SECOND EUROPEAN CONFERENCE ON EARTHQUAKE ENGINEERING
AND SEISMOLOGY, ISTANBUL AUG. 25-29, 2014

DETECTION OF NONLINEARITY IN VIBRATIONAL SYSTEMS USING THE SECOND TIME DERIVATIVE OF ABSOLUTE ACCELERATION

Masaki WAKUI[1] and Jun IYAMA[2] and Tsuyoshi KOYAMA[3]

ABSTRACT

This paper shows a criteria to detect nonlinearity in the restoring force using the 2nd time derivative of the absolute acceleration, which is often referred to as snap. In order to investigate the capability of snap in detecting changes in the stiffness of the system, earthquake response analysis was conducted. The analysis results show that discontinuous jump like signals, which coincide with the yielding locations, can be observed in the snap time histories. Thus one can say that in general, it is possible to detect damage due to change in stiffness through observation of the snap time history.

INTRODUCTION

Much research has been conducted in the area of structural health monitoring and damage detection, in order to develop methods which are capable of detecting aging deterioration and damage due to earthquakes as well as estimating the maintenance time.

When a steel structure experiences a strong earthquake, damage such as yielding and fracture occur in structural members. Such damage appears as degradation in stiffness and strength. Most developed methods estimate this type of damage through the dynamic transfer function of the structure, which requires either measurement of ground motion or microtremor input as well as the response of the structure. Such methods are complicated and costly, since the input and output must be synchronized. Therefore it is advantageous to be able to detect damage of structures and their components only through the output.

This paper investigates a method which is capable of detecting damage due to nonlinearity in the restoring force using the 2nd time derivative of the absolute acceleration, which is often referred to as snap. Such a method is possible when the degradation of the stiffness occurs abruptly, resulting in discontinuities in the time history of snap. The method is cost effective, since snap can be obtained numerically as the 2nd time derivative of the measured absolute acceleration, a quantity easily measured. The physical significance of snap is discussed in this paper as well as its applicability to the detection of damage and nonlinearity in the stiffness of structural members.

DETECTING CHANGES IN THE STIFFNESS THROUGH SNAP

In this paper, the time derivative of the jerk is referred to as snap.

[1]Graduate Student, The University of Tokyo, Tokyo Japan, wakui@stahl.arch.t.u-tokyo.ac.jp
[2] Assoc. Prof., The University of Tokyo, Tokyo Japan, iyama@arch.t.u-tokyo.ac.jp
[3] Asst. Prof., The University of Tokyo, Tokyo Japan, koyama@arch.t.u-tokyo.ac.jp

A single degree of freedom dynamical system with nonlinear restoring force $Q(x)$ can be represented as,

$$m\ddot{x} + c\dot{x} + Q(x) = -m\ddot{x}_0 \,. \tag{1}$$

Here, m is the structural mass and c is the damping coefficient. The terms $\ddot{x}$, $\dot{x}$, and x represent the relative acceleration, velocity, and relative displacement; $\ddot{x}_0$ is the ground acceleration. When there is no damping, $c = 0$, one obtains the following equation.

$$m(\ddot{x} + \ddot{x}_0) = -Q(x) \,. \tag{2}$$

By defining the absolute acceleration $a = \ddot{x} + \ddot{x}_0$, the equation above can be rewritten as follows,

$$a = -\frac{Q(x)}{m} \,. \tag{3}$$

A time derivative of this equation yields,

$$\dot{a} = -\frac{1}{m}\frac{\mathrm{d}Q(x)}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} \,, \tag{4}$$

where $\dot{a}$ is the jerk. By defining the tangent stiffness of the restoring force $k(x) = \mathrm{d}Q(x)/\mathrm{d}x$, Eq.(4) becomes,

$$\dot{a} = -\frac{1}{m}k(x)\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{m}k(x)\dot{x} \,. \tag{5}$$

Another time derivative of this equation yields the snap $\ddot{a}$ as,

$$\ddot{a} = -\frac{1}{m}\left\{\frac{\mathrm{d}k(x)}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}\dot{x} + k(x)\frac{\mathrm{d}\dot{x}}{\mathrm{d}t}\right\} = -\frac{1}{m}\left\{k'(x)\dot{x}^2 + k(x)\ddot{x}\right\}. \tag{6}$$

Here, $k'(x)$ is defined as the derivative of $k(x)$ with respect to the relative displacement.

In the case that a system behaves linear elastically with constant initial stiffness k_0, Eq.(6) reduces to,

$$\ddot{a} = -\frac{1}{m}k_0\ddot{x} \,. \tag{7}$$

Therefore, in a non-damaged system with constant initial stiffness, snap will harmonically oscillate proportionally to the relative acceleration. On the other hand, in a system with variable stiffness, such as an elasto-plastic system, k' can become non-zero. Eq.(6) shows that a sudden change in the stiffness can result in an instantaneous large value of snap. If this change in snap can be properly observed, it can lead to a damage detection method for systems in which the stiffness is affected by the accumulated damage. However, it is not an easy task, since the first term in Eq.(6)includes both k' and the square of the relative velocity; when the relative velocity is almost zero, the contribution of the first term is small compared to the second and difficult to detect.

When a system is vibrating linearly elastically, $Q(x)$ is less than Q_y, the yield force of the system. In such a case, the following inequality can be derived from Eq.(2),

$$m\left|\ddot{x} + \ddot{x}_0\right| \le Q_y \,. \tag{8}$$

Using the triangle inequality, the equation above can be rewritten as follows,

$$|\ddot{x}| \le \frac{Q_y}{m} + |\ddot{x}_0| . \tag{9}$$

If the predominant period of the ground acceleration is nearly identical to the natural period of the system, $\ddot{x}_0$ will be much smaller than $\ddot{x}$. In such a case, Q_y / m in Eq.(9) can be considered an approximate upper bound of $|\ddot{x}|$, and an estimate of the maximum absolute acceleration of the undamaged system in Eq.(7) can be given as,

$$\ddot{a}^*_{\max} = \frac{k_0 Q_y}{m^2} . \tag{10}$$

The numerical approximation to snap can be computed from the 2nd order backward difference of the absolute acceleration using the following formula.

$$\ddot{a}(t) \approx \frac{a(t) - 2a(t-\Delta t) + a(t-2\Delta t)}{\Delta t^2} . \tag{11}$$

It is possible to detect nonlinearity in vibrational systems by comparing the threshold value calculated from Eq.(10) with the snap time history calculated from the measured absolute acceleration time history through Eq.(11).

VALIDATION OF SNAP WITH EARTHQUAKE RESPONSE ANALYSIS

In order to investigate the actual capability of snap in detecting changes in the stiffness of the system, earthquake response analysis was conducted. The model is a single degree of freedom elasto-plastic vibrational system which has a predominant period of 1.0 seconds and a damping coefficient of 0%.

The three types of restoring force characteristics shown in Fig.1 are selected for the single degree of freedom system. The horizontal axis represents the displacement and the vertical axis force. δ_y , Q_y, and Q_u represents the yield displacement, the yield force, and the maximum force.

As shown in Fig.2, these three types differ in the way the stiffness decreases to zero after yielding, where Case.1 is discontinuous, Case.2 is linearly varying, and Case.3 is parabolically varying. In Fig.2, the horizontal axis represents the restoring force and the vertical axis the stiffness.

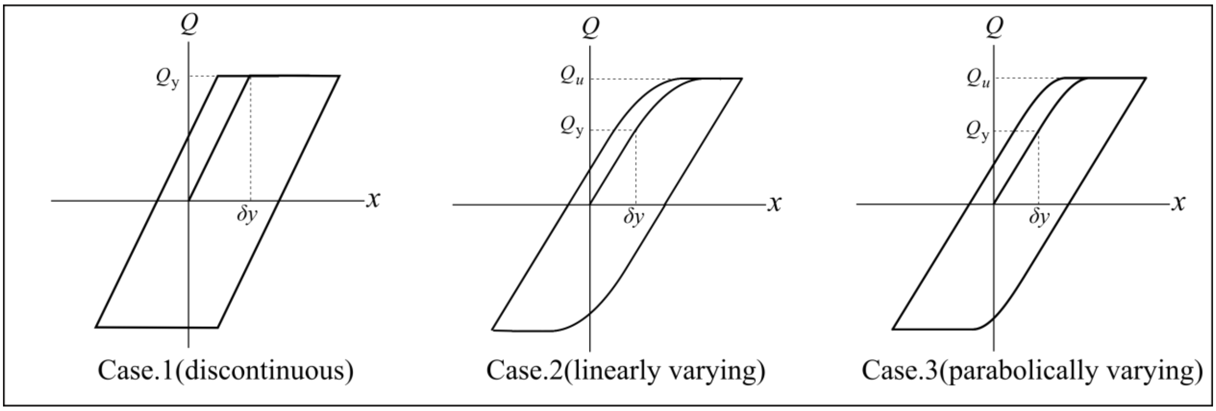


Figure 1. Restoring force characteristics

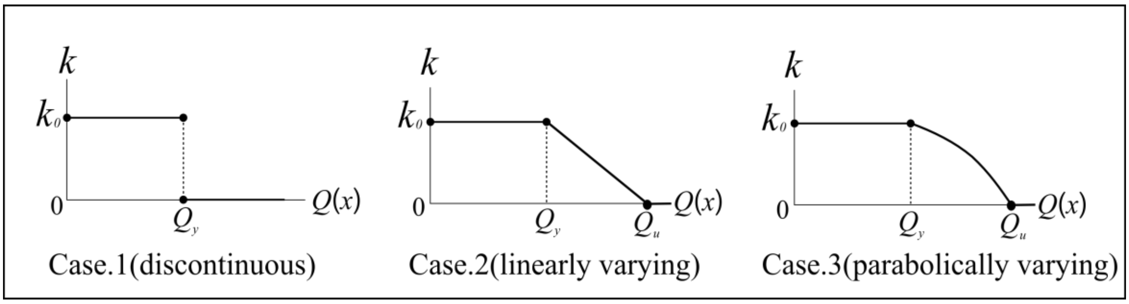


Figure 2. The relation between the restoring force and the stiffness

Case.1(discontinuous) : The stiffness of the system is initially k_0, and discontinuously drops to zero when the restoring force reaches the yield force. Thus k' will be infinite at this point and so will the value of snap.

Case.2(linearly varying) : The stiffness of the system is initially k_0, and linearly decreases after the restoring force reaches the yield force. At the maximum restoring force, the stiffness will become zero. The maximum restoring force is defined as Q_u, and is related to the yield force through the equation $Q_y = \beta Q_u$. k' can be represented as,

$$k' = -\frac{k_0^2}{2(1-\beta)Q_u}. \tag{12}$$

k' is a constant which depends on k_0, Q_u, and β. Because k' is discontinuous, the snap is also discontinuous in this restoring force model.

Case.3(parabolically varying) : The stiffness of the system is initially k_0, and parabolically decreases after the restoring force reaches the yield force. In this case, k' linearly decreases, and k'' is a constant. At the maximum restoring force, the stiffness will become zero. The maximum restoring force is defined as Q_u, and is related to the yield force through the equation $Q_y = \beta Q_u$. k'' can be represented as,

$$k'' = -\frac{8k_0^3}{9(1-\beta)^2 Q_u^2}. \tag{13}$$

k'' is a constant which depends on k_0, Q_u, and β. Since k' is continuous, the snap is also continuous in this restoring force model.

The maximum force Q_u is defined as,

$$Q_u = 0.4Q_e. \tag{14}$$

Here, Q_e is the maximum elastic force response of the system under the given ground motion. β, which determines the upper bound for the rate at which the stiffness decreases after yielding for Case.2 and Case.3, is fixed as 0.9. A value close to 1 implies that the rate is extremely large. The Newmark Beta method is used to integrate the equations of motion with a fixed time step of $\Delta t = 0.004$ seconds.

INPUT EARTHQUAKE MOTION

Since the method presented above is developed in the case where the predominant period of the ground acceleration is nearly identical to the natural period of the system, an artificial ground motion which has the desired time-frequency characteristic is generated and used. The Fourier amplitude spectrum is the given as the “Kanai-Tajimi spectrum”, where the predominant period is set to 1.0 seconds and the coefficient *hg* determining the spectral shape to 0.3. The standard deviation of the Fourier phase difference distribution is $0.125\times 2\pi$, which corresponds to the Fourier phase spectrum of an intermediate-field earthquake. The artificial earthquake motion is generated by an inverse Fourier transform of the Fourier phase and Fourier amplitude spectrum. The earthquake data is sampled at 50Hz, and the number of data points is 4096. This results in a ground motion with a duration of 81.92 seconds. Fig.3 shows the Fourier amplitude spectrum and the artificial ground motion. The earthquake data is linearly interpolated to match the time step in the earthquake response analysis.

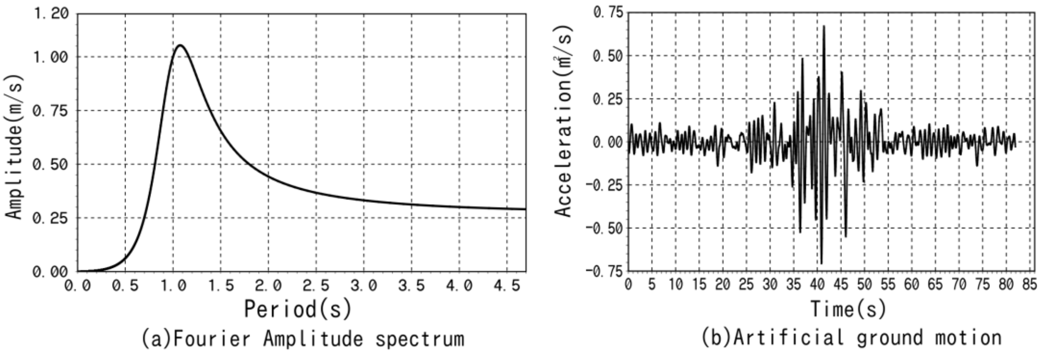


Figure 3. Artificial ground motion

NUMERICAL RESULTS

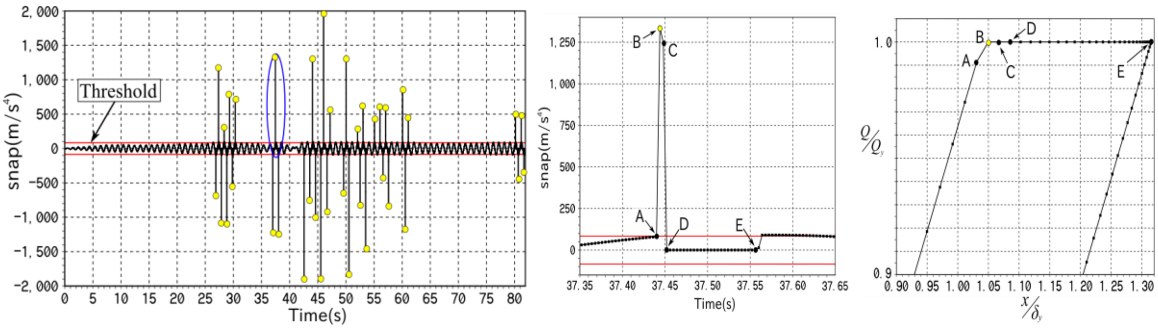
Fig.4 shows the snap time history calculated from Eq.(11) using quantities obtained from the time history response analysis of the three cases. The horizontal axis represents the time and the vertical axis the snap. Moreover, the red line shows the threshold calculated from Eq.(10), and the yellow circles denote yielding, i.e., change in the value of the stiffness. An enlarged figure of the portion of the time history enclosed by the blue ellipse is shown beside the time histories. The alphabet (A~E) shows the correspondence between the time history of snap and the load deformation curve.

Fig.4(a), which corresponds to Case.1(discontinuous), shows discontinuous jump like signals. These coincide with the yielding locations, such as point B. As previously explained, k is discontinuous at the onset of yielding, and the snap instantaneously increases or decreases. At the yield force, the stiffness and its derivative k' are zero, so the value of snap also becomes zero from Eq.(6) at point D. The value of snap will remain zero from point D until unloading at point E. At the onset of unloading, the stiffness k and relative acceleration are both nonzero, resulting in a nonzero value of snap, as can be seen from Eq.(6)

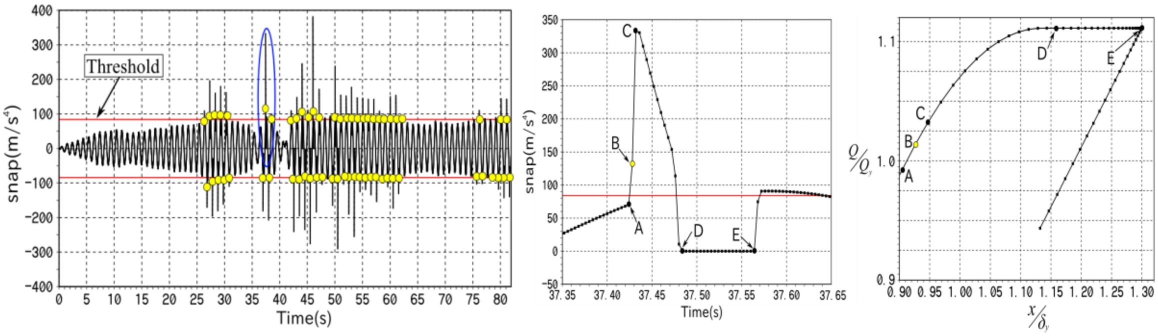
In Fig.4(b), which corresponds to Case.2(linearly varying), discontinuous jump like signals, which coincide with the yielding locations, such as the point B, can be observed in the snap time histories. At the maximum force, the stiffness and its derivative k' are zero, so the value of snap also becomes zero from Eq.(6) at point D. The value of snap will remain zero from point D until unloading at point E. This is similar to Case.1. The difference between Case.1, is the gradual decrease of snap from point C to point D. This is attributed to the fact that the decrement of the relative velocity is larger than the increment of the relative acceleration.

In Fig.4(c), which corresponds to Case.3(parabolically varying), k' is continuous, and thus so is snap. k' decreases after yielding at the point B, and the value of snap increases from point B to point C. After reaching the maximum force, the value of snap becomes zero, similar to the two previous cases. The value of snap remains zero from point D until unloading at point E.

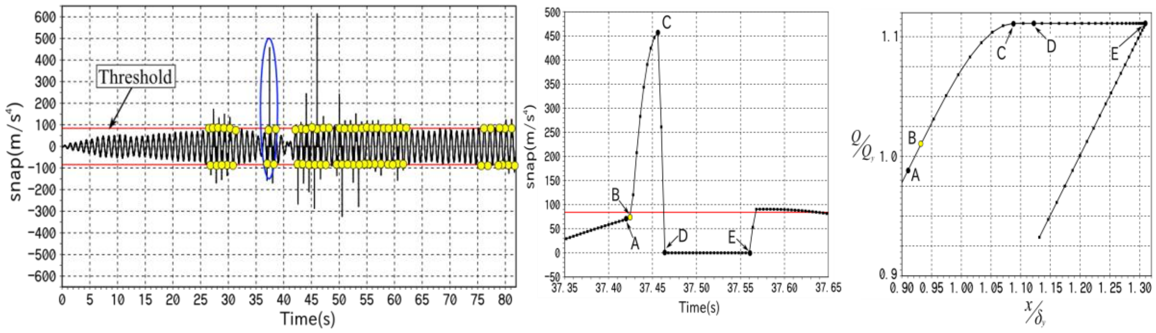
For all three cases, the value of snap exceeds the threshold value calculated from Eq.(10) whenever yielding occurs, i.e., the stiffness changes. This change in stiffness or nonlinearity in the restoring force can be detected through the snap time history not only when k and k' are discontinuous, such as Case.1 and Case.2, but also in the case where they are continuous, such as Case. 3. This claim holds true as long as the change in stiffness is rapid. When the change in stiffness as well as the relative velocity is small, the spiky behavior of snap also becomes small and the nonlinearity becomes difficult to detect.



(a)Case.1(discontinuous)



(b)Case.2(linearly varying)



(c)Case.3(parabolically varying)

Figure 4. The numerical results

CONCLUSIONS

This paper presents a method to detect damage due to change in the stiffness of structures using snap, the second time derivative of the absolute acceleration. Since the method requires measurement of only the absolute acceleration, it can be easily and cost effectively implemented.

In the case of a single degree of freedom elasto-plastic system without damping, the expression for calculating snap includes an additive term involving the product of “the derivative of the tangent stiffness” and “the square of the relative velocity”. At the onset of yielding, if the change in stiffness of the system is abrupt or nearly discontinuous, the value of snap will instantly increase and result in sharp peaks. By specifying an appropriate threshold, these peaks can be detected as the points at which yielding supposedly occur. In this paper, the proposed method is validated for single degree of freedom systems with 3 types of restoring force characteristics, which differ in the way the stiffness decreases to zero after yielding, where Case.1 is discontinuous, Case.2 is linearly varying, and Case.3

is parabolically varying. Case.1 and Case.2 show discontinuous jump like signals in the snap time history since k and k' are discontinuous at the onset of yielding. In Case.3, k and k' are continuous at the onset of yielding, and the snap time history is also continuous. For all three cases, the value of snap exceeds the threshold value whenever yielding occurs, so in the particular case presented in this paper, it is possible to detect nonlinearity in the vibrational systems. This is because the ability of snap in detecting nonlinearity does not depend solely on the discontinuity of the stiffness but on its rapid variation.

In future work, the effect of noise arising in actual measured accelerations on the proposed method will be investigated. Such noise can be severely troublesome in computing snap through numerical differentiation and a rational method must be developed to eliminate them. Another issue that must be addressed is the dependence of the ability of snap in detecting nonlinearity on the speed of variation of k with respect to displacement. In this paper β is fixed as 0.9, but the appropriate value for β in real structures is an open issue.

REFERENCES

Julien C S (1997) "Some simple chaotic jerk functions", *Am. J. Phys. 65(6), June*:537-543

Hiroshi T (1960) "A statistical method of determining the maximum response of a building structure during an earthquake", *Proceedings of the 2nd World Conference on Earthquake Engineering,* Tokyo, Japan:781-797

Yoshihiro I and Hitoshi K (2003) "Earthquake input motions in performance evaluation of steel building structures (Part2 Spectral Characteristics of Typical Recorded Earthquake Motions)", *Proc., 73th Archival Research Meeting, Chapter AIJ, 2012*:133-136 (in Japanese)