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INFLUENCE OF NONLINEAR VISCOUS DAMPERS ON THE PROBABILISTIC SEISMIC RESPONSE OF LINEAR STRUCTURES

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ABSTRACT

The paper aims at evaluating the influence of the damper properties on the probabilistic seismic response of structural systems equipped with nonlinear viscous dampers. For this purpose, a linear single-degree-of-freedom system with an added linear or nonlinear viscous damper is considered, and the seismic response statistics are evaluated for a set of natural records describing the ground motion uncertainty. A dimensional analysis of the seismic problem is carried out first to identify the minimum set of characteristic parameters describing the system properties and controlling the seismic response. An extensive parametric study is then carried out to estimate the influence of the damper properties on the statistics of the main response quantities of interest (i.e., maximum displacements, accelerations and damper forces) for a wide range of values of the characteristic parameters. Finally, a set of case studies is investigated in order to show some interesting issues concerning the influence of the damper nonlinear behaviour on the evaluation of the system reliability and to highlight some limitations of current deterministic approaches neglecting the probabilistic properties of the response.

INTRODUCTION

Viscous dampers are dissipation devices that permit to efficiently enhance the performance of structures exposed to seismic hazard by reducing both the displacement and force demand in the structures (Soong and Dargush 1997, Christopoulos and Filiatrault, 2006) through conversion of the seismic input energy into heat. Experimental studies (Symans and Constantiou 1998, Lee and Taylor 2001, Hwang et al. 2006) have shown that the force-velocity relationship of viscous dampers can be analytically described by a velocity power law involving two parameters: the damping constant (c_N) and the velocity exponent (α), controlling respectively the damper's size and nonlinear behaviour. Many works in the literature (Peckan et al.1999, Lin and Chopra 2002, Martinez-Rodrigo and Romero 2003, Diotallevi et al. 2012, Zhang 2012, Bahnasy and Lavan 2013) analyzed the steady-forced and earthquake response of frames equipped with linear or nonlinear viscous dampers by focusing on the sensitivity of the response to the damper exponent. In general, it was observed that nonlinear viscous dampers are more advantageous than linear dampers because they permit to achieve the same displacement reduction with lower damper forces.

In these above studies, which were mainly oriented to provide information useful for the damper design and the selection of the optimal damper properties, the seismic input was described by selecting a set of natural ground motions with different characteristics, and the seismic response was evaluated by averaging the results of the nonlinear time-history analyses for the different records. This approach is coherent with the prescriptions of several codes (Eurocode 8, FEMA-368, NZS 1170.5, ASCE 7), which allow to consider only the mean values of the response parameters of interest for the

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performance assessment, provided that an adequate number of records is employed to describe the seismic input. However, this performance measurement, often referred to as "deterministic" (Bradley 2011), has a number of limitations since it does not account for the response dispersion due to the input variability and does not consider that certain response parameters may be more sensitive to the input variability than others (i.e. have a higher dispersion). A more rigorous evaluation of the seismic performance of an engineered system should be carried out on a probabilistic basis and should aim at computing the effective statistical distribution of the various response parameters that affect the system reliability (Bradley 2011, Aslani and Miranda 2005). This general statement is explicitly acknowledged in modern performance-based earthquake engineering (PBEE) frameworks such as the PEER framework (Porter 2003, Zhang et al. 2004) and the SAC-FEMA method (Cornell et al. 2000), in latest seismic codes (Hamburger 2006), and also in recent works on various structural damped systems (Marano 2007, Güneyisi and Altay 2008, Freddi et al. 2013, Lavan and Avishur 2013).

This study is finalized to evaluate the influence of the damper properties on the probabilistic seismic response of structural systems equipped with linear or nonlinear viscous dampers, with an approach consistent with the context of PBEE. For this purpose, the uncertainty in the seismic input is described by introducing a seismic intensity measure, and by considering a set of natural ground motions records characterized by a different duration and frequency content which reflect the record-to-record variability. Then, the statistics of the main response parameters are built in order to evaluate the influence of the damper nonlinear behaviour on the seismic performance. In particular, the structural system considered in this paper consists of a single degree of freedom (SDOF) model coupling a linear visco-elastic term, representing the structural frame, and a linear or nonlinear purely viscous term, describing the added dissipative system. First, a nondimensionalization of the governing equation of seismic motion is applied to find the minimum set of characteristic parameters that control the problem. Successively, a parametric study is carried out by varying these characteristic parameters within a range of interest for the design. For each combination of the characteristic parameters' values, the statistic of the response parameters of interest for the performance assessment (such as displacements, accelerations, and damper forces) is built based on the response samples corresponding to a set of natural records describing the record-to-record variability. A lognormal model, widely employed in PBEE, is assumed to describe the probability distribution of the output variables and the influence of the damper nonlinear behaviour and dissipation capacity on the probabilistic response is evaluated based on the comparison of the geometric mean and dispersion of the response parameters obtained for the different values of the characteristic parameters.

Finally, the results of the parametric study are used to analyze a family of case studies consisting of the same structural system equipped with dampers having different properties (different values of c_N and α) ensuring the same deterministic performance objective. The influence of the damper nonlinear behaviour on the seismic performance is investigated for different seismic intensity levels, and the reliabilities of linear and non linear solutions are analyzed based on the comparison of the probability of exceedance of design values of the main response parameters of interest.

PARAMETRIC STUDY

Non dimensional formulation

The equation of motion governing the seismic response of a single-degree-of-freedom (SDOF) system equipped with a nonlinear viscous damper can be expressed as:

$$m\ddot{u}(t) + c_L\dot{u}(t) + c_N\left|\dot{u}(t)\right|^{\alpha} sgn\left(\dot{u}(t)\right) + ku(t) = -m\ddot{u}_g(t) \qquad (1)$$

where $u(t)$ is the relative displacement of the mass to the ground, m, k , and c_L denote respectively the system mass, stiffness, and viscous (inherent) damping constant, c_N is the damping constant of the added non linear viscous damper, $sgn(\cdot)$ is the sign function, $\ddot{u}_g(t)$ the ground motion input and the dot denotes differentiation over time. The differential problem is completed by the initial conditions, assumed homogeneous in the following. In order to reduce the equation to its non-dimensional form, the following dimensionless variables are introduced:

$$\begin{aligned}\psi &= u / u_0 \\ \tau &= t / t_0\end{aligned} \tag{2a,b}$$

where u_0 and t_0 are characteristic units measuring respectively the length and the time. The seismic input can be expressed in terms of the product of a constant scale factor a_0, whose dimension is an acceleration, and of a non-dimensional function $l(t)$, describing its variation over time:

$$\ddot{u}_g(t) = a_0 l(t) = a_0 \lambda(\tau) \tag{3}$$

where $\lambda(\tau)$ is obtained from $l(t)$ by scaling the time t by the factor $1/t_0$, according to Eq.(2a) After substituting Eqns. (2a) and (3) into Eq.(1) and rearranging, one obtains:

$$\ddot{\psi}(\tau) + \frac{c_L t_0}{m}\dot{\psi}(\tau) + \frac{c_N t_0^{2-\alpha}}{m u_0^{1-\alpha}}\operatorname{sgn}\left(\dot{\psi}(\tau)\right)\left|\dot{\psi}(\tau)\right|^{\alpha} + \frac{k t_0^2}{m}\psi(\tau) = -\frac{t_0^2}{u_0}a_0\lambda(\tau) \tag{4}$$

Finally, by choosing the time scale $t_0 = 1/\omega_0$, where $\omega_0 = \sqrt{k/m}$ denotes the system undamped circular frequency, and the length scale $u_0 = a_0 t_0^2 = a_0/\omega_0^2$, Eq.(4) can be simplified to:

$$\ddot{\psi}(\tau) + \frac{c_L}{m\omega_0}\dot{\psi}(\tau) + \frac{c_N}{m a_0^{1-\alpha}\omega_0^{\alpha}}\operatorname{sgn}\left(\dot{\psi}(\tau)\right)\left|\dot{\psi}(\tau)\right|^{\alpha} + \psi(\tau) = -\lambda(\tau) \tag{5}$$

Eq.(5) reveals that the non-dimensional displacement response of the system, $\psi(\tau)$, to the input $\lambda(\tau)$, is a function of only three non-dimensional parameters characteristic of the system:

$$\begin{aligned}\Pi_{c_L} &= \frac{c_L}{m\omega_0} \\ \Pi_{c_N} &= \frac{c_N}{m a_0^{1-\alpha}\omega_0^{\alpha}} \\ \Pi_{\alpha} &= \alpha\end{aligned} \tag{6a,b,c}$$

Parameter $\Pi_{C_L} = 2\xi$ describes dissipation capacity of the linear system and it is related to the inherent damping ratio ξ (Lin and Chopra 2002), parameter Π_{C_N} describes the damper dissipation capacity, and parameter $\Pi_{\alpha} = \alpha$ describes the damper non linearity. It is noteworthy that the choice of the dimensionless parameters is not unique. In fact, Π_{C_L} can be interchanged with ξ and Π_{C_N} can be substituted by other parameters measuring the total amount of nonlinear dissipation as well. For example, in Lin and Chopra (2002) the damper dissipative property is described by the supplemental damping ratio ξ_d. This parameter is proportional to the ratio between the energy dissipated by the damper in an ideal cycle with amplitude equal to the peak displacement response u_{max} and circular frequency ω_0, and the maximum elastic energy stored in the spring, and it can be expressed as

$$\xi_d = \frac{\lambda_\alpha}{\pi}\frac{c_N}{2m\omega_0}\omega_0^{\alpha-1}u_{\max}^{\alpha-1} \tag{7}$$

where $\lambda_\alpha = \frac{2^{2+\alpha}\Gamma^2(1+\alpha/2)}{\Gamma(2+\alpha)}$ and where $\Gamma(\cdot)$ denotes the gamma function. The parameter λ_α equals π for α = 1 and 4 for α = 0. As already observed in Diotallevi et al. (2012), the definition of the parameter ξ_d involves not only the system parameters, but also the problem solution. Thus, differently from Π_{CN}, ξ_d cannot be considered as a system characteristic parameter, but it can be useful to estimate dissipation properties exhibited for a particular seismic response. It is noteworthy that for α =1, $\xi_d = \Pi_{C_N}/2$ is independent on the response. Due to its physical meaning, the non-dimensional parameter $\Pi_{C_N}^* = \frac{\lambda_\alpha}{2\pi}\Pi_{C_N}$ will be used instead of Π_{C_N} in the parametric study discussed in the next section . It is important to observe that the normalized response of the dynamic system undergoing free vibrations or subjected to an impulsive input depends only on Π_{C_L} , Π_{C_N} , and Π_α . Differently, the seismic response depends also on the function $\lambda(\tau)$. Having assumed $\omega_0 = 1/t_0$ as time scale, it follows that the expression of $\lambda(\tau)$ corresponding to a seismic input $\ddot{u}_g(t)$ imposed to a system with circular frequency ω_0 changes with ω_0 itself. Thus, the same seismic input $\ddot{u}_g(t)$ yields different non-dimensional response histories $\psi(\tau)$ and solutions, for systems with different frequency ω_0. This observation has the important effect that also the system frequency ω_0 (or period $T=2\pi/\omega_0$) has to be considered and varied in the following parametric study.

Seismic response evaluation

With reference to the seismic input description, in general an earthquake is characterized in terms of intensity, frequency content, and duration. These characteristics exhibit a significant degree of variability from record to record at a site, and need to be properly described and addressed. Coherently with the performance-based earthquake engineering (PBEE) approach, this study separates the uncertainties related to the seismic input intensity from those related to the characteristics of the record (record-to-record variability) by introducing a scale factor, a_0, i.e. an intensity measure, through Eq. (3). By this way, the randomness in the seismic intensity can be described by an hazard curve, whereas the ground motion randomness for a fixed intensity level can be described by selecting a set of ground motion realizations characterized by a different duration and frequency content, and by scaling these records to the common a_0 value. In this study, the spectral pseudo-acceleration, $S_A(T,5\%)$, at the fundamental period of the system, $T = 2\pi/\omega_0$, and for ξ = 5% (i.e., Π_{C_L} =10%), is assumed as intensity measure. The spectral pseudo-acceleration is related to the spectral displacement $S_d(T,5\%)$ by the relation $S_A(T,5\%) = \omega_0^2 S_d(T,5\%)$. It is worth to note that, in general the assumed intensity measure is more efficient than the peak ground acceleration, thus it permits to reduce the response dispersion for the same number of ground motion considered and to obtain more confident response estimates for a given number of records employed. Moreover, in this specific study, the choice of the assumed intensity measure is motivated by the fact that if all the records are normalized to $S_A(T,5\%)$, then the displacement response of a SDOF system with period T, damping ratio ξ =5%, and without the supplemental damper becomes a constant, i.e., it is not affected by the record-to-record variability. Thus, the systems with no added damper can be assumed as reference cases for evaluating the influence of the added damper properties on the response dispersion.

By repeatedly solving Eq.(5) for the set of ground motions records considered a set of samples is obtained for each output variable that represents the response variability. In this paper, a probabilistic model based on a lognormal distribution, widely employed in PBEE, is used to describe the response, by evaluating its accuracy through statistical testing. The assumption of lognormal distribution permits to estimate, even with a limited number of samples, the response at different percentile levels, which is very useful for the system reliability assessment. It also permits to obtain a closed-form analytical estimate of the risk (Cornell et al. 2002). A lognormal distribution can be fitted to the generic response parameter D by estimating the sample geometric mean, $GM(D)$, and the sample lognormal standard deviation $\sigma_{\ln}(D)$, or dispersion $\beta(D)$, defined as follows:

$$GM(D) = \sqrt[N]{d_1 \cdot \ldots \cdot d_N} \tag{9}$$

$$\beta(D) = \sigma_{\ln}(D) = \frac{\left(\ln d_1 - \ln\left[GM(D)\right]\right)^2 + \ldots\ldots + \left(\ln d_N - \ln\left[GM(D)\right]\right)^2}{N-1} \tag{10}$$

where d_i denotes the i-th sample value, N is the total number of samples. The sample geometric mean provides an estimate of the median of the response and its logarithm coincides with the lognormal sample mean $\mu_{\ln}(D)$. For small values, e.g., below 0.3, the dispersion $\beta(D)$ is approximately equal to the coefficient of variation of the distribution (Cornell et al. 2002). In thid study, a set of response parameters relevant to the performance of the system components is considered in this study. This includes the peak relative displacement u_{max} (related to internal forces in the structural frame, the stroke demand in the damper as well as to eventual displacement-sensitive non-structural components), the peak absolute acceleration a_{max} (related to global forces on the system, i.e. the base shear, as well as to possible acceleration-sensitive non-structural components), and the peak internal force in the damper $f_{d,max}$. These response parameters can be expressed in non-dimensional form as:

$$\begin{aligned} \eta_u &= \frac{u_{\max}\omega_0^2}{S_A(\omega_0,5\%)} = \frac{u_{\max}}{S_d(\omega_0,5\%)} \\ \eta_a &= \frac{a_{\max}}{S_A(\omega_0,5\%)} \\ \eta_{f_d} &= \frac{f_{d,\max}/m}{S_A(\omega_0,5\%)} \end{aligned} \tag{11}$$

where η_u can be interpreted as the reduction factor of the 5% damped displacement response spectrum.

Parametric study results

In the parametric study, the system period T is varied in the range between 0s and 4s, the parameter Π_α in the range between 0.15 and 1, whereas a constant value of 10% is assumed for Π_{C_L} (i.e., $\xi = 5\%$). The parameter Π_{C_N} is varied in order to obtain values of $\Pi^*_{C_N}$ in the range between 0 and 0.30. It is recalled that $\Pi^*_{C_N}$ coincides with the supplemental damping ratio ξ_d for α =1.

A set of 28 ground motions is considered in the parametric study to describe the record-to-record variability. The list of ground motions are reported in Tubaldi et al. (2014). For each value of the parameters varied in the parametric study, the differential equation of motion corresponding to Eq.(5) has been repeatedly solved for the different ground motion considered scaled to the common value of $S_A(T,5\%)$, and the probabilistic response properties have been evaluated by estimating the geometric mean *GM* and the dispersion β through Eqns. (9) and (10). Figures 1 and 2 shows the values of *GM* and β of the response parameters obtained for the different values of $\Pi^*_{C_N}$ and of *T*, and for the two extreme values Π_α=1 (linear case) and Π_α = 0.15 (non linear case). For what concerns the geometric mean, similar trends of variation with $\Pi^*_{C_N}$ of the normalized displacement are observed in the linear and non linear cases, but for a given $\Pi^*_{C_N}$ value, higher displacement response reductions are achieved by nonlinear dampers. Differently, the normalized damper forces exhibit different trends of variation with $\Pi^*_{C_N}$ in the linear and non linear case. Finally the normalized accelerations are not significantly influenced by variations of Π_α. With reference to the response dispersion, $\beta(\eta_u)$ increases by increasing $\Pi^*_{C_N}$ (both in the linear and non linear casee) and it significantly increases for decreasing Π_α. Differently, the dispersion of the normalized forces, $\beta(\eta_{f_d})$, follows an opposite trend, since it significantly decreases for decreasing Π_α. Finally, the dispersion of the absolute accelerations, $\beta(\eta_a)$, similarly to the geometric mean, does not vary significantly by varying Π_α. Complete results of the parametric study and relevant comments may be found in Tubaldi et al. (2014). By assuming that the response parameters follow a lognormal distribution, the knowledge of the geometric mean and of the lognormal standard deviation is sufficient to fully characterize their probability distribution function. In a lognormal distribution, the relation between the mean (μ_D) and the *k*-th percentile (D_k) of the generic demand *D* can be expressed as:

$$D_k = \mu_D \exp\left[f(k)\beta(D) - \beta(D)^2/2\right] \tag{12}$$

where $f(k)$ is a function assuming the values $f(50) = 0$, $f(84) = 1$ and $f(16) = -1$.

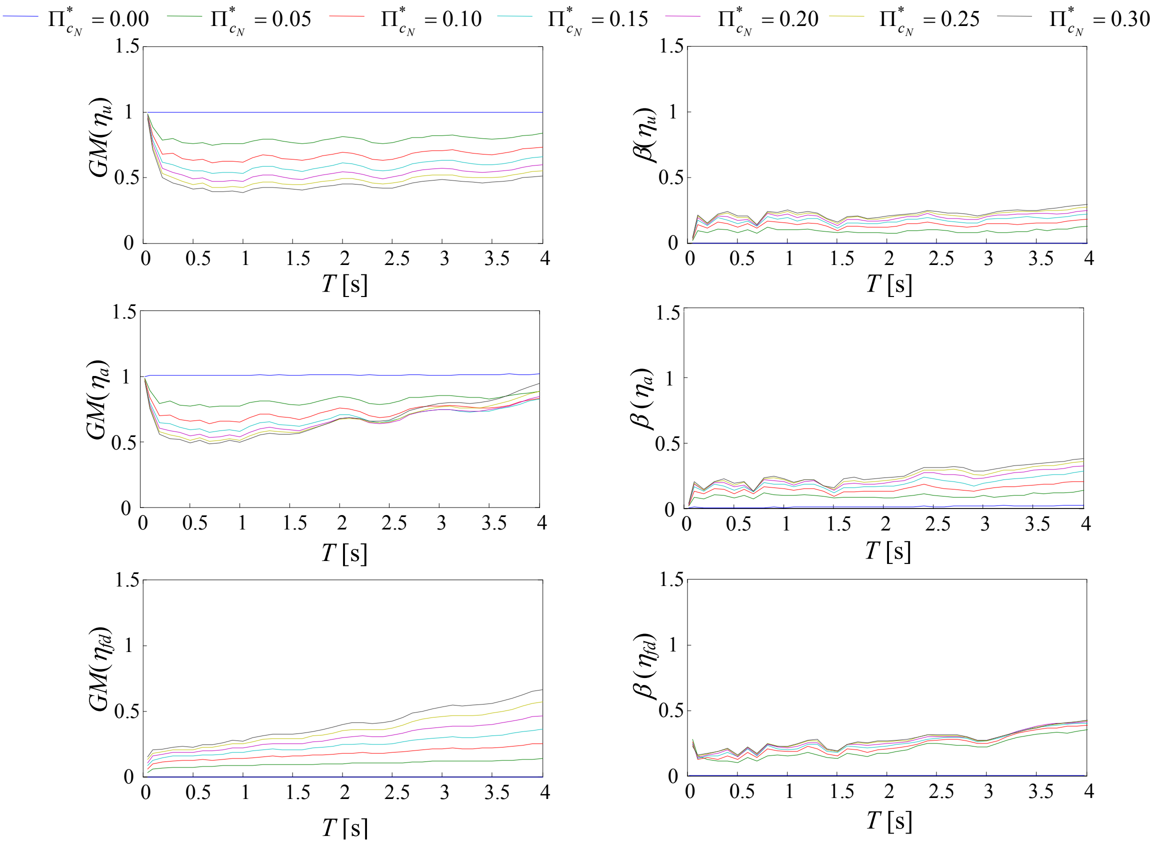

Figure 1. Sample geometric mean (a) and lognormal standard deviation (b) for Π_α =1.

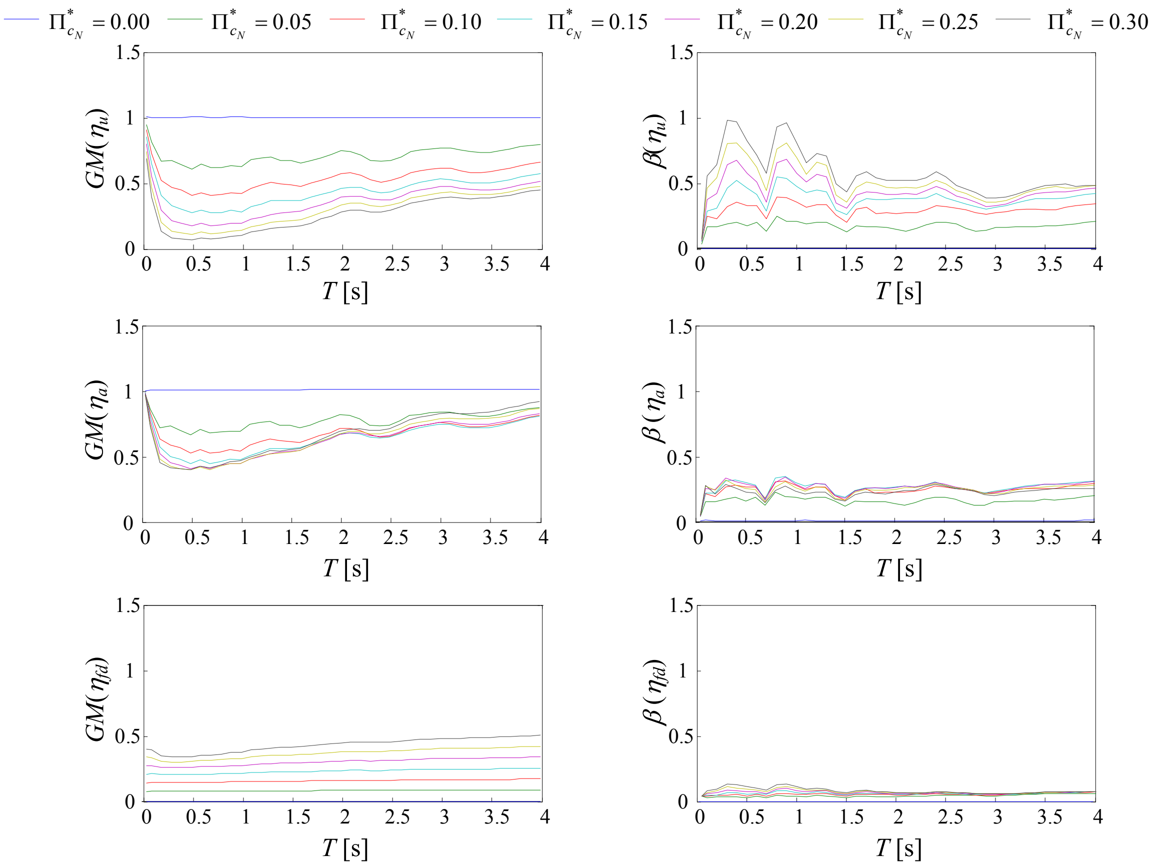

Figure 2. Sample geometric mean (a) and lognormal standard deviation (b) for $\Pi_\alpha = 0.15$.

RELIABILITY OF SYSTEMS EQUIPPED WITH NONLINEAR VISCOUS DAMPERS

In this section, the parametric study results are employed to evaluate and compare the reliabilities of a family of case studies involving viscous dampers with different nonlinear behavior designed to achieve the same performance target coherently with the approach of current seismic codes (Eurocode 8, FEMA-368, NZS 1170.5, ASCE 7).

Case study and seismic input description

The case studies considered in this section consist in a SDOF elastic system with natural vibration period T = 1 s and damping ratio ξ = 5%, equipped with linear or nonlinear viscous dampers. The design earthquake action is characterized by an intensity which corresponds to a pre-fixed exceedance probability or return period (uniform hazard acceleration spectrum) as defined in Collins et al. (2005). A simplified hazard curve is assumed for the seismic input, as described by the following expression:

$$\nu\left(a_0\right) = 0.1046 \cdot a_0^{-2.8571} \qquad (8)$$

where $\nu\left(a_0\right)$ denotes the mean annual frequency of exceeding the seismic intensity value a_0. The hazard curve described by Eq.(8) has been derived by following the procedure reported in Tubaldi et al. 2012, and is such that the value of the ultimate limit state (ULS) seismic intensity (with a probability of exceedance of 10% in 50 years) is $a_{0,\text{ULS}}$ = 0.4g, whereas the value of the damage limitation state (DLS) seismic intensity (with a probability of exceedance of 10% in 10 years), is $a_{0,\text{DLS}}$ = 0.23g. The definition of these two limit states is in accord with Eurocode 8.

The viscous dampers are designed so that the mean peak displacement corresponding to the records of previous section scaled to the ULS intensity does not exceed the limit value of 0.04m. Since in the case of no added damper the mean peak displacement demand at the intensity $a_{0,\text{ULS}}$ is equal to 0.1m, the design objective corresponds to a target mean value of the displacement reduction factor η_u equal to 0.4. This design objective is achieved by considering different values of the damper velocity exponent α in the range 0.15-1. Results in terms of $\Pi^*_{C_N}$ and ξ_d may be found in (Tubaldi et al. 2014). The values of $\Pi^*_{C_N}$ that ensure the design objective for α =1 and α =0.15 are respectively 0.292 and 0.113, corresponding to values of the normalized damper constant respectively equal to c_N/m = 3.669 and c_N/m =0.785. Both the cases are caracterized by very similar values of ξ_d (about 0.28).

Influence of damper non linearity on the probabilistic seimic responce

In Figure 3, the response properties obtained for the different damper nonlinearity levels are compared by plotting, vs. α, the sample mean, median, and the 84th and 16th percentiles of the normalized response parameters of interest at the design condition, as obtained via Eq.(10). For what regards the "deterministic" performance, described by the mean response values, it can be observed that the normalized mean displacement is equal to 0.4 for all the α values considered. The mean normalized accelerations assume values of about 0.5 almost constant with α (the ratio between the absolute accelerations for α = 0.15 and α=1 is 1.06). Differently, the mean normalized force decreases significantly by reducing α. The ratio between the mean damper forces for α = 0.15 and α =1 is about 0.60. These "deterministic" results, also observed in other studies (Lin and Chopa 2002, Martinez-Rodrigo and Romero 2003), confirm that the nonlinear viscous dampers permit to obtain on average displacement reductions similar to that achieved with a linear viscous damper while limiting significantly the damper force and without increasing significantly the absolute accelerations.

For what regards the probabilistic performance, synthetically described by the median and the 84th and 16th percentiles of the normalized response parameters of interest, it can be observed in Figure 3 that in general the median response values are close to the corresponding mean values, with normalized differences below 15%. This is expected, given the reduced response dispersion. Differently, the 16th and 84th percentiles are significantly different from the corresponding median values, and the difference varies with α and with the response parameter considered. In particular, the difference between the 16th and 84th percentile of the displacement response and the median value increases by increasing α, in consequence of the increased dispersion, as measured by β_u. A similar trend is observed for the absolute accelerations percentiles, whereas an opposite trend is observed for the 16th and 84th percentiles of the damper force. In fact, the force percentiles tend to the median value when α decreases in consequence of the decreasing dispersion β_{f_d}. This result may impact the damper sizing, which is usually governed by the stroke (proportional to u) and the force that have to be withstood. For example, for $\alpha = 1$, the 84th percentile of the normalized displacement is $\eta_{u,84} =$ 0.498 (i.e., 1.25 times the corresponding mean value), whereas the 84th percentile of the normalized damper force is $\eta_{fd,84} = 0.336$ (i.e., 1.22 times the corresponding mean value). For α = 0.15, $\eta_{u,84} =$ 0.549 (i.e., 1.37 times the corresponding mean value) and $\eta_{fd,84} = 0.175$ (i.e., 1.05 times the corresponding mean value). Given their importance in ensuring the safety and reliability of the whole system, it is fundamental to design the dampers (both the damper components and the connections to the structure) with a know level of reliability, which could be even higher that the reliability level of the structure to be protected. This goal may be achieved by proposing amplifying factors depending on the exponent α of the constitutive law.

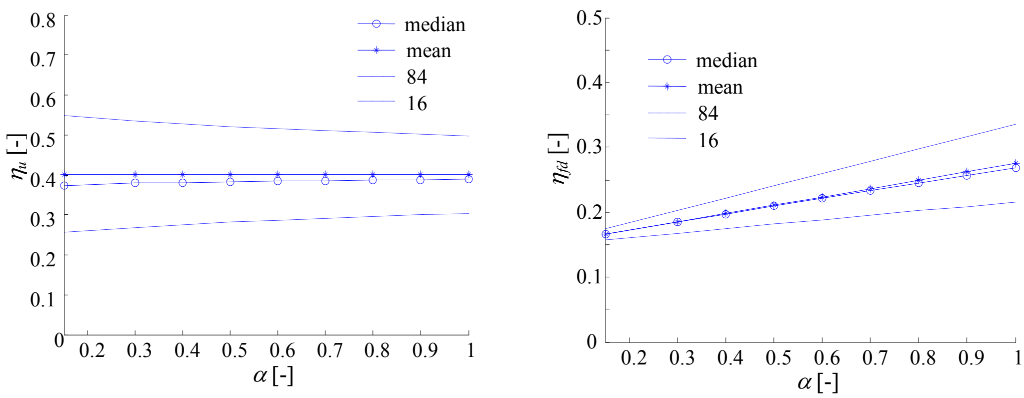

Figure 3. Normalized response parameters for different values of α.

Although the viscous dampers are usually designed so that the protected structure satisfies a specific performance objective for a given seismic intensity, the response of the system at different (smaller and larger) seismic intensities needs to be evaluated. With reference to the case studies analyzed in this section, the response parameters of interest (mean, median and 84th and 16th percentiles values) are evaluated also for seismic intensities other than the ULS intensity $a_{0,\mathrm{ULS}}$ considered for the damper design. As observed previously, this requires recalculating the values of the non-dimensional characteristic parameters reported in Eq. 6 and exploiting the parametric study results. Figure 4 plots the variation with a_0 of the system displacement response and of the damper force corresponding to the damper exponents α = 0.15 and α = 1. The response parameters are here reported in dimensional form so that the plots of Figure 4 may also be interpreted as summarized incremental dynamic analysis curves. Obviously, the mean displacement curves for α = 0.15 and α =1 pass through the design displacement $u_d = 0.04$ m at the design intensity $a_{0,\mathrm{ULS}} = 0.4$g, whereas the mean values of the damper force are significantly different and they are $f_d = 490.64$ kN and $f_d = 295.16$ kN in the linear and nonlinear case respectively.

As expected, in the case corresponding to α = 1 the response varies linearly with a_0. Thus, the mean displacement and damper force demand reduce by a factor 0.23/0.4 = 0.575 by passing from the ULS to the DLS intensity. On the other hand, in the nonlinear case corresponding to α = 0.15, the mean displacement demand increases more than linearly with a_0 whereas the mean damper force demand increases less than linearly. This implies that for low seismic intensities, the displacement demand obtained with the nonlinear damper is smaller than that obtained with the linear damper, while the value of the damper force normalized with respect to the design ULS value is higher. In particular, at the DLS intensity level, $a_{0,\mathrm{DLS}}$, the displacement for α =1 is about 0.023m, whereas for α =0.15 it is about 0.01 m. On the other hand, the ratio between the mean damper force at the DLS and ULS intensity (0.575 in the linear case) is 0.88 in the nonlinear case. Opposite considerations hold for seismic intensities larger than the design ULS intensity.

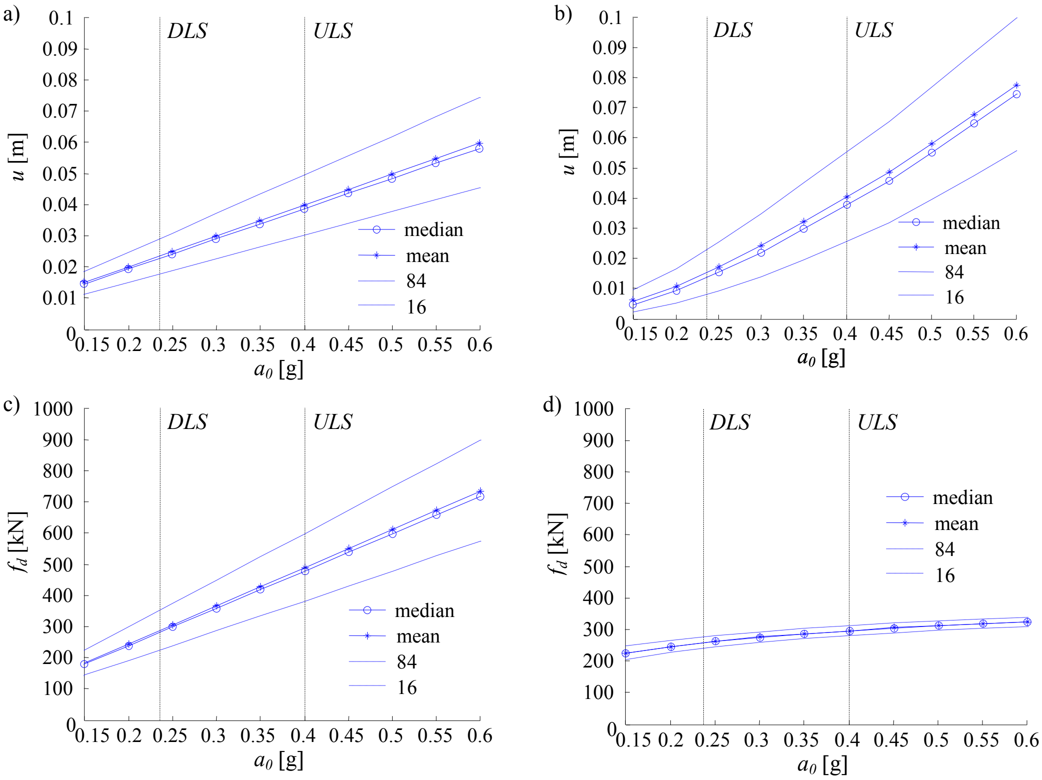

Figure 4. Variation with a_0 of the displacement demand for $\alpha = 1$ (a) and $\alpha = 0.15$ (b) and of the force demand for $\alpha = 1$ (c) and $\alpha = 0.15$ (d).

With regard to the probabilistic response, while in the linear case the response dispersion is constant, in the nonlinear case it varies significantly with a_0, as it can be better inferred from Figure 5 where the variation with a_0 of the displacement and damper force dispersion is reported. In particular, the displacement response dispersion is very high for low a_0 values, and it decreases for increasing a_0, whereas the dispersion of the damper forces is in general very low and it also decreases with a_0. At the intensity level $a_{0,\text{DLS}}$, in the linear case the displacement and force dispersions are the same as those evaluated at $a_{0,\text{ULS}}$, whereas in the nonlinear case the displacement dispersion is significantly higher and the force dispersion is slightly lower than the corresponding dispersion at $a_{0,\text{ULS}}$.

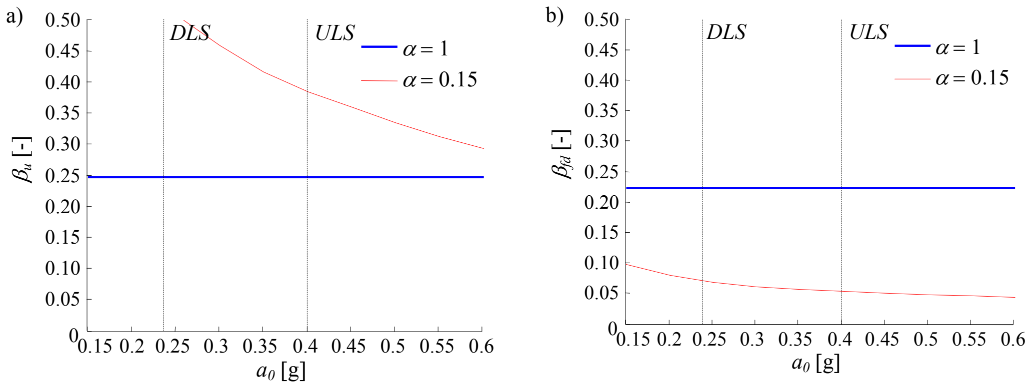

Figure 5. Variation with a_0 of the displacement and damper force dispersion.

Influence of damper non linearity on the seismic risk

In order to shed light on the effects of the damper nonlinear behaviour and of the response dispersion on the seismic reliability of the system, the risk of exceeding reference values of the displacements and of the forces during a life-time of T_L=50 yrs is computed. In particular, in order to highlight possible limitation of current deterministic approaches, the mean values of the response parameter of interest are assumed as reference values and both the deterministic (mean) and the probabilistic response are considered as demand models. More specifically, the reference value of the displacement is assumed equal to the target displacement of 0.04m for both the cases of the linear and nonlinear damper. The reference value of the force is assumed equal to the mean value of the damper force obtained for the various records at the seismic intensity $a_{0,\mathrm{ULS}}$, i.e., 490.64 kN for α= 1, and 295.16 kN for α= 0.15. The risk P_{D,T_L} that the uncertain demand D of displacement/force exceeds the corresponding reference value $d*$ during the time T_L is obtained by assuming a poissonian occurrence of the exceedance events as follows:

$$P_{D,T_L} = P\left(D \geq d^*\right) = 1 - \exp\left[v\left(D \geq d^*\right) T_L\right] \tag{9}$$

where $v\left(D \geq d^*\right)$ is the mean annual frequency of exceedance of the demand, expressed as:

$$v_D\left(D \geq d^*\right) = \int_0^{\infty} P_D\left(a_0\right) \cdot \left| dv\left(a_0\right)\right| \tag{10}$$

The expression of $v\left(a_0\right)$ is given by Eq.(8), and $P_D\left(a_0\right) = P\left(D \geq d^* \middle| a_0\right)$ denotes the probability of exceedance conditional on the seismic intensity level a_0. The displacement and force exceedance probabilities conditional to a_0 are plotted as continuous lines in Figure 6 and they can be interpreted to as fragility or vulnerability curves. These curves are derived by comparing, for each value of a_0, the lognormally distributed demand (Fig. 4) with the corresponding reference value. In Figure 6 also the stepwise fragility curve corresponding to the deterministic demand model is reported for comparison. It is worth to observe that, since for a lognormally distributed variable the mean value is always larger than the median value (see Fig. 4), at the design seismic intensity $a_{0,\mathrm{ULS}}$ the conditional probabilities of failure obtained by adopting a probabilistic demand model are lower than 0.5. This implies a local difference at $a_{0,\mathrm{ULS}}$ between the fragility curves of the linear and non linear case, as more evident in the case of damper forces (Fig. 6b). For what concerns the displacements, the difference between the fragility curves (Fig.6a) is notable only for a_0>0.4g and this is mainly due to the displacement demand in the nonlinear case that increases more than linearly by increasing the seismic intensity (see Fig.4b).

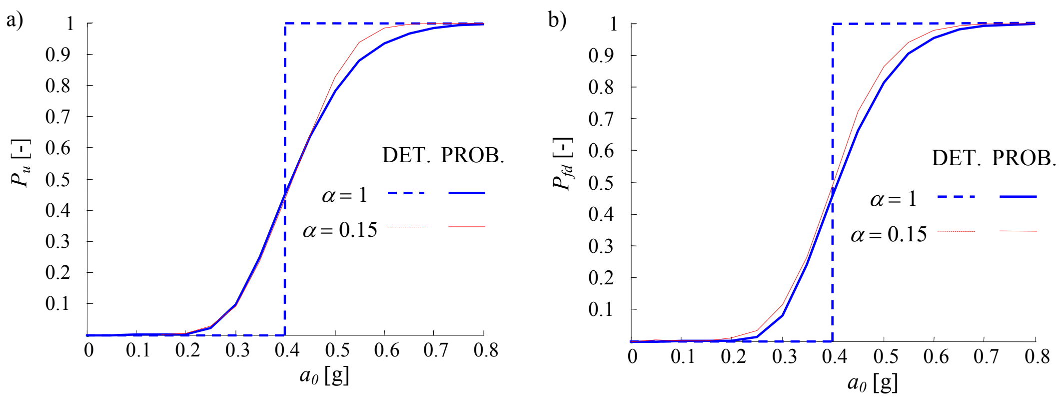

Figure 6. Variation with a_0 of the probability of exceeding the design value of the displacement (a) and of the damper force (b) according to the deterministic and the probabilistic approach.

Table 1 reports the probabilities of exceeding the reference values of the displacement and of the force during T_L=50 yrs according to the deterministic and the probabilistic demand model, for both the cases of the linear and nonlinear damper. It is noteworthy that according to the deterministic approach, the probability of exceeding the reference values of the demand is equal to the probability of exceeding the design intensity $a_{0,\mathrm{ULS}}$.

Table 1. Risk estimates according to the deterministic and probabilistic approach

Risk	Probabilistic approach		Deterministic approach	
	α = 1	α = 0.15	α = 1	α = 0.15
$P_{u,50}$ [-]	0.1138	0.1212	0.1000	0.1000
$P_{f_d,50}$ [-]	0.1112	0.1478	0.1000	0.1000

The risk estimates obtained with the probabilistic approach are always higher than the corresponding estimates obtained with the deterministic demand model based on the mean response values. In particular, the risk increases up to 20% for the displacements and to 50% for the damper forces by passing from the deterministic to the probabilistic approach. This is mainly the effect of the response dispersion, which increases the exceedance probability at low seismic intensities characterized by a high probability of occurrence.

Also some differences can be observed between the risk estimates obtained for the case of the linear and nonlinear damper by employing the probabilistic approach. In particular, the risk of exceeding the displacement reference value is only slightly higher for the nonlinear damper. In fact, the fragility curve for α = 0.15 assumes higher values compared to the fragility curve for α= 1 only at high seismic intensities (Fig. 4), which have a low probability of occurrence and thus provide a negligible contribution to the risk. On the other hand, the risk of exceeding the damper force reference value is significantly higher for the nonlinear case than for the linear case because of the higher vulnerability observed in Fig. 4 for all the a_0 values.

CONCLUSIONS

This paper analyzes the probabilistic characteristics of the seismic response of structural systems equipped with linear/nonlinear viscous dampers by considering a single-degree-of-freedom linear model for the structure with added damper. First, the characteristic parameters that control the system dynamic behaviour and seismic response are made explicit through the non-dimensionalization of the equation of motion. Then, an extensive parametric study considering a wide range of variation of these characteristic parameters is carried out to evaluate the influence of the damper properties on the probabilistic response of the system under a set of natural ground motions describing the record-to-record variability. The parametric study results are finally used to study the influence of the damper nonlinear behavior (as described by the damper velocity exponent), on the seismic reliability of structures equipped with viscous dampers. To this purpose, the probabilistic responses of a family of case studies, involving dampers with different velocity exponents and designed according to the same deterministic performance target, are evaluated and compared to each other. This performance target coincides with the mean displacement demand at the ultimate limit state (ULS) seismic intensity.

Based on the results of this investigation, the following conclusions can be drawn: *i*) at the ULS intensity, the family of case studies considered is characterized by a displacement dispersion higher in the nonlinear case than in the linear case, whereas the damper force dispersion is significantly lower in the nonlinear case than in the linear case (the dispersion of the normalized accelerations does not significantly depend on the damper nonlinearity level); *ii*) for increasing seismic intensities, in the case of nonlinear dampers the displacement demand increases more than linearly, whereas the damper forces increase less than linearly, *iii*) the dispersion of the displacement response decreases for increasing intensities, whereas the dispersion of the damper forces is in general very low and it decreases for increasing intensities; *iv*) at the damage limit state seismic intensity the nonlinear dampers provide higher reduction of the mean displacement response than the linear damper and also the response dispersion is higher for the nonlinear damper; *v*) the deterministic code approach for the seismic assessment/design of structures yields risk estimates that are lower than the corresponding estimates obtained through a probabilistic approach, and the safety levels observed in solutions obtained through a conventional deterministic design vary by varying the damper exponent and the response parameter considered.

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