



## KINEMATIC BENDING OF FIXED-HEAD PILES IN NON-HOMOGENEOUS SOIL

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### ABSTRACT

Kinematic bending of piles in inhomogeneous soil is explored in static and dynamic regime. The system under consideration consists of a fixed-head pile embedded in a continuously inhomogeneous viscoelastic soil layer resting on a rigid base. A generalized parabolic function is employed to describe the variable shear modulus in the inhomogeneous stratum. The problem is treated numerically by means of rigorous elastodynamic finite-element analyses and Beam-on-Dynamic-Winkler-Foundation (BDWF) formulations. A design formula for kinematic pile-head moments is derived both for static and dynamic loading by employing a characteristic pile wavenumber. A new normalization scheme for dynamic pile bending is proposed by means of a single dimensionless frequency parameter governing kinematic pile-head moments. A numerical example is also provided.

### INTRODUCTION

Piled foundations may be subjected to large curvatures during earthquakes due to deformations developing in the surrounding soil, even in absence of forces applied at the top. This interaction mechanism is known as “kinematic interaction”. Evidence on kinematically-stressed piles has been identified in post-earthquake observations (Tazoh et al. 1984, Tazoh et al. 1987, Mizuno 1987, Nikolaou et al. 2001) in soils that have not experienced large movements such as those induced by liquefaction. The above field data, in conjunction with analytical evidence to be discussed below has revealed the possibility of pile damage close to the pile head or near interfaces separating soil layers with sharply different stiffness. Pile-soil kinematic interaction has been the subject of systematic research (e.g. Margason 1975, Flores-Berrones and Whitman 1982, Kavvas and Gazetas 1993). Thus, a number of design-oriented scientific works (Mylonakis 2001, Nikolaou et al. 2001, Maiorano et al. 2009, Di Laora et al. 2012) provided simplified solutions for kinematic pile moments at the interface between two consecutive layers with sharply differing stiffness, considered as a critical condition by modern seismic codes (CEN 2003, Norme Tecnica per le Costruzioni, 2008). However, pile-head kinematic moments may be equally important for soils with small stiffness near surface where kinematic forces tend to dominate over inertial ones, especially for large-diameter piles (Di Laora and Mandolini 2011). With reference to a two-layer soil, the issue has been investigated recently by means of Finite-Element analyses (Dezi et al. 2010; de Sanctis et al. 2010), leading to approximate correlations between pile-head kinematic moments and maximum acceleration at the surface. Later, Di Laora et al. (2013) investigated the interplay between interface and pile-head kinematic moments, concluding that pile-head curvature is approximately equal to soil curvature at

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surface in the case of deep interfaces, whereas it assumes lower values for low interface depths, due to the restraining action provided by the deeper and stiffer soil layer.

Along these lines, kinematic bending moments at the pile-head are explored in this paper referring to a long pile embedded in a continuously inhomogeneous layer over a rigid base. The variation of soil stiffness with depth is described by a generalized parabolic function accounting both for zero and finite shear modulus at the surface. The problem is treated numerically by means of rigorous Finite-Element analyses in static and dynamic regime. A simplified Beam-on-Dynamic-Winkler-Foundation (BDWF) model in conjunction with a layer transfer-matrix approach known as the Haskell-Thompson technique (Thompson 1950) is also employed to elucidate the role of Winkler modulus on kinematic pile-head bending. The scope of the study is: (a) to elucidate the role of key dimensionless parameters of the problem; (b) to propose a new formulation for the active length of piles in continuously inhomogeneous soils; (c) to introduce the notion of an effective soil curvature being equal to pile-head kinematic curvature both in static and dynamic regime and (d) to implement a unique dimensionless frequency governing dynamic pile-head kinematic bending.

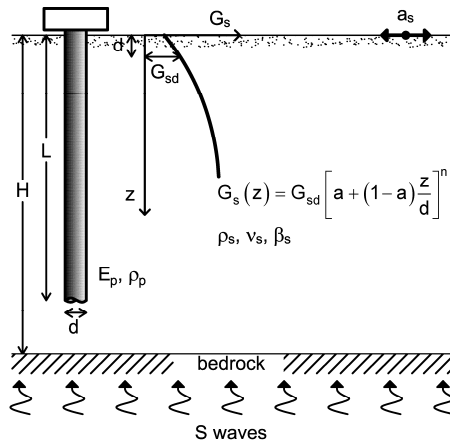


Figure 1. Problem under consideration: a single elastic fixed-head pile embedded in a continuously inhomogeneous layer over rigid rock.

## PROBLEM STATEMENT

The system under consideration consists of a fixed-head pile embedded in a continuously inhomogeneous viscoelastic soil layer on a rigid base (Fig. 1). The pile is modelled as a linearly elastic cylindrical solid beam of diameter  $d$ , length  $L$ , elastic modulus  $E_p$  and mass density  $\rho_p$ . Soil mass density,  $\rho_s$ , Poisson's ratio,  $\nu_s$ , and hysteretic damping ratio,  $\beta_s$ , are considered constant with depth, whereas shear modulus  $G_s(z)$  is assumed to increase according to the generalized power law function:

$$G_s(z) = G_{sd} \left[ a + (1-a) \frac{z}{d} \right]^n \quad (1)$$

where  $a = (G_{so} / G_{sd})^{1/n}$  and  $n$  are dimensionless inhomogeneity factors,  $G_{so}$  being the shear modulus at ground surface ( $z = 0$ ) and  $G_{sd}$  referring to the shear modulus at the depth of one pile diameter ( $z = d$ ). In this manner, the adopted stiffness variation allows study of a wide range of soils not covered in the pile dynamics literature. For a soft or moderately over-consolidated cohesive soil, a linear relationship between stiffness and depth is generally adequate. Cohesionless soils follow a less-than-linear distribution of shear modulus with depth, associated with  $n < 1$ . Naturally, for values of the inhomogeneity factor  $n$  close to zero or  $G_{so} / G_{sd}$  ratio close to 1, Eq. 1 describes a homogeneous medium (i.e.  $G_{so} = G_{sd}$ ), whereas for  $n = 1$  a Gibson-type soil is modeled having either zero or finite stiffness at the free surface depending on the value of parameter  $a$ . Representative  $G_s(z)$  profiles normalized by the shear modulus of the inhomogeneous layer at the depth of one pile diameter ( $G_{sd}$ ) are plotted in Fig. 2a, referring to a uniform ( $n = 0$ ), a proportional-with-depth ( $a = 0, n = 1$ ), a

parabolic ( $a = 0, n = 0.5$ ) and a linear ( $a = 0.5, n = 1$ ) distribution of soil stiffness. The pile-soil system is subjected to harmonic S-waves having different frequencies propagating in the soil mass.

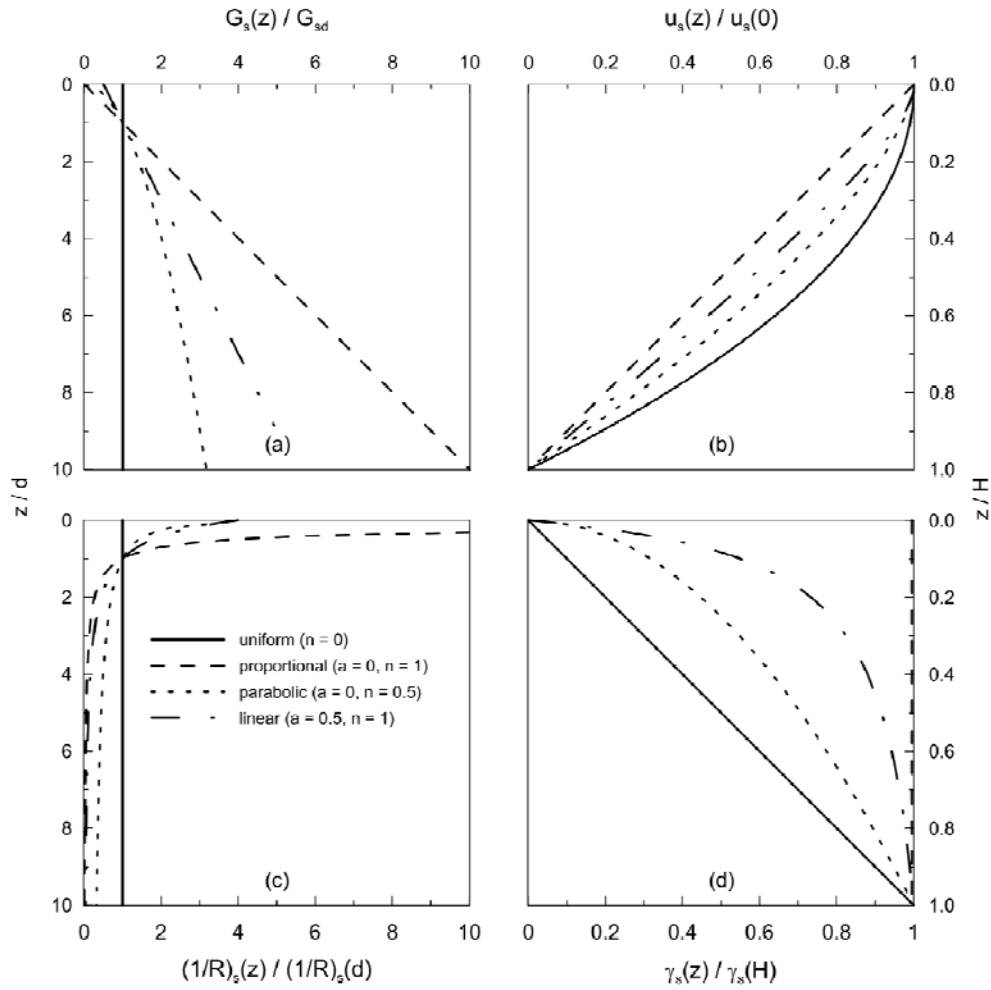


Figure 2. Representative soil profiles referring to a uniform ( $n=0$ ), proportional ( $a = 0, n = 1$ ), parabolic ( $a = 0, n = 0.5$ ) and linear ( $a = 0.5, n = 1$ ) distribution of soil stiffness (a) and corresponding free-field response in terms of normalized (b) displacement, (c) curvature and (d) shear strain with depth.

## STATIC BEHAVIOR

### One-dimensional soil response

Under constant ground acceleration ( $a_s$ ), equilibrium of an one-dimensional soil column with constant mass density  $\rho_s$  and variable shear modulus,  $G(z)$ , is described by the differential equation:

$$\frac{d\tau(z)}{dz} = a_s \rho_s \quad (2)$$

in which the product ( $a_s \rho_s$ ) represents the body force imposed to an infinitesimal soil element of height  $dz$  and  $\tau(z)$  is the shear stress related to shear strain  $\gamma(z)$  [ $= du_s(z) / dz$ ] through:

$$\tau(z) = G(z) \frac{du_s(z)}{dz} \quad (3)$$

where  $u_s(z)$  is soil displacement. Accordingly, Eq.2 takes the form

$$\frac{d}{dz} \left[ G(z) \frac{du_s(z)}{dz} \right] = a_s \rho_s \quad (4)$$

Upon integrating twice and imposing the boundary conditions of zero shear stress at soil surface  $[\tau(0) = 0]$  and zero soil displacements at the base  $[u_s(H) = 0]$ , soil lateral displacement is obtained as:

$$u_s(z) = \frac{a_s \rho_s d^2}{G_{sd} (1-a)^2 (n-2)(n-1)} \left\{ \left[ a + (1-a) \frac{H}{d} \right]^{1-n} \left[ a + (1-a)(n-1) \frac{H}{d} \right] - \left[ a + (1-a) \frac{z}{d} \right]^{1-n} \left[ a + (1-a)(n-1) \frac{z}{d} \right] \right\} \quad (5)$$

The solution in terms of soil shear strain and soil curvature is expressed by:

$$\gamma_s(z) = \frac{a_s \rho_s z}{G_{sd}} \left[ a + (1-a) \frac{z}{d} \right]^{-n} \quad (6)$$

$$\left( \frac{1}{R} \right)_s(z) = \frac{d\gamma_s(z)}{dz} = \frac{a_s \rho_s}{G_{sd}} \left[ a - (1-a)(n-1) \frac{z}{d} \right] \left[ a + (1-a) \frac{z}{d} \right]^{-n-1} \quad (7)$$

Equations 5, 6 and 7 are plotted in dimensionless form in Fig. 2b, 2d and 2c, respectively, referring to the inhomogeneous profiles shown in Fig. 2a.

### Effective soil curvature $(1/R)_{s,eff}$

For homogeneous soils, a common kinematic interaction factor for evaluating head bending is represented by pile-soil curvature ratio, defined as pile-head curvature over soil curvature at soil surface. In the case of an inhomogeneous soil with stiffness increasing proportionally with depth, such as that described by Eq. 1 for  $a = 0$  and  $n = 1$ , the above ratio cannot be employed since soil curvature at ground surface is infinite (Fig. 2c) as evident from Eq. 7. An alternative representation of kinematic bending was reported in Mylonakis (2001) by implementing a strain transmissibility function  $(\epsilon_p/\gamma_s)$ , relating peak pile bending strain  $(\epsilon_p)$  at the outer fiber of the pile section and soil shear strain  $(\gamma_s)$ . Accordingly, the above kinematic index is not applicable at the level of the pile head since shear strain at ground surface can be either zero or infinite, depending on the rate of soil stiffness distribution with depth (Rovithis et al. 2011), whereas a fixed-head pile will always experience a finite curvature.

Towards the definition of a physically-based interaction factor to be applied in any subsoil condition, one could think that pile-head curvature  $(1/R)_p$  should depend on the whole distribution of free-field deformations along the pile (Fig. 3). Hence, in this study the notion of an effective soil curvature  $(1/R)_{s,eff}$  is introduced as:

$$(1/R)_{s,eff} = \frac{\gamma_s(z_{eff})}{z_{eff}} \quad (8)$$

where  $z_{eff}$  stands for an effective depth of soil contributing to kinematic pile-head bending (whose value will be identified in the ensuing) and  $\gamma_s(z_{eff})$  is the corresponding shear strain of soil computed at  $z = z_{eff}$  by means of Eq. 6:

$$\gamma(z_{eff}) = \frac{a_s \rho_s z_{eff}}{G_{sd}} \left[ a + (1-a) \frac{z_{eff}}{d} \right]^{-n} \quad (9)$$

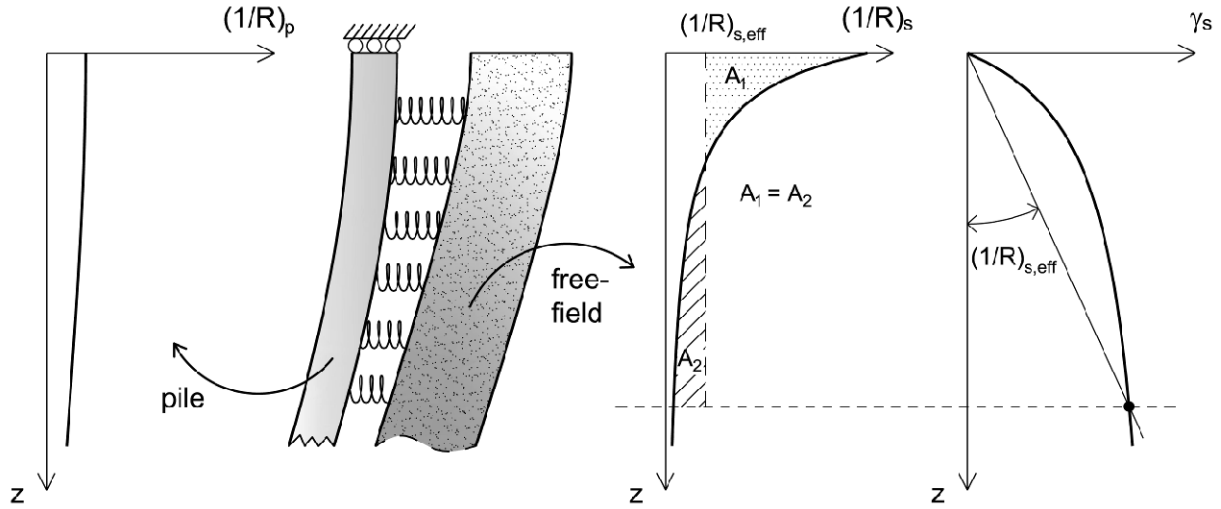


Figure 3. Distribution of free-field and pile curvatures along depth and definition of soil effective curvature.

Table 1. Parameters values employed in the parametric study.

$\rho_p / \rho_s$	$E_p / E_{sd}$	$a$	$n$	$\nu_s$
1.25	600	0	0	0.3
1.5	3000	0.25	0.25	0.499
		0.5	0.5	
		0.75	0.75	
		1	1	

$(1/R)_{s,eff}$  in Eq. 8 may be viewed as an average soil curvature along  $z_{eff}$  which always has a finite value and reflects the physics of the interaction phenomenon (Fig. 3).

The ratio of pile-head curvature to effective soil curvature  $[(1/R)_p / (1/R)_{s,eff}]$  will be employed in the ensuing to describe pile-head kinematic bending in inhomogeneous soils.

### Pile-head curvature: numerical analyses

With reference to long piles [i.e. piles with length larger than the active one (Randolph 1981)], the main parameters affecting pile-soil interaction in static regime are: pile diameter  $d$ , Young's modulus  $E_p$  and density  $\rho_p$ , and soil properties expressed by density  $\rho_s$ , coefficients  $a$  and  $n$ , Young's modulus at one diameter of depth  $E_{sd}$  and Poisson's coefficient  $\nu_s$ .

In light of dimensional analysis, it is straightforward to show that three out of the eight parameters mentioned above are dimensionally independent. Thus,  $(1/R)_p / (1/R)_{s,eff}$  ratio may be expressed as a function of five dimensionless parameters:

$$\frac{(1/R)_p}{(1/R)_{s,eff}} = f\left(\frac{\rho_p}{\rho_s}, \frac{E_p}{E_{sd}}, a, n, \nu_s\right) \quad (10)$$

A parametric investigation was performed to evaluate kinematic pile-head bending by means of rigorous finite-element analyses. The specific values of the dimensionless parameters in Eq. 10 combined within the parametric study are summarized in Table 1 leading to a total of 136 cases.

### FE model

Following Wilson (1965), the original three-dimensional soil-pile system may be conveniently reduced to a two-dimensional one, taking advantage from the axisymmetric geometry and the anti-

symmetric load. Numerical analyses were conducted using the commercial FE code ANSYS (Ansys Inc. 2005). Four-noded axisymmetric 2D elements were used to mesh soil and pile, having a vertical dimension equal to  $d/4$  and an horizontal dimension varying from  $d/6$  (at pile-soil interface) to  $1.5d$  (at the lateral boundaries). Vertical displacements were merely restrained at the lateral boundaries of the model, located far enough from the pile, to allow diffracted waves to attenuate due to the soil material damping. More specifically, lateral boundaries were placed 400 pile diameters away from pile axis to model accurately low soil damping. Base nodes were restrained against both horizontal and vertical motion to model a rigid bedrock. The height of the soil layer was set equal to 30 pile diameters, whereas pile length was considered equal to 25 pile diameters.

The analyses performed in the parametric study were carried out in the frequency domain, where soil possesses a constant hysteretic (frequency-independent) damping ratio equal to 5%. The load consists of a horizontal acceleration and is applied in the form of a body force imposed to the elements.

### Effective soil curvature as a measure of pile-head curvature

The effective depth  $z_{eff}$  introduced in Eq. 8 may be derived by considering that the effective portion of soil controlling pile head bending is proportional to a characteristic wavelength of the pile-soil system. To this end, an average wavenumber  $\mu$  may be introduced as (Mylonakis 1995):

$$\mu = \frac{1}{L_a} \int_0^{L_a} \lambda(z) dz \quad (11)$$

where the active pile length  $L_a$  may be taken equal to 10 pile diameters, as a first approximation, for typical values of pile-to-soil stiffness ratio, based on the corresponding expression proposed by Randolph (1981), and  $\lambda(z)$  is the Winkler wavenumber:

$$\lambda(z) = \left[ \frac{k_x(z)}{4E_p I_p} \right]^{1/4} \quad (12)$$

Two assumptions are made to compute  $\mu$  (measured in units of 1/Length). First,  $\mu$  and  $\lambda$  are treated as real-valued functions implying low-frequency loading. The validity of this assumption has been demonstrated by Krishnan et al. (1983) and Gazetas and Dobry (1984). Second, it is assumed that the Winkler springs modulus  $k_x(z)$  varies with depth with the same law as soil Young's modulus  $E_s(z)$  does (Mylonakis and Roubas, 2001):

$$k_x(z) = k_d \left[ a + (1-a) \frac{z}{d} \right]^n \quad (13)$$

In the above equation,  $k_d$  refers to the spring coefficient at one pile diameter depth given by the corresponding Young's modulus of soil ( $E_{sd}$ ) times a proportionality coefficient  $\delta$  [i.e.  $k_d = \delta E_{sd}$ ] (Roesset 1980, Dobry et al. 1982, Gazetas and Dobry 1984). The effect of  $\delta$  on kinematic pile-head bending is discussed in the sequel as part of a sensitivity analysis. The solution of the integral in Eq. 11 may be expressed as:

$$\mu = \frac{4\lambda_d}{d^{\frac{n}{4}} L_a (4+n)(a-1)} \left[ (ad)^{\frac{4+n}{4}} - (ad + L_a - aL_a)^{\frac{4+n}{4}} \right] \quad (14)$$

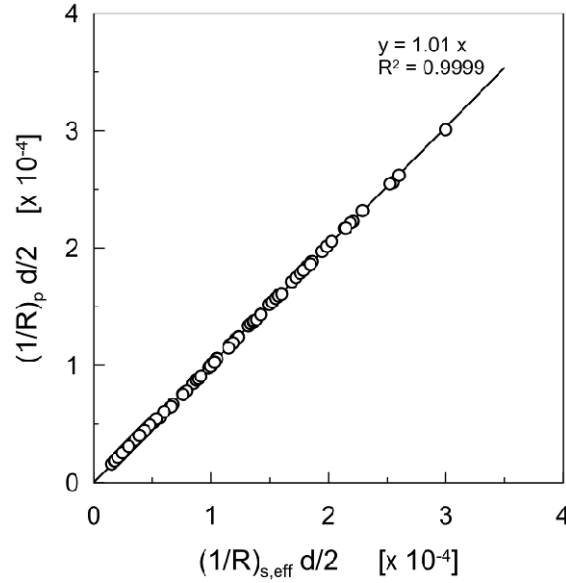


Figure 4. Correlation between pile-head curvature (obtained from Finite Element analysis) and soil effective curvature (computed by means of Eqs 8 and 16)

where  $\lambda_d$  corresponds to the (static) wavenumber of a pile in a homogeneous layer with Young's modulus equal to  $E_{sd}$ :

$$\lambda_d = \left[ \frac{k_d}{4E_p I_p} \right]^{1/4} \quad (15)$$

In Fig. 4, finite-element results in terms of pile-head curvature are compared to the effective soil curvature computed by means of Eq. 8, where  $z_{eff}$  is assumed as equal to:

$$z_{eff} = \frac{1.25}{\mu} \quad (16)$$

Each circle point in Fig. 4 corresponds to a different soil-pile configuration of Table 1. The excellent correlation between the above parameters is evident.

### Active pile length in inhomogeneous soils

Under kinematic loading, active pile length  $L_a$  is defined in this study as the length beyond which a pile behaves as infinitely long in terms of pile-head curvature, meaning that further increase in pile length does not exert any significant effect on pile-head bending.

Based on the above definition, a set of finite element analyses was performed to derive a simple expression of  $L_a$  that may be readily employed in design and analysis procedures. For this reason, a cylindrical pile of increasing length embedded in the inhomogeneous soil profiles shown in Fig. 2a was employed. Pile-head curvature to effective soil curvature  $(1/R)_p/(1/R)_{s,eff}$  ratios are plotted in Fig. 5a against the dimensionless pile length  $\mu L$ . It is observed that for values of  $\mu L$  larger than 2.5, the curves converge leading to  $(1/R)_p/(1/R)_{s,eff}$  ratios equal to 1. The above value may be interpreted as an active dimensionless pile length, leading directly to a simple expression for  $L_a$ :

$$L_a = \frac{2.5}{\mu} \quad (17)$$

Since the exact value of  $L_a$  can be derived only iteratively (recall that  $\mu$  depends on  $L_a$ ), the value of  $L_a = 10d$  mentioned above is suggested as a starting point for calculating  $\mu$  in Eq. 14; however, even a single iteration leads to quite accurate results. A comparison between Eq. 17 and earlier definitions of  $L_a$  reported in Davies and Budhu (1986), Budhu and Davies (1987) and Gazetas (1991) is given in Fig. 5b as a function of pile-soil stiffness ratio  $E_p/E_{sd}$ .

Upon combining Eqs 1, 8, 9, 16 and 17, the following design formula for kinematic pile-head bending is obtained:

$$(1/R)_p = \frac{a_s \rho_s}{G(z_{eff})} = \frac{a_s \rho_s}{G(L_a/2)} \quad (18)$$

The above expressions imply that the effective depth  $z_{eff}$  has the following properties: (a) it is half of the active pile length; (b) the soil shear modulus  $G(z_{eff})$  represents the stiffness of an equivalent homogeneous soil that leads to equal kinematic pile-head bending as in the inhomogeneous case. With reference to short piles (i.e.  $\mu L < 2.5$ ), Fig. 5a allows a simple expression for the kinematic head curvature to be employed for practical purposes:

$$\frac{(1/R)_p}{(1/R)_{s,eff}} = 0.3(\mu L)^2 \quad (19)$$

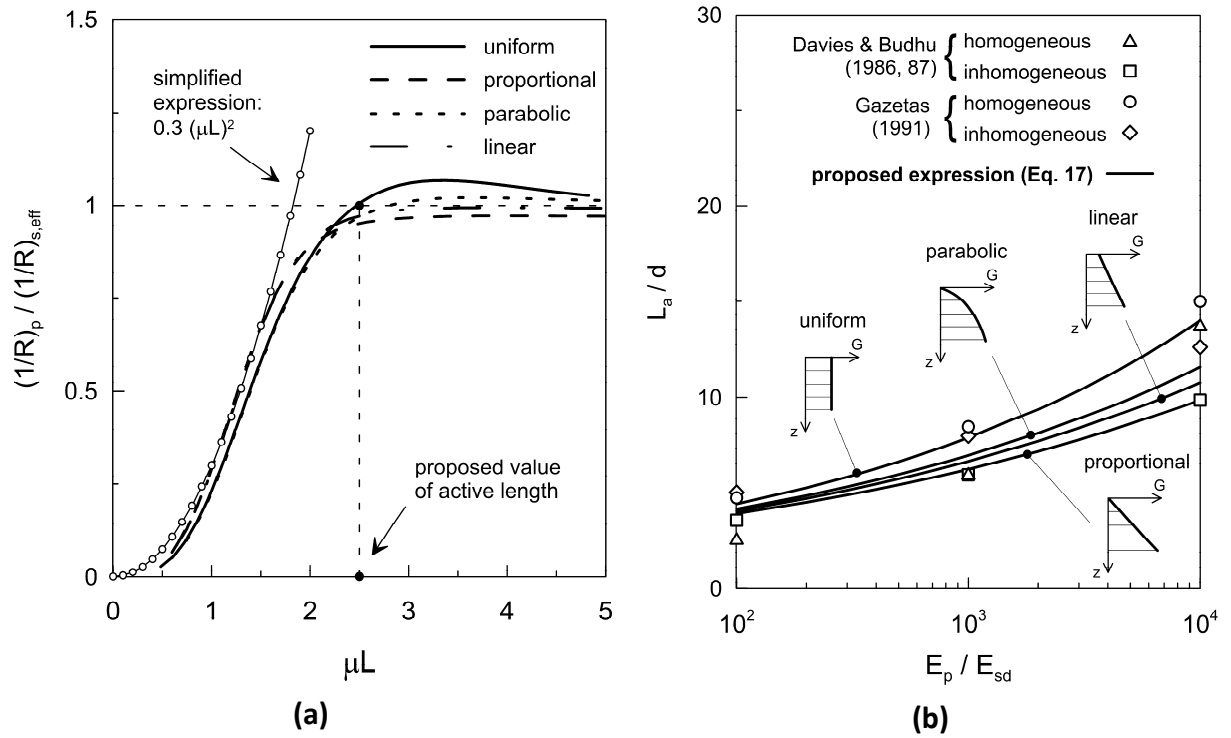


Figure 5. (a) Pile-head curvature to effective soil curvature ratio as function of pile mechanical slenderness  $\mu L$ . (b) Comparison between proposed formulation for active pile length and existing formulae.

### Winkler spring stiffness effect

Mention has already been made to the stiffness ( $k_x$ ) of the Winkler springs representing soil reaction, defined by the product of soil Young's modulus times a proportionality coefficient  $\delta$ . Various formulations have been proposed to compute  $\delta$  under inertial or kinematic action. Following the early work of Blaney et al. (1976), Roesset (1980) proposed the value:



$$\delta = 1.2 \quad (20)$$

regardless of the mechanical parameters involved in the interaction problem. Improvements over the above formula have been presented in Dobry et al. (1982), Gazetas and Dobry (1984) and Syngros (2004), referring to pile-head loading. For kinematic pile bending of long piles in a two-layer soil, Mylonakis (2001) simplified the original expression proposed by Kavvadas and Gazetas (1993) by relating  $\delta$  to pile-to-soil stiffness ratio according to the equation:

$$\delta \cong 6 \left( \frac{E_p}{E_{s1}} \right)^{-1/8} \quad (21)$$

where  $E_{s1}$  is the Young's modulus of the upper layer.

A sensitivity analysis was performed to investigate the effect of  $\delta$  on kinematic pile-head bending. A hybrid numerical-analytical solution was adopted in the realm of a Beam-on-Dynamic-Winkler-Foundation (BDWF) model in conjunction with a layer transfer-matrix approach known as the Haskell-Thompson technique. Further details may be found in Rovithis et al. (2013). The spring stiffness of the Winkler medium was defined by means of Eqs 20 and 21 (by replacing  $E_{s1}$  with the characteristic soil Young's modulus  $E_{sd}$ ) whereas  $\delta = 2$  was also examined as a third case.

Comparative results between FE and Winkler analyses in terms of pile-head curvature are shown in Fig. 6a, referring to the complete set of the examined soil-pile configurations (Table 1). It is observed that Winkler spring stiffness has a minor effect on kinematic bending. However, a value of  $\delta$  close to 2 may be considered suitable when pile-head curvature is to be matched, in agreement with results obtained by Anoyatis et al. (2013).

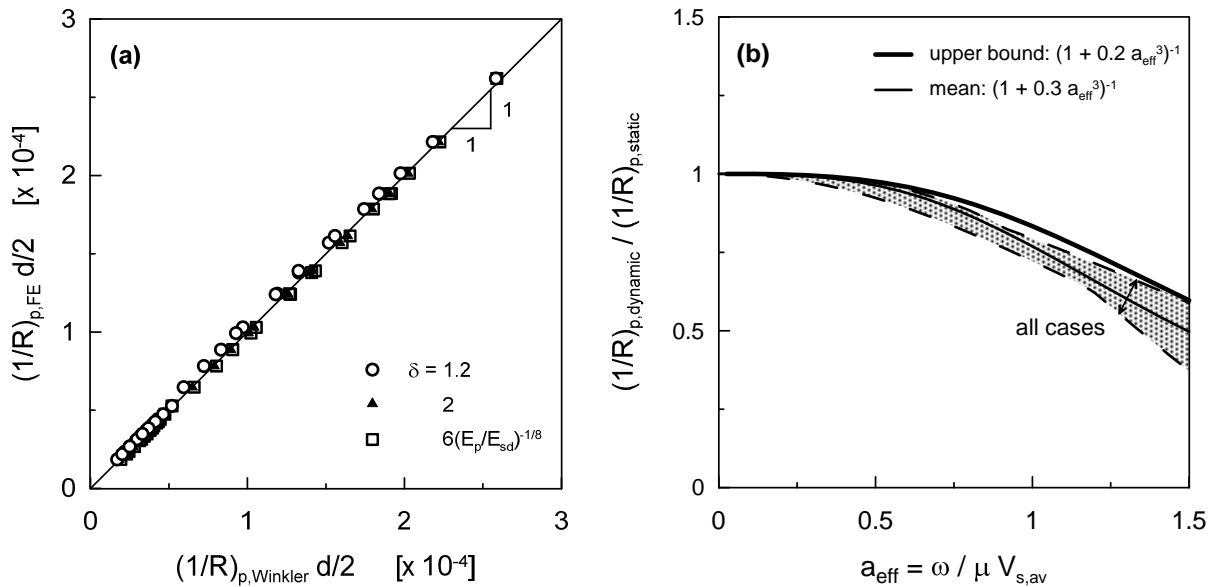


Figure 6. (a) Effect of spring stiffness coefficient  $\delta$  on pile-head curvature (b) Dynamic reduction of pile-head curvature as function of the dimensionless frequency parameter  $a_{eff}$ .

## DYNAMIC BEHAVIOR

Towards an identification of the key parameters involved in kinematic pile-head bending under dynamic action, an average shear wave velocity  $V_{s,av}$  is defined as:

$$V_{s,av} = \frac{z_{eff}}{\int_0^{z_{eff}} \frac{dz}{V_s(z)}} \quad (22)$$

providing equal travel times between a homogeneous soil with  $V_s=V_{s,av}$  and an inhomogeneous soil. Upon introducing Eq. 1, the above expression may be rewritten as:

$$V_{s,av} = \frac{z_{eff}(a-1)(n-2)}{2d \left\{ -a^{1-n/2} + \left[ a + (1-a) \frac{z_{eff}}{d} \right]^{1-n/2} \right\}} \quad (23)$$

In this manner, a new normalization scheme for dynamic pile bending is proposed by means of the dimensionless frequency parameter  $a_{eff}$ :

$$a_{eff} = \frac{\omega}{\mu V_{s,av}} \quad (24)$$

$(1/R)_{dyn.}/(1/R)_{st}$  ratios for the complete set of inhomogeneous soils under investigation are plotted in Fig. 6b against  $a_{eff}$ . The evident similarity of the curves indicates that kinematic pile-head moments in inhomogeneous soil are essentially governed by the single dimensionless frequency parameter given in Eq. 24. Approximate mean and upper-bound curves are also suggested as a useful manner to determine dynamic effects in kinematic pile-head bending.

## APPLICATION EXAMPLE

The case of a fixed-head solid cylindrical concrete pile embedded in normally-consolidated clay is employed as an application example of the proposed analysis. The pile has diameter  $d = 0.8$  m and Young's modulus  $E_p = 30$  GPa. The evaluation of kinematic demand is performed under the conservative assumption of low frequency excitation. Soil shear modulus varies linearly with depth according to the law  $G(z) = 500 + 1500z$ , where  $G(z)$  is expressed in kPa and  $z$  in meters, corresponding to  $a = 0.6$  and  $n = 1$  in Eq. 1. Shear modulus  $G_{sd}$  at a depth of one pile diameter is, therefore, equal to 1700 kPa. Poisson coefficient and mass density were set at 0.5 and 1.8 Mg/m<sup>3</sup>, respectively, corresponding to undrained conditions. The design acceleration at soil surface is  $a_s = 0.3g$ ;  $g$  being the acceleration of gravity.

Under the reasonable assumption  $\delta = 2$ , the wavenumber  $\lambda_d$  is obtained from Eq. 15 as:

$$\lambda_d = \left[ \frac{2 \cdot 1700 \cdot 3}{4 \cdot 30000000 \cdot \pi \cdot 0.8^4 / 64} \right]^{1/4} = 0.255 \text{ m}^{-1} \quad (27)$$

The characteristic pile wavenumber  $\mu$  may then be calculated from Eq. 14, by taking  $L_a = 10d$ , which is sufficiently accurate as a first approximation:

$$\mu = \frac{4 \cdot 0.255}{0.8^4 (10 \cdot 0.8) 5 (0.3-1)} \left[ (0.3 \cdot 0.8)^{\frac{4+1}{4}} - (0.3 \cdot 0.8 + 10 \cdot 0.8 - 0.3 \cdot 10 \cdot 0.8)^{\frac{4+1}{4}} \right] = 0.343 \text{ m}^{-1} \quad (28)$$

resulting in an effective depth  $z_{eff} = L_a/2 = 1.25/\mu = 3.642$  m. Soil shear modulus may be calculated as:

$$G(z_{eff}) = 500 + 1500 \cdot 3.642 = 5963 \text{ kPa} \quad (29)$$

Accordingly, the kinematic bending moment  $M_{kin}$  at the pile head is:

$$M_{kin} = E_p I_p \frac{a_s}{G(z_{eff})} = 30000000 \cdot (\pi \cdot 0.8^4 / 64) \cdot \frac{0.3 \cdot 9.81}{5963} = 536 \text{ kNm} \quad (30)$$

Note that if a homogeneous soil of equal shear modulus with the inhomogeneous soil at surface had been adopted, the corresponding value of kinematic bending moment would have been equal to 6391 kNm resulting evidently in a conservative design of pile reinforcement (10 times larger).

## CONCLUSIONS

Kinematic pile-head bending in a continuously inhomogeneous soil with shear modulus described by a generalized parabolic function was explored in both static and dynamic regime. The main findings of the investigation can be summarized in the following points:

- A simple expression for the active pile length (i.e., the length beyond which a pile behaves as an infinitely long element) in inhomogeneous soil (Eq. 17) was derived based on an average pile wavenumber;
- The notion of an effective soil curvature has been introduced (Eq. 8) as an average index of soil shear deformations over an effective depth which was found to be one half of the active pile length. The above response parameter leads to excellent predictions of kinematic pile-head moments for long piles in both static and dynamic regime, thus allowing a direct implementation in pile design practice;
- Kinematic head bending of short piles may be computed by means of a simple approximate expression (Eq. 19);
- A new normalization scheme for dynamic pile bending is proposed by means of a single dimensionless frequency parameter governing kinematic pile-head bending moments. Approximate mean and upper-bound curves for dynamic-to-static pile-head curvature ratios are suggested that can be used for pile analysis or design;
- With reference to Winkler modelling issues, kinematic bending shows a minor dependence on the stiffness of the Winkler springs. A value of the dimensionless parameter  $\delta$  close to 2 may be suitable when pile-head curvature is of the main concern.

The importance of soil stiffness profile in assessing kinematic pile head bending is proven by a application example of the proposed method.

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