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REVIEW OF PLAN REGULARITY CRITERIA OF BUILDING STRUCTURES. NEW APPROACH BASED ON MODAL DATA.

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ABSTRACT

Seismic performance of building structures is largely influenced by the plan regularity, as asymmetric distributions of stiffness and/or mass negatively affect its dynamic behaviour. The most recent European standard (Eurocode 8, EN 1998-1) states that a building structure can be classified as regular in plan if it meets six conditions, some of these qualitative others quantitative. Among the quantitative conditions, 4.1 a) states that the structural eccentricity should be limited to 0.3rx (or ry), rx and ry being the torsional radii, and 4.2 b) states that the torsional radii should be larger than the radius of gyration of the floor mass in plan. However, the recommended procedure for the verification of the former conditions involves the application of numerous loadings in the numeric model that is used for the analysis of the building, both in terms of forces and moments, and has many conceptual difficulties in what concerns multi-storey buildings.

The aim of this paper is to propose an alternative formulation for the verification of structural regularity in plan, using the output of numeric models, more precisely modal properties of structures such as periods and corresponding modal participation mass factors, thus making it less time-consuming and more consensual and accepted in the technical and scientific community.

1. INTRODUCTION

The influence of the regularity of buildings on the structural response to severe ground motions has been widely proven, both by scientific research and direct observation of damages on irregular structures. It is widely accepted that regular structures, without large variations of stiffness, both in plan and in elevation, have a much better behaviour than others where these conditions are not met, when designed and detailed according to the same criteria and using the same codes. In particular, the torsional behaviour of building structures and its effect on the distribution of demands throughout the building has been the subject of research over the past decades, as exemplified by the publications by Newmark (1969), Ridell and Vásquez (1984), Takizawa (1984), Boroschek and Mahin (1992), Consenza et al. (2000), Fahjan et al. (2006) and Tabatabaei (2011).

The current version of Eurocode 8 establishes guiding principles of conceptual design of building structures located in seismic hazard areas, which are based on the examination of buildings with good behaviour when subjected to seismic motions, and replicated below:

-structural simplicity;
-uniformity, symmetry and redundancy;
-bi-directional resistance and stiffness;

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-torsional resistance and stiffness;
-diaphragmatic behaviour at storey level;
-adequate foundation.

These guiding principles are of high importance, as many of the problems that can occur in building structures during seismic motions can be avoided in a conceptual design stage. At the same time, all of these principles converge in the structural regularity concept.

If we focus on regularity in plan, the distinction between regular and irregular structures in Eurocode 8 is achieved by guaranteeing that the building structure is approximately symmetrical both for mass and stiffness distributions with respect to two orthogonal axes, that the in-plane stiffness of the floor is sufficiently large, that the plan configuration is compact and that the slenderness is not higher than 4. Additionally, two other conditions are stated, following the work of Cosenza et al. (2000), limiting structural eccentricity to 0.3rx, where rx is the torsional radius with respect to the centre of stiffness, and requiring that the minimum value for the torsional radius is equal to the radius of gyration. This classification has consequences on the structural model to be used (simplified planar or spatial) and the method of analysis (simplified or modal response spectrum analysis). Since the criteria for torsionally flexible structures also uses the minimum value for the torsional radius condition, the value of the behaviour factor q could also be affected, as it shall be decreased for torsionally flexible structures.

To verify the last two conditions, which represent the torsional behaviour of a building structure, one must compute the structural eccentricity, which is the distance between the centre of stiffness and the centre of mass, the torsional radius and the radius of gyration of the floor mass in plan. For one-storey buildings, exact definitions of the centre of stiffness and of the torsional radius can be obtained, however for multi-storey buildings only approximate definitions of these quantities can be obtained, as one cannot define a single set of coordinates for the centre of stiffness or a single value for the torsional radius.

By using the components of the stiffness matrix of a building to define the torsional radius, one can assume that a multi-storey building where all the floors have the same centre of mass and the same radius of gyration and all lateral load resisting systems run without interruption from the foundations to the top of the building, has a constant structural eccentricity.

Also, most of the building designs in Europe use the standard method and model of analysis as defined in Eurocode 8 (spatial modal response spectrum analysis), so great benefits can arise from using the standard outputs of modal analysis of buildings, such as frequencies and modal participation mass factors, to verify the criteria for regularity in plan. Since the modal properties of buildings depend on the stiffness matrix elements and the radius of gyration (through the mass matrix), it is expectable that a relation between modal properties, structural eccentricity and torsional radius can be obtained.

Therefore, the purpose of this paper is to provide a fast and practical mean of verifying the criteria for regularity in plan, avoiding the use specific load cases in the numeric model for the determination of the structural eccentricity and of the torsional radius.

Standard methods for verifying regularity in plan are presented, followed by the derivation of the governing equations of regularity for single-storey buildings where only one of the horizontal directions is coupled with torsional vibrations. This procedure is then extended to single-storey buildings where the two horizontal directions are coupled with torsional vibrations and for multi-storey buildings with the same characteristics. A study for distributed mass cantilevered systems is also presented, and finally a comparison between the standard method and the proposed alternative method is provided.

2. OVERVIEW OF EXISTING PROCEDURES

Eurocode 8 defines the centre of stiffness in single storey buildings as the centre of lateral stiffness of all primary seismic members, i.e., the point in which an external applied force causes no rotations. For multi-storey buildings, if all lateral load resisting systems, such as cores, structural walls, or frames, run without interruption from the foundations to the top of the building and the deflected shapes of the individual systems under horizontal loads are not very different, the same definition can be applied.

For a single-storey building with three degrees of freedom (two orthogonal horizontal translations and one vertical rotation) where the vibrations in the horizontal directions are uncoupled but with coupled horizontal and torsional vibrations, if a force vector F with a unit force in the x direction is applied, the equilibrium equation with respect to the centre of mass, making use of the stiffness matrix [k] and of generalized displacements vector [d], is:

$$[k][d]=[F]\Leftrightarrow\begin{bmatrix}k_x & 0 & k_{\theta x}\\ 0 & k_y & k_{\theta y}\\ k_{\theta x} & k_{\theta y} & k_\theta\end{bmatrix}\begin{bmatrix}d_x\\ d_y\\ \theta\end{bmatrix}=\begin{bmatrix}1\\ 0\\ 0\end{bmatrix} \quad (1)$$

The solution for the previous system yields the following value for the rotation about a vertical axis:

$$\theta=\frac{-k_{\theta x}}{k_x\left(k_\theta-\frac{k_{\theta x}^2}{k_x}-\frac{k_{\theta y}^2}{k_y}\right)}\quad (F_x=1) \quad (2)$$

The equilibrium equation for a unit moment M about a vertical axis is:

$$[k][d]=[F]\Leftrightarrow\begin{bmatrix}k_x & 0 & k_{\theta x}\\ 0 & k_y & k_{\theta y}\\ k_{\theta x} & k_{\theta y} & k_\theta\end{bmatrix}\begin{bmatrix}d_x\\ d_y\\ \theta\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix} \quad (3)$$

The corresponding rotation about a vertical axis is:

$$\theta=\frac{1}{k_\theta-\frac{k_{\theta y}^2}{k_y}-\frac{k_{\theta x}^2}{k_x}}\quad (M=1) \quad (4)$$

The denominators in expressions (2) and (4) are similar, so to counterbalance a rotation caused by a unit force, one must apply a moment of

$$M=1\times\frac{k_{\theta x}}{k_x} \quad (5)$$

Therefore, using the definition of centre of stiffness provided in Eurocode 8, the structural eccentricity is

$$e_y=-\frac{k_{\theta x}}{k_x}=\frac{\theta(F_x=1)}{\theta(M=1)} \quad (6)$$

This means that in order to find the centre of stiffness coordinate with respect to the centre of mass in a given direction, one must apply a unit force and a unit moment in the centre of mass and then find the ratio of the obtained rotations for each one of the loadings.

The radius of gyration of the floor mass, the floor mass and centre of mass can be easily obtained through:

$$M_\theta = \gamma \times \int_A r^2 dA = \gamma \times \int_A (x^2 + y^2) dA \tag{7}$$

$$M = \gamma \times \int_A dA \tag{8}$$

$$cm = \frac{\gamma \times \int_A y dA}{\gamma \times \int_A dA} \tag{9}$$

where γ is the weight per unit area of the floor, cm is the centre of mass y coordinate, M is the mass of the floor and M_θ is the polar moment of inertia of the floor mass in plan.

By writing the stiffness matrix with respect to the centre of stiffness, obtained with the combination of structural eccentricity and the centre of mass coordinate, the equilibrium equation for an applied unit moment and applied unit forces in each direction is

$$[k][d] = [F] \Leftrightarrow \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_\theta \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ \theta \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{10}$$

Torsional and translational stiffness's can be obtained from:

$$k_\theta = \frac{1}{\theta(M = 1)} \tag{11}$$

$$k_x = \frac{1}{d_x(F_x = 1)} \tag{12}$$

where k_θ is the torsional stiffness, k_x is the translational stiffness, θ is the rotation about a vertical axis and dx is the horizontal displacement. The torsional radius and the radius of gyration are obtained from the following expressions, respectively:

$$r_x = \sqrt{\frac{k_\theta}{k_y}} \tag{13}$$

$$l_s = \sqrt{\frac{M_\theta}{M}} \tag{14}$$

For multi-storey buildings, the described method requires that a total number of 6n loading cases, where n is the number of storeys, is applied to the numerical model, as well as the computation of the floor mass and polar moment of inertia, as described in Bisch et al. (2012).

Another of the possible procedures that can be used in frames and in systems of slender walls with prevailing flexural deformations is to compute the position of the centre of stiffness using the moments of inertia of the cross-sections of the vertical elements, as mentioned in Eurocode 8. Shear deformations should also be accounted for, if relevant to the structural response of the building. This procedure is less reliable than the aforementioned loading of the structural model, because it neglects the horizontal elements (beams and slabs) contribution to the global stiffness of the structure.

3. PROPOSED PROCEDURE – SINGLE STOREY BUILDINGS WITH ECCENTRICITY IN ONE DIRECTION

The free vibrations of a multi-degree of freedom building structure are analysed using the following equation:

$$[K]\{v\} - p^2[M]\{v\} = 0 \tag{15}$$

where $[K]$ is the stiffness matrix, p is the modal frequency, $[M]$ is the mass matrix and $\{v\}$ is the modal configuration vector. For equation (15) to be true, one must write:

$$|K - p^2 M| = 0 \tag{16}$$

For the simplest case, it is assumed that only one of the horizontal directions vibrations is coupled with torsional vibration and that there is no coupling between horizontal directions. The dynamic equilibrium equation (15) is written with respect to the centre of mass, meaning that the mass matrix is diagonal and the stiffness matrix is non-diagonal.

Under this set of conditions, it is possible to expand equation (16) to the characteristic equation of frequencies, in order to find the equations of regularity for such a system:

$$\begin{vmatrix} K_{xx} - p^2 M_{xx} & 0 & 0 \\ 0 & k_{yy} - p^2 M_{yy} & k_{y\theta} \\ 0 & k_{y\theta} & k_{\theta\theta} - p^2 M_{\theta\theta} \end{vmatrix} = 0 \tag{17}$$

This determinant yields two solutions:

$$p_x = \sqrt{\frac{K_{xx}}{M}} \tag{18}$$

$$p^2 = \frac{k_{yy}}{M} \frac{\left(l_s{}^2 + r_{x,CM}{}^2\right) \pm \sqrt{\left(l_s{}^2 - r_{x,CM}{}^2\right)^2 + 4 l_s{}^2 e^2}}{2 l_s{}^2} \tag{19}$$

In the previous expression $r_{x,CM}$ denotes the torsional radius with respect to the centre of mass of the floor and e denotes the structural eccentricity as defined by equation (6).

Considering only the two coupled modes (y and θ), the modal configuration vector is:

$$\underset{\sim}{v} = \begin{Bmatrix} 0 \\ 1 \\ \theta \end{Bmatrix} \quad \text{with} \quad \theta = \frac{p^2 M - k_{yy}}{k_{\theta y}} \tag{20}$$

The percentage of effective modal mass in y direction is

$$\%P_y = \frac{P_y{}^2}{M} = \frac{1}{1 + l_s^2 \theta^2} \tag{21}$$

Assuming that the translational mode has a lower frequency than the rotational mode, and introducing the corresponding solution presented in equation (19) in equation (20), and then rewriting equation (21), one obtains

$$\%P_y = \frac{1}{1 + l_s^2 \left(\frac{\frac{\left(l_s{}^2 + r_{x,CM}{}^2\right) - \sqrt{\left(l_s{}^2 - r_{x,CM}{}^2\right)^2 + 4 l_s{}^2 e^2}}{2 l_s{}^2} - 1}{e} \right)^2} \tag{22}$$

At this moment it is useful to define the limiting value of eccentricity of Eurocode 8 when the stiffness matrix and the torsional radius are written with respect to the centre of mass, as well as the ratio of torsional radius to the radius of gyration:

$$e = \frac{0.30}{\sqrt{1 + 0.3^2}} \cdot r_{x,CM} = 0.287 \cdot r_{x,CM} \tag{23}$$

$$\alpha = \frac{r_{x,CM}}{l_s} \tag{24}$$

Dividing the two roots of equation (19), assuming $r_x \geq l_s$, and using the limiting value of eccentricity of Eurocode 8, the following equation can be written:

$$\left(\frac{p_y}{p_\theta}\right)^2 = \frac{\left(1 + \alpha^2\right) - \sqrt{1 - 2\alpha^2 + \alpha^4 + 4 \times 0.287^2 \alpha^2}}{\left(1 + \alpha^2\right) + \sqrt{1 - 2\alpha^2 + \alpha^4 + 4 \times 0.287^2 \alpha^2}} \tag{25}$$

Rearranging equations (22) and (25) leads to

$$\%P_y = \frac{1}{1 + \frac{1}{0.287^2 \alpha^2} \left(\left(\frac{p_y}{p_\theta}\right)^2 \frac{1}{2} \chi - 1 \right)^2} \tag{26}$$

with

$$\chi = \left(1 + \alpha^2\right) + \sqrt{1 - 2\alpha^2 + \alpha^4 + 4 \times 0.287^2 \alpha^2} \tag{27}$$

$$\alpha^2 = \frac{-\left(\frac{4 \times \left(0.287^2 - 1\right)}{\xi} - 2 \right) + \sqrt{\left(\frac{4 \times \left(0.287^2 - 1\right)}{\xi} - 2 \right)^2 - 4}}{-2} \tag{28}$$

$$\alpha^2 = \frac{-\left(\frac{4 \times \left(0.287^2 - 1\right)}{\xi} - 2 \right) - \sqrt{\left(\frac{4 \times \left(0.287^2 - 1\right)}{\xi} - 2 \right)^2 - 4}}{-2} \tag{29}$$

$$\xi = \frac{\left(\left(\frac{p_y}{p_\theta} \right)^2 - 1 \right)^2}{\left(\left(\frac{p_y}{p_\theta} \right)^2 + 1 \right)^2} - 1 \tag{30}$$

Equation (28) should be used for $\frac{r_{x,CM}}{l_s} < 1$ and equation (29) for $\frac{r_{x,CM}}{l_s} \geq 1$.

If $r_{x,CM} < l_s$ the frequencies on the left side of equation (25) should be inverted, and the corresponding changes to equations (26) to (30) should be made.

Equation (26) represents the regularity in plan, expressing the percentage of effective modal mass as a function of the ratio of the fundamental translational frequency to the fundamental torsional frequency, by using equations (27) trough (30).

Analysis of the structural eccentricity (condition 4.1 a) of Eurocode 8) and minimum torsional radius (condition 4.1 b) of Eurocode 8) is performed using these relations.

A plot of the equations of regularity in plan is presented in Fig. 1 for several eccentricities, including the limiting value of Eurocode 8. It becomes obvious that one can determine the structural eccentricity only knowing the values of the percentage of effective modal mass and of the ratio of frequencies.

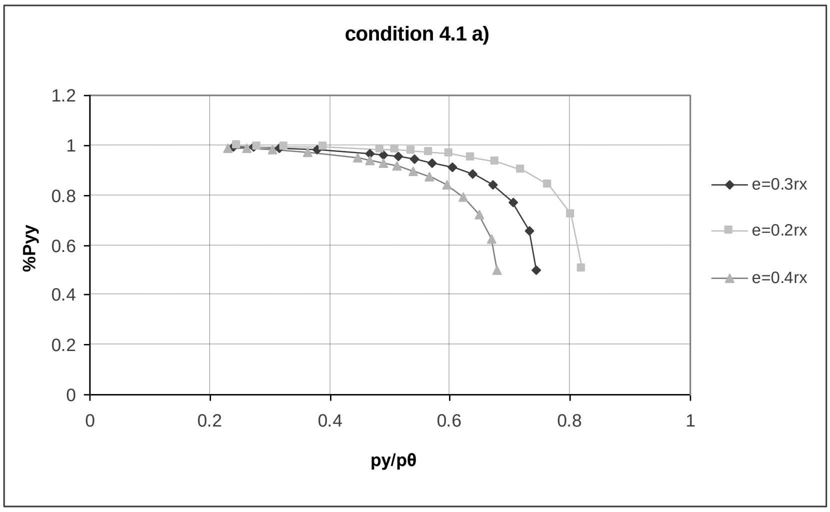

Figure 1 – Plot of regularity equations for various eccentricities.

The torsional radius can be computed using equations (28) and (29). For the purpose of verifying condition 4.1b) of Eurocode 8, only equation (29) should be used. As a result of the previous expressions, it is also possible to conclude that if $\frac{p_y}{p_\theta} < 1$ then $r_x \geq l_s$ (condition 4.1 b) of Eurocode 8), regardless of the value of the structural eccentricity.

4. PROPOSED PROCEDURE – SINGLE STOREY BUILDINGS WITH ECCENTRICITY IN TWO DIRECTIONS

Unlike the previous case, the regularity conditions were not obtained in the form of equations, rather in the form of charts, similar to the one presented in Fig. 1, by varying the different intervening parameters.

It was assumed that the two horizontal directions are coupled with torsional vibration and that there is no coupling between horizontal directions. The dynamic equilibrium equation (15) is written with respect to the centre of mass, meaning that the mass matrix is diagonal and the stiffness matrix is non-diagonal.

Under this set of conditions, the characteristic equation of frequencies can be written in its determinant form as:

$$\begin{vmatrix} K_{xx} - p^2 M_{xx} & 0 & k_{x\theta} \\ 0 & k_{yy} - p^2 M_{yy} & k_{y\theta} \\ k_{x\theta} & k_{y\theta} & k_{\theta\theta} - p^2 M_{\theta\theta} \end{vmatrix} = 0 \quad (31)$$

It is useful to define a new variable:

$$\delta = \sqrt{\frac{k_{xx}}{k_{yy}}} \quad (32)$$

Using several eccentricities, it is possible to plot the percentage of effective modal mass in the x and y direction as a function of the ratio $\frac{p_y}{p_\theta}$ for different values of δ just by varying α. Therefore, a parametric variation of α (1 to 4), δ (1 to 2) and e_i (0 to 0.6r$_i$) was made in order to obtain the regularity curves for this case.

Plots of these scenarios are presented below in Fig. 2 and Fig. 3, for different values of δ of 1 and 1.4.

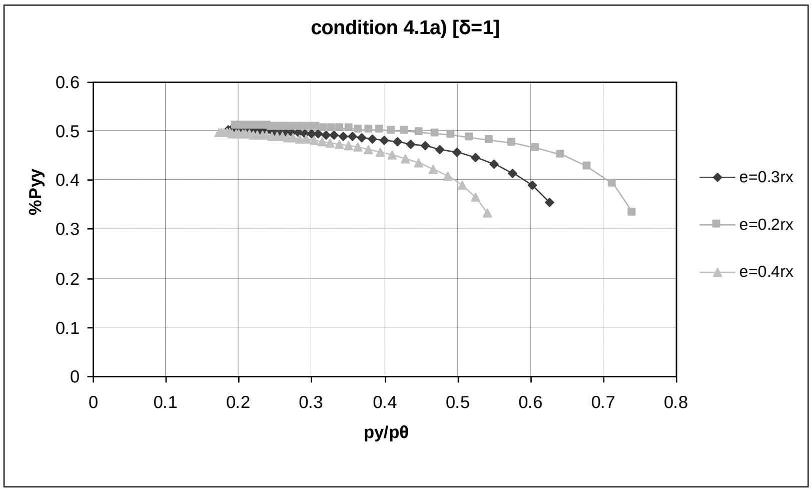

Figure 2 – Plot of regularity conditions for δ =1 and various eccentricities.

The torsional radius is one of the varying parameters when obtaining the regularity conditions and the structural eccentricity charts. Once again, for the sake of verifying the compliance with condition 4.1 b) of Eurocode 8, it can be seen that if $\frac{p_y}{p_\theta} < 1$ then $r_x \geq l_s$. The only difference with respect to the former presentation is that when $r_x = l_s$ and δ=1 the percentage of effective modal mass in the y direction is equal to 0.33, as shown in Fig. 2.

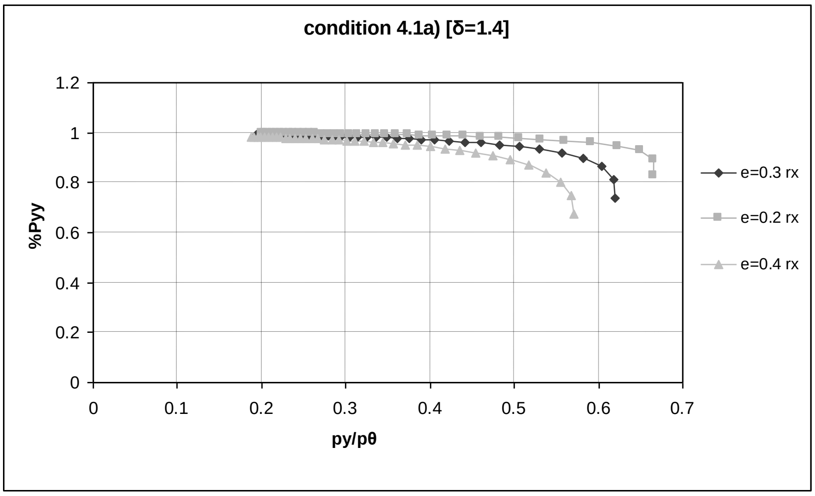

Figure 3 – Plot of regularity conditions for δ =1.4 and various eccentricities.

5. PROPOSED PROCEDURE – MULTI-STOREY BUILDINGS WITH ECCENTRICITY IN TWO DIRECTIONS

For this case, the same hypotheses adopted in the previous example were followed. Additionally, six other base hypotheses were assumed:

- The sections of all horizontal load carrying elements remain constant in elevation;
- The building has three degrees of freedom per storey, two orthogonal translations and one rotation about a vertical axis;
- The mass of all floors is the same, as well as the polar moment of inertia of the floor mass in plan;
- The eccentricity varies proportionally to the square root of the ratio $\frac{k_\theta}{k_i}$ in both directions, although as the ratio $\frac{k_i}{k_j}$ also varies, eccentricity is not the same in the two directions;
- The eccentricity, as defined in chapter 2, is constant in elevation;
- All degrees of freedom are referred to the mass centre, and as a result the mass matrix is diagonal.

A numerical approach was followed to obtain the regularity conditions for multi-storey buildings. A program with dynamic multi-dimensional arrays was developed, and variations of four different parameters (number of storeys, translational stiffness, eccentricity and the polar moment of inertia of the floor mass in plan) was studied.

The square root of the ratio of translational stiffness's δ was assumed to be varying from 1 to 2, while values for structural eccentricity ranged from 0 to 0.6rx. The polar moment of inertia of the floor mass in plan varied with α=1 to α=4.

It is not necessary to compute the torsional radius for verification of condition 4.1 b) of Eurocode 8, because as shown before when $\frac{p_y}{p_\theta} < 1$ the torsional radius $r_x \geq l_s$.

Similarly to the previous examples, it is possible to plot the percentage of effective modal mass in the y direction as a function of the ratio $\frac{p_y}{p_\theta}$ for different values of δ just by varying α, for a given number of storeys.

As expected, the increase in the number of storeys forces the percentage of effective modal mass to be lower, with minimum values rising with the increase of δ.

As an example, a plot of the structural eccentricity conditions is presented below in Fig. 4 for a building with five storeys.

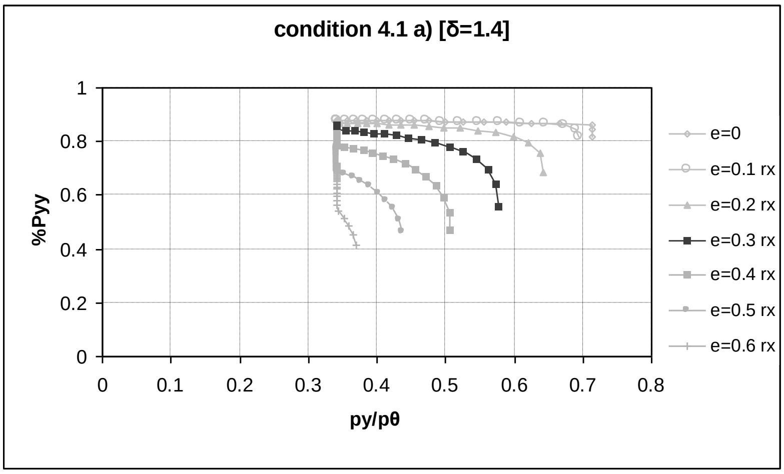

Figure 4 – Plot of regularity conditions for δ=1.4 and various eccentricities for a five storey building.

6. CASE STUDY – SIX STOREY BUILDING

A case study was developed for the application of the alternative formulation for the verification of regularity in plan for a six storey building, based on the example provided in Bisch et al. (2012) and using the commercial program SAP2000®, but eliminating basement levels and establishing a constant storey height of 3.0m.

The modelled building has beams with 0.25mx0.50m, exterior columns with 0.30mx0.70m and interior columns with 0.50mx0.50m. Slab thickness is 0.18m, exterior walls have a thickness of 0.30m and the interior core walls have a thickness of 0.25m. Concrete grade is C25/30 and the permanent loading, besides self-weight, is 2.0 kN/m^2. An exploration load of 3.0 kN/m^2 was used and a combination factor of 0.4 was adopted for the determination of the floor masses. The following picture shows the typical floor plan.

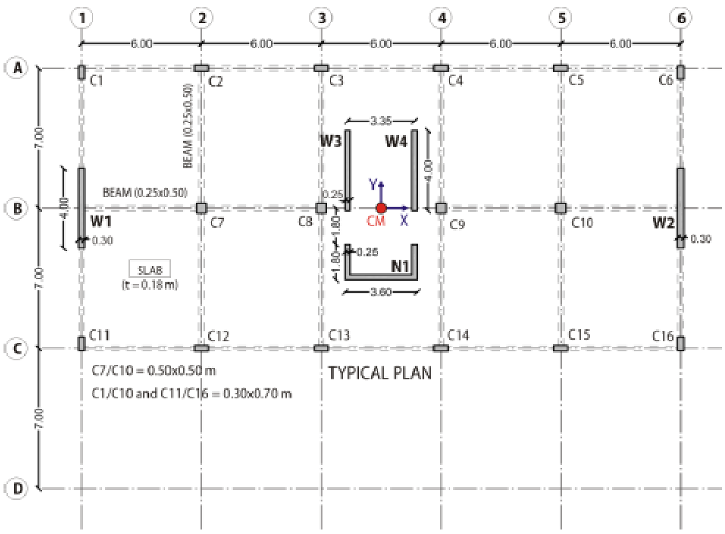

Figure 5 – Typical floor plan. Adapted from Bisch et al. (2012).

The modal properties of the structure are listed in the table below:

Table 1 – model properties of the building

Mode	T [s]	%PTx	%PTy	%PRz
1	0.71	70.85	0.00	15.22
2	0.50	0.00	70.39	42.72
3	0.48	2.88	0.00	12.76

Applying the procedure specified in chapter 5 to the building, condition 4.1 b) is automatically satisfied by examination of the fundamental periods and condition 4.1 a) of Eurocode 8 can be evaluated graphically, as presented in Fig. 6.

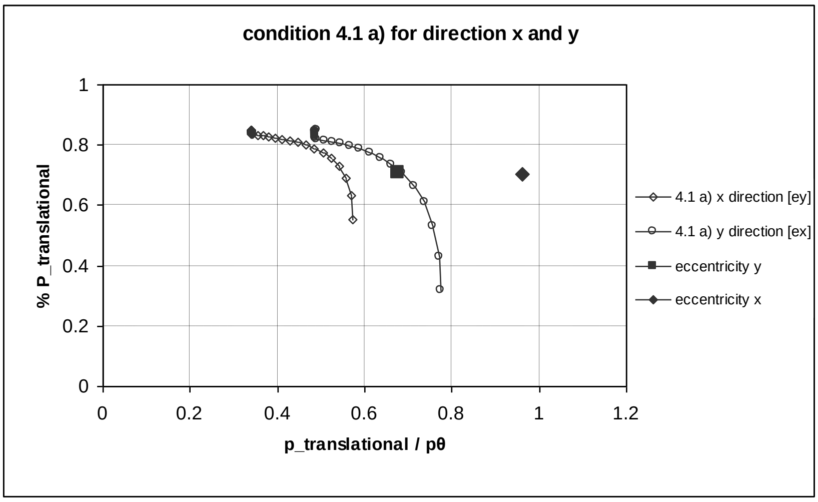

Figure 6 – Verification of regularity in plan.

7. CONCLUSIONS

An alternative method for verifying structural regularity in plan was presented, using only modal properties of the structure for the verification of conditions 4.1 a) and 4.1 b) of Eurocode 8. The method was presented for three types of buildings: single storey buildings with only one translational mode coupled with the torsional mode, single storey buildings with both translational modes coupled with the torsional mode and multi-storey buildings with both translational modes coupled with the torsional mode.

Regularity conditions were presented in the form of charts, expressing the percentage of effective modal mass of the translational modes as a function of the ratio of frequencies of the translational mode to the rotational mode. This allows designers to verify the torsional behaviour of a structure using only the standard outputs of structural analysis programs, such as the percentages of effective modal masses and the frequencies of each mode. The amount of work required to verify these conditions is minimal, when comparing to the existing procedures presented in chapter 2.

The proposed method appears to be suitable for wide use, allowing a fast and clear verification of the regularity in plan, and avoiding the mass usage of the irregular structure classification by designers, in face of all the work that the existing procedures require.

8. ACKNOWLEDGMENT

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