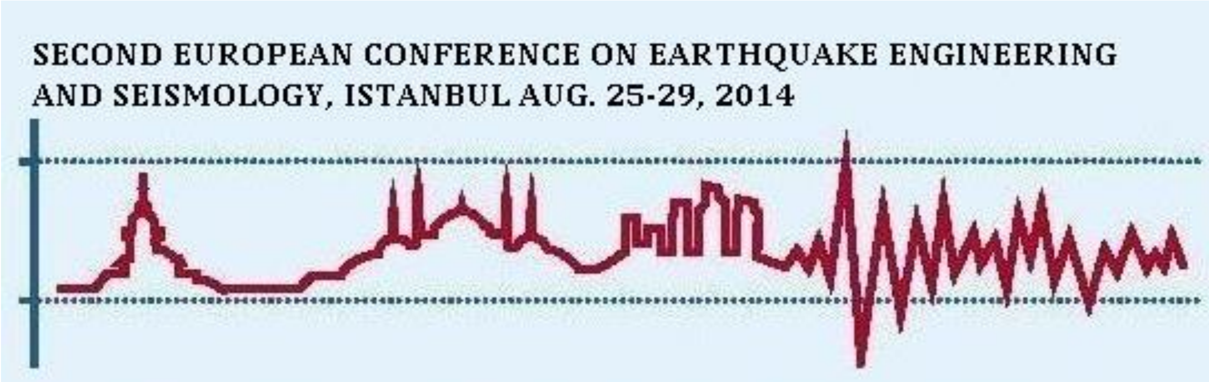


FLAT-BOTTOM SILOS FILLED WITH GRAIN-LIKE MATERIAL: REFINEMENTS OF THE SILVESTRI ET AL. (2012) THEORY

Luca PIERACCINI[1], Stefano SILVESTRI[2], Tomaso TROMBETTI[3]

ABSTRACT

Seismic behavior of squat flat-bottom silos containing grain-like material still presents strong uncertainties and current design codes tend to provide too conservative formulations.

Over the years, many researchers focused on the overall dynamic behavior of such silos mainly through numerical investigations. Only recently, Silvestri et al. (2012) obtained the analytical expression of the pressures exerted by the ensiled grain on the silo walls in accelerated conditions, by means of plain dynamic equilibrium considerations.

In the present paper, refinements to the original theory proposed by Silvestri et al. (2012) are presented. In detail, the static and the dynamic actions on the silo walls (as effect of the ensiled material) are idealised in a more consistent way, as far as the distribution of the vertical normal pressure is concerned. A direct comparison with the consolidated Janssen and Koenen (1895) theory for design of silos is also performed in order to check the theoretical model in static conditions.

Once again, the findings confirm that, in case of squat silos, i.e. characterized by low, but usual height/diameter slenderness ratios, the portion of ensiled material that interacts with the silo walls turns out to be noticeable smaller than the total mass of the grain and the *effective mass* suggested by EC8 for seismic design.

INTRODUCTION

Seismic behavior of squat flat-bottom silos containing grain-like material still presents strong uncertainties and current design codes tend to provide too conservative formulations. The assessment of the dynamic response of such structures with ensiled granular materials is a challenging task from a rigorous theoretical point of view, especially for what regards shear and bending moment on the walls since common Finite Element approaches cannot be used.

Recently, Silvestri et al. (2012) proposed a first attempt for a complete theoretical formulation on the seismic behavior of squat silos containing grain-like material. This theory is grounded on the conservative assumption on the frictional forces (fully exploited) of Janssen and Koenen (1895) and on other basic assumptions of Eurocode 8 (EN 1998-4 2006), except for the one regarding the horizontal shear forces among consecutive grains. The *effective mass* and the stresses distributions on the external walls are calculated by means of plain dynamic equilibrium equations. Although the simplifications introduced and the rough slant of the original theory, an in-depth comparison between

[1] PhD., Department DICAM, University of Bologna, Italy, luca.pieraccini5@unibo.it

[2] Assistant Professor, Department DICAM, University of Bologna, Italy, stefano.silvestri@unibo.it

[3] Associated Professor, Department DICAM, University of Bologna, Italy, tomaso.trombetti@unibo.it

the proposed analytical formulation and the experimental evidences provided by shaking-table tests on in-scale specimen showed good agreement, even if some mathematical and geometrical limits of validity of the theory were not satisfied (Foti et al., 2013).

This has strongly encouraged further improvements on the theoretical and analytical formulation in order to delve deeper into the issue. In fact, the main intent of the theory is to provide a efficient, but still conservative formulation of the present assessment for the widest as possible area of applicability. Therefore, the introduced refinements are aimed at improving the theoretical framework increasing the accuracy in terms of bending moment predictions and at extending the application range for what regards mathematical and geometrical restrictions.

In this paper, a general overview on the theoretical frameworks and analytical formulations at the basis of the static and seismic silo behavior is presented. In order to better contextualize the actual research work into the state of the art, a brief summary of the Janssen and Koenen (1895) theory for the evaluation of the gravity load effects on the silo walls and of the original formulation proposed by Silvestri et al. (2012) is provided. In accordance with the purpose of increasing the overall accuracy of the base shear and bending moment predictions, refinements to the analytical expression of Silvestri et al. (2012) are then provided. By means of a rigorous analytical developments and following the same logic organization of the previous research work, an integral mathematical formulation for pressures distributions, shear and bending moment on the silo walls is obtained. A numerical comparison of the new proposed formulation with the Janssen and Koenen (1895) theory and the original theory of Silvestri et al. (2012) is performed in static and in accelerated conditions. Finally, regarding with base bending moment, a comparison between the proposed analytical formulations (original and refined) and Eurocode 8 (EN 1998-4 2006) provisions is performed.

EUROCODE 8 PROVISIONS

The seismic design of silos containing granular materials is usually performed on the basis of the identification of an *effective mass* that interacts with the silo walls under seismic excitation, i.e. which pushes on the silo walls. Among the current design codes, Eurocode 8 Part 4 (EN 1998-4 2006) provides two methods: (i) simplified method, as given in point (4) of §3.3, and (ii) a more accurate one, as given in point (5)-(12) of §3.3, here reported by Eqs. (1) – (4).

For common values, *effective mass* results in the range $75\% \div 95\%$ of the total ensiled content and base shear and base bending moment in flat-bottom circular silos depends only on the slenderness ratio (height/diameter) of the circular silo.

$$T_{EC8,simplified} = a_{eh0} \cdot \gamma \cdot \pi \cdot R^2 \cdot H \cdot 0.8 \tag{1}$$

$$T_{EC8} = a_{eh0} \cdot \gamma \cdot \pi \cdot R^2 \cdot H \cdot \left(1 - \frac{R}{6H}\right) \tag{2}$$

$$M_{EC8,simplified} = a_{eh0} \cdot \gamma \cdot \pi \cdot R^2 \cdot H \cdot 0.8 \cdot \frac{H}{2} \tag{3}$$

$$M_{EC8} = a_{eh0} \cdot \gamma \cdot \pi \cdot \frac{R^2}{2} \cdot \left(H^2 - \frac{R^2}{27}\right) \tag{4}$$

STATE OF THE ART: THEORY BY JANSSEN & KOENEN (1895) IN STATIC CONDITIONS

A first idealized model of actual distribution of the vertical and horizontal pressures on the walls of silos containing grain-like materials was originally proposed by Janssen and Koenen (1895). With the purpose of evaluating the effective mass of grain that leans against the walls and providing conservative design indications for the static case, the vertical pressures $p_{v,GG}(z)$ at the base of a grain portion, at a generic height *z*, are assumed to be equally distributed over the whole cross-section surface (Fig. 1). In particular, this model leads to a conservative evaluation of the forces on the walls in that the frictional vertical stresses along the grain-wall contact surface are fully exploited, whilst the actual frictional stresses are likely to be lower. Eq. (5) provides the proposed analytical closed-form of the normal horizontal pressure $p_{h,GG}(z)$ that insists on the silo walls:

$$p_{h,GG}(z) = \frac{\gamma \cdot R}{2\mu_{GW}}\left[1 - e^{-2\frac{z \cdot \mu_{GW} \cdot \lambda}{R}}\right] \tag{5}$$

where λ and μ_{GW} represent the pressure ratio between horizontal and the vertical pressures and the grain-wall friction coefficient, respectively; γ expresses the specific weight of the ensiled material; *R* is the radius of the silo and *z* is the height from the top free-surface of the ensiled content of a generic grain layer.

Many experimental evidences, among which Tatko and Kobielak (2008), validated such expression.

On the other hand, the application of the model proposed by Janssen and Koenen results not suitable in dynamic and seismic conditions. Since the hypothesis of axial-symmetry of the system decays under accelerated conditions, the analytical resolution of the assessment results more complex and requires strong basic assumptions on the pressures distributions.

STATE OF THE ART: THEORY BY SILVESTRI ET AL. (2012) IN ACCELERATED CONDITIONS

Regarding with the dynamic response of such particular structures, an analytical investigation concerning with the effective behavior of grain flat-bottom silos during earthquake was recently proposed by Silvestri et al. (2012). An integral evaluation of the global forces that the grain produces and exchange with the silo was carried out by considering the grain-like material as a set of overlapped layers of infinitesimal height dz (continuous approach).

A physical idealization was consistently developed with the one identified by Janssen and Koenen in the static case (the grain-wall friction along the vertical direction is supposed to be fully exploited) and accounted for the pressure variations on the walls in the seismic case. On the other hand, unlike supposed by Janssen and Koenen, the proposed idealized model considered each grain layer as subdivided into two “equivalent” portions composed of (i) grain completely leaning against the layers below (central portion, disk D) and (ii) grain completely sustained by the walls through friction (external torus, element E). Therefore, only disk D presented equally distributed vertical pressures $p_{v,GG}(z)$, as also assumed by the Janssen and Koenen idealized model.

A new physically-based evaluation of the *effective mass* of the grain that horizontally pushes on the silo walls under earthquakes was obtained starting from the same basic assumptions of Eurocode 8 (EN 1998-4 2006), except for the one regarding the horizontal shear forces among consecutive grains.

With the purpose of providing a solid and conservative formulation of the seismic behavior of such structures, the most pejorative conditions were considered. Thus, due to the many theoretical uncertainties, a cautionary formulation of the grain-silo interactions necessarily needed to account the envelope of the pressures distributions referring to different limit conditions. Consequentially, the portion of grain volume which leans against the walls, as identified by the thickness $s(z,\vartheta)$ from the silo walls of the external torus in accelerated conditions, depending on the horizontal and vertical equilibrium equations, resulted the greatest possible.

In addition, as cautionary assumption time constant vertical and horizontal accelerations simulate the earthquake ground motion investing the silo were supposed to reach simultaneously their peak values.

Analytical developments were carried out by means of simple plain dynamic equilibrium equations and, consecutively, as derivative results, pressure distributions on the walls in accelerated conditions were achieved (static distributions resulted accordingly).

$$p_{h,GW}(z,\vartheta) = \frac{\lambda \cdot \gamma \cdot z}{\nu_0 \cdot (1 - \nu_0 \cdot \mu_{GW} \cdot a_{eh0} \cdot \cos\vartheta)} \tag{6}$$

$$\tau_{h,GW}(z,\vartheta) = \mu_{GW} \cdot a_{eh0} \cdot \sin\vartheta \cdot \frac{\lambda \cdot \gamma \cdot z}{1 - \nu_0 \cdot \mu_{GW} \cdot a_{eh0} \cdot \cos\vartheta} \tag{7}$$

$$p_{h0}(z) = \frac{\lambda \cdot \gamma \cdot z}{\nu_0} \tag{8}$$

where H means the height of the ensiled material, a_{eh0} represents the horizontal constant acceleration, $\nu_0 = 1/(1 + a_{ev0})$ expresses the vertical acceleration factor and ϑ is the latitude with respect to the earthquake direction.

Consequentially, the base shear was given by the integral, on the lateral surface of the walls, of the projection of additional pressures $\Delta p_{h,GW}(z,\vartheta)$ and horizontal tangential stresses $\tau_{h,GW}(z,\vartheta)$ towards x (namely, the direction of the horizontal acceleration). On the other hand, the analytical expression of the bending moment accounted the height from the silo foundation into the integral.

$$T = a_{eh0} \cdot \gamma \cdot \pi R H^2 \cdot \left(\frac{\lambda \cdot \mu_{GW}}{\sqrt{1 - \nu_0^2 \cdot \mu_{GW}^2 \cdot a_{eh0}^2}} \right) \tag{9}$$

$$M = \frac{1}{3} a_{eh0} \cdot \gamma \cdot \pi R H^3 \cdot \left(\frac{\lambda \cdot \mu_{GW}}{\sqrt{1 - \nu_0^2 \cdot \mu_{GW}^2 \cdot a_{eh0}^2}} \right) \tag{10}$$

Such analytical expressions showed good agreement with the experimental outputs provided by shaking-table tests on in-scale specimen performed at the EQUALS (University of Bristol, UK), even if when some mathematical limits of validity of the original theory were not satisfied.

This has encouraged a complete revision and refinement of the theoretical framework in order to delve deeper into the issue, leading to a new analytical development of the original theory as proposed by Silvestri et al. (2012).

As first result of this current refinement work, the contribution of the frictional vertical stresses on the bending moment, which was neglected in the original theory, is here taken into account in order to increase the accuracy of the base bending moment estimation. The expression of the base bending moment provided by Eq. (10) is completed by adding the contribution related to the frictional vertical stresses:

$$M_{completed} = a_{eh0} \cdot \gamma \cdot \pi R H^2 \cdot \left(\frac{\lambda \cdot \mu_{GW}}{\sqrt{1 - \nu_0^2 \cdot \mu_{GW}^2 \cdot a_{eh0}^2}} \right) \cdot \left(\frac{H}{3} + \mu_{GW} \cdot \frac{R}{2} \right) \tag{11}$$

NEW ASSUMPTIONS OF THE REFINED THEORY

Based on precautionary and conservative assumptions, a relatively comfortable formulation for predicting seismic behavior of grain flat-bottom silos was provided by Silvestri et al. (2012) through the simplification of the complex analytical treatise.

With the purpose of providing a less conservative and more physically consistent formulation with respect to the one originally proposed, the same idealized physical model presented in the previous research work by Silvestri et al. (2012) is considered and the system of hypothesis at base is refined. Some of the original assumptions are kept in order to simplify complex aspects of the present assessment, whilst other ones are removed.

In these terms, the assumption of a fully exploitation of the grain-wall friction coefficient ($\tau_{v,GW}(z,\vartheta) = \mu_{GW} \cdot p_{h,GW}(z,\vartheta)$) overall the vertical lateral surface of the grain is kept, as assumed by Janssen and Koenen (1895) and Silvestri et al. (2012) as well.

On the other hand, the role played by the vertical normal pressures $p_{v,GG}(z)$ is revolutionized, leading to a more consistent evaluation of the physical interaction between disk *D* and element *E*.

In order to simplify the analytical developments assumption on the shape of $C_{A_D}(z)$, i.e. the perimeter of the disk D on the horizontal plain, is considered in the current research work. In fact, it is assumed that $C_{A_D}(z)$ keeps enough regular and close to a circumference in accelerated condition.

Finally, the cautionary assumption regarding to the simultaneous peak values reached by time constant vertical and horizontal accelerations is kept.

At the light of the present idealization of the mechanisms inside the ensiled content, pressures distributions on the silo walls in accelerated condition derives from a unique, consistent, integral evaluation of the plain dynamic equilibrium equations of disk D and element E.

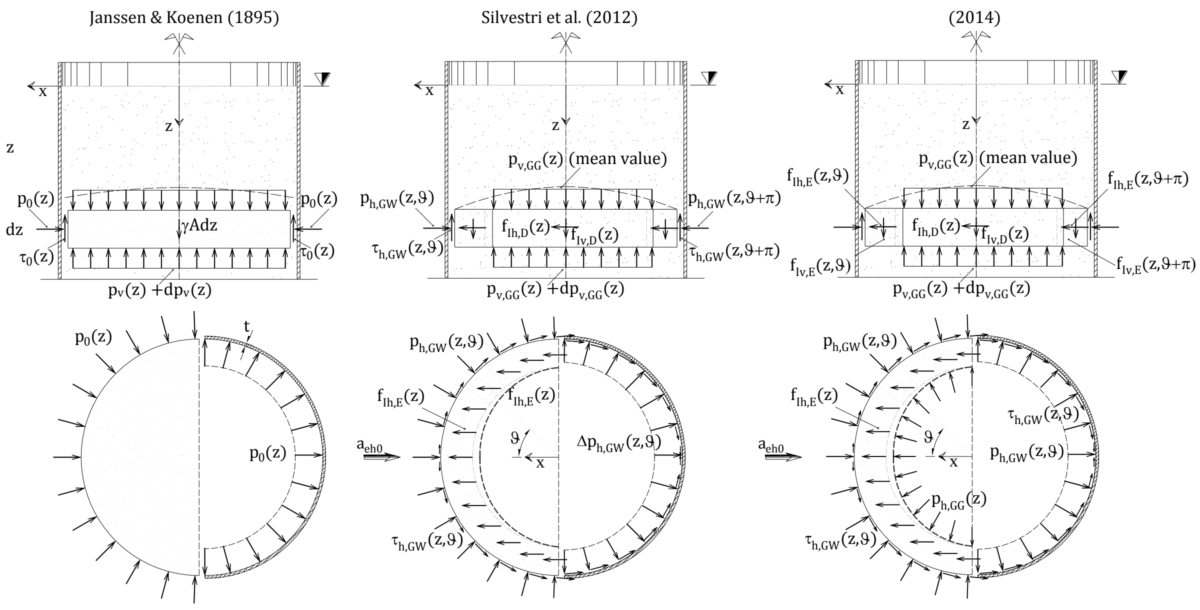


Figure 1. Physical idealized models: (a) Janssen and Koenen (1895) theory for static conditions, (b) the Silvestri et al. (2012) theory and (c) the refined theory (2014) for accelerated conditions.

NEW ANALYTICAL DEVELOPMENTS

The analytical study of the seismic behavior of silos containing grain-like material can be now developed for the refined theoretical framework. As performed in the original theory, the equilibrium in accelerated condition accounts for the additional dynamic effects generated by the two constant acceleration components a_{eh0} and a_{ev0}, respectively along the horizontal and the vertical directions. Then, through simple dynamic equilibrium equations, pressures distributions exchanged between silo and grain are analytically defined, then as derivative results, base shear and base bending moment are calculated by means of opportune integrations.

The details will be given in an upcoming paper presenting an overall analytical presentation of the current assessment.

Eqs. (12) – (14) report the thickness of the external torus of the sustained material, the normal pressures, the horizontal frictional stresses on the silo walls in accelerated conditions respectively.

$$s(z,\vartheta)=\frac{\omega(z)+\beta_0(\vartheta)\cdot R-\sqrt{\omega(z)^2+\beta_0(\vartheta)^2\cdot R^2}}{\beta_0(\vartheta)} \tag{12}$$

$$p_{h,GW}(z,\vartheta)=\frac{\lambda\cdot\gamma\cdot z}{\nu_0\cdot\left(1-\nu_0\cdot\mu_{GW}\cdot a_{eh0}\cdot\cos\vartheta\right)}\cdot\frac{R-s(z,\vartheta)}{R} \tag{13}$$

$$\tau_{h,GW}(z,\vartheta)=a_{eh0}\cdot\sin\vartheta\cdot\gamma\cdot\left[1-\frac{s(z,\vartheta)}{2R}\right]\cdot s(z,\vartheta) \tag{14}$$

where $\omega(z)=\mu_{GW}\cdot\lambda\cdot z$ and $\beta_0(\vartheta)=1-\nu_0\cdot\mu_{GW}\cdot a_{eh0}\cdot\cos\vartheta$ (here named "static function" and "dynamic function" respectively).

Considering $a_{eh0}=0$, Eqs. (8) and (15) express the normal pressures in absence of horizontal acceleration, i.e. in static condition.

$$p_{h,GW,st}(z)=\frac{\lambda\cdot\gamma\cdot z}{\nu_0}\cdot\frac{R-s_{st}(z)}{R} \tag{15}$$

In analogy with what done on Eq. (9), the refined shear action $T_{xx}(z)$ (namely, along the direction of the horizontal acceleration) is given by Eq. (16):

$$T_{xx}(z)=\frac{1}{2}\gamma\cdot a_{eh0}\cdot\int_0^z\left(\int_0^{2\pi}\left[2R\cdot s(z,\vartheta)-s(z,\vartheta)^2\right]\cdot d\vartheta\right)\cdot dz \tag{16}$$

With respect to the original formulation, the refined global bending moment $M_{yy}(z)$ (namely, along the direction perpendicular to the earthquake) is composed by two different contributions: $M_{yy,1}(z)$ and $M_{yy,2}(z)$, accounting the frictional vertical stresses contribution. Obviously, the former accounts for the distributed inertial load $q_{xx}(z)=\gamma\cdot a_{eh0}\cdot A_{E,din}(z)$ contribution and derives from the integration of the internal shear action $T_{xx}(z)$, whilst the latter derives from the integration, on the lateral surface of the walls, of the frictional vertical stresses $\tau_{v,GW}(z,\vartheta)$ multiplied by the correspondent lever arm with respect to the direction perpendicular to the earthquake ground motion (namely along the *y*-axis).

The total bending moment in Eq. (17) simply results from the sum of the two contributions.

$$\begin{aligned}M_{yy}(z)=&\frac{1}{2}\gamma\cdot a_{eh0}\cdot\int_0^z\left\{\int_0^z\left[\int_0^{2\pi}\left(2R\cdot s(z,\vartheta)-s(z,\vartheta)^2\right)\cdot d\vartheta\right]\cdot dz\right\}\cdot dz+\\&+\int_0^z\int_0^{2\pi}\tau_{v,GW}(z,\vartheta)\cdot R^2\cdot\cos\vartheta\cdot d\vartheta\cdot dz\end{aligned} \tag{17}$$

HORIZONTAL PRESSURES BETWEEN THE GRAIN AND THE SILO WALLS IN STATIC AND ACCELERATED CONDITIONS

At the light of the present findings, a direct comparison of the static pressures ($a_{eh0}=0$ and $a_{ev0}=0$) exchanged by grain and silo walls, as provided by the consolidated theory of Janssen and Koenen (1895), by the original theory of Silvestri et al. (2012) and the here proposed refined formulation can be performed. As an illustrative example, the following dimensions and characteristics have been considered for the silo: $R=10m$, $H=20m$, $\gamma=9000N/m^3$, $\mu_{GW}=0.40$, $\lambda=0.50$. Fig. 3 reports the distributions of the static pressures $p_{h,GG}(z)$, $p_{h0}(z)$ and $p_{h,GW,st}(z)$ over the height of the silo, from the free surface of the grain up to the bottom of the silo.

It can be noted that the refined formulation of the static pressures results noticeable lower (around -30% on the bottom) than the one provided by Silvestri et al. (2012). Thus, the refined analytical expression accurately fits the exponential formulation by Janssen and Koenen (1895), both from a qualitative and a quantitative point of view.

In order to visualize the results given by Eqs. (6) and (13), the pressures distributions against the silo walls in accelerated conditions are evaluated for the same silo by considering the acceleration components $a_{eh0}=0.40$, $a_{ev0}=0.15$.

Fig. 4 plots the pressures distributions $p_{h,GW}(z,\vartheta)$ on the front side of the silo with respect to the versus of the horizontal acceleration. Also in this case, the distributions show a different trend, and the refined formulation provides a lower magnitude of the normal pressures with respect to the original one.

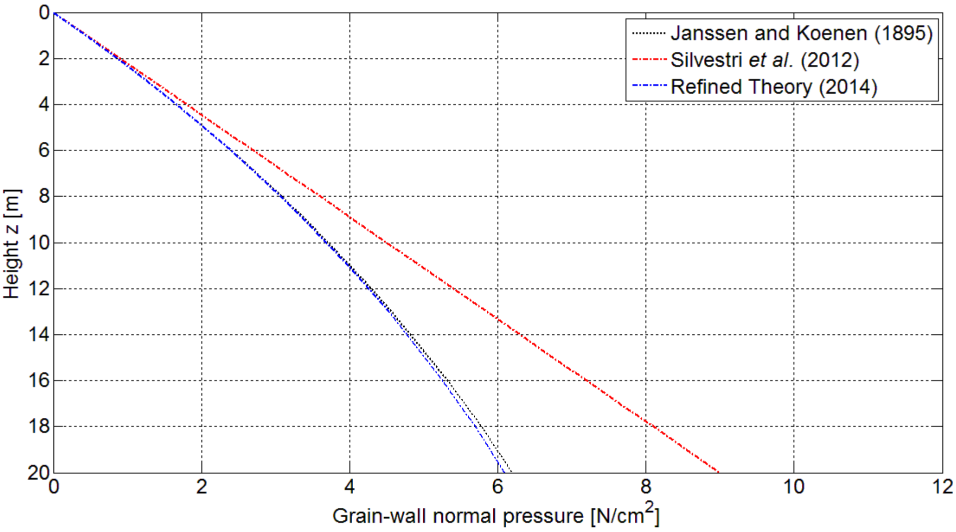


Figure 3. Plots of the grain-wall normal pressures for Janssen and Koenen (1985) theory, Silvestri et al. (2012) theory and the refined one (2014) in static conditions

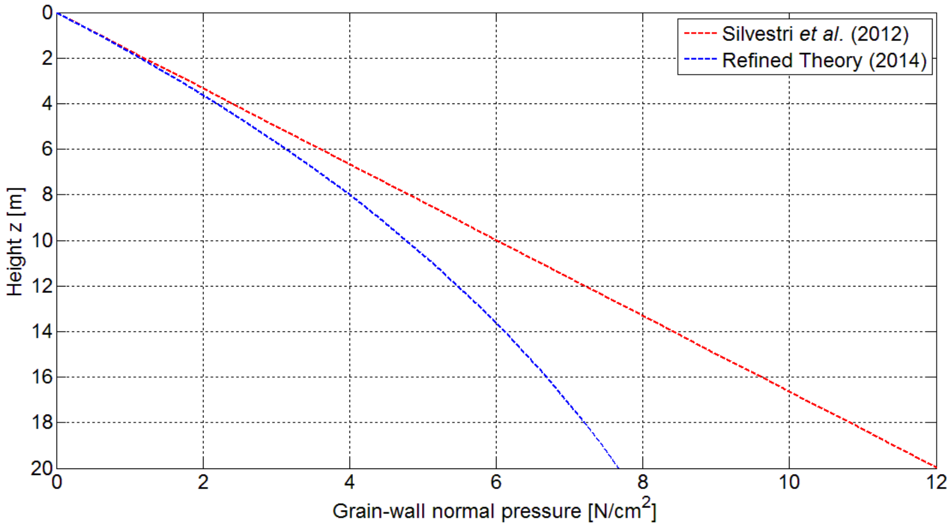


Figure 4. Plots of the grain-wall normal pressures for Silvestri et al. (2012) theory and the refined theory (2014) in accelerated conditions at the front side ($\vartheta = 0$) of the silo.

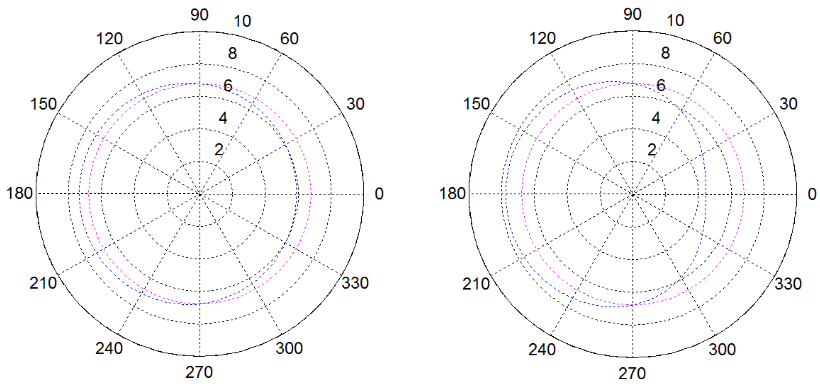


Figure 5. Comparison between static and dynamic thickness at the bottom of the silo for different magnitude of the horizontal acceleration 0.80 (left side) and 1.60 (right side) with constant vertical acceleration 0.15.

VOLUMES OF THE GRAIN PORTIONS IN ACCELERATED CONDITIONS

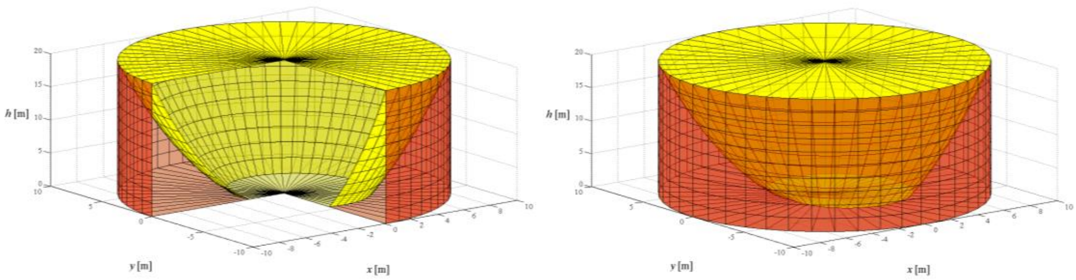
Equation (12) provides the thickness $s(z,\vartheta)$ of the portion of grain that actually pushes on the silo walls in accelerated conditions. Therefore, two volume arise inside the whole granular content, characterized by different dynamic behavior: $V_{E,din}(z)$ and $V_{D,din}(z)$. The former individuates the amount of grain that is completely sustained by the lateral silo walls, whilst the latter is the amount of grain leaning against the lower portion of the material up to the silo foundation without interacting with the silo walls. As presented by Silvestri et al. (2012), from a geometrical point of view, $V_{E,din}(z)$ and $V_{D,din}(z)$ can be respectively visualized as a vertical-axis cylindrical annulus with thickness $s(z,\vartheta)$ and a vertical-axis truncated cone solid of radius $r(z,\vartheta) = R - s(z,\vartheta)$.

In the new refined formulation, from a mathematical point of view, the volumes occupied by the disk *D* and element *E* are expressed as follows:

$$V_{D,din}(z) = \pi R^2 H - \frac{1}{2} \cdot \int_0^z \left(\int_0^{2\pi} \left[2R \cdot s(z,\vartheta) - s(z,\vartheta)^2 \right] \cdot d\vartheta \right) \cdot dz \tag{18}$$

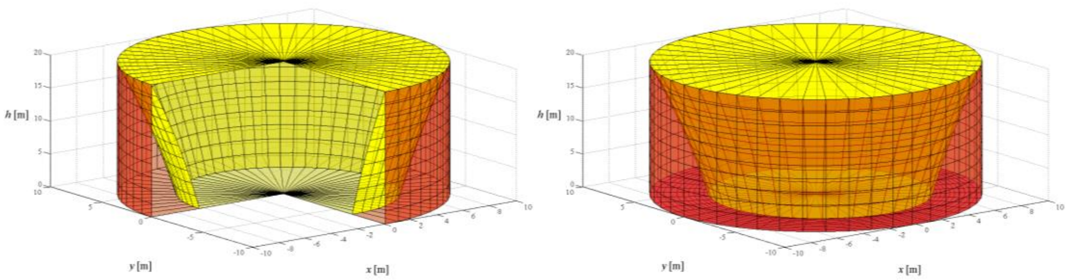
$$V_{E,din}(z)=\frac{1}{2}\cdot\int_{0}^{z}\left(\int_{0}^{2\pi}\left[2R\cdot s(z,\vartheta)-s(z,\vartheta)^{2}\right]\cdot d\vartheta\right)\cdot dz \qquad (19)$$

Mathematical integration of both Eqs. (18) and (19) involves many difficulties. Thus, a closed-form cannot be now provided. In any case, it has to be noted that the quantities expressed by integrals inside such equations represent positive values in order to satisfy the mass balance of the granular content inside the silo.



(a) (b)

Figure 6. Three-dimensional view of portion D (in yellow) and of portion E (in red) of the flat-bottom silo for the Silvestri et al. (2012) theory: (a) sectioned view and (b) overview



(a) (b)

Figure 7. Three-dimensional view of portion D (in yellow) and of portion E (in red) of the flat-bottom silo for the here proposed refined theory (2014): (a) sectioned view and (b) overview

SHEAR AND BENDING MOMENT IN ACCELERATED CONDITIONS

At the light of the findings, evaluation and comparison of shear and bending moment as provided by the completed original formulation represented by Eqs. (9) – (11), and by the refined theory represented by Eqs. (18) and (19), are performed through numerical integration in those cases where an analytical closed-form is not available. The incidence of the frictional vertical stresses contribution on the total bending moment is investigated as well.

A sample is here performed by choosing same physical and dynamic features as adopted previously. Figs. (8) and (9) show the original and refined shear along the height of the silo and the original and refined bending moment with and without the frictional vertical stresses contribution from the free surface of the ensiled material up to the bottom, respectively.

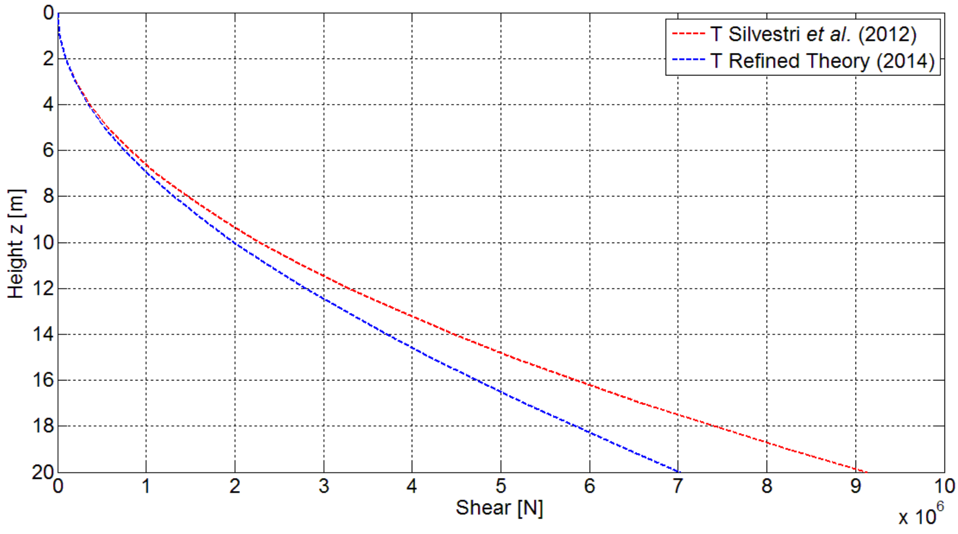


Figure 8. Plot of the shear for the Silvestri et al. (2012) theory (red dash line) and the refined theory (2014) (blue dash line) along the height *z* of the silo.

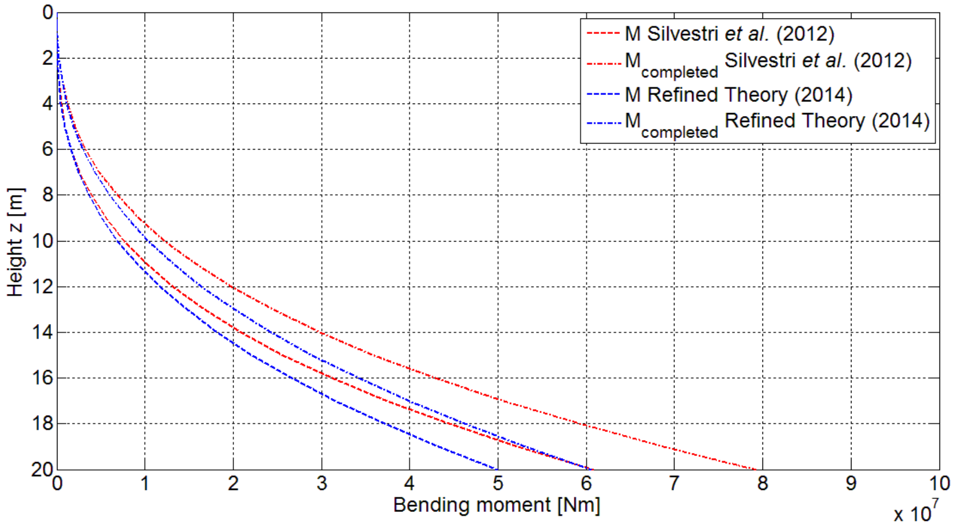


Figure 9. Plot of the bending moment accounting for the frictional vertical stresses contribution (dash-point line) and without (dash line) for the Silvestri et al. (2012) theory (red) and the refined heory (2014) (blue) along the height *z* of the silo.

Regarding to the shear, the original formulation (red color) provides higher values than the refined one (blue color) at each height z. Especially at the base of the silo, the original theory provides a 30% greater magnitude for the shear with respect to the refined one.

Regarding to the bending moment, the complete original formulation (red color) turn out to be more conservative than the refined one (blue color) along the whole height of the walls. Especially at the base of the silo, the original theory provides a 33% greater magnitude for the bending moment with respect to the refined one.

As aimed by the current research work, the refined formulation appears as a lower Upper Bound with respect to the Silvestri et al. (2012) theory in terms of both shear and bending moment predictions.

Both for the original and the refined formulations, the contribution of the frictional vertical stresses to the bending moment, here expressed as $M_{yy,2}(z)$, results not negligible with respect to

$M_{yy,1}(z)$, i.e. the one obtained considering only the pressures effects on the silo walls. Especially at the base of the silo, this bending contribution results around the 18% and the 23% of the total bending moment $M_{yy}(z)$, respectively for the refined and the original formulas.

Even if the base bending moment provided by Eq. (17) results slightly lower than to the one given by the original expression through Eq. (10), from an engineering point of view, such formulations appear to express the same value.

THE ANALYTICAL RESULTS AND THE EC8 PRESCRIPTIONS

At the light of the analytical developments provided on both the original and the refined frameworks, base shear and base bending moment on the walls of silos containing grain-like material in accelerated conditions can be predicted.

Fig. 10 displays a comparison between the original and the refined formulations of the base bending moment under increasing horizontal accelerations for the case in study.

In accordance with what underlined in Fig. 9, the completed formulations of the base bending moment provides greater values than the refined one. Thus, as aimed by the current research work, the refined theory demonstrates to be a lower Upper Bound with respect to the one proposed by Silvestri et al. (2012). One again, the contribution provided by the frictional vertical stresses to the base bending moment results not negligible both for the theories and appears to increase with higher horizontal accelerations. In particular, it can be noted that the refined expression and the original formulation of the base bending moment, as provided by Eq. (17) and Eq. (10) respectively, show the same trend within the considered acceleration range. Therefore, Fig. 10 suggests the possibility to conciliate the simplicity of the original formulation proposed by Silvestri et al. (2012), with the mathematical consistence and the physical robustness of the refined formulation. After experimental validation, the former could present the remarkable advantage to be handy and suitable for further engineering applications, whilst the latter could provide a more solid and more robust theoretical base with respect to the original analytical framework.

Finally, Fig. 10 shows that both the EC8 base bending moment provisions (simplified and accurate methods) appear noticeable greater than the proposed estimations (original and refined ones), suggesting that the activated mass results noticeably lower than the *effective mass* proposed by the Eurocode 8 prescriptions.

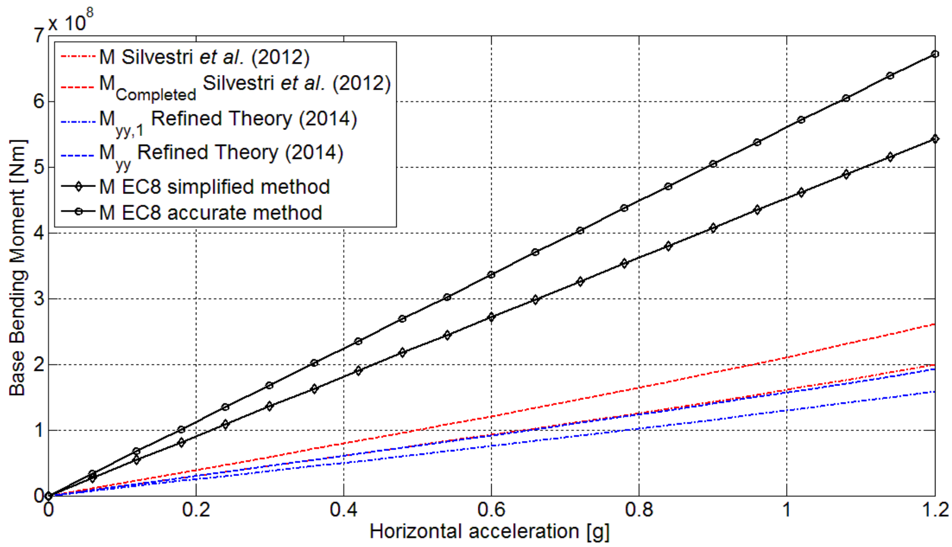


Figure 10. Analytical comparison between the base bending moment provided by: Eurocode 8 provisions (simplified and accurate methods), the Silvestri *et al.* theory (2012) (red lines) and the refined one (2014) (blue lines) with the contribution of the frictional vertical stresses (dashed lines) and without (dashed-pointed lines).

CONCLUDING REMARKS

This research work provides a relevant improvement of the overall analytical formulation proposed by Silvestri et al. (2012) for the assessment of the seismic actions on the walls of flat-bottom silos filled with grain-like material.

First, the Eurocode 8 prescriptions, the considalted theory by Janssen and Koenen (1895) for the static design of silos and the original theory by Silvestri et al. (2012) have been briefly summarised. Second, refinements to the original theory have been applied: the dynamic actions on the silo walls (as effect of the ensiled material) are idealised in a more consistent way, as far as the distribution of the vertical normal pressure is concerned. New closed-form expressions have been obtained for the grain-wall pressures, for the volume of grain that is completely sustained by the lateral silo walls, for the volume of grain that leans against the lower portion of the material up to the silo foundation, and for the shear and bending moment at the base of the silo, in accelerated conditions. Finally, a comparison between the two (the original one and the refined one) analytical predictions and the Eurocode 8 provisions is carried out.

It has been shown that both analytical formulations lead to an activated mass which is noticeably lower than the *effective mass* proposed by Eurocode 8 provisions. Also, the simplicity of the original formulation proposed by Silvestri et al. (2012) can be conciliated with the mathematical consistence and physical robustness of the refined formulation: the remarkable advantage to be handy and suitable for further engineering applications of the former can be then joined with a more solid and more robust theoretical framework as provided by the latter.

REFERENCES

EN 1998-4 (2006) Eurocode 8 “Design of structures for earthquake resistance” *Part 4 Silos, tanks and pipelines*, CEN, Brussels

Foti D, Ivorra S, Trombetti T, Silvestri S, Gasparini G, Di Chiacchio L (2013) *“ASESGRAM Assessment of the seismic behavior of flat-bottom silos containing grain-like materials”* SERIES seismic engineering research infrastructures for European synergies (http://www.series.upatras.gr/ASESGRAM)

Janssen HA (1895) “Versuche über getreidedruck in silozellen” *Zeitschr des Vereines deutscher Ingenieure*, 39(35), 1045-1049 (in German)

Koenen M (1896) “Berechnung des Seiten-und Bodendrucks in Silozellen” *Centralblatt der Bauverwaltung*, 16, 446-449 (in German)

Silvestri S, Gasparini G, Trombetti T, Foti D (2012) “On the evaluation of the horizontal forces produced by grain-like material inside silos during earthquakes” *Bulletin of Earthquake Engineering*, 10(5), 1535-1560

Tatko R, Kobielak S (2008) "Horizontal bulk material pressure in silo subjected to impulsive load" *Shock and Vibration*, 15(5): 543-550