because for higher intensities the complete collapse of the building governs the trend and therefore the loss ratio approximates to 1. Analysing the bottom plot in Figure 7 one can observe a few outlier records that yielded lower loss ratios than the average at higher intensity levels. This was most likely caused by records that despite the intensity level don't excite the structure as much as others. The influence of these records on the general trend of the mean loss ratio is however negligible given that the majority of the analysis still leads to a unit loss ratio. It is worth mentioning that this observation wasn't as clear in Figure 5 as it was in Figure 7 due to the overestimation of losses given by the repair costs proposed by Haselton *et al* (2008).

Taking the average value of the DDI given by the thirty ground motion records for each IML and plotting it against the average loss ratio per element one can trace a relationship between them. Plotting the natural logarithm of both quantities one observes that a trend appears that can be very accurately reproduced by a third degree polynomial function, as depicted in Figure 8 (bottom). For this case study the polynomial regression is only good for abscissa values up to 2. At this point the function reaches its inflection point and starts to deviate from the data set. This however should not pose any problem because this abscissa value is equivalent to DDI of about 7 which translates to a ductility level that very few elements can reach and in fact most of the elements are expected to have collapsed.

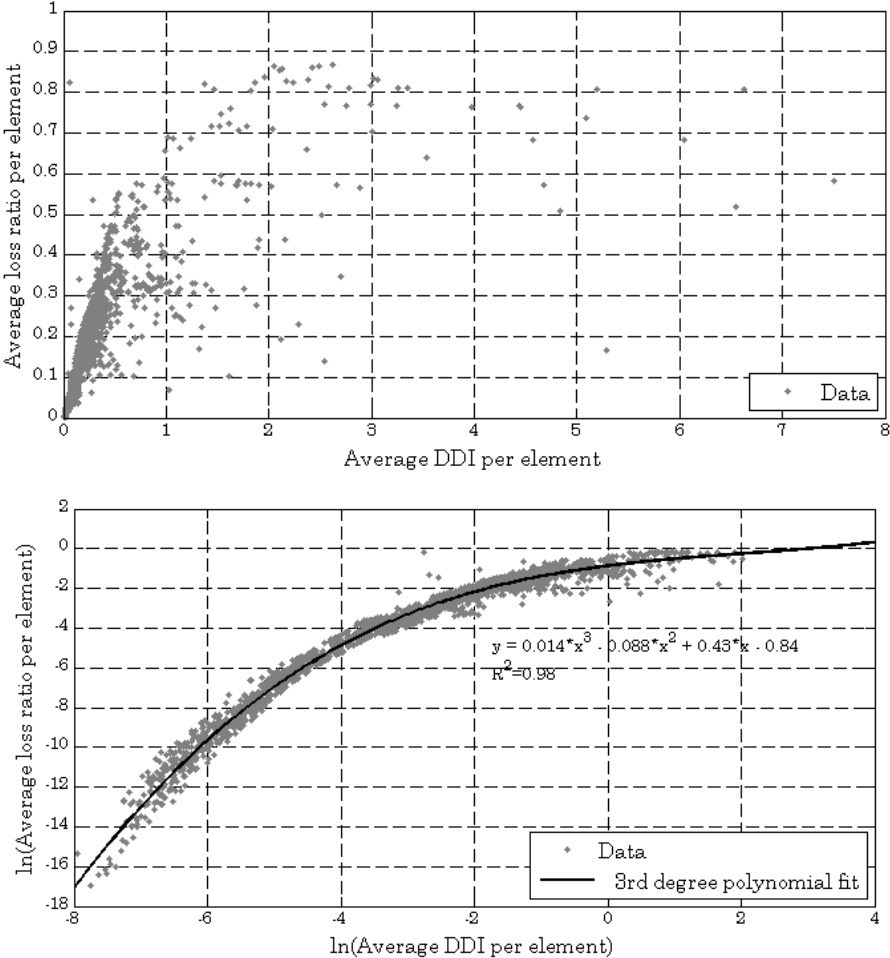


Figure 8. DDI-loss ratio relationship. Top) Linear scale plot; Bottom) Log-log space plot.

**CONCLUSIONS**

The results presented in this paper are part of an ongoing research on damage-to-loss functions for reinforced concrete structures. In this paper an analytical vulnerability assessment of a real Portuguese dwelling building through analytical methods was presented.

Two different approaches for fragility assessment were applied, one based upon the global building performance and another that focuses on the performance of individual structural elements. This analysis allowed to determine the probability of collapse for a given intensity level and the distribution of damage per structural element sorted, by element type and floor level that were later used on the vulnerability assessment.

In order to assess the structural vulnerability, two different techniques were presented. Firstly a methodology proposed by Haselton *et al* (2008) was applied to determine the distribution of loss ratios for a given intensity level. This method was deemed to lead to excessive losses for the lower end of the ground shaking intensity level. This methodology has been calibrated for Californian building stock and as previously stated the usage of existing damage-to-loss models that have not been adjusted to the type of construction being assessed may lead to a misvaluation of the structural vulnerability.

The second technique presented to assess structural vulnerability assumes the replacement cost of each structural element as a percentage of overall building replacement cost and takes into account the cost of each repair job needed to restore the damaged element to its previous undamaged condition. Although this technique accounts for eventual differences in replacement costs between elements because it uses the replacement cost of the building as a common denominator to determine them, which makes its adjustment to other realities very straightforward, provided that the average construction cost is known, its major drawback is knowing the cost of each required repair techniques. Therefore this methodology would benefit greatly from further investigation on this topic. The main objective of this ongoing research will thus be to accurately assess the repair costs for reinforced concrete elements in Portugal.

A relationship between the natural logarithm of the modified version of the Park & Ang damage index (so-called DDI) and the natural logarithm of the average expected loss ratio was presented. It has been observed that a third degree polynomial offer a good fit to the data in the entire range of interest for the damage index. Establishing a relationship directly between an EDP and the expected loss provides an easy, simple and quick way to estimate losses directly from nonlinear dynamic analysis without the need for fragility assessment. If applied to a large enough set of structures, this methodology may in the future prove itself useful for the development of EDP-to-loss models suitable for an entire building typology.

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