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P-DELTA EFFECTS IN DISPLACEMENT BASED ASSESSMENT OF R.C. HINGED FRAMES

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ABSTRACT

In the last two decades, several performance based design approaches were developed and recently applied to the seismic assessment of existing structures. Among these procedures, increasing attention was placed on displacement based assessment methodology, which considers structural displacements, in terms of inter-story and roof drift, and material strain limits as the main seismic vulnerability indicators. This approach is based on the substitute structure theory and the structural response is evaluated by means of an equivalent single degree of freedom (SDOF) system, which accounts for the inelastic behaviour of the building using an effective stiffness and an Equivalent Viscous Damping (EVD). In this context, the appropriate definition and response evaluation of the equivalent SDOF system is fundamental, as it significantly affects the results.

During an earthquake, the horizontal displacements achieved by the structure can significantly increase P-Δ effects, especially in the case of laterally deformable buildings, as the R.C. hinged frames considered herein. The displacement based assessment procedure should take into account the contribution of second order effects, being P-Δ effects influencing the SDOF system both in terms of shear-displacement relationship and in terms of EVD. In the literature, several EVD formulations are available, accounting for the non-linear force-displacement SDOF response associated to various hysteretic models: in the case of R.C. frame structures the Takeda model is commonly adopted with a post-yield stiffness ratio of 0.05 typically selected. Considering that P-Δ effects reduce the post-yield stiffness of the force-displacement curve, the aforementioned formulations could underestimate both the equivalent viscous damping values and the effective stiffness at maximum displacement adopted in the assessment procedure.

A parametric study is carried out to investigate the effective EVD accounting for P-Δ effects. The study is based on the evaluation of the dynamic response of a series of non-linear SDOF systems changing the post-yield stiffness ratio, while keeping constant the displacement ductility μ_Δ and the effective period T_{eff}. According to the parametric analyses results, new formulations are proposed in order to include P-Δ effects in the common displacement based assessment procedure.

INTRODUCTION

In past years a lot of effort was put in investigating the influence of P-Δ effects on the structural performance of buildings under seismic type excitation (Bernal 1987, Fenwick et al. 1992, Priestley et al. 1996, Pettinga and Priestley 2008 among others). P-Δ effects are basically second order bending moments associated to the vertical gravity load equilibrium with respect to the structural deflected

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shape (Fig.1). As a consequence the same lateral displacement is reached at lower lateral forces when P-Δ effects are included; in other terms the structure generally experiences higher lateral displacements under the same earthquake if P-Δ effects are considered (Fig.2a).

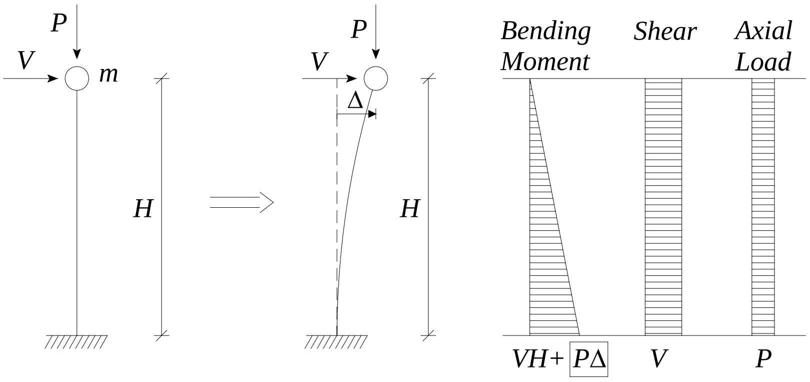
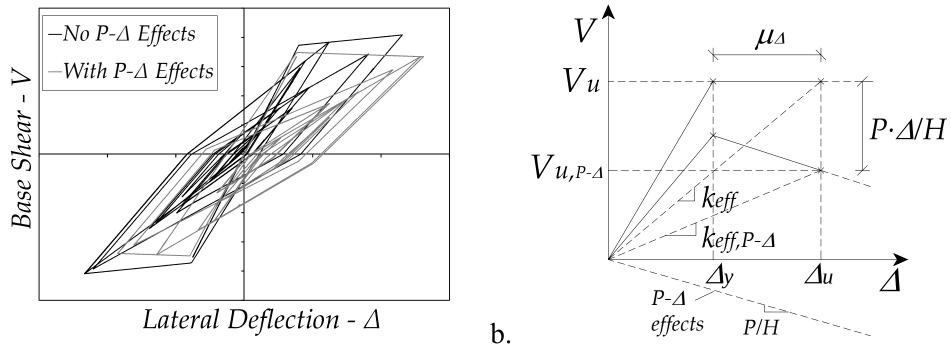


Figure 1. Increase of bending moment demand due to P-Δ effects



a. b.

Figure 2. a. Lateral displacement increase due to P-Δ effects
b. V-Δ curve modification due to P-Δ effects

P-Δ effects are significantly influenced by the type of hysteresis of the lateral force resisting system structural elements. It has been shown (Priestley et al. 1996) that for elasto-plastic behaviour there is a tendency to accumulate residual displacements in a preferential direction which could cause structural instability and failure. This does not happen for instance in the case of Takeda degrading stiffness model.

Considering a nonlinear behaviour of the SDOF system of Fig.1 characterized by the development of a flexural plastic hinge at the base, it is observed that the system rotation/displacement associated to a selected limit state is the same including or not including P-Δ effects. In addition the moment-rotation relationship is not affected by P-Δ, only the force-displacement loops change in shape due to P-Δ but maintain the same hysteretic energy. For a given lateral deflection (Δ_u), the equilibrium of the system is reached with a lower base shear (Fig.2b). The reduced base shear including P-Δ ($V_{u,P\text{-}\Delta}$) is calculated equating the base moment of the system considering ($M_{u,P\text{-}\Delta}$) and not considering (M_u) P-Δ effects.

$$M_u = M_{u,P-\Delta} \;\Rightarrow\; V_u \cdot H = V_{u,P-\Delta} \cdot H + P \cdot \Delta \;\Rightarrow\; V_{u,P-\Delta} = V_u - \frac{P \cdot \Delta}{H} \qquad (1)$$

In recent years seismic design and assessment procedures have been developed considerably, moving the attention to a Performance Based approach rather than solely comparing the element moment/force capacity with the seismic demand. These new approaches allow to associate limit states to seismic events with a defined probability of occurrence. The purpose of this research is the development of an effective way to include P-Δ effects in the assessment of existing structures. The assessment procedure

followed herein, Displacement Based Assessment (DBA), is based on the Direct Displacement Based Design (DDBD) procedure developed by Priestley (Priestley et al. 2007).
After introducing the basis of the Displacement Based Assessment, the paper considers how to specifically take into account P-Δ effects in the procedure. The DBA procedure including P-Δ is applied to a three story R.C. hinged frame structure and validated by means of nonlinear dynamic analyses. The attention is placed on R.C. hinged frames due to their higher lateral flexibility, therefore leading to high P-Δ effects, compared to traditional R.C. frames.

DISPLACEMENT BASED ASSESSMENT PROCEDURE

As stated before, the Displacement Based Assessment (DBA) procedure considered herein is directly derived from the Direct Displacement Based Design (DDBD) procedure developed by Priestley (Priestley et al. 2007). The first DBA step is the definition of the structural deflected shape resembling the fundamental inelastic vibration mode. The deflected shape allows the definition of the parameters of an elastic SDOF substitute structure with stiffness equal to the secant stiffness of the original structure at a selected target displacement. Pushover analysis represents the most efficient way to take into account structural nonlinearities in the inelastic deflected shape definition. The pushover force-displacement curve is bilinearized and the deflected shape at yielding ($\Delta_{y,i}$) is used to evaluate the substitute structure yield displacement ($\Delta_{y,ss}$):

$$\Delta_{y,ss} = \frac{\sum m_i \cdot \left(\Delta_{y,i}\right)^2}{\sum m_i \cdot \Delta_{y,i}} \tag{2}$$

Where m_i and $\Delta_{y,i}$ are the mass and lateral displacement at yield corresponding to the i^{th} floor respectively. The ratio between the selected target displacement, corresponding to a chosen limit state, and the yield displacement, both identified in the pushover curve, corresponds to the displacement ductility μ_Δ used to calculate the substitute structure target displacement ($\Delta_{u,ss}$):

$$\Delta_{u,ss} = \Delta_{y,ss} \cdot \mu_\Delta \tag{3}$$

The effective mass m_{eff}, stiffness k_{eff} and period T_{eff} of the SDOF substitute structure are:

$$m_{eff} = \frac{\sum m_i \cdot \Delta_{u,i}}{\Delta_{u,ss}}; \quad k_{eff} = \frac{V_u}{\Delta_{u,ss}}; \quad T_{eff} = 2 \cdot \pi \cdot \sqrt{\frac{m_{eff}}{k_{eff}}} \tag{4, 5, 6}$$

The point corresponding to T_{eff} and $\Delta_{u,ss}$ lies on the damped displacement spectrum ($S_{D,in}$). The elastic displacement spectrum is obtained from Eq.(7) (CEN 2005) once the Equivalent Viscous Damping (EVD), ξ_{eq}, is defined. Different EVD formulations are available in the literature, as for instance in Grant et al. (2004) or in Priestley et al. (2007).

$$\eta = \frac{S_{D,in}(T_0, \mu_\Delta)}{S_{D,el}(T_{eff})} = \sqrt{\frac{10}{5+\xi_{eq}}} \tag{7}$$

The return period (T_R) associated to the considered limit state is obtained from Eq.(8): interpolation between two known T_R – PGA (Peak Ground Acceleration) couples (T_{R1}-PGA_1 and T_{R2}-PGA_2) once the PGA associated to the elastic displacement spectrum, from Eq.(7), has been defined.

$$T_R = T_{R1} \cdot e^{\left[\ln\left(\frac{T_{R2}}{T_{R1}}\right) \Big/ \ln\left(\frac{PGA_2}{PGA_1}\right)\right] \cdot \left[\ln(PGA) - \ln(PGA_1)\right]} \tag{8}$$

ACCOUNTING FOR P-Δ EFFECTS IN DBA

To account for P-Δ effects in the DBA procedure, the first step is the inclusion of second order moments in the development of the substitute structure capacity curve. This is accomplished directly in the pushover analysis of the Multi Degree of Freedom (MDOF) system. The chosen method to bilinearize the capacity curve needs to allow for negative post-yield stiffness.

The target displacement of the substitute SDOF system including P-Δ is characterized by a lower effective stiffness ($k_{eff,P-\Delta}$ in Fig.2b) compared to the case without P-Δ (k_{eff} in Fig.2b), which leads to an increase of the effective period T_{eff}.

The second step is the evaluation of the EVD associated to T_{eff} and μ_Δ. It is worth mentioning that the available formulations (Grant et al. 2004, Priestley et al. 2007) adopted in the definition of the EVD have been calibrated based on the force-displacement response of inelastic SDOF systems with positive post yield stiffness ratio (r), typically r=0.05. Therefore if these formulations are directly applied, the considered SDOF system will be the one referred to as Curve B in Fig.3 instead of Curve A, which represents the actual SDOF system including P-Δ. This leads to a net hysteretic energy underestimation, and consequently EVD underestimation, for Curve B SDOF system compared to Curve A.

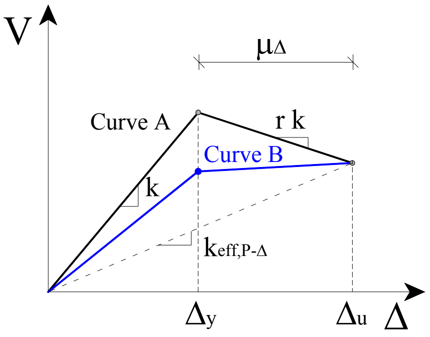


Figure 3. Different SDOF systems associated to EVD evaluation including P-Δ effects

A parametric study has been carried out in order to determine the relationship between EVD values associated to Curve A and Curve B. The study is based on the evaluation of the response of a series of non-linear SDOF systems, Takeda hysteresis rule, with the same displacement ductility μ_Δ and effective period T_{eff}, but different values of post-yield stiffness ratio r (Fig.3).

The analyses involve the comparison of the dynamic response of two types of SDOF systems: the non-linear SDOF system including or not including P-Δ effects (Curve A and Curve B, Fig.3) and the elastic SDOF system with stiffness equal to $k_{eff,P-\Delta}$ (Fig.3). The time history analyses are performed considering 5% tangent stiffness Rayleigh damping for the non-linear systems and 5% relative damping for the elastic systems.

The EVD evaluation procedure is subdivided in the following 4 steps.

Step 1

The EVD of a system with r=0.05 is first evaluated (Curve B, Fig.3). The nonlinear SDOF system is subjected to a selected ground motion and the maximum shear and displacement, $V_{D,in}$ and $S_{D,in}$, are recorded. Subsequently, a linear elastic SDOF system with stiffness equal to $V_{D,in}/S_{D,in}$ (secant stiffness at maximum displacement) is subjected to the same ground motion and the maximum displacement $S_{D,el\ Step\ 1}$ is recorded. The ratio between $S_{D,in}$ and $S_{D,el\ Step\ 1}$ is equal to (CEN 2005):

$$\eta_{Step\ 1} = \frac{S_{D,in}}{S_{D,el\ Step\ 1}} = \sqrt{\frac{10}{5+\xi_{eq,z\ Step\ 1}}} \tag{9}$$

From this equation it is possible to determine $\xi_{eq,z\ Step\ 1}$.

Step 2

In this step a nonlinear SDOF system with negative post yield stiffness, i.e. including P-Δ effects, is considered. The nonlinear SDOF system is subjected to a scaled version of the ground motion used in Step 1 in order to obtain a maximum displacement equal to $S_{D,in}$ (Step 1). The scaled ground motion is applied to the linear elastic SDOF system of Step 1 and the maximum displacement $S_{D,el\ Step\ 2}$ is recorded. The ratio of the inelastic and elastic displacements, $S_{D,in}$ and $S_{D,el\ Step\ 2}$, allows to calculate $\xi_{eq,z\ Step\ 2}$.

$$\eta_{Step\ 2} = \frac{S_{D,in}}{S_{D,el\ Step\ 2}} = \sqrt{\frac{10}{5+\xi_{eq,z\ Step\ 2}}} \tag{10}$$

Step 3

The EVD values found in the previous steps are combined in a new parameter λ:

$$\lambda = \frac{\xi_{eq,z\ Step\ 2}}{\xi_{eq,z\ Step\ 1}} \tag{11}$$

Step 1 to 3 are repeated in order to obtain λ associated to different values of displacement ductility, effective period and post-yield stiffness ratio.

Step 4

The influence of P-Δ effects in the EVD can be introduced directly in Eq.(7) as:

$$\eta_{P-\Delta} = \sqrt{\frac{10}{5+\lambda\cdot\xi_{eq\ r=0.05}}} \tag{12}$$

Where $\xi_{eq,r=0.05}$ is the equivalent viscous damping obtained from a system with r equal to 0.05. In this way it is possible to continue using EVD formulations available in the literature and defined for r=0.05, such as Grant et al. (2004) expressions.

The EVD evaluation procedure including P-Δ effects just described has been applied to a Takeda "thin" hysteretic system (Priestley et al. 2007) with the following properties T_{eff}=[1.5; 2.0; 2.5]s, μ_Δ=[2; 3; 4; 5] and r=[0.05; 0; -0.04; -0.08; -0.12; -0.16; -0.20]. A set of 7 natural ground motions from the European Strong-Motion Database (Ambraseys et al. 2004) were selected. The records have been scaled in order to be spectrum-compatible with the EN 1998-1:2005 (CEN 2005) Type 1 displacement spectrum, for soil type C and PGA=0.3g, in the range of periods between 1.5 s < T < 4 s (Fig.4). The selected ground motions are reported in Table.1.

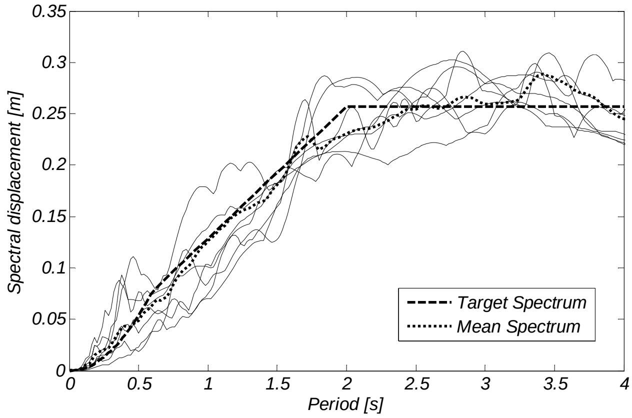


Figure 4. Elastic displacement spectra for the selected ground motions

Table 1. Selected ground motions for P-Δ analyses

Waveform ID	Earthquake Name	Date	M_w	PGA (m/s^2)	Epicentral distance (km)	Scale Factor
000244xa	Valnerina	19/09/1979	5.8	0.386	39	1.455
000302ya	Campano Lucano	23/11/1980	6.9	0.155	92	1.565
000359xa	Umbria	29/04/1984	5.6	0.497	17	1.47
000377ya	Lazio Abruzzo	07/05/1984	5.9	0.751	49	1.428
005270xa	Mt. Vatnafjoll	25/05/1987	6	0.302	25	1.519
005791ya	Gulf of Akaba	22/11/1995	7.1	0.138	345	1.518
005815xa	Kalamata	13/10/1997	6.4	0.278	73	1.484

The results of the procedure, in terms of the average of the 7 records, are shown in Fig.5 and Fig.6, subdivided in constant effective period T_{eff} and constant ductility μ_Δ groups. The graphs are presented in terms of λ and post-yield stiffness ratio r. Fig.7 shows the average of the results.

The values of the coefficient λ are clearly more dependent on the ductility compared to the effective period. It is worth noting that in Eq.(12) the constants 10 and 5 are dependent on the selected set of ground motions. However, the evaluation of the actual constants for the considered ground motions leads to negligible errors in the calculation of $\eta_{P-\Delta}$.

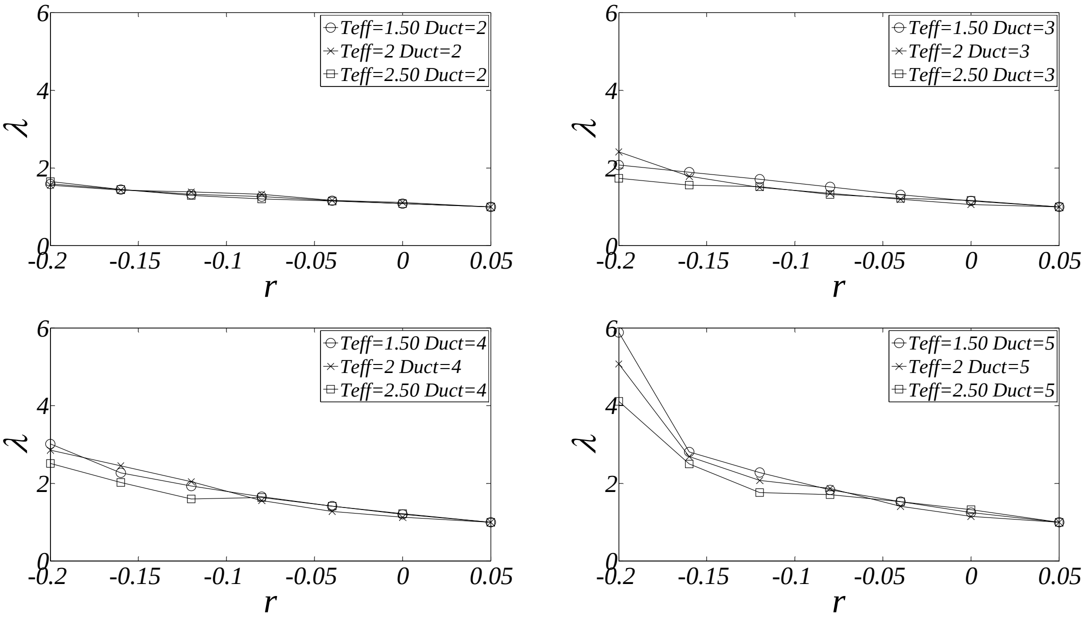


Figure 5. EVD procedure results in terms of constant ductility

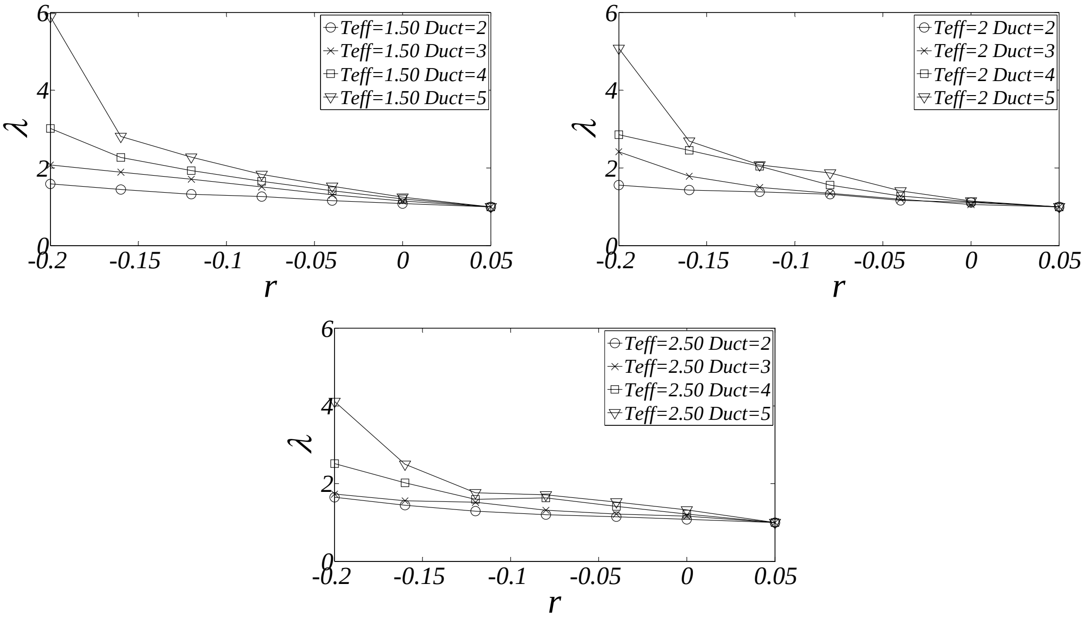


Figure 6. EVD procedure results in terms of constant effective period

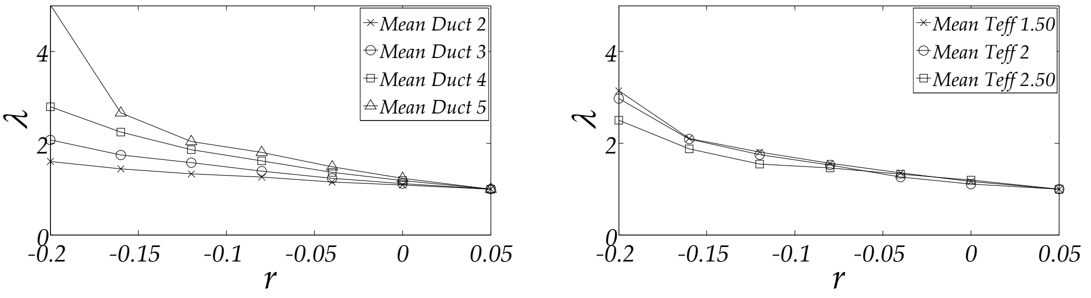


Figure 7. EVD procedure results in terms of mean ductility and mean effective period

DBA OF A R.C. HINGED FRAME CONSIDERING P-Δ EFFECTS

The DBA procedure including P-Δ effects is applied to a R.C. hinged frame with a layout typical of multi-storey precast concrete buildings in Europe. To account for precast structure peculiarities, special care should be placed on evaluating the mechanical characteristics of connections between precast elements, being the strain and deformation demand concentrated at the connections due to their lower stiffness compared to the connected elements (Belleri et al. 2013).

Regarding column-foundation connections, it is possible to consider the seismic behaviour of different typologies, as grouted sleeve connections (Belleri and Riva 2012), by acting directly on the yield curvature definition, as reported in Belleri et al. 2012. Regarding beam-column connections, it is possible to define force-displacement (Ferreira and El Debs 2000) and moment-rotation relationships accounting for displacement and rotation compatibility between adjacent elements, as reported in Belleri et al. 2012. For sake of clarity the beam-column connections of the selected case study are considered as perfect hinges, being the purpose of the research the inclusion of P-Δ effects in the DBA procedure rather than investigating the effects of different connection types.

The considered frame is shown in Fig.8. The columns are made of R.C. precast elements with cross section 50x50cm and 2.1% longitudinal reinforcement ratio. The material properties selected for the analyses are: f_c = 40 MPa for concrete strength and f_y = 450 MPa for steel yield stress.

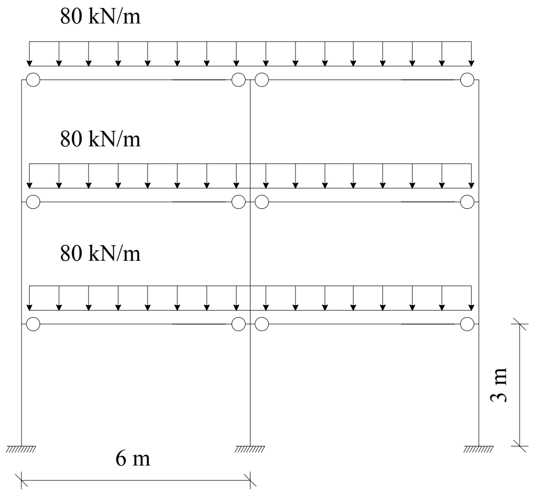


Figure 8. Considered hinged frame case study

The maximum beam-column rotation before connection failure is assumed as 0.04 rad (Belleri et al. 2012). The columns are considered fixed at the ground level and their hysteretic behaviour is included in the pushover analysis by means of a lumped plasticity model with Takeda hysteretic rule. The MDOF pushover capacity curve is shown in Fig.9, along with the SDOF capacity curve obtained from the DBA procedure. The target limit state is associated to the column flexural failure.

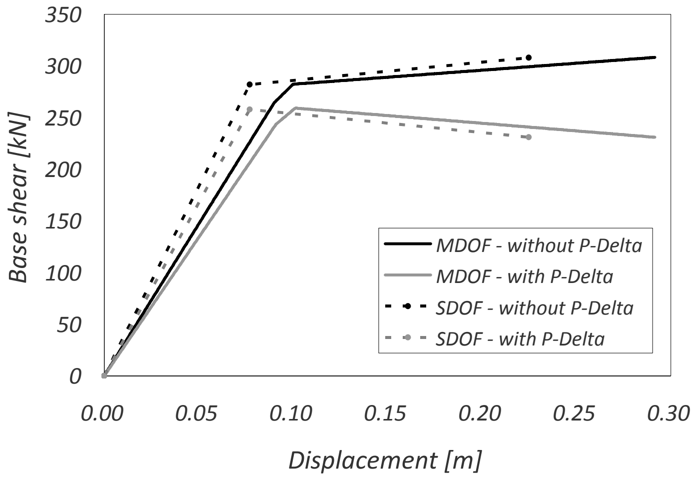


Figure 9. Pushover capacity curve for the SDOF and MDOF systems

The yield and ultimate points of the MDOF system capacity curve including P-Δ are Δ_y= 0.10 m V_y= 258 kN and Δ_u= 0.29 m V_u= 231 kN respectively. The displacement ductility (Δ_u/Δ_y) is 2.9 and the yield and target displacements of the SDOF substitute structure are:

$$\Delta_{y,ss} = \frac{\sum m_i \left(\Delta_{y,i}\right)^2}{\sum m_i \Delta_{y,i}} = 0.077\ m\ ;\ \Delta_{u,ss} = \Delta_{y,ss}\mu_\Delta = 0.225\ m \qquad (13,\ 14)$$

The SDOF substitute structure effective stiffness, mass and period are:

$$k_{eff} = \frac{V_u}{\Delta_{u,ss}} = 1045\frac{kN}{m}\ ;\ m_{eff} = \frac{\sum m_i \Delta_{u,i}}{\Delta_{u,ss}} = 234 ton\ ;\ T_{eff} = 2\pi\sqrt{\frac{m_{eff}}{k_{eff}}} = 2.97\,\mathrm{s} \qquad (15,\ 16,\ 17)$$

The EVD adopted in the DBA procedure is based on Grant et al. (2004) formulation, with a, b, c, d equal to 0.183, 0.588, 0.848 and 3.607 respectively, according to Takeda "thin" hysteretic model:

$$\xi_{eq} = 0.05 + a\left(1 - \frac{1}{\mu_{\Delta}^{b}}\right)\left(1 + \frac{1}{\left(T_{eff} + c\right)^{d}}\right) \tag{18}$$

The EVD is combined with Eq.(7) according to Borzi et al. (2001) to account for a damping reduction for periods beyond the elastic displacement spectrum corner period T_D:

$$\begin{cases} \eta = \eta_{T_D} & \text{if } T_{eff} \le T_D \\ \eta = \dfrac{1}{1 + \left(\dfrac{1}{\eta_{T_D}} - 1\right)\left(T_D / T_{eff}\right)} & \text{if } T_{eff} > T_D \end{cases} \tag{19}$$

Where η_{T_D} is the elastic displacement spectrum reduction, Eq.(7), at the corner period T_D. According to Eq.(18) and Eq.(19), the η value at T_{eff} is 0.80 which corresponds to a EVD equal to 10.6%.

To account for P-Δ effects, the EVD is multiplied by $\lambda = 1.26$. This value is derived from Fig.7. The resulting EVD is 13.4% which leads to $\eta = 0.74$.
The SDOF substitute structure considered displacement, $\Delta_{u,ss}$, belongs to the damped displacement spectrum. To obtain the corresponding point in the un-damped displacement spectrum $\Delta_{u,ss}$ is divided by η. The PGA associated to the considered limit state is related to the elastic displacement spectrum passing through the point (T_{eff}, $\Delta_{u,ss}/\eta$). The obtained PGA is 0.356g, which corresponds to a return period T_R equal to 623 years. The return period, Eq.(8), is defined based on the T_R-PGA points of the hazard curve reported in Table.2.

Table 2. Main points of the considered hazard curve

Return period (years)	PGA (g)
30	0.092
50	0.119
72	0.143
101	0.166
140	0.192
201	0.223
475	0.319
975	0.425
2475	0.599

The results were validated by means of nonlinear incremental dynamic analyses (Vamvatsikos and Cornell 2002). The same finite element model used in the pushover analysis was adopted. The ground motions selected for the analyses have been scaled in order to be spectrum-compatible with the EN 1998-1:2005 (CEN 2005) Type 1 displacement spectrum, for soil type C and PGA=0.3g, in the range of periods between 1.5 s < T < 4 s (Table.3). The scaled ground motions were taken as reference for the incremental dynamic analyses. The damping is defined in accordance to tangent stiffness Rayleigh damping (ξ=0.05 for T_1 = 1.6s and T_2 = 3.0s).

Table 3. Selected ground motions for the time history analyses

Waveform ID	Earthquake Name	Date	M_w	PGA (m/s^2)	Epicentral distance (km)	Scale Factor
000343xa	Urmiya	23/07/1981	5.8	0.480	50	1.250
000472xa	Vrancea	30/05/1990	6.9	0.373	162	0.978
000644xa	Umbria Marche (aftershock)	14/10/1997	5.6	0.538	29	0.867
000707ya	Friuli (aftershock)	11/09/1976	5.3	0.916	8	1.182
001769ya	Cerkes (aftershock)	14/08/1996	5.6	1.002	13	1.234
003802xa	SE of Tirana	09/01/1988	5.9	1.113	7	0.822
006960ya	Izmit (aftershock)	13/09/1999	5.8	0.494	27	0.827

As mentioned before the selected limit state is associated to the column flexural failure. Fig.10 shows the column plastic hinge rotation time series corresponding to flexural failure.

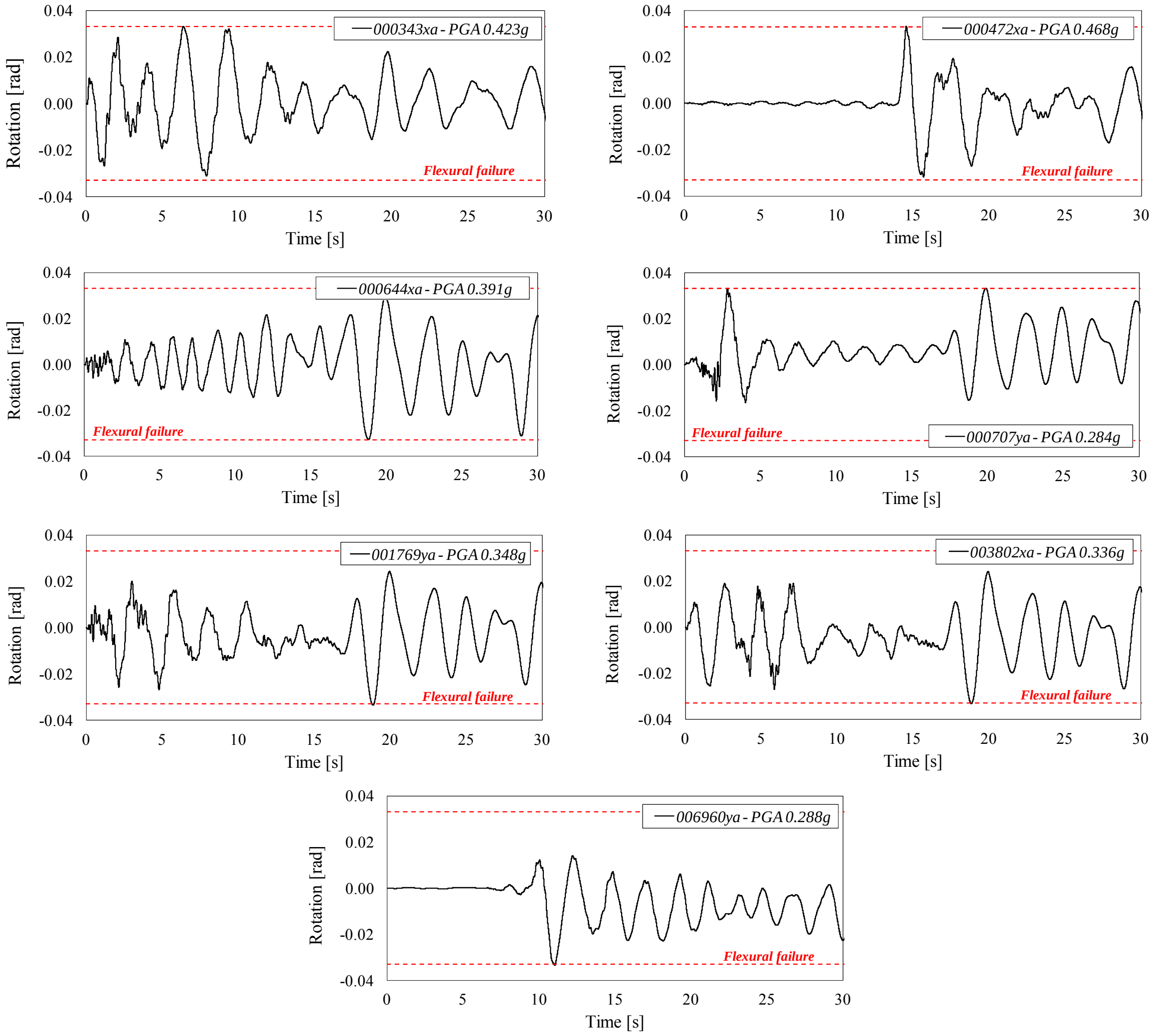


Figure 10. Column base rotation time series corresponding to the selected limit state

The PGA related to the considered limit state is equal to 0.363±0.068g which is in accordance to the PGA obtained from the DBA procedure. The return period associated to 0.363g is 657yr.

CONCLUSIONS

The paper investigated the inclusion of P-Δ effects in the Displacement Based Assessment (DBA) procedure. P-Δ effects reduce the lateral load associated to a selected deflected shape in the substitute structure capacity curve leading to a decrease of the effective stiffness and consequently to an increase of effective period. In addition the available formulations of Equivalent Viscous Damping (EVD) do not account for negative post-yield stiffness which could arise when P-Δ effects are considered.
The paper proposed a procedure to define relationships between available EVD formulations, typically associated to post yield stiffness ratio r = 0.05, and EVD including P-Δ effects. The proposed procedure was applied to SDOF systems with Takeda “thin” hysteresis, with post yield stiffness ratio in the range -0.20 to 0.05, effective period in the range 1.5 to 2.0s and displacement ductility in the range 2 to 5.
The DBA procedure including P-Δ was applied to a selected case study constituted by a three storey R.C. hinged frame typical of European multi-storey precast concrete buildings and validated by means of nonlinear time history analyses. The DBA procedure including P-Δ allowed to correctly estimate the PGA associated to the selected limit state. The attention was placed on R.C. hinged frames due to their higher lateral flexibility, therefore high P-Δ effects, compared to traditional R.C. frames. The proposed procedure is general and can be applied to other structural typologies and other hysteretic rules.

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