



A NONLINEAR MACRO MODEL FOR NUMERICAL SIMULATION OF EXTERIOR RC JOINTS WITHOUT TRANSVERSE REINFORCEMENT

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ABSTRACT

In the assessment of the performance of typical existing buildings, seismic collapse safety might be significantly affected by the non-linear behaviour of the joints that are involved in the failure mechanisms because of poor structural detailing, as the lack of an adequate transverse reinforcement in the joint panel or deficiencies in the anchorage due to the absence of any capacity design principle.

Few reliable approaches for modelling all sources of nonlinearity are available for poorly designed beam-column joints because of relatively poor information from experimental tests. Many nonlinear joint models are available, but most of them may be unsuitable for the assessment of older concrete buildings, either because they were developed and calibrated for confined joints or because they are complicated to use.

In this study, the attention is focused on external joints with no transverse reinforcement and a preliminary exterior joint macro model is proposed. First, an experimental database of tests available in literature on joints without transverse reinforcement (and substantially homogeneous for beam bars anchorage type) that were subjected to a range of displacement histories and joint shear stress demands, and which exhibited different modes of failure (J-mode and BJ-mode failure), is collected and analyzed. Second, the joint panel constitutive parameters are defined to reproduce the experimental joint shear stress-strain relationships, when they were available. Then, bond-slip is taken into account by introducing a slip spring whose properties are calculated using a bond-slip model. Finally, the proposed joint model is validated using some of the experimental tests included in the database.

INTRODUCTION

Damage observed after the most recent earthquakes and experimental investigations on the seismic performance of existing Reinforced Concrete (RC) buildings highlighted the vulnerability of the beam-column joint region. The behavior of beam-column joints is a critical issue in the assessment of seismic performance of RC moment resisting frames; therefore, within the context of Performance-Based Earthquake Engineering, a growing attention should be addressed to the modelling of RC beam-

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column connections and the influence of failure of joints on the seismic performance of RC buildings.

In particular, in the assessment of the performance of typical existing buildings, seismic collapse safety might be significantly affected by the non-linear behaviour of the joints that are involved in the failure mechanisms because of poor structural detailing, as the lack of an adequate transverse reinforcement in the joint panel or deficiencies in the anchorage due to the absence of any capacity design principle. Such unreinforced joints are vulnerable to brittle shear failure under seismic action due to insufficient shear reinforcement in the joint region, especially for exterior joints.

In literature there is not yet a commonly accepted approach for the determination of the shear strength and for nonlinear modeling of RC beam-column joints in moment resisting RC frames. Many nonlinear joint models are available, but most of them may be unsuitable for modelling all sources of nonlinearity for the assessment of older concrete buildings, either because they were developed and calibrated for confined joints or they are complicated to implement. Moreover, the very poor dataset from experimental tests on unconfined joints makes it difficult to calibrate a comprehensive and simple nonlinear model.

In this study, the attention is focused on external joints containing no transverse reinforcement and a preliminary exterior joint macro-model is proposed. First, an experimental database of tests available in literature on joints without transverse reinforcement (and substantially homogeneous for beam bars anchorage type) that were subjected to a range of displacement histories and joint shear stress demands, and which exhibited different modes of failure (J-mode and BJ-mode failure), is collected and analyzed. Second, the joint panel constitutive parameters are defined to reproduce the experimental joint shear stress-strain relationships, when they were available. Then, bond-slip is taken into account by introducing a slip spring whose properties are analytically calculated using a bond-slip model. Finally, the proposed joint model is validated using some of the experimental tests included in the database.

STATE OF ART: JOINT MODELING FOR NUMERICAL SIMULATION

As far as the seismic behavior of RC beam-column joints under seismic loads is concerned, first of all, joint flexibility contributes significantly to the overall story drift, especially in nonlinear range. Basically, two contributions to overall deformability due to beam-column joints cannot be neglected: (i) the shear deformation of the joint panel and (ii) the contribution of the fixed-end-rotation due to the slip of the longitudinal bars anchored into the joint (e.g., Cosenza et al. 2006).

Moreover, under lateral seismic loading, high shear forces are generated in the joint core. Beam-column joints bear horizontal and vertical shear forces that are usually much larger than those acting within the adjacent beams and columns. Thus, joints can experience shear failures which should be avoided by an appropriate design to ensure a ductile response of the whole frame. However, such a design does not regard typical existing structures designed for gravity loads only. In particular, exterior unreinforced RC joints often experience brittle failure under horizontal actions. Basically, two main different modes of failure can be identified: (i) J-failure, namely joint failure occurs prior to yielding of beam longitudinal reinforcement; (ii) BJ-failure, namely joint failure occurs after yielding of beam longitudinal reinforcement.

Beam-column joint shear strength

As far as code prescriptions are concerned, ASCE-SEI 41 (2007) proposes recommendations for the shear strength of unreinforced joints for seismic rehabilitation purposes based on the pre-standard developed in FEMA 273 (1997) and FEMA 356 (2000). According to ASCE 41, nominal joint shear strength is defined according to Eq. (1), independent of the failure mode and the axial load ratio. The values of γ_n for joint shear strength are provided depending on the joint typology and the transverse reinforcement ratio ρ , as shown in Table 1.

$$V_n = \gamma_n \sqrt{f_c} b_j h_c \quad (1)$$

Table 1. Joint shear coefficient according to ASCE-SEI 41 (2007)

γ_n (MPa) ^{0.5}					
ρ	Interior joint with transverse beams	Interior joint w/o transverse beams	Exterior joint with transverse beams	Exterior joint w/o transverse beams	Knee joint with or w/o transverse beams
< 0.003	1.0	0.8	0.7	0.5	0.3
\geq 0.003	1.7	1.2	1.2	1.0	0.7

Several authors have proposed analytical or empirical models to predict joint shear strength. Thus, in literature there are different possible approaches to evaluate such a strength.

Models based on the Modified Compression Field Theory (MCFT) by Vecchio and Collins (1986), e.g. Lowes and Altoontash 2003, appeared to underestimate the shear strength of lightly reinforced joints (Shin and LaFave 2004); thus they are unsuitable for prediction of shear strength of unreinforced joints.

Other strength models proposed in literature are based on an empirical or semi-empirical approach (Bakir and Boduroglu 2002, Celik and Ellingwood 2008, Hassan 2011). Bakir and Boduroglu (2002) included the beam reinforcement ratio and the joint aspect ratio in the proposed equation for evaluating the contribution of concrete to joint shear strength, and anchorage details were included by an empirical factor. Based on the analysis of an experimental database from literature, Celik and Ellingwood (2008) proposed to evaluate the maximum joint shear strength as the joint stress corresponding to the beam or column capacity; such a strength is reduced so as not to exceed the maximum joint shear strength (statistically defined through a dataset of experimental results in a range of values with a uniform distribution), when shear failure of joint occurs before beams or columns reach their capacities. Hassan (2011) proposed an empirical strength model including the effect of joint aspect ratio and axial load ratio for J-mode of failure.

However, most of the empirical models proposed in literature have been developed based on statistical regression analysis with large scatter or small size of experimental data sets.

Other models proposed in literature for evaluating joint shear strength are the principal-tensile-stress-based models. Pauley and Priestley (1992) first proposed a comparison between the average principal tensile stress of the unconfined joint with some critical values, namely shear cracking and shear failure. Eurocode 8 (CEN 2005) and Italian provisions (D.M. 2008) adopt this approach for unreinforced joints. Other authors proposed a similar approach, introducing an upper bound of such critical values in the case of ineffective anchorage (Sharma et al. 2011) or in the case of plain bars as longitudinal reinforcement of the adjacent beam and bar end hooks (Pampanin et al. 2002). Such approaches allows to explicitly take into account the (beneficial) effect of the axial load ratio on joint shear strength, but the analysis of the experimental results – carried out on specimens with identical geometrical and mechanical properties and different axial load ratio level – showed that such an influence is quite negligible.

Moreover, the deterioration of the shear strength of beam-column joints under cyclic displacement was experimentally observed. The diagonal tension cracking of the joint core in alternative directions during seismic loading causes the reduction of the diagonal compressive strength of the concrete; therefore, the joint shear strength may degrade with the increase in ductility demand in the adjacent members during cyclic loading. Some models in literature attempted to capture this effect proposing a joint shear strength that decreases with increasing the ductility demand in the adjacent beam (Park 1997, Hakuto 2000). The relationship between the reduction of joint shear strength and the ductility factor is empirically proposed in these models, but they cannot be accurately generalized because the ductility factor is uncertain and it takes also into account the deformation of the members adjacent to the joint.

Successively, experimental tests conducted on exterior unreinforced beam-column connections aimed at the definition of the main parameters having the greatest influence on joint shear strength. Park and Mosalam (2012a) investigated the effects of three main parameters, namely (i) joint aspect ratio, (ii) beam longitudinal reinforcement ratio, and (iii) column axial load, and confirmed that joint aspect ratio and beam longitudinal tension reinforcement ratio and its strength mainly influence joint shear strength. Park and Mosalam (2012b), based on the results of parametric studies, proposed another approach to evaluate joint shear strength and its degradation after beam yielding without any ductility factor; their model directly provides a definition of the failure mode (J or BJ failure mode)

and it is formulated using a mechanical approach based on the strut-and-tie mechanism (Park and Mosalam, 2012b). The strut-and-tie approach was already proposed and adopted in other literature studies (Vollum and Newman 1999, Tsonos 2007). Most of the “strut-and-tie-based” models have a conceptual limitation, because the average equilibrium and compatibility equations they are based on are not suitable to reproduce the real behaviour of unreinforced beam-column joints – for which the joint shear failure is generally localized. Moreover, the accuracy of the strut-and-tie approach highly depends on the estimation of the diagonal strut area that strictly affects the joint shear strength. Nevertheless, the shear strength model proposed by Park and Mosalam (2012b) is based on a *modified* strut-and-tie approach, it is independent on the diagonal strut area and it shows a good agreement with experimental tests (see Table 2).

Beam-column joint modeling

ASCE/SEI 41 first suggests that beam-column joint in monolithic construction should be represented as a stiff or rigid zone having horizontal dimensions equal to the column cross-sectional dimensions and vertical dimension equal to the beam depth. Successively, ASCE 41 suggests a complete backbone curve for joint shear stress-strain modeling in nonlinear analyses, but recommended values for joint shear strength coefficient and plastic shear strain for joint modeling appear to be quite conservative. Moreover, ACI 369-R11 defines an implicit beam-column joint using centerlines models with semi-rigid joint offsets: only a portion of the beam and column, or both, within the joint panel zone, is defined as rigid.

Many researchers have attempted to model the behavior of beam-column joints with different approaches, basically finite element simulations, lumped plasticity approach or multi-spring models. Some models proposed in literature may be unsuitable for older concrete building assessment, either because they were developed and calibrated for confined beam-column joints or because they are complicated to implement.

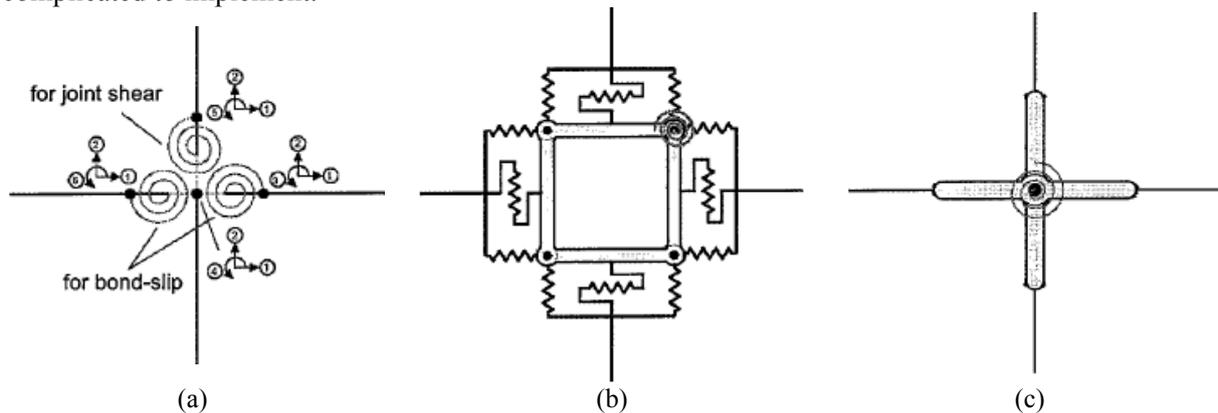


Figure 1. Biddah and Ghobarah (1999) multi-spring model (a); Lowes and Altoontash (1999) multi-spring model (b); Alath and Kunnath (1995) scissors model (c)

As far as multi-spring models are concerned, Biddah and Ghobarah (1999) modeled the joint with separate rotational springs for joint shear and bond-slip deformations (Figure 1a). The shear stress-strain relationship of the joint was simulated using a tri-linear idealization based on a softening truss model, while the cyclic response of the joint was captured with a hysteretic relationship with no pinching effect. Within the context of multi-spring modeling approach, Lowes and Altoontash (2003) proposed a 4-node 12-degree-of-freedom joint element (Figure 1b). This macro-model is constituted by eight zero-length translational springs which simulate the bond-slip response of beam and column longitudinal reinforcement, a zero-length rotational spring that simulates the shear deformation of the joint, and four zero-length shear springs that simulate the interface-shear deformations. Shear stress-strain relationship of the panel zone is defined through the MCFT and, thus, joints with no transverse reinforcement were not taken into account.

Moreover, the multi-spring approach is often too difficult to implement and it requires the knowledge of multiple different contributions, which often are difficult to calibrate.

One of the simplest models, the so-called *scissors model*, may be very easy to implement: it is composed by a rotational spring with rigid links that take into account the finite size of the joint panel.

Such a model was first suggested by Alath and Kunnath (1995) (Figure 1c) and then it was adopted by Pampanin et al. (2002), Celik and Ellingwood (2008) and Park and Mosalam (2013) for interior and exterior unconfined RC beam-column joints.

In particular, the model proposed by Celik and Ellingwood (2008) is based on a statistical analysis carried out on ten experimental tests – as far as external joints are concerned – non homogeneous for anchorage conditions and all related to J-failure mode. Celik and Ellingwood (2008) suggested that the shear stress-strain backbone curve for the panel zone in typical existing RC beam-column joints can be defined through four key points, which correspond to joint shear cracking, reinforcement yielding, joint shear strength/adjoining beam or column capacity, and residual joint strength, respectively. The ordinates of the backbone points were scaled if the shear failure of the joint occurs before beams or columns reach their capacities. Shear failure of the joint is assumed to depend on the kind of joint (internal/external) and the anchorage conditions. Bond-slip was taken into account through a reduced envelope for the joint shear stress-strain relationship. As far as strain capacity is concerned, they proposed a range of values for each characteristic point with a uniform probability distribution (Celik and Ellingwood, 2010). However, from the analysis of the experimental dataset they adopted, it can be observed that the proposed joint shear strain values for the four key points are related to internal joints tests only; such values are very high if compared with shear strain values obtained from experimental tests on external beam-column joints.

A multi-linear backbone curve to represent the moment-rotation relationship of an unreinforced corner beam-column joint was also proposed by Park and Mosalam (2013). The modeling parameters of such a backbone curve were estimated on the basis of experimental results of four corner joint specimens tested by the authors with different joint aspect ratio and beam longitudinal reinforcement ratio. For strength prediction in the developed backbone curve, a simplified version of an analytical model proposed by Park and Mosalam (2012a) was adopted. Joint shear strain, γ_{xy} , and rotation at the beam joint interface, θ_s , due to bar slip and/or crack opening were separately measured during the tests and the rotation of the backbone joint spring, θ_j , was defined as the sum of them, that is, $\theta_j = \gamma_{xy} + \theta_s$.

Thus, the stress-strain relationship was often calibrated on the basis of statistical regression analysis with large scatter or small size of experimental data sets. Therefore, a strong effort, especially towards a better calibration of the panel zone deformability, is still necessary.

EXPERIMENTAL DATABASE

Various experimental tests have been performed and presented in literature by researchers in the past. Thus, first the collection and the analysis of an experimental database are carried out based on tests available in literature on joints without transverse reinforcement that were subjected to a range of displacement histories and joint shear stress demands, and which exhibited different modes of failure (J-mode and BJ-mode of failure) – see Table 2. Experimental database includes only planar unreinforced joints substantially homogeneous for beam bars anchorage type, namely no loss of bond can be imputed to an anchorage failure. No tests on 3D-corner joints specimens which include influence of slab were considered.

PROPOSED MODEL

The proposed model is based on the introduction of rigid links spreading through the panel zone dimensions and the adoption of two rotational springs in series: the first one is located in the centreline of the joint panel and represents the shear behaviour of the joint panel; the other one is located at the interface between the joint panel and the adjacent beam and it represents the bond-slip contribution. The joint panel constitutive parameters are defined to reproduce the experimental joint shear stress-strain relationships, on the basis of tests for which they were available. Then, bond-slip rotational spring properties are calculated using an analytical bond-slip model.

Table 2. Experimental database of planar unreinforced RC beam-column joints

Reference	ID specimen	ν	failure mode	γ_n (MPa) ^{0.5}	axial failure	$\tau-\gamma$	Loading element	$V_{max,b}$ (kN)	b_{col} (mm)	h_{col} (mm)	b_{beam} (mm)	h_{beam} (mm)	f_c (MPa)	$f_{y,beam}$ (MPa)	$f_{y,col}$ (MPa)	$A_{s,beam}$ (mm ²)	Lb mm	Lc mm	V_j ASCE/exper	V_j P&M/exper	
Clyde et al. (2000)	2	0.10	BJ	1.07	yes	yes	beam	290	304.8	457.2	304.8	406	46.2	454	470	2565	1270	1080	0.46	0.98	
	6	0.10	BJ	1.11	yes	yes	beam	279	304.8	457.2	304.8	406	40.1	454	470	2565	1270	1080	0.45	0.95	
	4	0.25	BJ	1.14	yes	yes	beam	290	304.8	457.2	304.8	406	41.0	454	470	2565	1270	1080	0.44	0.93	
	5	0.25	BJ	1.14	yes	yes	beam	275	304.8	457.2	304.8	406	37.0	454	470	2565	1270	1080	0.44	0.93	
Pantelides et al. (2002)	3	0.10	J	0.83	yes	yes	beam	187.8	406.4	406.4	406.4	406	34.0	459	470	2565	1490	1397	0.60	1.21	
	4	0.25	J	0.97	yes	yes	beam	211.4	406.4	406.4	406.4	406	31.6	459	470	2565	1490	1397	0.51	1.03	
	5	0.10	J	0.89	yes	yes	beam	194.0	406.4	406.4	406.4	406	31.7	459	470	2565	1490	1397	0.56	1.13	
	6	0.25	J	0.91	yes	yes	beam	197.6	406.4	406.4	406.4	406	31.0	459	470	2565	1490	1397	0.54	1.09	
Wong (2005)	JA-NN03	0.03	BJ	0.57	no	no	beam	82.0	300	300	260	400	44.8	520	520	628	1500	1350	0.87	0.91	
	JA-NN15	0.15	BJ	0.60	no	no	beam	87.2	300	300	260	400	46.0	520	520	628	1500	1350	0.83	0.86	
	JB-NN03	0.03	BJ	0.57	no	no	beam	58.2	300	300	260	300	47.4	520	520	628	1500	1400	0.87	0.94	
	BS-L	0.15	J	0.73	no	no	beam	100.5	300	300	260	450	30.9	520	520	942	1500	1325	0.68	1.07	
	BS-L300	0.15	BJ	1.11	no	no	beam	95.4	300	300	260	300	34.1	520	520	942	1500	1400	0.45	0.78	
	BS-L600	0.15	J	0.61	no	no	beam	134.6	300	300	260	600	36.4	520	520	942	1500	1250	0.80	1.02	
	BS-U	0.15	J	0.79	no	no	beam	111.4	300	300	260	450	31.0	520	520	942	1500	1325	0.62	0.97	
BS-L-LS	0.15	J	0.79	no	no	beam	110.4	300	300	260	450	31.6	520	520	942	1500	1325	0.63	0.99		
Di Ludovico et al. (2012)	TC2	0.21	J	0.72	no	no	beam	80.5	300	300	300	500	19.2	460	460	603	1650	1450	0.72	1.05	
	TC		J	0.69	no	no	beam	76.5	300	300	300	500	16.4	460	460	603	1650	1450	0.70	1.02	
Hwang et al (2005)	OTO ¹⁰	0.02	BJ	0.57	no	no	beam	192	420	420	320	450	67.3	430	421	2564	1900	1190	0.88	1.21	
Tsonos and Papanikolaou (2003)	F1	0.17	J	1.00	no	no	beam	60	200	200	200	300	20.0	520	520	462	900	550	0.50	0.78	
	F2	0.17	J	0.71	no	no	beam	53	200	200	200	300	31.0	530	530	383	900	550	0.70	1.05	
	L1	0.17	BJ	0.77	no	no	beam	60	200	200	200	300	34.0	520	520	616	900	550	0.65	1.02	
Antonopoulos and Triantafillou (2003)	C1	0.04	J	0.78	no	no	beam	31.3	200	200	200	300	19.5	585	585	462	1000	500	0.64	1.01	
	C2	0.04	J	0.70	no	no	beam	31.1	200	200	200	300	23.7	585	585	462	1000	500	0.71	1.12	
Genesio (2012)	JT1-1	0.00	J	0.67	no	no	beam	76.9	300	350	300	400	25.4	560	560	829	1875	1300	0.74	1.18	
	JT5-1	0.00	BJ	0.60	no	no	beam	61.5	300	350	300	400	24.5	540	540	452	1875	1300	0.82	0.78	
Uzumeri (1977)	SP1	0.52	J	0.64	no	no	beam	102	381	381	304.8	508	30.8	347	332	1283	2350	1270	0.78	1.14	
	SP2	0.51	J	0.61	no	no	beam	97.9	381	381	304.8	508	31.1	349	335	1283	2350	1270	0.82	1.20	
	SP5	0.61	J	0.69	no	no	beam	102	381	381	381	508	26.3	352	390	1283	2350	1270	0.72	1.15	
Karayannis (2008)	A0	0.05	J	0.38	no	no	beam	25	200	200	200	300	31.6	580	580	157	1000	600	1.32	1.01	
	B0	0.05	J	0.58	no	no	beam	58	200	300	200	300	31.6	580	580	471	1000	600	0.86	1.22	
	C0	0.05	J	0.65	no	no	beam	65	200	300	200	300	31.6	580	580	452	1000	600	0.77	1.06	
																			Mean	0.69	1.02
																			CoV	0.27	0.12

Calibration of shear stress-strain relationship of the joint panel

The joint panel zone model was calibrated through tests well documented in the literature, different for the failure mode they exhibited, namely J-failure (by Pantelides et al., 2002) and BJ-failure (by Clyde et al., 2000), for which experimental stress-strain relationships for joint panel were available.

Pantelides et al. (2000) performed cyclic tests on six full-scale models of exterior beam–column joints with two different axial load ratio levels (10% and 25% of compressive strength of concrete f_c) and no transverse reinforcement within the joint core. All specimens had the same dimensions. Reinforcement bars in beam and columns were designed to prevent yielding, forcing a shear mode of failure in the joint (J-failure). Two specimens (test units 1 and 2) were designed with typical gravity load detailing and the bottom beam reinforcement bars did not have adequate embedment inside the joint. Such specimens have been excluded from the analyzed database because a loss of bond due to anchorage failure occurred. The other four specimens (test units 3, 4, 5 and 6) had both top and bottom beam bars bent into the joint. Test units 5 and 6 presented a U-hook type of anchorage for both top and bottom beam bars.

Clyde et al. (2000) performed cyclic tests on four half-scale exterior unreinforced RC beam-column joints, with two different levels of axial load on the column, namely 10% and 25% of the compressive strength of concrete (f_c). All beam-column specimens had exactly the same dimensions and detailing. Both bottom and top beam reinforcements were bent up and down, respectively, into a hook in the joint. The yielding of beam longitudinal reinforcement bars before joint failure was documented by the authors (BJ-failure).

The proposed joint panel zone model is a scissors model. It can be implemented by defining duplicate nodes, node A (master) and node B (slave), with the same coordinates at the center of the joint panel. Node A is connected to the column rigid link and node B is connected to the beam rigid link. A zero length rotational spring connects the two nodes and allows only relative rotation between them through a constitutive model which represents the shear deformation of the joint panel zone. Such a rotational spring is defined as a quadrilinear Moment (M_j) – Rotation (γ_j) spring characterized by four points for J- and BJ-mode of failure, separately: cracking, pre-peak, peak and residual points (as shown in Figure 2).

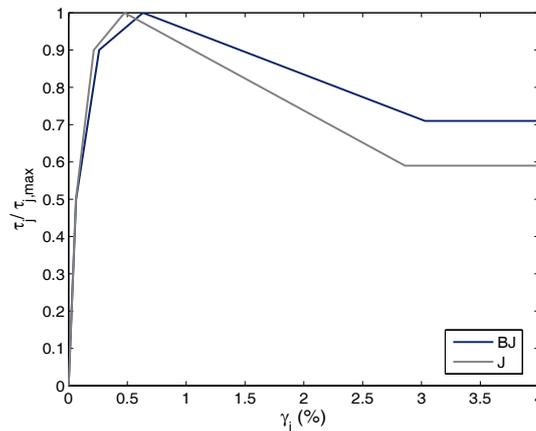


Figure 2. Stress-strain relationship for joint panel (for J- and BJ-mode of failure)

Table 3 shows the values of shear strain and the shear stress-to-peak strength ratios ($\tau_j / \tau_{j,peak}$) related to the experimental tests used for the calibration of the panel zone constitutive relationship for each characteristic point of the proposed backbone. A summary of the coordinates of the characteristic points of the proposed backbone is reported in Table 4.

Cracking point

The cracking point represents the onset of hairline cracks in the joint panel. Experimental tests adopted in this study to calibrate the stress-strain shear behavior of the joint panel suggested that the corresponding shear strain (γ_j) is the same for J- and BJ-mode of failure and equal to 0.06%. The

cracking strength is evaluated in accordance to Uzumeri (1977) – which provided a good agreement with experimental data – as reported in Eq. (2):

$$\tau_{j,cr} = 0.29\sqrt{f_c} \sqrt{1 + 0.29 \frac{P}{A_j}} \text{ (MPa)} \quad (2)$$

Pre-peak point

In the case of BJ-failure the pre-peak point corresponds to the yielding of longitudinal beam bars, thus the pre-peak strength is calculated as the joint stress corresponding to the achievement of yielding in the adjacent beam. The corresponding joint shear strain obtained from tests is assumed equal to its mean value (0.26%).

In the case of J-failure, the pre-peak point corresponds to the widening of the main diagonal cracks and the developing of other cracks in the joint panel. The corresponding strength is assumed equal to 0.9 times the peak strength (defined below), in accordance with the observation of experimental tests by Pantelides et al (2002) and other models proposed in literature, e.g. Park and Mosalam (2013). The corresponding joint shear strain obtained from tests is assumed equal to its mean value (0.21%).

Peak point

In the case of BJ-failure the peak point corresponds to the achievement of the flexural capacity in the adjacent beam; in the case of J-failure this point corresponds to the achievement of the maximum shear strength specifically inherent to the joint, independently on the stress demand in the adjacent beam. As explained in the previous section, the shear strength model proposed by Park e Mosalam (2012b) directly provides a shear strength which depends on the failure typology and shows a very good agreement with experimental tests. In Table 2 a comparison between models by Park and Mosalam (2012b) and ASCE SEI-41/06 versus experimental results is reported in terms of joint strength for the experimental tests belonging to the database. It can be verified that the mean of the model-to-experimental strength ratio is closer to 1 and the CoV is lower when model by Park and Mosalam is adopted to predict shear strength with respect to the adoption of the code provision. The strength model proposed by Park e Mosalam (2012b) is adopted in this study to define the ordinate of the peak point and reported in Eq. (3):

$$V_n = k \left[\sqrt{f_c} b_j h_c \frac{\cos \theta}{\cos(\pi/4)} \right] \text{ (MPa)} \quad (3)$$

where θ is a function of the joint aspect ratio (beam height/column height), b_j and h_c are the effective joint width and the column cross-sectional height, respectively, f_c is the concrete compressive strength and k is a strength factor accounting for the effect of the beam longitudinal reinforcement ratio.

The joint shear strain corresponding to the peak point obtained from tests is assumed equal to its mean value, i.e. 0.63% for BJ-failure and 0.48% for J-failure.

Residual point

The softening branch is obtained by a straight line connecting the peak point and the ultimate point provided by the authors for each experimental test. In this study, a residual strength equal to 60% or 70% of the peak strength for J-failure mode and BJ-mode is assumed, respectively. The joint shear strain corresponding to the residual point obtained from tests is assumed equal to its mean value, i.e. 3.03% for BJ-failure and 2.86% for J-failure. However, it is worth to highlight that there are very poor data regarding the achievement of this limit state and they are not always reliable because of the uncertainties in the experimental measurements for large displacement demand.

For each characteristic point of the backbone, from simple equilibrium equation, the moment transferred through the rotational spring M_j can be obtained as a function of the joint shear stress τ_j through the Eq. (4):

Table 3. Stress-strain relationship for joint panel for BJ- and J-mode of failure

	Clyde et al. (2000)								BJ-failure					
	test 2		test 6		test 4		test 5		min	max	mean	min	max	mean
	γ_j	$\tau_j/\tau_{j,peak}$	γ_j	$\tau_j/\tau_{j,peak}$	γ_j	$\tau_j/\tau_{j,peak}$	γ_j	$\tau_j/\tau_{j,peak}$	γ_j			$\tau_j/\tau_{j,peak}$		
cracking	0.07%	0.52	0.03%	0.31	0.11%	0.68	0.05%	0.44	0.03%	0.11%	0.06%	0.31	0.68	0.49
pre-peak	0.31%	0.94	0.18%	0.69	0.34%	1.02	0.21%	0.88	0.18%	0.34%	0.26%	0.69	1.02	0.88
peak	0.72%	1.00	0.48%	1.00	0.85%	1.00	0.48%	1.00	0.48%	0.85%	0.63%	1.00	1.00	1.00
residual	2.87%	0.53	0.75%	0.83	2.64%	0.71	5.85%	0.75	0.75%	5.85%	3.03%	0.53	0.83	0.71
	Pantelides et al. (2002)								J-failure					
	test 5		test 6		test 3		test 4		min	max	mean	min	max	mean
	γ_j	$\tau_j/\tau_{j,peak}$	γ_j	$\tau_j/\tau_{j,peak}$	γ_j	$\tau_j/\tau_{j,peak}$	γ_j	$\tau_j/\tau_{j,peak}$	γ_j			$\tau_j/\tau_{j,peak}$		
cracking	0.06%	0.54	0.08%	0.69	0.05%	0.65	0.06%	0.59	0.05%	0.08%	0.06%	0.54	0.69	0.62
pre-peak	0.23%	0.84	0.27%	0.95	0.19%	0.93	0.18%	0.92	0.18%	0.27%	0.21%	0.84	0.95	0.91
peak	0.61%	1.00	0.65%	1.00	0.33%	1.00	0.31%	1.00	0.31%	0.65%	0.48%	1.00	1.00	1.00
residual	1.96%	0.81	3.05%	0.46	4.37%	0.42	2.07%	0.68	1.96%	4.37%	2.86%	0.42	0.81	0.59

Table 4. Summary of the proposed backbone for the joint panel

	J-failure		BJ-failure	
	τ	γ	τ	γ
cracking	From Eq. (2)	0.06%	From Eq. (2)	0.06%
pre-peak	$0.9 \tau_{peak}$	0.21%	$\tau (M_{yielding,beam})$	0.26%
peak	From Eq. (3)	0.48%	From Eq. (3)	0.63%
residual	$0.6 \tau_{peak}$	2.86%	$0.7 \tau_{peak}$	3.03%

Table 5. Comparison between the proposed model and other models from literature in terms of γ_j

	Cracking		Pre-peak		Peak		Residual	
	J	BJ	J	BJ	J	BJ	J	BJ
mean (proposed model)	0.06%		0.21%	0.26%	0.48%	0.63%	2.86%	3.03%
Celik and Ellingwood (2008)	0.01-0.13%		0.2-1.0%		1-3%		3-10%	
Genesio (2012)	0.10%		-	-	0.70%	0.50%	-	-
Park and Mosalam (2012a)	-	-	-	-	$2\varepsilon_1^{(+)}$	$2\varepsilon_1^{(+)}$	-	-
Hassan (2011)*	downward	$\tau_{j,i}/G_{0i}$	0.2%	0.02%	τ_{j3}/G_{03}	0.02%	$\tau_{j3}/G_{03}+0.02$	$\tau_{j3}/G_{03}+0.025$
	upward		0.25%	τ_{j2}/G_{02}		τ_{j3}/G_{03}	3%	3%

⁽⁺⁾ ε_1 is the principal tensile strain and it depends on the joint aspect ratio.

* $\tau_{j,i}$ is the shear stress related to the i^{th} characteristic point; G_{0i} is the secant stiffness to the i^{th} characteristic point.

$$M_j = \tau_j A_j \frac{1}{\frac{1-h_c/L_b}{jd_b} - \frac{1}{2L_c}} \quad (4)$$

where L_b and L_c are the beam and column length, respectively, from the inflection point to the centerline of the joint panel, jd_b is the beam level arm, h_c the column height and A_j the effective joint area.

As far as the abscissa of the backbone is concerned, the rotation of this spring is defined equal to the joint panel rotation γ_j , because the joint rotation resulting from beam bar slip is explicitly

defined by a separate zero-length rotational slip spring element attached between the beam-joint interface section and the end of the beam rigid link (as explained below).

A comparison with other models proposed in literature in terms of joint panel deformability is carried out and reported in Table 5. It can be noted that proposed values for shear strain are generally lower with respect to values proposed by other literature models for each characteristic point of the backbone.

Slip spring

There are several techniques to represent bond-slip rotation in an analytical model of RC beam-column joints. One of the techniques proposed in literature to account for slip deformation is the reduction of the effective stiffness of beams and columns as recommended by ASCE/SEI 41 supplement. The most direct approach is to introduce a slip spring whose properties are calibrated directly from tests or analytically calculated using a bond-slip model. In the present study, an explicit quadrilinear slip spring is introduced at beam-joint interface and its properties are calculated based on the bond-slip model by Sezen and Setzler (2008).

A reinforcing bar embedded in concrete can be modeled by assuming linear elastic behavior and uniform bond stress u_b over the development length l_d of the bar. The development length can be easily calculated from equilibrium equation, thus obtaining Eq. (5):

$$l_d = \frac{f_s d_b}{4u_b} \quad (5)$$

where d_b and A_b are the diameter and the area of the reinforcing bar, respectively, and f_s represents the steel stress. By assuming that the bar stress decreases linearly from f_s to zero through the development length (and in the hypothesis that the embedment length is longer than l_d), the slip can be obtained integrating the strain ε_s over l_d , as reported in Eq. (6):

$$slip = \int_0^{l_d} \varepsilon_s(x) dx \quad (6)$$

The development length can be considered as the sum of the development length for elastic and inelastic portion of the bar, i.e. l_d' and l_d'' , respectively.

The relationship between the bond stress and the local slip at each location along the embedded length of the bar can be obtained from stress-slip literature models calibrated on the basis of experimental tests by several authors. The bond-slip model by Sezen and Setzler (2008) – adopted in this study – assumes a stepped function for bond stress between the concrete and reinforcing steel over the embedment length of the bar. Based on experimental observations (Sezen 2002), the bond stress is assumed equal to $1.0\sqrt{f_c}$ MPa for elastic steel strains and $0.5\sqrt{f_c}$ MPa for inelastic steel strains. Thus, assuming bond stress uniform in each range, the strain distribution will be bilinear and the integration in Eq. (6) leads to the following expressions (Eqs (7)):

$$slip = \frac{\varepsilon_s \cdot l_d'}{2} \dots \text{for } \varepsilon_s \leq \varepsilon_y$$

$$slip = \frac{\varepsilon_y \cdot l_d'}{2} + \frac{(\varepsilon_s + \varepsilon_y) \cdot l_d''}{2} \dots \text{for } \varepsilon_s > \varepsilon_y \quad (7)$$

where l_d' and l_d'' can be obtained from Eq. (5) assuming the yielding of the bar ($f_s = f_y$) and the difference $f_s - f_y$ (with $f_s > f_y$) as bar stress, respectively.

The development length calculated as the sum ($l_d' + l_d''$) can be larger than the provided embedment length. In this case the end of the bar slips in order to mobilize bond strength and satisfy equilibrium (Eligehausen et al. 1983). A uniform or stepped bond stress model cannot directly account for slip at the unloaded end; therefore, other approaches can be adopted, for example increasing the development length adopted in the calculation of slip. Because of the quite low percentage contribution of the slip at the unloaded end and for the sake of simplicity, in this study the bar is assumed to exhibit zero slip at the end of the straight portion of the bar.

Once the slip of a reinforcing bar is determined, the rotation due to slip, θ_s , can be calculated. Assuming that slip will occur in bars under tension only, and that the rotation will be about the neutral axis (Sezen and Setzler 2008), the slip rotation θ_s can be calculated by dividing the bar slip by the distance between the bar under tension and the neutral axis, which is the difference between the

section effective depth and the neutral axis depth. In this study, θ_s is approximated as suggested by Otani and Sozen (1972), neglecting the slip involving the compression rebar, as reported in Eq. (8):

$$\theta_s = \frac{slip}{d - d'} \quad (8)$$

where $(d-d')$ is the distance between the tensile and the compressive steel bars.

As far as the definition of the quadrilinear slip spring is concerned, the first point of the spring is identified by the beam cracking and no slip deformation is assumed; therefore, the first branch of the slip spring is a rigid branch. The second and third points are defined by beam yielding and beam flexural capacity, respectively; the corresponding θ_s can be calculated according to Eq. (7) and (8) assuming a bar stress which corresponds to the yielding or the flexural capacity of the beam, respectively. The fourth point identifies the softening branch and it is defined from the beam softening branch. However, both in the cases of J- and BJ-failure mode, the flexural capacity of the beam is not reached; therefore, the definition of the softening branch of the slip spring is not influent.

It is worth to highlight that the fixed-end rotation becomes predominant in the joint response mainly when smooth rebars are used. Some efforts were already addressed to the detailed analysis of the force-slip relation for straight plain rebars and for different end anchorage details (Verderame et al. 2009). Nevertheless, currently, very poor experimental data about joint sub-assembly with plain bars are available (e.g. Pampanin et al. 2002).

MODEL VALIDATION

The proposed joint model is validated using some of the experimental test included in the database. In particular eight of the tests conducted by Wong (2002) are simulated and reported in this Section: the four specimens (denominated JA-NN03, JA-NN15, JB-NN03 and BS-L300) are characterized by a BJ-failure mode; the remaining ones (denominated BS-L, BS-L600, BS-U and BS-L-LS) by a J-failure mode. The classification of the failure mode is carried out according to Park and Mosalam (2012a). Moreover, a comparison between the proposed model and other models from literature (Celik and Ellingwood 2008, Park and Mosalam 2013) is carried out; the comparison is also performed with respect to the rigid joint model and the ASCE SEI-41 model (Figure 4 and Figure 5).

The finite element analyses of the specimens were performed using OpenSees (McKenna and Fenves, 2006). The fiber approach was adopted. Figure 3 shows the structural model developed for analysis by OpenSees. Concrete and steel properties were obtained from the analyzed test reports. *Concrete01* and *ReinforcingSteel* uniaxial materials were adopted in the modeling of nonlinear beam/column elements. The $M-\gamma$ relationship for joint panel were represented through the constitutive model given in Figure 2, and implemented in OpenSees through the *Pinching4* uniaxial material in a zero-length element located in the centerline of the joint. Another zero-length element was introduced at beam-joint interface for the slip spring.

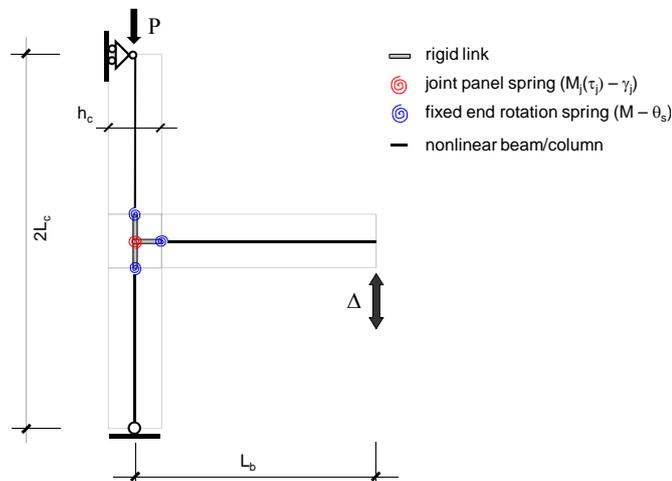


Figure 3. Numerical model for beam-column joint sub-assemblages simulations

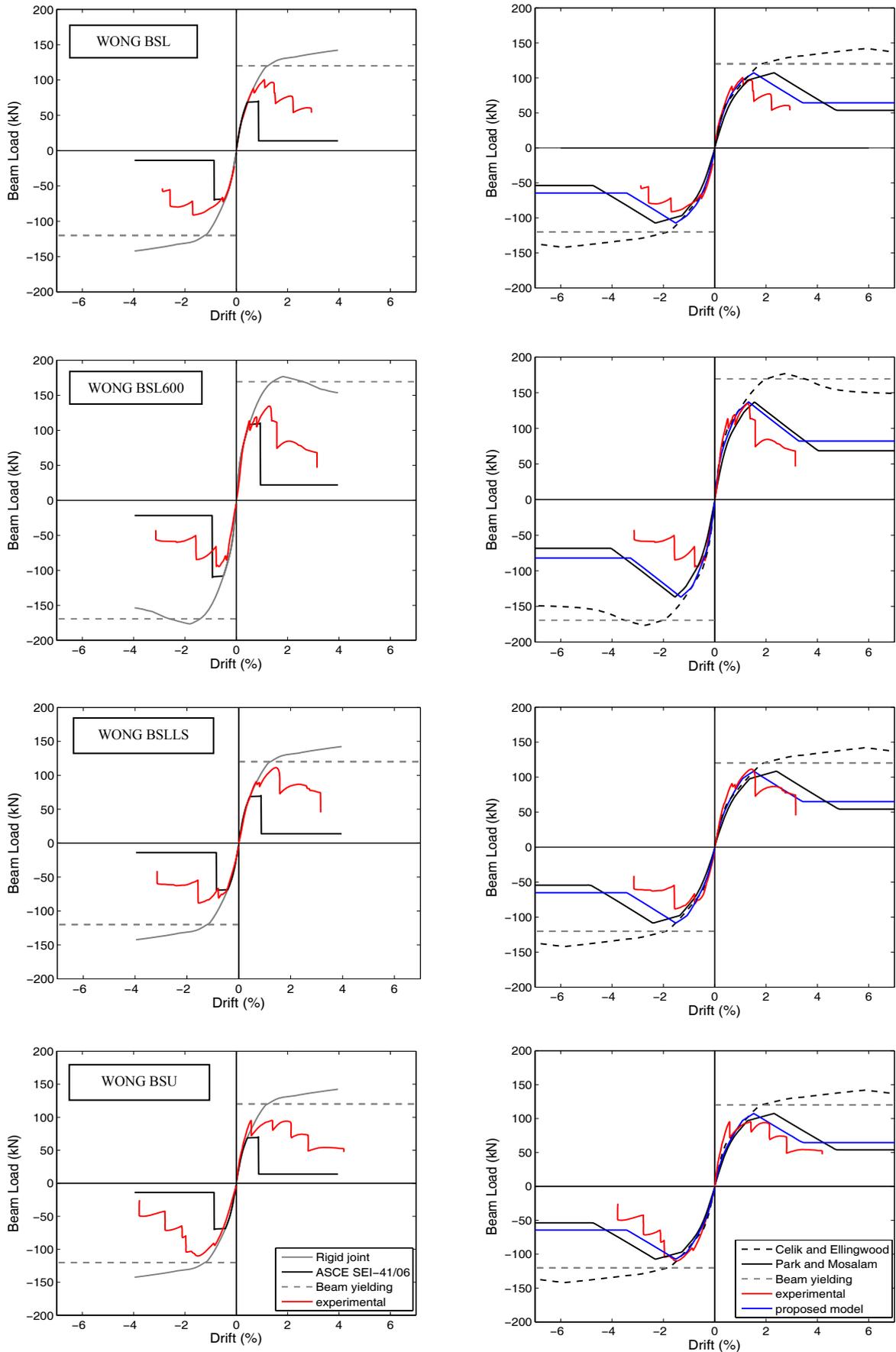


Figure 4. Comparisons between experimental and numerical results – J failure mode

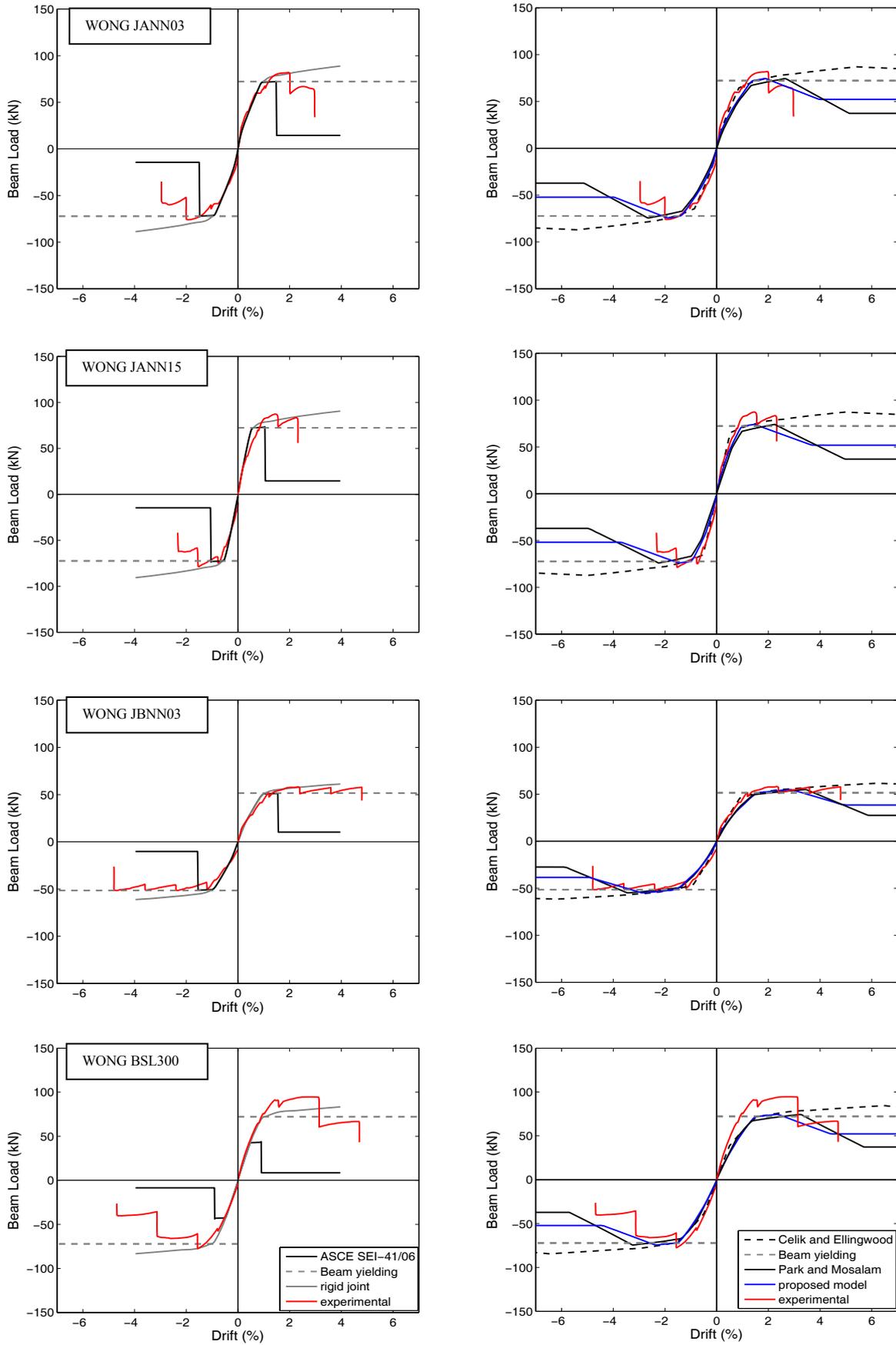


Figure 5. Comparisons between experimental and numerical results – BJ failure mode

In Table 6 experimental values of elastic stiffness (k_{el}), peak force (F_{peak}) and peak drift (Δ_{peak}) are reported for the analyzed tests; the errors related to the proposed model, and model by Celik and Ellingwood (2008) and Park and Mosalam (2013) are also reported for the positive and the negative loading directions. It can be observed that the rigid joint model generally overestimates the sub-assembly strength, especially for J-failure mode. In this model there is no limitation of strength due to joint failure and thus the adjacent elements can develop their flexural capacity (if shear failure is prevented, as in these cases). On the contrary, for the selected tests, ASCE SEI-41 underestimates the strength (as can be verified also in Table 2) and the displacement capacity of the sub-assembly, thus this code-approach appears to be very conservative. Celik and Ellingwood's model was applied using the mean values of the proposed range of values for each characteristic point. This model generally overestimates maximum strength (especially in the cases of J-failure mode) and displacement capacity for the analyzed experimental tests; the proposed joint strength ($0.83-1.00\sqrt{MPa}$ for exterior joints) seems to be too high. Moreover by comparing such a maximum strength with the joint stress corresponding to beam flexural capacity all of the tests should be classified as BJ-failure mode, even if some of them experienced a J-failure mode (i.e. no yielding in the adjacent beam occurred). With respect to other simulations, such a model provides also higher peak drifts because of the high values of joint drift capacity (mean of 2% at peak point) and because it allows the adjacent beam to completely develop its flexural deformability contribution until its peak strength is reached. Finally, model by Park and Mosalam captures quite well the strength of the sub-assemblages, as shown also in Table 2, but it seems to overestimate their deformability. By comparing the mean errors in Table 6, it can be observed that the proposed model conducts to lower errors in terms of initial stiffness, strength and drift capacity.

Table 6. Experimental values and percentage errors (e) of elastic stiffness (k_{el}), peak force (F_{peak}) and peak drift (Δ_{peak}) for the proposed model (Prop.), models by Celik and Ellingwood (C&E) and Park and Mosalam (P&M)

Test	failure mode		$k_{el, \text{ exper}}$ (kN/m)	$F_{peak, \text{ exper}}$ (kN)	$\Delta_{peak, \text{ exper}}$ (-)	e [k_{el}] (%)			e [F_{peak}] (%)			e [Δ_{peak}] (%)		
						Prop.	C&E	P&M	Prop.	C&E	P&M	Prop.	C&E	P&M
BS-L	J	Pos	17180	100.50	1.1%	-31.2	-26.2	-51.3	6.8	44.8	6.8	38.9	473.8	112.7
		Neg		-91.30	-1.7%				17.5	59.4	17.5	-10.7	268.9	36.7
BS-L600	J	Pos	13300	134.58	1.3%	69.4	99.7	4.6	1.7	31.2	1.7	1.1	105.2	17.9
		Neg		-94.80	-0.7%				44.4	86.3	44.4	93.1	292.0	125.3
BS-U	J	Pos	10690	111.35	1.4%	10.7	18.7	-21.2	-3.5	27.3	-3.5	7.5	318.2	64.2
		Neg		-88.26	-1.5%				21.8	60.7	21.8	-1.6	282.7	50.2
BS-L-LS	J	Pos	18010	95.12	1.4%	9.4	17.2	-22.2	14.1	49.2	14.1	10.9	321.1	69.3
		Neg		-110.35	-1.5%				-1.7	28.6	-1.7	0.7	282.2	53.7
JA-NN03	BJ	Pos	7110	81.96	2.0%	71.3	79.3	17.9	-9.2	11.7	-9.2	-3.2	276.9	36.5
		Neg		-76.11	-1.9%				-2.2	20.3	-2.2	-0.4	287.9	40.5
JA-NN15	BJ	Pos	8522	87.16	1.4%	62.7	71.4	6.9	-15.3	5.4	-15.3	2.6	358.1	55.2
		Neg		-78.59	-1.5%				-6.1	16.9	-6.1	-2.3	336.4	47.9
JB-NN03	BJ	Pos	5829	58.18	2.3%	-31.1	-28.8	-48.2	-5.4	5.9	-5.4	24.5	167.7	48.8
		Neg		-51.57	-4.7%				6.7	19.5	6.7	-38.7	31.7	-26.8
BS-L300	BJ	Pos	6915	95.40	2.7%	-25.6	-21.7	-39.6	-21.4	-11.1	-21.4	-12.9	141.3	19.5
		Neg		-77.57	-1.6%				-4.0	8.7	-4.0	51.5	319.9	108.0
mean error					Pos	16.9	26.2	-19.1	-4.0	20.6	-4.0	8.7	270.3	53.0
					Neg				9.6	37.5	9.6	11.4	262.7	54.4

CONCLUSIONS

In this study, a preliminary macro-model for exterior unconfined joints with no anchorage failure has been proposed. First the adopted experimental database was presented. Second, the joint panel constitutive parameters were defined to reproduce the experimental joint shear stress-strain relationships, when they were available from tests. The proposed model for joint panel is a scissors model characterized by a quadrilinear Moment (M_j) – Rotation (γ_j) spring characterized by four points for J- and BJ-mode of failure, separately: cracking, pre-peak, peak and residual points. The peak

strength is evaluated according to the model by Park and Mosalam (2012a), which directly provides a shear strength depending on the failure typology and shows a very good agreement with analyzed experimental tests. Bond-slip has been taken into account by introducing an explicit slip spring (at the beam-joint interface) whose properties are analytically calculated using a simplified bond-slip model. Finally, the proposed joint model has been validated using some of the experimental tests included in the database, and a comparison between the proposed model and other models from literature and code provisions (ASCE SEI-41, Celik and Ellingwood 2008, Park and Mosalam 2013) has been carried out. It was observed that the proposed model conducts to the lower errors in terms of initial stiffness, strength and peak drift, if compared with the other models.

Further investigations shall be conducted in order to specify the proposed model in the cases of anchorage failure due to an insufficient embedment of longitudinal rebars into the joint core. Moreover, the study will be extended to the calibration of the cyclic behavior of external joints, starting from the cyclic experimental shear stress-strain response of the joint panel, when they are available. Joint axial failure should be better investigated and introduced into the model; some theoretical friction models have been already proposed in literature (Hassan, 2011); nevertheless, currently there are only a few experimental tests which reach axial failure.

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