



CONTROLLING TORSIONAL RESPONSES OF STRUCTURES UNDER ONE AND TWO DIRECTIONAL EXCITATIONS USING DAMPERS

M.R MANSOORI¹ and A.S MOGHADAM²

ABSTRACT

The past researches show that supplemental dampers have a great effect on improving the seismic performance of structures. Further, suitable distribution of such devices can control undesired torsional responses of asymmetric structures, efficiently. In this paper the suitable distribution of viscous dampers are determined to control torsional seismic responses of asymmetric structures considering several affecting parameters.

The first part of the investigation focuses on controlling torsional responses of one-story asymmetric buildings with one directional stiffness asymmetry by using viscous dampers. Nonlinear behavior of structural elements is considered in each analysis and optimum damper distribution is proposed. The sensitivity of the results is investigated for structures with different damper capacities and structures with different torsional stiffness. In the next step, suitable damper distribution for controlling torsion of mass asymmetric structures is investigated. The results show that suitable distribution of dampers for controlling a response may change in different cases. Finally, the effects of two directional asymmetry and two directional excitation are studied on the obtained damper distributions. The results of this part show that the suitable distribution of viscous dampers obtained for controlling torsion of two directional asymmetric structures has not a considerable difference compared to the case of one directional asymmetry.

INTRODUCTION

Studying the effects of past earthquakes on the asymmetric buildings shows that damage due to structural torsion is one of the most important causes of failure in such buildings. This is mainly because of the uneven lateral deformation demand among the resisting elements which results in concentration of inelastic deformations in specific parts and may lead to local or global instability. Also structural asymmetry is usually inevitable because of architectural constraints, torsional components of earthquake records and uncertainties in material which lead to undesired yielding of some parts of the structures. Thus, controlling torsional response of asymmetric buildings has always been one of the research topics in the earthquake engineering.

During the past decades, there was a trend to improve buildings performance in the earthquakes by using energy dissipating devices such as dampers. Such devices absorb most of the earthquake energy and consequently decrease the formation of plastic hinges in the structural elements which leads to a higher performance level of the structures. Among different types of dampers, viscous dampers are very effective in controlling response of buildings considering the following

¹ Assistant Professor; Department of Earthquake Engineering; Islamic Azad University, Science and Research Branch, Tehran, Iran, m.mansoori@srbiau.ac.ir

² Assistant professor, International Institute of Earthquake Engineering and Seismology (IIEES), Tehran, Iran, moghadam@iiees.ac.ir

specifications: first, their forces are out of phase with other forces applied to the building. Second, the low velocity loads such as thermal loads do not lead to continuous stress as the damper forces are velocity dependent. Third, after an earthquake the structure returns to its initial positions (for example a structure with frictional dampers may not return to its initial position because of the plastic deformations).

It is obvious that torsional responses of asymmetric structures can be controlled efficiently by using a suitable distribution of viscous dampers as indicated in many previous studies. In one of the first researches, the effect of governing parameters of structures with dampers such as damping eccentricity, damping coefficient and damping radius of gyration on the responses of a one-story elastic building with one-directional stiffness asymmetry has been studied (Goel, 1998). The results show that if the center of supplemental damping (C_{sd}) is located at a distance equal to the stiffness eccentricity on the opposite side with respect to the center of mass (CM), the lateral displacement is controlled efficiently. Another investigation (Goel, 2000) shows that in a one-story building, the damping ratio of the first mode is increased as the damping eccentricity is increased on the flexible side. Since the displacement of the flexible edge is controlled by the first mode, such distribution of dampers may impose the most decrease on the displacement of the flexible edge.

The studies on the nonlinear behavior of structures equipped with viscous dampers have been performed by Goel et al. (2001). The results for controlling lateral displacement were the same as results obtained in the linear behavior. Also the study shows that optimum damping eccentricity obtained for the linear behavior will impose the least ductility demand of the elements located on the flexible side. The possibility of controlling multiple seismic responses of a structure by using suitable distributions of viscous dampers has been studied by Mansoori and Moghadam (2009) considering nonlinear behavior of the structures. The results of this study show that in a one-story structure with small values of stiffness eccentricity, if C_{sd} is located at the opposite side of stiffness center (CS) with respect to CM in a way that damping eccentricity is equal to stiffness eccentricity, lateral displacement, lateral acceleration and diaphragm torsion could be controlled efficiently.

An innovative concept called "Torsional Balance" has been introduced for controlling of structures with viscous dampers (De La Llera et al., 2005). Torsional balance is a property of an asymmetric structure that leads to a similar deformation demand in structural members equidistant from the geometric center of the diaphragm (GC). In this concept, it is tried to equalize the mean square values of the deformation of elements equidistant from the GC by using a suitable damper distribution. The results show that using the distribution obtained by this concept may control the rotation and lateral displacement of the diaphragm efficiently. This concept has also been studied for the structures equipped with tuned mass dampers which lead to some suitable distributions of such devices in the asymmetric structures. (Almazán et al. 2012).

Studying the previous researches show that finding an optimum distribution of damper is dependent to several parameters including type of response (lateral displacement, lateral acceleration, etc.), type of asymmetry (mass asymmetry, stiffness asymmetry, etc) and earthquake excitation (one directional or two directional excitations). In this paper the effects of different parameters on the suitable distribution of viscous dampers are studied. First part of this investigation concentrates on suitable distribution of viscous dampers to control torsional response of one-directional asymmetric structures considering nonlinear behavior of structural elements. In the next step, the effects of two-directional asymmetry and two-directional excitation are studied on the obtained distributions.

EFFECT OF VISCOUS DAMPERS ON DAMPING MATRIX

In a structure equipped with supplemental viscous dampers, damping matrix consists of two parts as following:

$$C = C_0 + C_d = (\alpha M + \beta K) + C_d \quad (1)$$

Where C_0 is the matrix of inherent viscous damping and α and β are Rayleigh coefficients. C_d is the supplemental damping matrix which is dependent to the capacity and distribution of viscous dampers.

Let consider a single story structure with two directional damping and stiffness asymmetry as presented in Fig.1. The displacement vector is defined by $u = [u_x \ u_y \ u_\theta]^T$. Assume c_{xi} and c_{yi} represent the damping coefficient for the i -th damper in x and y direction and y_{di} and x_{di} are the distance of the i -th damper from center of mass (CM) in x and y direction respectively. The translational and torsional damping coefficients with respect to CM are obtained as:

$$c_x = \sum_i c_{xi} \quad c_y = \sum_i c_{yi} \quad c_\theta = \sum_i c_{xi} y_{di}^2 + \sum_i c_{yi} x_{di}^2 \quad (2)$$

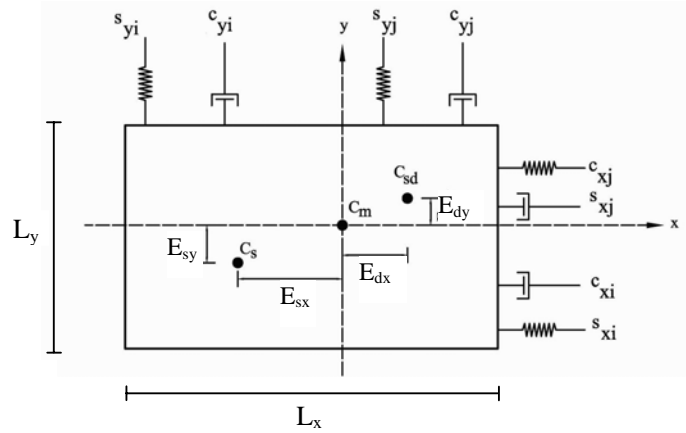


Figure 1. one-story structure with two directional damping and stiffness asymmetry

In a system equipped with viscous dampers, damping eccentricity is defined as the distance between the centroid of damper forces (damping center or C_{sd}) and CM when the system is subjected to a uniform translational velocity in the direction under consideration. Mathematically the Normalized damping eccentricities in the x and y directions are defined as:

$$e_{dx} = \frac{E_{dx}}{L_x} = \frac{l}{L_x c_y} \sum_i x_{di} c_{yi} \quad e_{dy} = \frac{E_{dy}}{L_y} = \frac{l}{L_y c_x} \sum_i y_{di} c_{xi} \quad (3)$$

Finally, the supplemental damping matrix for the system is obtained by:

$$C_d = \begin{bmatrix} c_x & 0 & -L_y^2 e_{dy}^2 c_x \\ 0 & c_y & L_x^2 e_{dx}^2 c_y \\ -L_y^2 e_{dy}^2 c_x & L_x^2 e_{dx}^2 c_y & c_\theta \end{bmatrix} \quad (4)$$

It is obvious from Eq.(1) and Eq.(4) that the damping matrix C is dependent to the distribution of dampers and consequently the structure is classified as a system with non-proportional damping. The modal analysis of such system is performed using the equations of motion in the state space as expressed by Foss (1958) and completed by Veletsos and Ventura (1986). Dynamic analysis of systems with non-proportional damping could be performed using the time integration method by changing the damping matrix in each step similar to stiffness matrix.

NUMERICAL MODEL SPECIFICATIONS

The system considered for the parametric study is a single story steel structure consisting of a rectangular rigid deck ($18m \times 15m$) supported by four moment-resisting frames in each of the two orthogonal directions. The height of the system is 3.2 m and the supplemental viscous dampers are located in the bracing system. It is assumed that the bracing system does not incorporate in the lateral stiffness and strength of the system. Fig.2 shows the plan view of the basic model.

The system is designed in the symmetric state (excluding all torsional effects) according to national Iranian building code and Iranian seismic code (BHRC, 2006) for high seismic risk area ($A=0.35$) and stiff soil ($T_s=0.5$ sec). Since the Codes let a reduction in the design base shear of structural systems with supplemental dampers, the modification factors are also considered (FEMA 450, 2003). The symmetric model is specified as model 1 in the analyses.

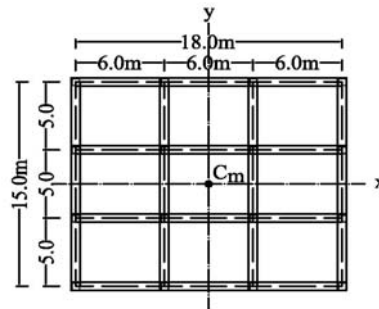


Figure 2. plan view of the basic model

ONE DIRECTIONAL STIFFNESS ASYMMETRIC MODELS

The stiffness asymmetric models (No. 2 to 7) are derived by changing the beam and column sections of the basic model. The stiffness and strength properties are asymmetric only about the y axis. The stiffness and strength asymmetry are generated by increasing the dimensions of the elements of two left frames and decreasing the dimensions of the elements of two right frames in a way that the total lateral strength of the system about y axis remains equal to the basic model. Several pushover analyses have been performed to obtain the strength, stiffness and yield displacements of the frames. The analyses are performed by OpenSees program (McKenna et al., 2000) using fiber elements for beam and columns and an strength hardening behavior for steel. Also the pushover curves are idealized as bilinear curves according to FEMA 356 (2000). Table.1 shows different parameters of models 1 to 7. In this table e_s and e_r represent stiffness and strength eccentricity, T_y and T_θ represents uncoupled lateral and torsional periods and T_1 and T_2 represents first and second periods of the structures respectively. Comparing T_y and T_θ specifies that all models are torsional stiff ($T_y > T_\theta$ or $\Omega = T_y / T_\theta > 1$).

Table 1. Static and dynamic parameters of models 1 to 7

Model No.	%es	%er	Y Strength (ton)	T_y (sec) (Uncoupled)	T_θ (sec) (Uncoupled)	T1(sec)	T2(sec)
1	0.0	0.0	152.8	0.3926	0.3074	0.3926	0.3074
2	-5.1	-5.0	153.2	0.3919	0.3061	0.3973	0.3036
3	-10.0	-9.9	152.8	0.396	0.3091	0.4146	0.3005
4	-10.2	-7.1	152.4	0.3895	0.3063	0.4112	0.297
5	-15.4	-11.1	152.8	0.386	0.3045	0.4331	0.2867
6	-20.2	-15.7	153.2	0.3817	0.3007	0.4607	0.2751
7	-25.2	-21.1	153.4	0.3655	0.2946	0.4944	0.2589

ONE DIRECTIONAL MASS ASSYMETRIC MODELS

Mass asymmetric models are produced by moving the CM from the Geometric center of the diaphragm (GC) along a desired direction. In this regard, the distance between the CM and GC is equal to mass eccentricity (E_m). If the diaphragm is considered as a plate with a mass density of σ , a one directional mass asymmetric model could be established by increasing the density of a division of one side of the diaphragm and decreasing the density of the equal division in the other side as shown in Fig.3. In this figure the density of a division with a width of λa in the left of diaphragm is changed to $\sigma(1-\eta)$ while the density of a similar division in the right is changed to $\sigma(1+\eta)$. Such changes in the diaphragm mass density lead to a normalized mass eccentricity of:

$$e_{mx} = \frac{E_{mx}}{a} = \eta\lambda(1-\lambda) \quad (0 < \lambda < 0.5, \quad -1 < \eta < 1) \quad (5)$$

Maximum mass eccentricity is obtained when $\eta = 1$ and $\lambda = 0.5$ which leads to $e_{mx} = 0.25$. The mass moment of inertia of the rectangular diaphragm with respect to CM in the symmetric and asymmetric cases are obtained by:

$$I_{sym.} = m \frac{(a^2 + b^2)}{12} = \sigma \cdot ab \frac{(a^2 + b^2)}{12} \quad (6)$$

$$I_{asym.} = m\lambda\left(1 - \frac{e_{mx}}{\lambda(1-\lambda)}\right) \left[\frac{(\lambda^2 a^2 + b^2)}{12} + \left(\frac{a}{2} - \frac{\lambda a}{2} + ae_{mx}\right)^2 \right] + m(1-2\lambda) \left[\frac{(a-2\lambda a)^2 + b^2}{12} + a^2 e_{mx}^2 \right] + m\lambda\left(1 + \frac{e_{mx}}{\lambda(1-\lambda)}\right) \left[\frac{(\lambda^2 a^2 + b^2)}{12} + \left(\frac{a}{2} - \frac{\lambda a}{2} - ae_{mx}\right)^2 \right] \quad (7)$$

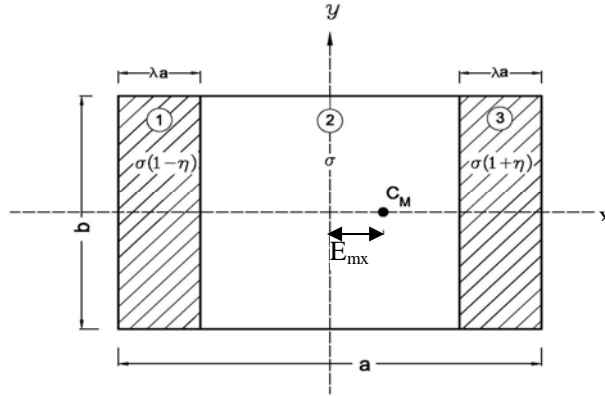


Figure 3. Method of creating one directional mass eccentricity

TWO DIRECTIONAL ASSYMETRIC MODELS

Two directional asymmetry is considered as a stiffness asymmetry in the y direction and a mass asymmetry in the x direction. All models have the same strength in the y direction according to Table.1. The x strength of the models are also the same and approximately equal to 145 ton.

DISTRIBUTION OF DAMPERS

For the one directional asymmetry case, viscous dampers are assigned to the models as a bracing system in y direction which leads to one directional damping asymmetry. Linear pure viscous behavior is considered for all dampers. For comparison between the responses of models in different cases the following assumptions are made:

1. The total lateral capacity of dampers in all models (c_y) is set to a fixed value of 100 ton.s/m which leads to a damping ratio of 20% for the lateral mode of the symmetric model. Using constant value of lateral damping capacity makes the suitable distributions to be based on damping equipment expenses which are related to the capacity and plays an important role in the design.
2. A linear distribution of dampers is considered between four frames to catch a desired damping eccentricity. For damping eccentricity equal to $e_{dx}=0.0$ all resisting y frames have the same damping capacity equal to one-fourth of the total capacity. For $+0.278 < e_{dx} < +0.389$, the damping capacity of the left frame is set to zero and the total damping capacity is assigned to the rest three frames in a linear pattern. For $+0.389 < e_{dx} < +0.5$, the damping capacities of the two left frames are set to zero and the total damping capacity is assigned to the rest two frames in a linear pattern. For $e_{dx}=+0.5$ all damping capacity is assigned to the right frame.

For the two directional asymmetry case, the distribution of dampers in the y direction is similar to above, but the distribution in the x direction is considered uniform. In other word, only one directional damping asymmetry is considered in this study.

MODLLING CHARACTERISTICS AND GROUND MOTIONS

In order to study nonlinear dynamic responses of structures, several time history analyses have been performed using OpenSees program (McKenna et al., 2000). Nonlinear effects are considered using fiber elements for beams and columns with a strength hardening behavior. The dampers are modeled as linear viscous zero length elements in the bays of the frames. Time history analyses have been performed using seven far-field earthquakes all recorded on stiff soil type B (according to NEHRP). Table 2 shows the specifications of the records.

One directional asymmetric models have been analyzed using the y direction of the records applied to the asymmetric direction (y direction), while two directional asymmetric models have been analyzed using both component of the records. In order to study the effect of sever earthquakes; all records are scaled in a way that the PGA values of the y components is set to 0.55g. Maximum of the torsional response of each model is calculated for each record and the final result is the average of the obtained values.

Table 2. Specification of ground motion records

	Earthquake	Year	Magnitude	Duration(sec)	PGA y (g)	PGA x (g)	Site	Distance
1	Chi-Chi	1999	7.6m	35	0.413	0.300	TCU047	33 km
2	Manjil	1990	7.4mw	25	0.184	0.131	Qazvin	49 km
3	Imperial Valley	1979	6.5m	40	0.169	0.157	Cerro Prieto	26.5 km
4	Kern county	1952	7.4mw	25	0.175	0.156	Taft	41 km
5	N. Palm Spring	1986	6m	20	0.228	0.222	San Jacinto	32 km
6	Northridge	1994	6.7m	20	0.256	0.222	LA-Century	25.4 km
7	San Fernando	1971	6.6m	20	0.324	0.268	Castaic	25 km

RESULTS FOR ONE DIRECTIONAL STIFFNESS-ASYMMETRIC MODELS

Fig.4 show values of the maximum diaphragm rotation versus damping eccentricity for the 7 one directional stiffness-asymmetric models. Damping eccentricity corresponding to the minimum point of each curve shows the optimum damping eccentricity for diaphragm rotation or e_{d0}^* for that model. For the symmetric model (model 1) e_{d0}^* is equal to zero as expected and by increasing stiffness

and strength eccentricity on the left side of the diaphragm, $e_{d\theta}^*$ increases on the right side (flexible side) with a higher rate.

As the figure shows, the sensitivity of the response to damper eccentricity (e_d) decreases as e_d changes from -0.5 to 0.5 . In other words, when the damper eccentricity is on the stiff side, the difference between the rotations of models is much more as compared to the case when damper eccentricity is on the flexible side. As presented in the figure, in $e_d = 0.5$ (all viscous dampers are concentrated on the flexible edge of the diaphragm), stiffness and strength asymmetries have little effect on the diaphragm rotation.

Coincidence of the results for models 3 and 4 which have the same stiffness eccentricities and different strength eccentricities shows that nonlinear behavior of the structure has little effect on the results. It could be attributed to the fact that even in a severe earthquake; the structures equipped with viscous dampers mainly remain in the elastic range of behavior.

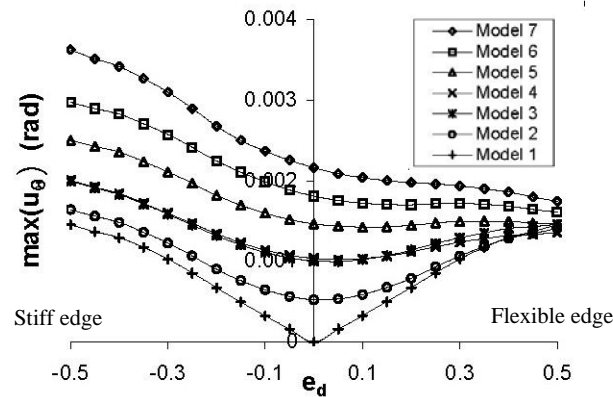


Figure 4. Maximum diaphragm rotation (radian) vs. damping eccentricity for one directional stiffness-asymmetric models

In order to study the sensitivity of the results for structures with different capacities of viscous dampers, Model 5 (with a stiffness eccentricity of -15.4% and strength eccentricity of -11.1%) is considered with three lateral damping capacities of $c_y=100$, $c_y=200$ and $c_y=300$ ton.sec/m. Fig.5 shows values of the maximum diaphragm rotation versus damping eccentricity in these three models. As shown in the figure, increasing the lateral damping capacity decrease the diaphragm rotation. By increasing the damping eccentricity, sensitivity of the diaphragm rotation to lateral capacity decreases so that in $e_d = \pm 0.5$ lateral capacity of dampers has no effect on diaphragm rotation.

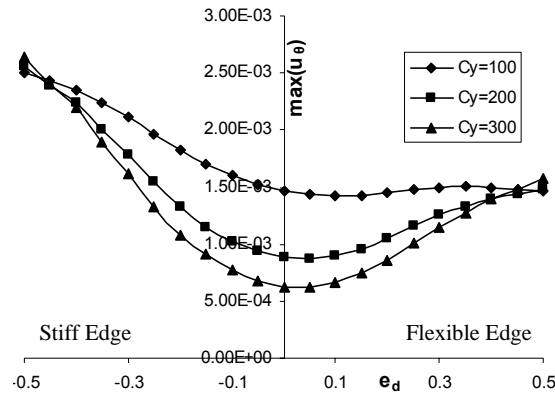


Figure 5. Maximum diaphragm rotation (radian) vs. damping eccentricity in model 5 with different damping capacities

The figure also shows that by increasing c_y , the minimum points of the curves which are the optimum damping eccentricities decrease from 0.15 for $c_y=100$ ton.sec/m to 0.05 for $c_y=300$ ton.sec/m. In other word, by increasing the lateral capacity, a more symmetric distribution of dampers is suitable for controlling the diaphragm rotation.

All the obtained results are attributed to torsional stiff structures since the ratio between uncoupled lateral to torsional period is more than unit. torsional coupled and torsional flexible structures have specific dynamic behavior which specifies them from torsional stiff structures. The behavior of such structures has been studied by some researchers and could be referred in the literature. One of the first studies on the elastic and inelastic behavior of such structures has been performed by Kan and Chopra (1976,1979) which shows the important role of torsional stiffness on the structural responses. In order to study the sensitivity of the results for structures with different torsional stiffness, the asymmetric Model 5 with $\Omega = 1.27$ is considered as model A. The lateral to torsional period ratio of this model is changed to $\Omega = 1.00$ (model B) and $\Omega = 0.79$ (model C) by changing only the diaphragm mass moment of inertia while other parameters are constant.

Fig.6 shows the values of maximum diaphragm rotation versus damping eccentricity for models A to C. The figure shows that diaphragm rotation in the torsional coupled model is a little more than torsional stiff and torsional flexible models which has been specified in the previous studies (Kan and Chopra 1979). Also the values of optimum damping eccentricity (the minimum points of the curves) have little variation in the three models. It could be concluded that torsional stiffness has little effect on the optimum distribution of dampers in structures equipped with viscous dampers.

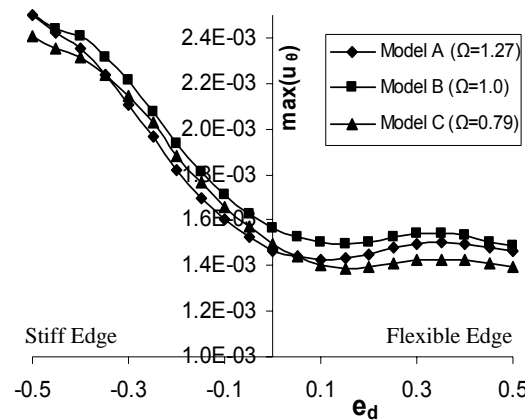


Figure 6. Maximum diaphragm rotation (radian) vs. damping eccentricity of model 5 with different uncoupled lateral to torsional periods

RESULTS FOR ONE DIRECTIONAL MASS ASYMMETRIC MODELS

One directional mass asymmetric models are produced by moving the CM of model 1 (the stiffness symmetric model) from the geometric center of diaphragm (GC) along the x direction. Mass eccentricities of $e_m = 0.0$ to $e_m = -0.25$ are considered using the indicated method which makes the left edge of the diaphragm as the flexible edge and the right edge of the diaphragm as the stiff edge. The values of mass moment of inertia in the asymmetric cases are calculated using Eq.(7).

Fig.7 shows the values of maximum diaphragm rotation versus damping eccentricity for model 1 with different mass eccentricities. The figure shows that the optimum damping eccentricity for each value of mass eccentricity (minimum of each curve) is located at the same side of mass eccentricity (flexible side of the diaphragm). Value of optimum damping eccentricity for each case is equal or more than mass eccentricity value. For example for model with $e_m = -0.05$, the value of $e_{d\theta}^*$ is equal to -0.05 while for model with $e_m = -0.25$ the value of $e_{d\theta}^*$ is equal to -0.5 . Also the sensitivity of the diaphragm rotation to mass eccentricity is decreased in the cases which damping center moves toward the flexible

edge (for $e_d = -0.5$, mass eccentricity has no effect on the response). Comparison between Fig.4 and Fig.7 shows that if the flexible and stiff edges of a diaphragm are determined correctly, there is no considerable difference between optimum distribution of viscous dampers in mass asymmetric and stiffness asymmetric structures.

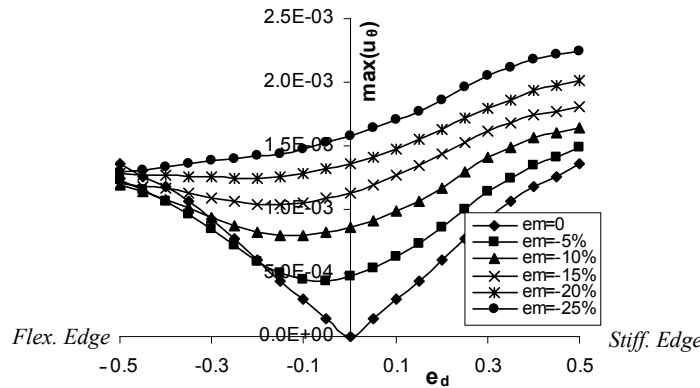


Figure 7. Maximum diaphragm rotation (radian) vs. damping eccentricity for one directional mass asymmetric models

RESULTS FOR TWO DIRECTIONAL ASYMMETRIC MODELS

Two directional asymmetric models are produced for each of the 7 one-directional asymmetric models by changing the mass eccentricity in the y direction. For each model, three mass eccentricities of $e_{my} = 0.0$, $e_{my} = 0.10$ and $e_{my} = 0.25$ are applied which leads to mass asymmetry in the x direction. Consequently, 27 two-directional asymmetric models (stiffness asymmetry in the y direction and mass asymmetry in the x direction) are analyzed.

Fig. 8 shows the maximum diaphragm rotation versus damping eccentricity in the x direction for two directional asymmetric models. As it was indicated that the distribution of dampers in the x direction is uniform ($e_{dx} = 0$), the results are expressed as a function of e_{dx} (similar to other diagrams).

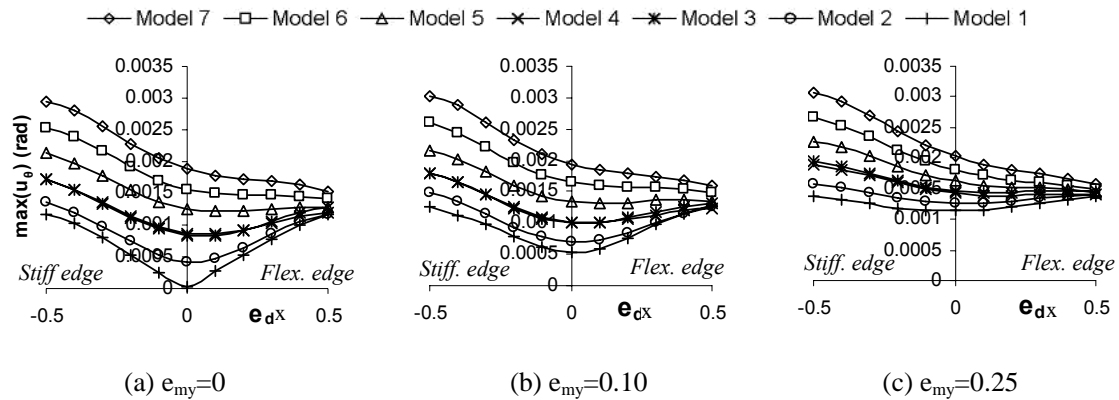


Figure 8. Maximum diaphragm rotation (radian) vs. damping eccentricity in the x direction for two directional asymmetric models

Fig. 8 shows that in models with small stiffness eccentricity (e_{sx}), increasing e_{my} leads to a higher diaphragm rotation while in models with more values of e_{sx} , increasing e_{my} has little effect on the rotation. Similar to one directional asymmetric models, by increasing stiffness and strength eccentricity on the left side of the diaphragm, optimum damping eccentricity (e_{d0}^*) increases on the

right side (flexible side) with a higher rate. Comparing Fig.8(a) with Fig.4 (that both of them are symmetric in the x direction), shows a decrease in the diaphragm rotation which is due to existence of viscous dampers in the x direction in the two directional asymmetric models. Such dampers increase the damping radius of gyration which is very effective in controlling torsion of the diaphragm.

CONCLUSIONS

The main contribution of this study is investigating the effect of different structural parameters including kinds of asymmetries and torsional stiffness of structures and also capacities of viscous dampers on the optimum distribution of dampers for controlling structural torsion in the earthquake. Several parametric analyses on the single-story and damper-equipped structures with one directional stiffness asymmetry; one directional mass asymmetry and two directional asymmetry have been performed considering nonlinear behavior of the structures. The summary of the results is as following:

1. In one directional stiffness-asymmetric structures, optimum damping center is located on the flexible side of the diaphragm with a damping eccentricity more than the stiffness eccentricity.
2. Increasing the capacities of dampers leads to obtain a more symmetric distribution of dampers for controlling the diaphragm rotation. In other word, optimum damping eccentricity in the flexible side decreases by increasing the capacities of dampers.
3. Torsional stiffness of the structures has little effect on the optimum distribution of dampers. However, more diaphragm rotations are usually happened in torsional coupled structures.
4. In mass asymmetric structures, optimum damping center is located on the flexible side of the diaphragm with a damping eccentricity more than the mass eccentricity. Such distribution is similar to optimum distribution of dampers in the stiffness asymmetric structures.
5. In two-directional asymmetric models, the optimum damping eccentricity in a desired direction is dependent to the mass or stiffness eccentricity in that direction similar to what obtained for one directional asymmetry case (results number 1 and 4). The results also show that the sensitivity of the diaphragm rotation to the orthogonal eccentricity only happens in small eccentricity of mass or stiffness in the direction under consideration.

REFERENCES

- Almazán JL, Espinoza G, Aguirre JJ (2012) "Torsional balance of asymmetric structures by means of tuned mass dampers", *Engineering Structures*, 42:308-328
- BHRC: Building and Housing Research Center (2006) .Iranian Code of practice for Seismic Resistant Design of Buildings (Standard 2800), third Edition, Tehran, Iran
- De La Llera JC, Almazan JL, Vial IJ (2005) "Torsional balance of plan- asymmetric structures with frictional dampers: Analytical results", *Earthquake Engineering and Structural Dynamics*, 34:1089-1108.
- FEMA, 450, The 2003 NEHRP Recommended Provisions For New Buildings And Other Structures, Federal Emergency Management Agency, Washington D.C
- FEMA, 356, The 2000 NEHRP Recommended Provisions For New Buildings And Other Structures, Federal Emergency Management Agency, Washington D.C
- Foss KA (1958) "Coordinates which uncouple the equation of motion of damped linear dynamic systems", *Journal of Applied Mechanics*, 25:361-364
- Goel RK (1998) "Effects of supplemental viscous damping on seismic response of asymmetric-plan systems", *Earthquake Engineering and Structural Dynamics*, 27(2):125-141.
- Goel RK and Booker CA (2001) "Effects of supplemental viscous damping on inelastic seismic response of asymmetric systems", *Earthquake Engineering and Structural Dynamics*, 30:411-430.
- Kan CL and Chopra, A.K (1976) "Coupled lateral torsional response of buildings to ground shaking", Report No. UCB/EERC-76-13, Earthquake Engineering Research Center, University of California, Berkeley, 175 pages.

- Kan CL and Chopra AK (1979) "Linear and nonlinear earthquake responses of simple torsionally coupled systems", Earthquake Engineering Research Center, University of California, Berkeley, 120 pages
- Mansoori MR and Moghadam AS (2009) "Using viscous damper distribution to reduce multiple seismic responses of asymmetric structures", Journal of Constructional Steel Research, 65(12):2176-2185
- McKenna F, Fenves GL, Scott MH and Jeremic B. Open system for earthquake engineering simulation (2000). Web page: <http://OpenSees.berkeley.edu>.
- Veletsos AS and Ventura CE (1986) "Modal analysis of non-classically damped linear systems", Earthquake Engineering and Structural Dynamics, 14:217-243