



PRACTICAL SUGGESTIONS FOR 2D FINITE ELEMENT MODELING OF SOIL-STRUCTURE INTERACTION PROBLEMS

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ABSTRACT

Finite element simulations are widely used in soil-structure interaction (SSI) problems. It is usual that the semi infinite soil medium becomes truncated to a manageable size through artificial boundaries with finite degrees of freedom. However ambiguous points still exist in different aspect of reasonable modeling of such boundaries. This ambiguity starts with free field motion analysis which normally precedes each SSI problem. Current practice suggests viscous or viscoelastic boundaries including spring and dashpot elements. Introducing input motion to the system through such boundary elements, should guarantee the correct free field motion in the medium before structural positioning. However, seismic loading from such borders, faces frequency content distortions which needs to be treated appropriately. At second stage, the boundaries should be able to properly simulate the outward propagation of waves, emanated by SSI, i.e. radiation damping. Although viscous boundary condition is capable of damping out most of the reflecting waves, due to the assumption of 1D wave propagation in setting damper coefficients, it is not able to absorb whole of the body waves with different angles of incident. This weakness becomes augmented in the case of surface waves. Here, existing limitations in seismic loading in boundaries are discussed. Then, practical solutions are suggested that can improve arrival and transmitting out of seismic waves from viscoelastic boundaries, properly.

INTRODUCTION

Finite element method is widely used in Soil Structure Interaction problems for its versatility. In this procedure only a finite domain is considered instead of unbounded media. The infinite domain can be truncated with special boundary conditions. Fixed boundary condition in static analysis can be used with appropriate distance from structure without sacrificing much in terms of accuracy. Using fixed boundary condition in dynamic analysis neglects the propagation of waves into infinity. Although considering wide soil domain with material damping can help vanishing of the reflected waves, it is computationally expensive. The radiation of waves from a finite domain can be achieved, applying a virtual artificial absorbing boundary condition. The essence of absorbing boundary is an ability to absorb the energy of scattering waves from the structure. Extensive studies on these boundaries have been made thus far. Lysmer and kuhlemeyer (1969) proposed an artificial boundary condition with viscous dampers to absorb the radiating waves. The coefficients of dampers are defined by the assumption of 1D wave propagation theory. The tangential and perpendicular dashpots were adjusted with shear and longitudinal wave velocity respectively. The coefficients are presented in equation 1 where ρ represents the unit mass. C_s and C_p are respectively shear and longitudinal wave velocities as well. A , is the total area of each element around the considered node on the boundary. Due to the

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assumption of 1D wave propagation in derivation of damping coefficients, multidimensional implementation of viscous boundary would cause large errors, especially in the case of surface waves absorption. Another drawback of models with viscous boundaries is that the entire model is statically unstable in space and can be shifted as a rigid body and produce permanent displacement at low frequencies for the case of inner source problem (Jingbo et al., 2006). Also because of absorbing half of the amplitude of input motion by viscous boundary it is suggested that for the case of external excitation the input motion must be doubled (Mejia and Dawson, 2006).

$$\begin{aligned} C_T &= \rho c_s A \\ C_N &= \rho c_p A \end{aligned} \quad (1)$$

A doubly asymptotic boundary element method based on integrating static and viscous boundaries was suggested by Underwood and Green (1981). On one hand this type of boundary is able to simulate the zero and infinite-frequency problems to a precise level of accuracy and on the other hand it approximated to the rest of frequencies problems with only first-order of accuracy. Other methods that can be pointed are a doubly asymptotic unbounded media-structure interaction analysis proposed by Wolf and Song (1995) and an integration of multi transmitting formula and static boundary by Jing and Liao (2000). The reasons made the previous ABCs approaches uncommon are too much complexity to be used in numerical simulation and lack of accuracy to be applied in common practice. Deeks and Randolph (1994) derived an absorbing boundaries for plane strain axisymmetric shear and dilation waves which consisting of a linear spring, a linear viscous dashpot and, in the case of longitudinal wave absorbing boundary, a lumped mass. It was shown that this boundary is more accurate than the viscous boundary. In that study, a circular shape of the main domain is considered and the coefficients of springs are calculated using the radius of domain.

Jingbo et al. (2006) proposed a 3D viscous-spring artificial boundary consists of a unit of typical spring-dashpot in each direction. They also recommended a set of modified coefficients for the spring parameters of rectangular domain. Equation (2) shows the modified coefficients for viscous-spring boundary suggested by Jingbo et al.(2006) where the 1 and 2 indices point to coordinates in tangential directions and index 3 refer to the normal direction. The distance between node point and the boundary is represented by R which takes the approximate value of the perpendicular distance from the load point to the boundary. Modification parameters, α_T and α_N , are presented in table (1). G and A represents shear modulus and the total area of each boundary element around the boundary node. Note that the proposed boundaries were examined only for linear homogenous unbounded space problems and internal excitation.

$$\left. \begin{aligned} K_1 &= K_2 = \frac{\alpha_T G}{R} A \\ C_1 &= C_2 = \rho c_s A \\ K_3 &= \frac{\alpha_N G}{R} A \\ C_3 &= \rho c_p A \end{aligned} \right\} \quad (2)$$

Table 1. Recommended value of α_T and α_N

Modification parameters	Value range	Recommended value
α_N	1.0-2.0	1.33
α_T	0.5-1.0	0.67

Dynamic loading is another important step of numerical modeling. External loading such as acceleration, velocity, displacement or force of a specific earthquake can be applied at the base of the model. Loading can be applied at inner or outer side of artificial viscoelastic elements located at

border lines. To model a homogeneous medium, acceleration, velocity or displacement of earthquake cannot be applied at the inner side of the absorbing boundaries since the effect of absorption would be nullified. The point that is not valid for stress and force loading. For rectangular soil domain, a horizontal rim at the base and two vertical edges at sides of the model, form the entrance gates for upcoming seismic waves. In the case of vertically propagating shear waves, the nodes at the base, receive same input motions. The input motions at side-edge nodes are also the same, but out of phase. This fact necessitates the assignment of input motion at different nodes separately. A common replacement of such multiple assignment of motions at side nodes, are free field columns which simulate 1D vertical propagation of shear waves at each side of the model. This method is recommended by Zienkiewicz et al. (1989) and Wolf (1988). Also the same concept has entered popular engineering software like ABAQUS and even finite difference based FLAC^{3D}. A representative model with absorbing boundaries at edges and free field motion columns is illustrated in Figure 1.

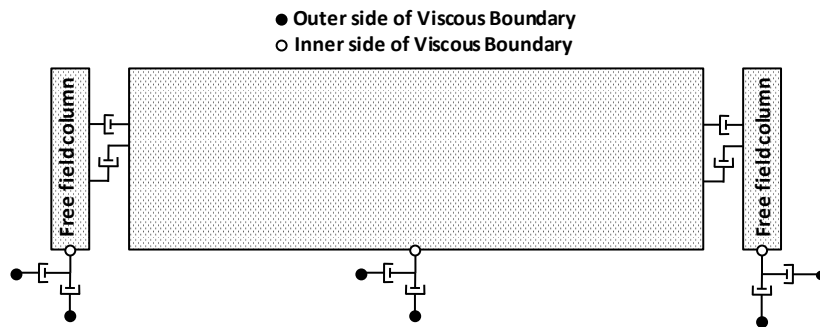


Figure 1. Dynamic soil domain with free field columns

Beside boundary configurations, there are comments on element size limitations to reach a reliable model. Kuhlmeier and Lysmer (1973) and Lysmer et al. (1975) proposed that the maximum element size which is controlled by the shear wave length L should satisfy Equation (3).

$$l_{\max} \leq \left(\frac{1}{8} \sim \frac{1}{5}\right)L \quad (3)$$

STATE OF THE PROBLEM

Introducing input motion to the system through artificial boundary elements, should guarantee the correct free field motion in the medium as a preceding step to SSI analysis. However, seismic loading from such borders, faces frequency content distortions due to new dynamic characteristics of the model induced by absorbing added elements. This problem needs to be treated appropriately before insertion of any structure in the model. After controlling the accuracy of free field motion, it should be investigated that to what extent would the reflected waves from boundaries affect the response of the soil-structure system. As mentioned before, 1D-based side dampers can't absorb the whole 2D or 3D scattered waves. In addition, the presence of free field columns weakens their performance, too. To give more insight on the severity of unwanted reflection of outgoing waves, a numerical model with free field columns is constructed using ABAQUS software. The model that constitutes of a single degree of freedom system rested on soil medium and its response to a short time impulse loading is investigated in Figure 2a and 2b, respectively. Viscous and viscoelastic boundaries are examined against the exact solution. In the case of reference model with artificial boundaries, soil medium of half-width 40m and depth 20m with shear wave velocity 120m/s is considered. That's while the benchmark model possess the dimensions of 200m half-width and depth. It can be seen that the foundation, receives the reflected body P and S waves from the base boundary, first, and then the reflected surface waves arrive. It should be noted that the model with viscous boundaries, experiences permanent displacement at the end of analysis.

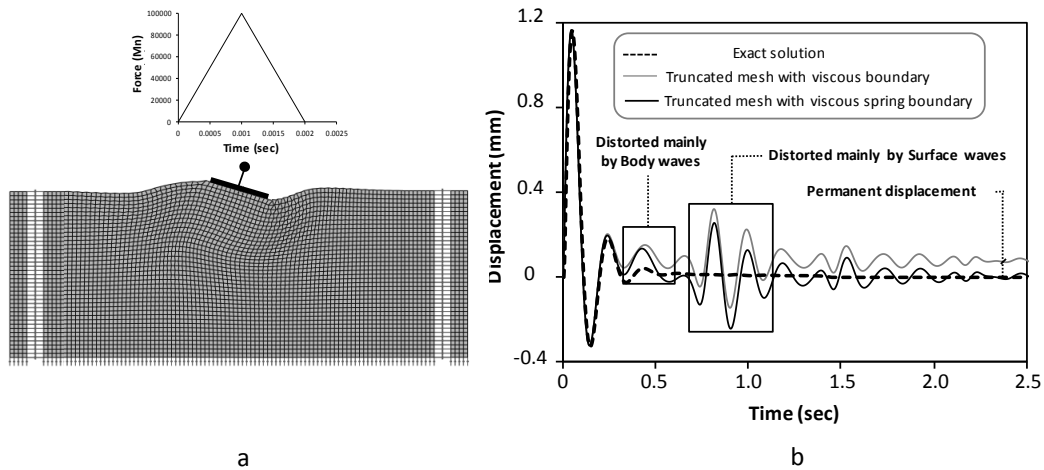


Figure 2. Comparison between responses of SDOF system rested on homogeneous soil medium and artificial boundaries against exact solution

In this paper, model characteristics for external seismic loading are studied in detail. Then the problem of input distortion through different types of boundaries is studied and a solution is suggested. Also a new scheme with a series of dampers at the surface of the model is proposed for the sake of absorbing the surface waves, propagated from structure to boundaries. The characteristics of surface dampers are optimized through a parametric study. All cases are examined in both free field conditions and presence of structure.

MODEL ADJUSTMENT FOR EXTERNAL LOADING

In this part the effectiveness of the common artificial boundaries in transmitting seismic input motion into the model is investigated. Numerical models are prepared using the finite element code ABAQUS. A linear finite domain of half-width 40m and depth 20m with quadrilateral plane strain elements of width 0.5m with shear wave velocity 100m/s, $\rho=1.65\text{ton/m}^3$ and $\nu=0.33$ is considered. Free field columns are attached at the sides of model through a viscous boundary condition suggested by lysmer and kuhlmeier (1969). It is important to constrain the translational degree of freedom of each side node with opposite corresponding node located at the other side of the domain. Similarly, the side nodes of each free field column should be constrained with the opposite corresponding nodes. These constraints would improve the performance of viscoelastic border elements. Seismic load can be applied in different types of acceleration, velocity, displacement or force. As mentioned before the imposed acceleration, velocity or displacement history of earthquake at the top of base boundaries dictates the history of translation in that degree of freedom and neglects the effect of reflecting waves from surface. The external acceleration, velocity and displacement motion are applied at outer side of base boundaries while the force history of motion is applied at inner side of base boundary. The presence of absorbing boundary at the loading point, changes the input motion. The level of variation depends on the type of the boundary. First the effect of viscous boundary on external loading is described. The main model is subjected to normally distributed impulse motion in horizontal direction which is shown in Figure 3. The four aforementioned types of input motions are implemented. It is expected that only pure shear (SH) waves propagate in the system because the input motion is applied in horizontal direction and the model is in free field condition. In this case, the boundary condition should be capable of damping out the whole incident waves reflected from ground surface. As the impulse reaches the ground free surface, based on theory of 1D wave propagation, it is expected to record two times amplified displacement of the input motion with appropriate delay (time lag from base to surface). Moreover the inner side nodes of base absorbing boundary should experience the external excitation and the reflected waves from surface correctly. These waves are reflected from surface with the delay which depends on the shear wave velocity of the soil. Figure 4a shows the

expected response of the free field model at the inner and outer side of base absorbing boundary and the ground surface nodes. Figure 4b indicates the response of the free field finite element model at the previously mentioned points in 4a. As the responses of different types of input motions are similar, only the response under displacement loading is presented in Figure 4b. Comparing the response of inner side of base dampers, Figure 4a and 4b illustrates that the input motion is decreased by the scale factor of 0.5 in the numerical model. The same conclusion was pointed by Mejia and Dawson (2006). As it is prospected, the affected input motion at the surface is amplified 2 times while it reaches the surface with the expected delay. Noticeably, the viscous boundary suggested by Lysmer and Kuhlmeyer (1969) is independent of frequency. Also the scale factor of the deamplification of input motion is constant among the different range of frequencies. Another model with half-width 40m and depth 20m with shear wave velocity 300 m/s, $\rho=1.65\text{ton/m}^3$ and $\nu=0.33$ is considered. Only a negligible difference in the magnitude of deamplification factor is observed. Furthermore similar outputs for the case of seismic loading are resulted.

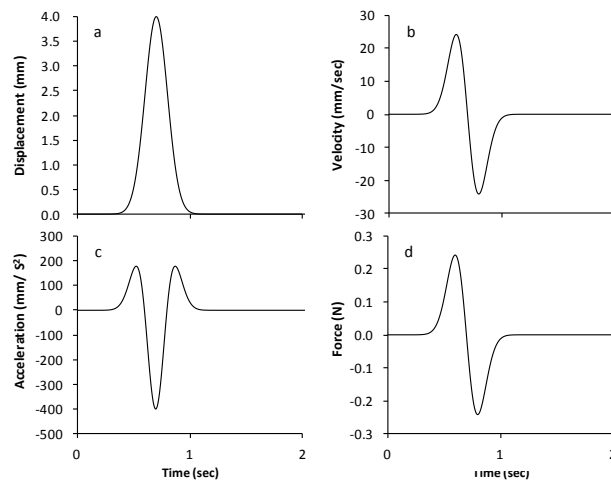


Figure 3. Normally distributed impulse in different types of: a) Displacement b) Velocity c) Acceleration d) Force

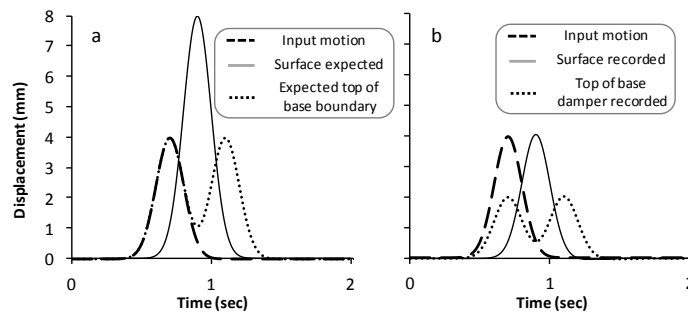


Figure 4. Comparison of exact and F.E response of a free field model A .a) exact response b) F.E. with uncorrected input motion

Viscous-spring boundaries suggested by jingbo (2006) have different effects on input motion. Figures 5a and 5c compare the input motion at the bottom of absorbing boundary and the recorded response at the surface of the model. A soil deposit with shear wave velocity 100m/s, $\rho=1.65\text{ton/m}^3$ and $\nu=0.33$ is considered. Figure 5b and 5d shows the frequency content of input motion and recorded motion at the surface which is calculated using Fourier Transform function. Clearly, the effect of viscous-spring boundary is different over the range of frequencies. To solve this problem, a transfer function between the base excitation and surface response is calculated (Figure 5e). This transfer function shows the effect of viscous-spring boundary on input motion over the range of frequencies. Using the following transfer function, it is possible to create a new input motion which leads to the

desired response at the surface. The expected response at the surface and the new input motion using the aforementioned transfer function is calculated (Figure 6a and 6b). The new record is imposed at the base of the previous model. The response of nodes located at the surface and the top of base boundary are illustrated in Figure 6c. As shown from Figure 6c, the new input motion results in the appropriate response at nodes of the surface and the inner side of absorbing boundary. The calculated transfer function is unique for each specific set of model characteristics. But, any variation in model geometry or material parameters would change this transfer function.

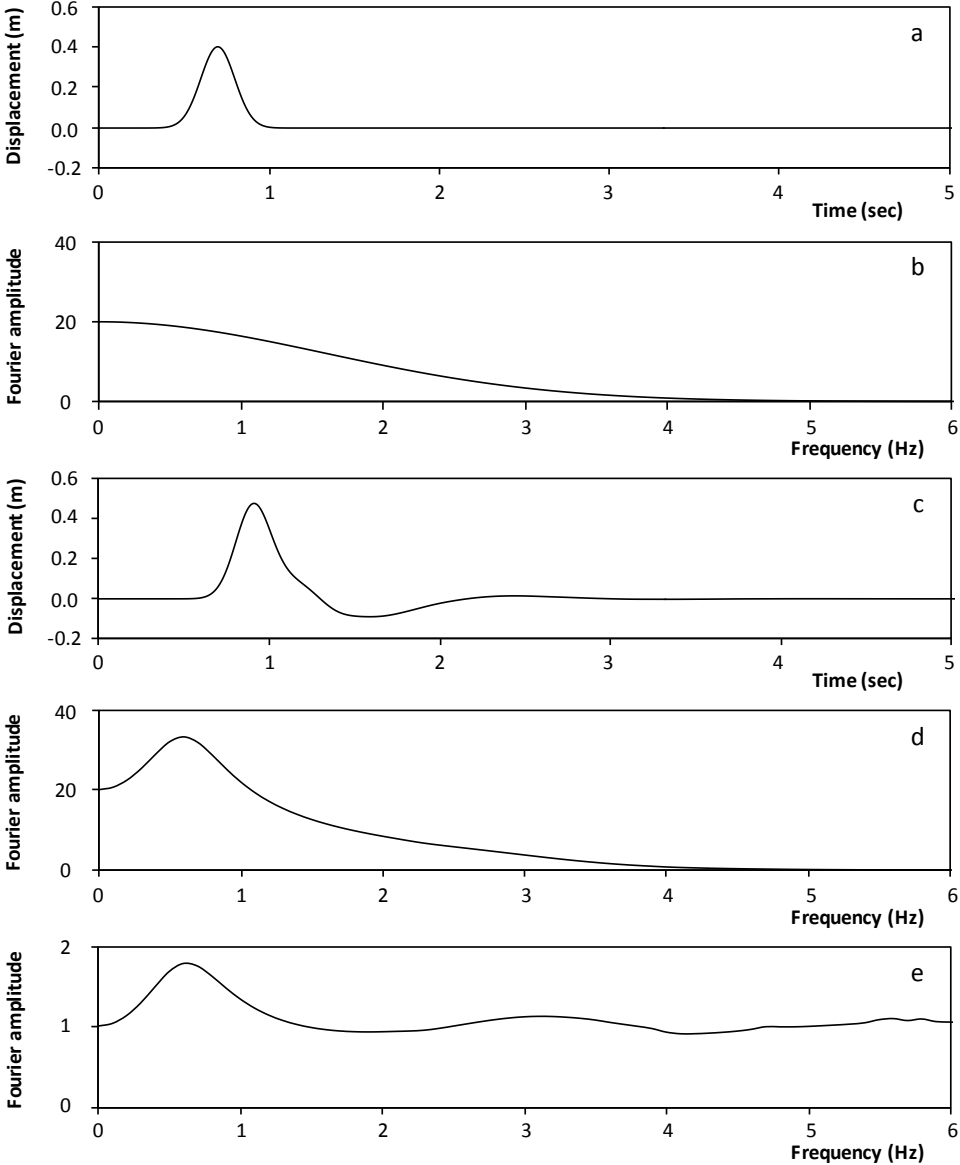


Figure 5. Transfer function calculation a)input motion at the base of the model b)Fourier amplitude of input motion at the base of the model c) Response at the surface of the model d)Fourier amplitude of response at the surface of the model e) Transfer function between base and surface

A new soil deposit with shear wave velocity 200m/s, $\rho=1.65\text{ton/m}^3$ and $\nu=0.33$ are applied in order to show the effectiveness of the defined method. The free field analysis is conducted and the transfer function is calculated then. To make it short, only the calculated transfer function which differs from the previous transfer function (Figure 5e) is presented in Figure 7.

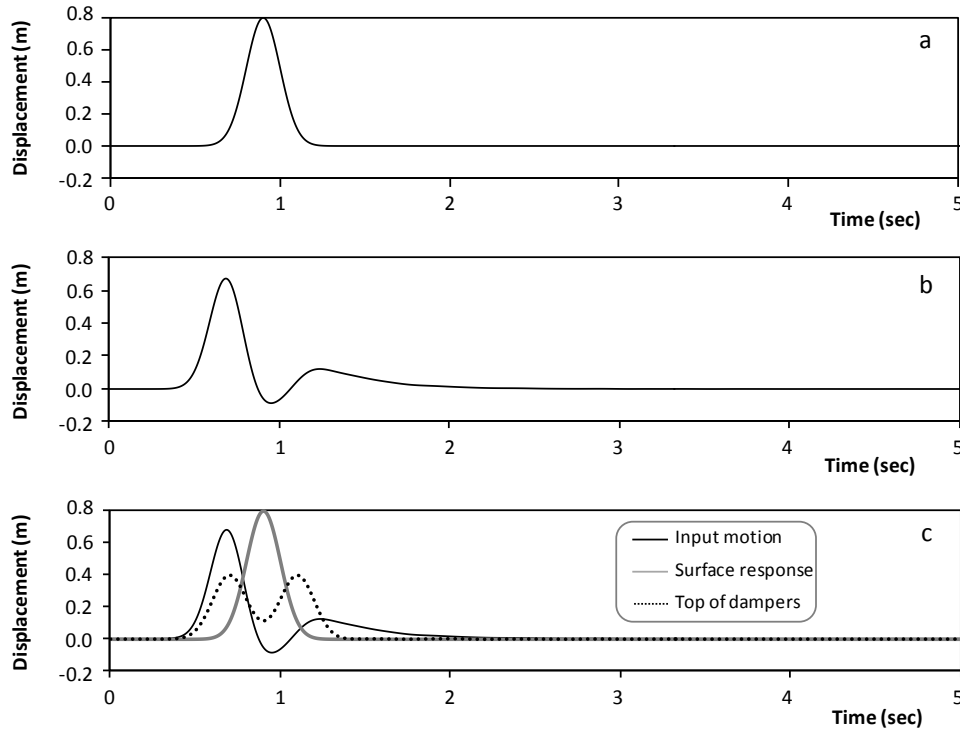


Figure 6. Calculation the new record with the expect response at surface. a) expected response at the surface b)New record calculated with transfer function c)Comparing the result of numerical model with new record

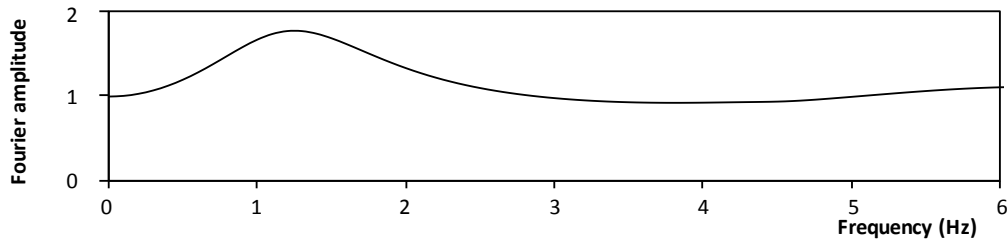


Figure 7. Calculated transfer function between surface and base excitation in model B

MODEL ADJUSTMENT TO ABSORB SURFACE WAVES

The input adjustment to capture free field condition was examined thus far. In this part, the effect of presence of a structure on the performance of boundaries is studied. The structure is assumed as a single degree of freedom system (Figure 8). Basically, the response of the soil-structure system depends on the size of the structure, dynamic characteristics of soil and structure as well as the applied excitation. Soil-structure interaction system is characterized by the following non-dimensional parameters:

1. A non-dimensional frequency as an index for the structure to soil stiffness ratio defined as equation 2.

$$a_{0,fix} = \frac{\omega r}{V_s} \quad (2)$$

Where ω is the circular frequency of the fixed base structure, V_s and r are the soil shear wave velocity and the radius of circular foundation respectively. The practical range of $a_{0,fix}$ for ordinary building type structures starts from 0 for fixed base conditions to 3, for cases with dominant soil-structure interaction effect.

2. The aspect ratio of the structure that is defined as h/r .

3. Structure to soil mass ratio index defined as equation 3.

$$\bar{m} = \frac{m_s}{\rho r^2 h} \quad (3)$$

In equation (3) structure mass, m_s , soil density, ρ , and effective structural height, h , are presented. Note that in 2D analysis r is substituted by half-width of foundation B .

Kinematic interaction between soil and structure complicates the propagation of waves in the system. Longitudinal and shear waves with different angles of incident propagate in the system. As mentioned before, the proposed boundary coefficients were calculated with the assumption of 1D wave propagation theory and they are not capable of absorbing most of the reflected body and surface waves. This problem is more conspicuous in systems with high radiation damping. Radiation damping in soil-structure systems with higher non-dimensional frequency and lower h/r ratio is more than other cases. A soil-structure system with the following non-dimensional parameters is assumed as a sample case to show the wave-transmission capability of the boundaries (Figure 8).

$$h/r = 0.5, \quad a_{0fix} = 3.0, \quad \bar{m} = 0.5 \quad (4)$$

The soil deposit with the shear wave velocity of 66.7 m/s with $\rho=1.65$, $\nu=0.4$ and zero material damping is assumed. A short time triangle shape impulse, shown in Figure 9, is applied at the mass point of the single degree of freedom system. After imposing the following impulse, the different types of longitudinal, shear and surface waves propagates from foundation of the structure toward the infinity. An exact solution without truncation effects is calculated using a finite element mesh extending to 200m in width and height, preventing any boundary reflection before 3.5sec. The mesh is then truncated at the distance of 40m in width and 20 m in depth with viscous boundary at sides and viscous-spring boundaries at base. Figure 10 compares the exact and truncated model results in horizontal displacement of mass point of SDOF structure. Deformed mesh of the aforementioned truncated model shows that the reflected surface waves is more important than the reflected body waves. Comparing the truncated solution and the exact one in Figure 10 shows that the truncated mesh is accurate until the reflected wave from the boundary returns to the structure at approximately 0.65sec. however, the major boundary reflection takes place for the case of surface waves which appears in the response at approximately 1.4, 2.8 and 4 sec. As an alternative to absorb the surface waves, a series of viscous dampers are implemented at the surface of the model. The surface dampers are implemented only in normal direction because dampers in tangential direction affect the free field motion of the model in the case of external loading. This would restrict the applicability domain of proposed method which is not desirable. Figure 11 illustrates schematically the region attributed to surface dampers. The coefficient, length and the depth of surface dampers play an important role in absorption of surface waves. The coefficients of dampers are increased by an appropriate function from the structure side to boundary. This strategy is taken to avoid abrupt confrontation of these elements with traveling surface waves which would behave as a new barrier and as well initiate a new set of reflections. A parametric study on the surface dampers is conducted and different functions are examined. These functions depend on the length of application of top dampers with reference to side boundary. Finally a polynomial function of the form equation 5 is considered.

$$Dashpot\ Coef = D(x - dashpotlength)^2 + C \quad (5)$$

As the model truncated at the width of 40m with absorbing boundaries, different dashpot lengths from 6 to 32m, D from 0.1 to 10000 are applied and the depth of the affected zone and C coefficient assumed 2m and 1 respectively. Two different indexes are introduced to compare the errors of the model with exact solution. First the time-vector displacement histories from 0 to 0.6sec are compared calculating the relative vector-angle (in degree). Second, the absolute norms of the vector histories from 0.6 to 2.5 sec are compared and the error is estimated. As D coefficient increases, the

amount of reflected waves reduced. At the same time high coefficients of dampers cause the incident waves, which propagate from the structure, new reflections while passing through surface dampers. This reflection occurs in less than 0.6sec. To distinguish between these two effects two different errors are introduced. The more the magnitudes of errors approach to zero, the more the accurate of the result. Figure 12 shows the proposed errors of the parametric study. Clearly the dashpot coefficient function with $D=100$ and dashpot length=8 m has somehow the best performance. Figure 13 compares the history of unrefined model with the refined model. The top dampers with the optimized function are implemented in the refined case. Due to the direction of surface dampers, the amplitude of reflected surface waves is decreased about 70% .

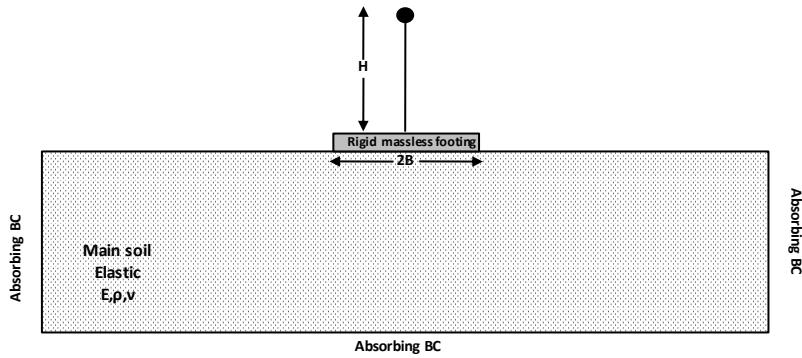


Figure 8. Single degree of freedom structure rested on elastic soil medium

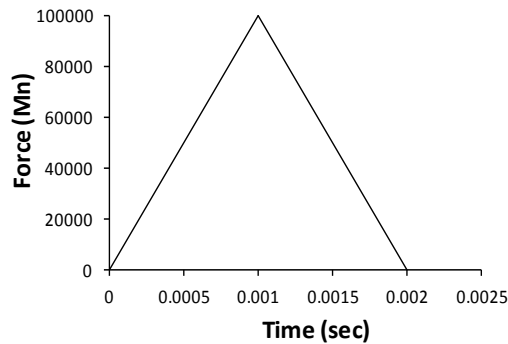


Figure 9. Impulse force used in the parametric study of top dampers

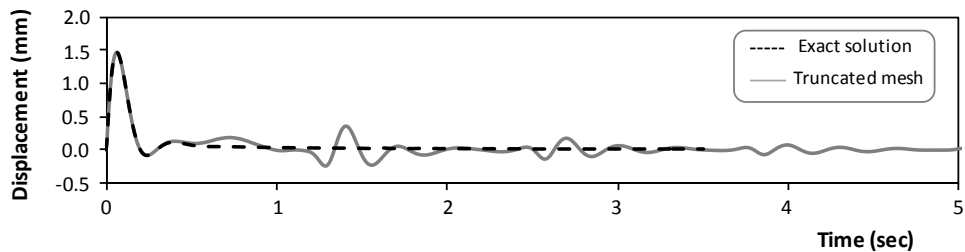


Figure10 . Comparison between exact response of the mass point of SDOF and the similar results for truncated model

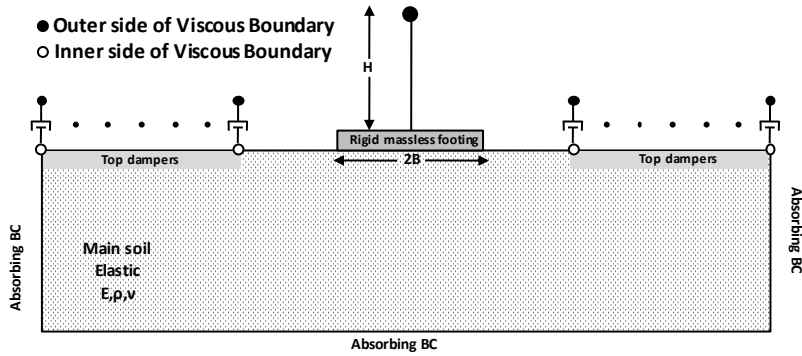


Figure 11. Single degree of freedom structure rested on elastic soil medium with the affected zones with surface dampers

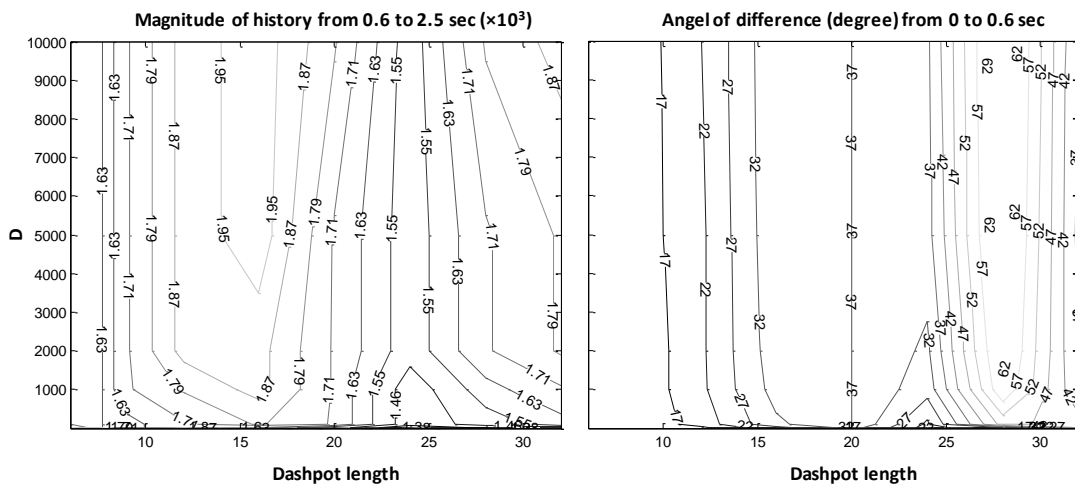


Figure 12. Errors of the parametric study

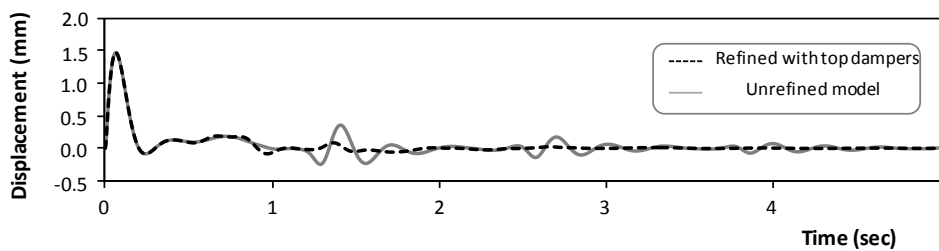


Figure13 . Comparison between response of model with optimized top damper and unrefined case

CONCLUSIONS

In this paper the characteristics of artificial boundaries in finite element modeling of soil structure interaction problems are studied. Several problems like make distortion in external input motions, inducing permanent displacement and unwanted reflections of body and surface waves are illustrated using a typical SDOF system resting on surface foundation under impulse loading. Then a set of solution for improving the capability of transmission of waves, both in and out, is proposed. First, the transmission capability of viscous and viscous-spring boundaries on external input motions is presented. Viscous boundaries reduce the input motion with the scale factor of 0.5 without dependency to frequency. The input motion should be multiplied with the scale factor 2 for the sake of recording the appropriate free field response on the ground surface. Viscous-spring boundaries change the input motion with different values over the range of frequencies. The difference is calculated using the

transfer function between the Fourier amplitudes of input motion and the recorded response at surface. Considering the calculated transfer function and the expected response at surface, it becomes possible to properly adjust the input motion. The study indicates that the suggested procedure is capable of leading to exact free field response at the surface. Second, the absorption potential of artificial boundaries to die out the internally generated surface waves is studied. Illustrating a sample case, it is shown that notable reflections may come to pass from such 1D-based absorbing border elements. As a solution a series of top dampers are implemented at two top corner sides of the model. The added elements were set such as not to affect the upcoming vertical shear wave free field motion, but absorb scattered surface waves, instead. For this purpose the length, Depth and finally the function of coefficients are studied parametrically. Then two vector-based indices of errors were introduced through which the optimized configuration of dampers is determined. It is shown that simple proposed devices would substantially facilitate transmitting out of surface waves. At last, it should be noted that, although the SSI problem may be simulated more accurately through the proposed procedures, their validity for more complex problems like embedded structures and nonlinear analysis needs further investigations.

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