



A COMPARATIVE STUDY OF ITALIAN CODE-BASED SEISMIC VULNERABILITY ASSESSMENT PROCEDURES FOR EXISTING BUILDINGS

Murathan PAKSOY¹, Elena MOLA², Franco MOLA³

ABSTRACT

The importance of seismic risk has recently been recognized by the public authorities and property owners in Italy. Older Italian reinforced concrete buildings form an important part of the building inventory and they represent a risk that must be identified. Most of these structures in Italy were designed without considering seismic-induced actions and seismic criteria for strength and ductility design. It is crucial to assess the seismic performance of these buildings under future earthquake actions.

Most of the earthquake countries have started to introduce performance-based approaches in their building codes. Recently, the Italian building code (D.M. 08) has been updated with significant changes regarding seismic design, assessment and retrofitting. The enhancements involved in different parts of the code mostly follow the recommendations provided in Eurocode 8.

In this paper, a comparative study is performed according to the Italian code-based linear seismic assessment methods and nonlinear procedures for a case study derived from an existing building, designed and built between 1965 and 1975 according to different regulations than today's. The present case study is slightly different from the assessed building regarding the geometrical configuration, seismic input parameters and because it was analysed both by using the linear response spectrum method and the nonlinear static procedure, as opposed to using the response spectrum method only. The case study building is a six-storey, non-seismically designed RC building with core walls. The building has been surveyed to confirm its as-built structural layout, member sizes and material properties. As mentioned above, the assessment study hereby presented is based on a thorough application of the recently declared Italian Building Code (D.M. 08), which allows both linear and non-linear analysis techniques for existing structures. Although the multi-modal response spectrum analysis is performed frequently by practitioners, the application of nonlinear static procedure for the assessment of seismic vulnerability of existing structures is also getting widely accepted in engineering office practice. Nevertheless, their success in predicting the response is extremely dependent on the building configuration. The main goal of the present study is thus to evaluate the seismic vulnerability of the building comparing the effectiveness of the linear dynamic and nonlinear static methods in estimating the seismic performance of the building in global and local response parameters.

The lateral force analyses and comparisons are carried out with ground motions representative of an hazard level of SLV defined in the current code with an approximately 1000 year return period, assuming that the building has critical importance for the users/owners.

¹Engineer, ECSD Consulting Engineering S.r.l., Milan, paksoy@ecsd.it

²Engineer, ECSD Consulting Engineering S.r.l., Milan, elena.mola@ecsd.it

³Full Professor, Department ABC, Politecnico di Milano, Milan, franco.mola@polimi.it

INTRODUCTION

There exist a large number of reinforced concrete buildings that are primarily designed for gravity loads because these structures were constructed in areas that were not considered seismic prone at the time of construction. Loads on these structures are low due to a low seismicity and expected to result in elastic structural response. However, under a seismic event the structure may also respond beyond its elastic limit.

By advancing capabilities of powerful software packages in design offices become more common, the linear dynamic procedure has become the standard analysis technique even for buildings for which codes allow the linear static procedure to be used to assess the structural response to lateral loads. The availability of fast computers has also made it possible for engineers to evaluate the expected seismic performance of a structure by employing nonlinear analysis. In the last decade, all of the major building codes have included nonlinear analysis within their method of analysis. All structural analysis techniques involve some approximations to true behaviour. The most advanced form of analysis is the non-linear time history analysis, which provides realistic predictions of demands on individual components loaded beyond their elastic range. However, this analysis has well documented drawbacks, in particular the selection of appropriate time history input. For this reason, as mentioned above, the most commonly used procedure among Italian practitioners for the evaluation of existing buildings is to implement a linear dynamic procedure (i.e. response spectrum analysis), followed, if more refined results are required, by the non-linear static procedure (i.e. pushover analysis). The latter is not as commonly adopted as the former, since it requires a deeper knowledge of structure, requires more adequate guidelines to implement and is considered as time-consuming.

The Italian seismic code defines four limit states related to structural and non-structural damage: Collapse prevention (SLC), Life safety (SLV), Immediate occupancy (SLD), Operational (SLO). As the Italian code does not enforce a fully performance based approach yet, each rehabilitation goal is required to represent one target performance level for a given earthquake hazard level. The objective of the current work is to compare the most common and most sophisticated assessment procedures performed in engineering offices in Italy.

CASE STUDY

A 22 m tall reinforced concrete building located in Northern Italy was assessed according to the D.M. 08. The dimensions of the building, typical plan and side view are shown in Figure 1. The building has six stories and a basement with typical story height 3.3 m except the 1st floor, which is higher. The building was designed according to the Building Code enforced in Italy in the 1970s.

The building is rectangular in plan; with approximate dimensions 17.5 x 43.5 m. It has both moment resisting frames and structural walls. All peripheral beams are 100 cm wide and 25 cm deep while the internal beams are 150 cm wide. The column sections keep the same dimensions above the 2nd floor with reduced reinforcement content along the height of the building. Structural walls are approximately 20 cm thick along the height of the building. The capacity of the existing RC walls is not clear since they were most probably not designed to contribute to the lateral load resistance but as core walls to accommodate the elevators and the stairs of the building. From the available design data and on site testing, the strength of concrete is 20 MPa and reinforcement is 420 MPa. The design seismic load is calculated based on D.M. 08 and defined for a soil category B with an intensity level 0.23g (Figure 2).

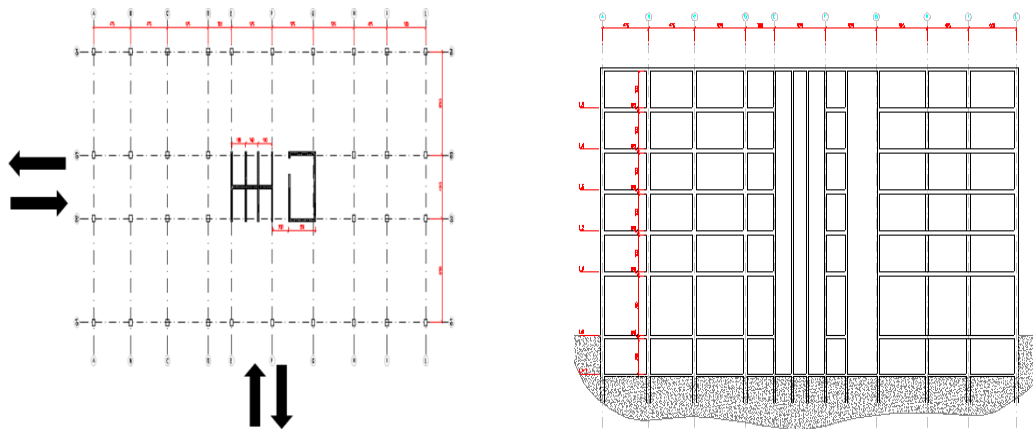


Figure 1 (a) Plan View, (b) Lateral View

SCREENING PHASE

When assessing an existing building dating back forty or fifty years in Italy, one of the most common drawbacks includes getting hold of the original design drawings and reports. For this reason, the work presented in this paper was carried out based on the assumption that the available information about the original design was not complete. In particular, the design drawings of the building were available, but information regarding material properties and the original calculations were missing. In order to evaluate the structure in the most representative way, a limited testing of the materials was thus required. In the present case study the requested samples were kept to a minimum, in order to perform field investigation as representative as possible of the common practice in Italy: in order to limit disruption of use and the costs of an experimental campaign, in fact, the owners and the practitioners usually agree on minimally invasive testing activities, even if, as a drawback, a limited knowledge of the material properties would result into the use of more severe reductive coefficients (i.e. confidence factors) when performing the assessment. Destructive examinations were limited to concrete only and conducted per floor to define the material properties. Pacometric tests were performed in order to evaluate reinforcement positions and accurately select the zones on which cores were performed. Since the extraction is invasive and the steel reinforcing bar characteristics were evaluated to be appropriate, steel reinforcement bar samples were not removed from the structure. In order to account for the uncertainties, related to geometry, amount and arrangement of bars, material properties etc., arising in the assessment of existing structures, D.M. 08 introduces confidence factors to reduce the mean material strengths. With the above mentioned tests and the available drawings, general condition of the building was assumed to be adequately known to adopt a confidence factor corresponding to Knowledge Level 2, as defined in D.M. 08. The value of confidence factor for this level is 1.2 and the coefficient is used to penalize nominal material properties when computing the capacity of the elements.

In the initial stages of the assessment of an existing building, it is also necessary to know how the design requirements were at the time of the design. In comparing the previous requirements and current standards, it is crucial to consider many interacting factors. For example, in Italy, in the 1970s, all the calculations were performed using the ‘allowable stress design’ method, so in order to confirm all of the missing details of reinforcements, the redesign was carried out according to the former regulations. A correct redesign is an important phase of the assessment process since it complements the information derived from the experimental tests.

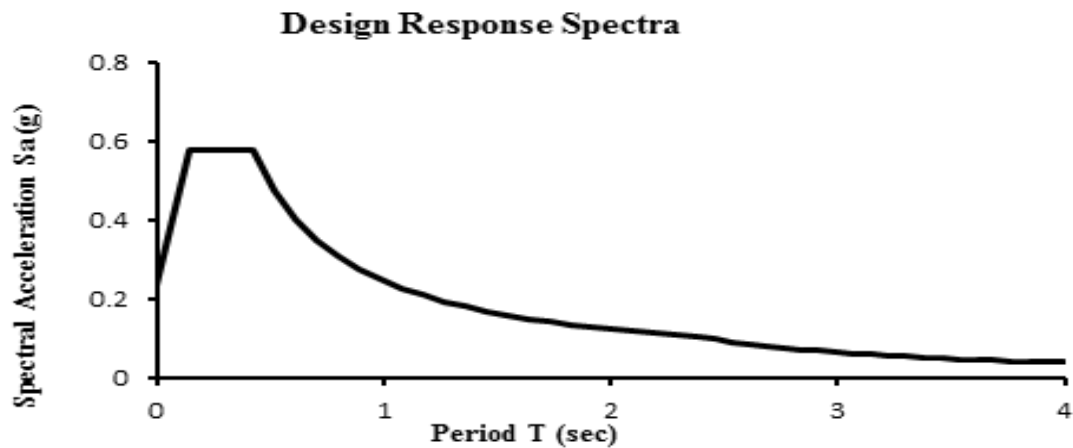


Figure 2 Response spectra for design earthquake

NUMERICAL ANALYSIS

The major phase of the assessment, after all of the available and collected data has been surveyed, is the numerical analysis. In the present case study, the structural analysis software used to perform both the response spectrum and the pushover analyses is Midas Gen v.13. The three dimensional model was built with a space frame model assuming centreline dimensions. Model I was created for linear elastic analysis and Model II for nonlinear inelastic analysis.

The modelling procedure is based on macro-modelling techniques, where the force-deflection function for a complete element is used to define the response. Flexural hinges at the extreme ends of each element were characterized by a moment-rotation relationship ($M-\Theta$). The definition of the hinge properties was derived from moment-curvature analysis of each element. In this study the ultimate curvature was defined as the curvature corresponding to the extreme compression fiber reaching the ultimate compressive strain. Following the definition of hinge properties of the elements, three levels of acceptance criteria were defined at yielding, $3/4\Theta_u$ and Θ_u . To account for the shear behaviour, shear hinges were added in parallel to each flexural hinge. In Model II, a similar refinement as would be used for linear elastic analysis was used to ensure that for an elastic level of response the procedure would remain as accurate as less sophisticated method of analysis.

Flexural elements were used to model beams and columns. These elements are assumed to be prismatic and yielding can occur at concentrated plastic hinges at element ends only. The axial load moment interaction was accounted for by means of modelling M-N yield surfaces. The beams and columns in the building are typical without irregularity so each beam and column was modelled with single elements. In Model II, the modelling technique that was selected for structural walls is known as wide column modelling. The web and flange sections were represented by vertical elements located at the centroid of the web and flange sections. These vertical elements were then connected by horizontal links along the weak axis of the sections. PMM hinges with axial force-moment interaction were assigned at the wall ends near floor levels and shear hinges were assigned at the mid-height level of the walls. On the other hand, in Model I, structural walls were modelled using 'wall' elements which are included in the finite element library of Midas Gen. Wall element is generated by the combination of rectangular 4 node plane stress element with beam elements at the boundaries to provide the rotational degree of freedom and stiffness about the axis perpendicular to the plane of the element. The element is capable to resist the in-plane forces and moments about the axis normal to the wall plane.

A compressive mean strength of 20 MPa was considered for the reinforced concrete frame while 32 MPa for the structural walls. To determine the force deformation relationship based on plastic hinge analysis, moment-curvature analysis were carried out on cross sections. The reinforcement was modeled in a bilinear way ignoring strain hardening. The confinement effect by the transverse reinforcement was calculated by the rules proposed by Mander et al. Since the calculations based on

Mander revealed the existing confinement is not effective, it was decided to neglect confinement effect. The presence of floors was modelled with diaphragm constraints. The basement of the building is assumed to be on stiff soil and is surrounded by basement wall along the perimeter of the building. The quality of the stiff soil at the back of the base wall was judged to be able to transfer the seismic forces near grade so at street level the columns were modelled with full restraint. No interaction with soil was taken into account.

As seen due to the shape of the wall plans, separate pushover analyses in positive and negative directions were needed. For seismic assessment, stiffness of uncracked member is irrelevant so in the analysis cracked member stiffnesses were employed. For all of the wall elements, cracked sections were assumed, with an effective stiffness equal to 50 % of the gross section. Considering that structural walls have a height to length ratio above 3, the lateral response of the walls is expected to be dominated by flexure.

The coupled shear wall system, shown in Figure 3, is a popular lateral load resisting form for multi-story buildings. In this system, generally two walls in the same plane are connected at the floor levels. Coupled shear wall systems resist lateral loads by cantilever bending action, which results in rotation of the wall cross-sections. The bending of the interconnected walls is resisted by the floor slabs. Due to the large width of the wall, considerable differential shear action is imposed on the slab where they are connected to the walls. It is noted that the moment is decreased to zero at the point of contra-flexure in the centre of the corridor opening from maximum value at the centre of the wall.

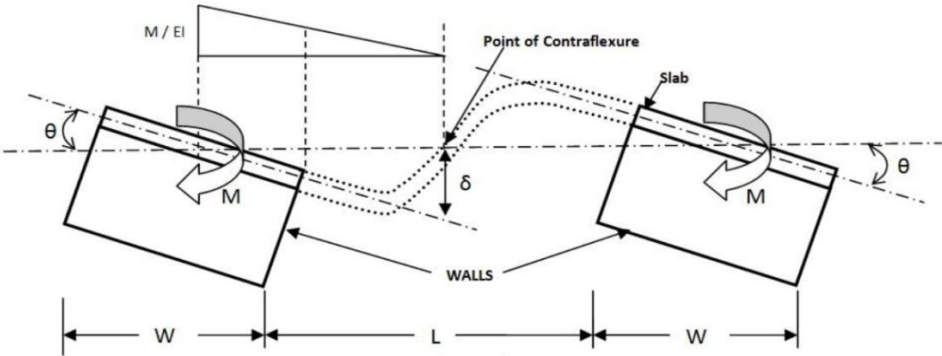


Figure 3 The response under lateral action at coupling elements

Keeping in mind the above mentioned resisting behaviour, three modes of failure in RC coupled shear wall structures can be identified, basically depending upon the interaction and the behaviour of the coupling slab beams. In this particular case, the relatively thin coupling beams with a small amount of main reinforcing steel will develop flexural cracks. Under increasing seismic load the cracking will progress into the slab and the deeper into the wall.

METHOD OF ANALYSIS: LINEAR ANALYSIS

In order to be able to use linear methods in the assessment procedure, the Italian Seismic Code requires conditions of applicability to be complied with. In particular, the distribution of the ratio of bending moment demands to the corresponding capacities was monitored in all the structural elements. If this condition does not satisfy a limit value, the load path will change significantly when nonlinear mechanisms develop and the information obtained from the linear analysis will not be accurate. In order to evaluate the flexural hinging hierarchy, all the elements of the model having $M_D/M_R \geq 2$ are evaluated and $\max(M_D/M_R)/\min(M_D/M_R)$ has to be verified to be less than 2.5.

The Italian Seismic Code also requires for each structural member, flexural hinging at the end of members to precede shear failure of the members. A brittle failure of any member may change all the load distribution in the building thus making the linear analysis results irrelevant. Considering each

end section of structural members, shear capacity of the section was compared to the corresponding shear demand and the building elements were verified to satisfy this condition.

According to the linear assessment procedure in D.M. 08, ductile modes should be checked in terms of chord rotation while the brittle modes should be checked in terms of shear. The ductile mechanisms are checked through the evaluation of the chord rotation demand and the capacity at the ends of each structural element. The capacity is evaluated with the mean values of concrete strength f_{cm} and reinforcement bar strength f_{ym} reduced by the confidence factor CF. As shown in Figure 4, the chord rotation is the angle between the chord connecting the end section of the member to the section at which $M=0$ and the tangent to the member axis at the end section. If M and V are the bending moment and shear demands at the considered node of the member, $L_s = M/V$.

The chord rotation demand for the column ends was calculated as the drift at the point of contraflexure. On the other hand, in beams, the chord rotation was defined as the nodal rotation at the member end. The sources of chord rotation in beams are gravity loads and seismic loads. As the seismic loading can reverse its sign, the overall chord rotation is dependent on the superposition of gravity and seismic rotations. In order to simplify the calculations, the chord rotation of the beams due to the lateral loading only was considered in the calculations since in comparison, the amount of rotation due to gravity turned out to be practically negligible.

$$\theta = \frac{\Delta}{L_s} \quad (1)$$

According to the Italian Code, the chord rotation capacity at the ‘significant damage’ limit state is evaluated as $\frac{3}{4}$ of the value determined at the collapse limit state. In order to compute the chord rotation capacity, the yield curvature and rotation have to be assessed. The condition of yielding of a reinforced concrete section is considered to correspond to the yielding of the tensile longitudinal reinforcement or in case of high axial loads yielding of concrete under compression at a strain limit of $\varepsilon = 0.002$. Two different approaches are proposed to evaluate the chord rotation capacity in the code. In this study, the theoretical approach is used and described by means of the following expression to state the elastic and plastic rotation.

$$\theta_y = \phi_y \frac{L_v}{3} + 0.0013(1 + 1.5 \frac{h}{L_v}) + 0.13\phi_y \frac{(d_b f_{ym}/CF)}{\sqrt{\frac{f_{cm}}{CF}}} \quad (2)$$

$$\theta_u = \frac{1}{1.5} [\theta_y + (\phi_u - \phi_y) L_{pl} (1 - \frac{L_{pl}}{2L_s})] \quad (3)$$

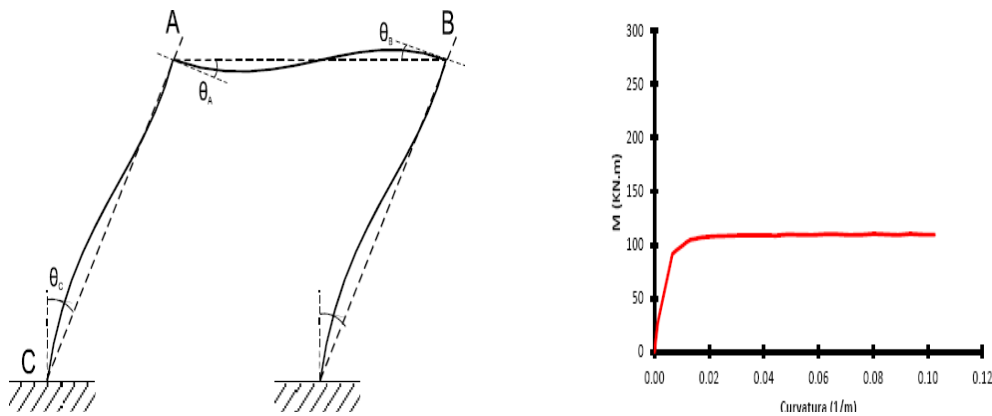


Figure 4 (a) Definition of chord rotation at member ends (b) Typical Moment curvature response for axially loaded member

The brittle mechanisms are assessed at a sectional level. The Italian Building code evaluates the shear capacity of sections as it does for the non-seismic case. According to the D.M. 08, the theory behind the shear resisting mechanism is based on Morsch-Ritter truss. V_R is therefore computed as the minimum of action that causes shear reinforcement to yield or the crushing of compressive strut.

$$V_R = \min(V_c + V_w, V_{\text{crushing}}) \quad (4)$$

In comparing the shear capacity to the shear demand at the section, care should be taken not to obtain V_D directly from the analysis results. Since, in the elastic analysis, action effects increase linearly, the nonlinear mechanism was taken into account manually. Once the moment demand at a section was larger than the section flexural capacity, the shear demand at that section was calculated through the equilibrium of the element.

METHOD OF ANALYSIS: NONLINEAR STATIC ANALYSIS

Nonlinear static procedure is a practical tool to perform nonlinear analysis. Academic studies in the literature show its good performance in regular buildings and frame buildings (Pinho R. et al., 2012). In the present paper, the procedure is used for a core wall building. The Italian seismic code states that nonlinear static analysis is the reference method in the assessment of existing buildings. D.M. 08 recommends the N2 method developed by Fajfar when a pushover analysis is performed.

The code requires lateral forces to be applied in one of the two ways: a uniform pattern, a modal pattern. As shown in Figure 1 along the two principal axis of the building, 8 different pushover analyses were thus performed, including positive and negative global directions for both kind of loading. The control node to monitor the displacement of the building was selected at the centre of mass of the top floor. The maximum expected control node displacement was evaluated. Midas provides the user two types of pushover analysis control options to manage the load increment: i.e., load control and displacement control. In the present paper, a displacement control based analysis was performed. With this method, nodal displacements increase step by step through a factor which is kept constant throughout the building. Then lateral forces to achieve those displacements are evaluated. The shortcoming of this control is that the displacement pattern cannot be updated during the analysis. Although, in theory, a more advanced control type exists for pushover analysis, the commercial software in use does not provide this option.

The case study building has a good and compact shape, however the location of the structural walls at the centre of the building increases the percentage of modal mass associated to the rotation around the vertical axis. It means Φ_i is not expected to be unidirectional. However, the modal pattern of lateral forces was taken as unidirectional, proportional to the displacements of the 1st mode of the considered direction. Although the nature of seismic action is bidirectional, the Italian seismic code suggests evaluating the results of 8 pushover analyses separately. This results in evaluating the internal actions for each end section of structural members 8 times. At the end of this screening procedure, the highest internal action was considered to govern the assessment. The deformation-controlled and force-controlled actions were checked for the SLV limit state at the target displacement. The deformation-controlled actions were checked with the $\frac{3}{4}$ of the ultimate rotation capacity at the member ends.

LINEAR ANALYSIS RESULTS

Dynamic linear analysis was performed by multi-mode response spectrum analysis, implemented for Model I. The first step in the modal response spectrum analysis was the computation of the natural mode shapes and periods of vibrations as shown in Figure 5. The modal properties of the analysed building are presented in Table 1, which shows the periods and effective modal mass percentages of the modes of interest, i.e. those necessary in order to get a cumulative mass at least

equal to 85% for translations and rotation around the vertical axis. The building has a fundamental mode of vibration of 0.91 s along the Y direction and a second mode of 0.66 s along the X direction.

The horizontal seismic action was represented by a 5% damped elastic pseudo-response spectrum. As mentioned in the previous sections, according to the regulations of D.M. 08, the level of knowledge for the project was identified as level 2. For the calculation of the capacity of ductile elements, an average value of the properties of existing materials was thus reduced by the confidence factor corresponding to knowledge level 2, i.e. 1.2. In the capacity evaluation of the fragile elements, in addition to the confidence factors the material properties were also reduced by the partial factors. To determine the likelihood of yielding, demand to capacity ratios (DCR) were computed for each element as shown in Figure 6. After the verifications, several observations were made regarding the expected earthquake response of the analysed building:

- The sequence of yielding progresses from lower-level beams to the upper levels.
- With the exception of the first level columns, the beams yield before the columns.
- Some of the ground storey columns develop flexural hinges.
- None of the elements develops shear hinges.
- All flexural elements except the coupling beams between structural walls satisfy the acceptance criteria in the significant damage limit state (SLV)

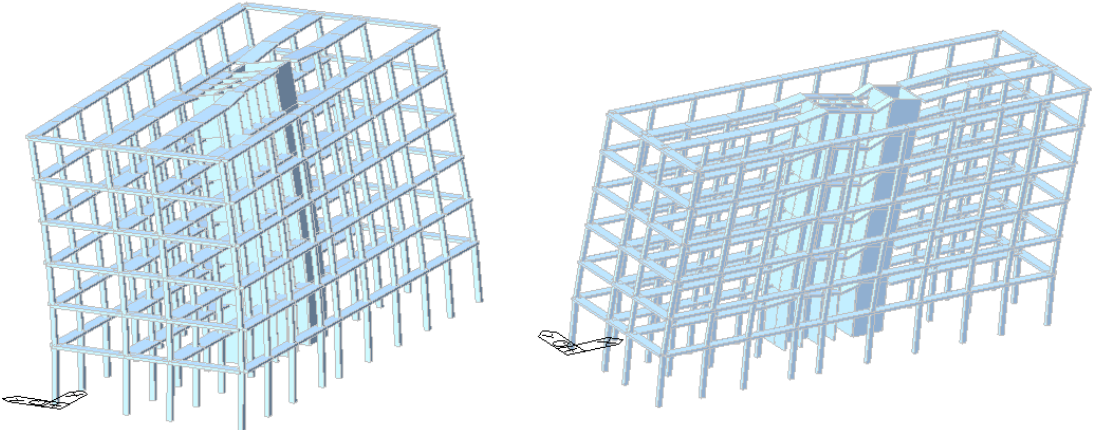


Figure 5 Translational vibration modes of the analysed building (a) 1st, (b) 2nd

Table 1 Period and effective modal mass percentages

Mode No	PERIOD	TRAN-X (%)	TRAN-Y (%)	ROTN-Z (%)
1	1.42	0.00	6.52	74.67
2	0.91	0.00	64.14	7.27
3	0.66	75.94	0.00	0.00
4	0.41	0.00	1.01	11.34
5	0.21	0.00	0.43	4.53
6	0.17	0.00	19.96	0.18
7	0.17	18.87	0.00	0.00

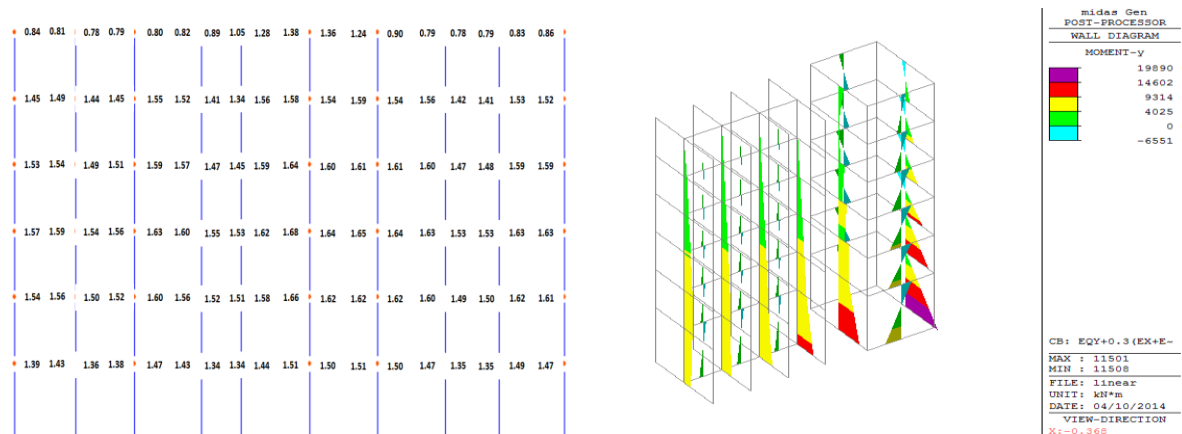


Figure 6 (a) DCRs of peripheral frame beams, (b) Core wall bending moments for seismic combination along Y

NONLINEAR ANALYSIS RESULTS

How reliable the response quantities computed by means of pushover analysis are, actually depends on how well the local deformations due to the assumed load vector correspond to the actual deformation pattern due to natural seismic action. In the case at hand, the results reveal that the accuracy of the response quantities estimated from the pushover procedure depends on the number of considered curves and that the difference between computed quantities can be remarkable.

As shown in Figure 1, two different load patterns were applied on the structure. The loads were applied independently in the global X and Y directions resulting in 8 different analyses. As the actual inelastic response in existing structure is unknown, it can only be said that this actual response lies between the results obtained from two different forcing patterns. For each pushover analysis, a corresponding capacity curve was thus obtained. A target displacement (Table 2) was then assigned to each capacity curve for the ‘significant damage’ limit state. The deformations and action effects at each member were finally evaluated at the target displacement for each curve. The comparative results are reported in Figure 7-9.

The procedure to estimate the maximum seismic response of the MDOF system loaded by uniform lateral forces is shown for X and Y directions as below. Different from the fundamental mode loading, in uniform loading eigenvector becomes equal to 1, so the SDOF and MDOF systems become equivalent.

$$F^* = \frac{F}{\Gamma_n} \quad (5)$$

$$\Delta^* = \frac{\Delta}{\Gamma_n} \quad (6)$$

$$\Gamma_n = \frac{\phi^T [M] \{1\}}{\phi^T [M] \phi} \quad (7)$$

Table 2 Response parameters used in capacity spectrum method

Parameters	Y direction	X direction
F_y^* (KN)	7632.49	9731.17
T^* (sec)	1.06	0.66
$S_e(T^*)$ (g)	0.231	0.375
d_{et}^* (m)	0.065	0.041
d_t^* (m)	0.065	0.041

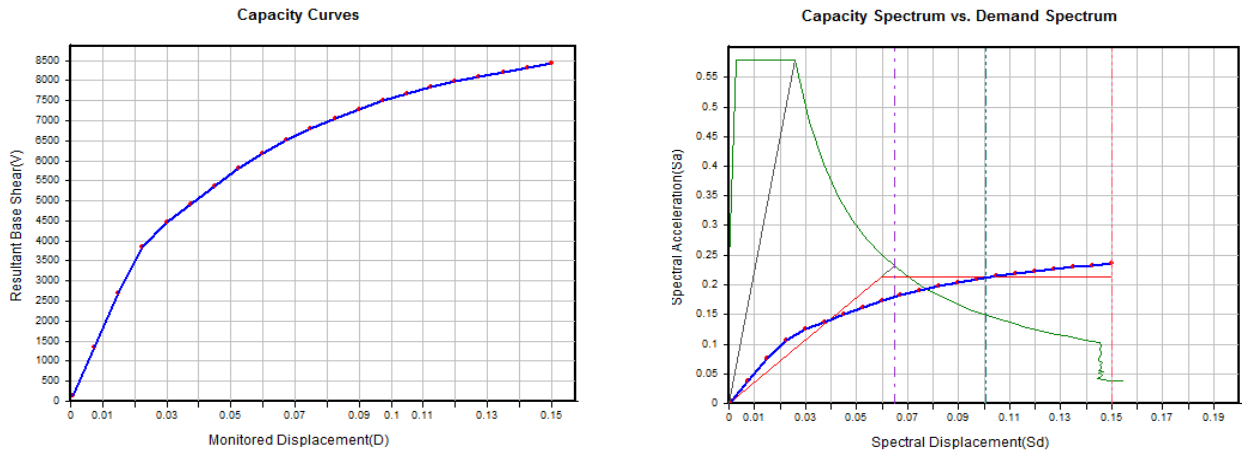


Figure 7 Pushover Y direction (a) Force deformation relationship of MDOF, (b) Capacity curve for SDOF and elastic design spectrum in SLV limit state

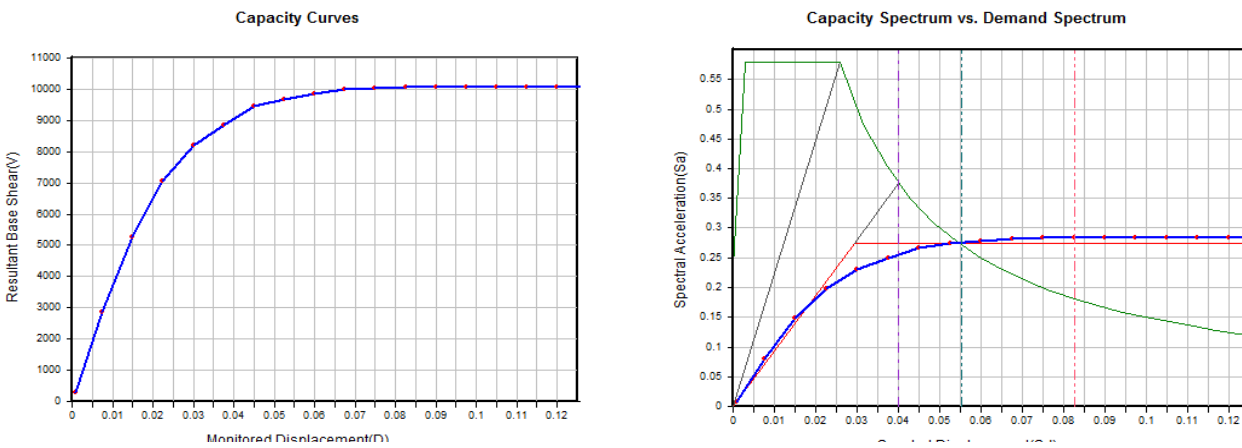


Figure 8 Pushover X direction (a) Force deformation relationship of MDOF, (b) Capacity curve for SDOF and elastic design spectrum in SLV limit state

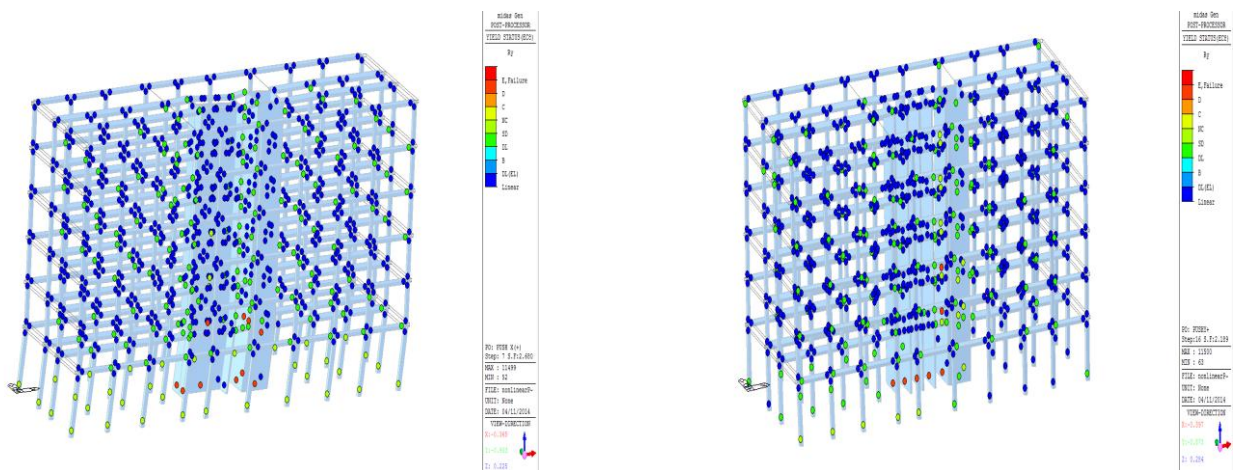


Figure 9 Maximum seismic response of the system when the control point reaches the target displacement (a) X direction under uniform load (b) Y direction under uniform load

According to the results obtained from the nonlinear analysis, the building did not exhibit a fragile failure. Through pushover analysis, the system behaviour was observed to undergo inelastic deformations however with its inherent limited ductility capacity the building was able to satisfy the

SLV limit state requirements with few local exceptions. By observing yielding of most of the vertical and horizontal elements at the ground story level, the results from the linear dynamic analysis were also confirmed. The analysis in elastic and inelastic phases has shown that the building performance under the considered excitation level is satisfactory.

CONCLUSION

In this paper, two different seismic assessment approaches suggested by the Italian Building Code D.M. 08 were applied to a case study building. The investigation was aimed at understanding the amount of additional information gained by employing a more sophisticated analysis method and to compare the level of conservatism in both of the methods for a medium height core wall building designed for gravity loads only.

In this particular study, the results of the linear dynamic procedure were more conservative compared to the pushover results. Though the number of primary seismic elements reaching significant damage state from both of the analyses was almost the same, the number of primary seismic elements above the ‘damage limitation’ state was estimated to be 15% higher than the one derived from the nonlinear static analysis. This may happen because of the higher modes contribution in the dynamic analysis, as most of these elements were detected at the edges of the building. Another issue which should be highlighted is how the pushover analysis results are evaluated and combined. In fact, even though earthquakes are never unidirectional, the nonlinear static procedure in the Italian code does not take this into account; as a result, 8 pushover curves are determined and for each of them all of the structural elements are evaluated, then the worst condition for each element is assumed to govern the failure. This might not be the most realistic way to take into account earthquake directionality effects in the nonlinear range of behaviour, even in roughly regular buildings.

Pushover procedures may be numerically expensive and may cause difficulties depending on the software availabilities and modelling approach of the engineer. A reliable pushover analysis is technically more complex than a linear dynamic analysis but it is also easier to implement compared to the nonlinear dynamic (i.e. time history) analysis. Moreover, in order to perform linear dynamic procedures the Italian code, similar to many other codes, requires the engineer to do extra calculations before starting the analysis to check its applicability. This results in additional man hours to complete the calculations. The case study building composed of frames and core walls of different lengths may cause the analyst to develop a very complicated model depending on the objective of the engineer. In fact for the case study building, the yielding instant of the elements, post-yield base shear distribution between the walls could be complicated to predict by lumped plasticity model however the system’s response was obtained accurately. For buildings having a similar configuration to the case study, it may be worth skipping the applicability checks of multimodal response spectrum analysis, by directly applying the linear dynamic methods and using the saved time to also apply a pushover analysis to have two sets of results complementing each other, resulting in a more thorough assessment procedure.

REFERENCES

- ATC (2005) FEMA440 Improvement of nonlinear static seismic analysis procedures, Applied Technology Council, Redwood, CA
- D.M. (2008) Italian Code for Structural Design, D.M. 14/01/2008, Official Bulletin no. 29 of 4/02/2008 (In Italian)
- EC8 Design of structures for earthquake resistance, European Committee for Standardisation Eurocode 8, 2005
- Fajfar P and Gasperic P “*The N2 method for the seismic damage analysis of RC buildings*” *Earthquake Engineering and Structural Dynamics* 1996, 25:23-67
- Kwan A [1993] “*Improved wide-column-frame analogy for shear/core wall analysis*” *Journal of Structural Engineering*, ASCE 119(2), 420–437
- Pinho R et al. (2012) “*Evaluation of Nonlinear Static Procedures in the Assessment of Building Frames*” *Earthquake Spectra*