



## SEISMIC RESPONSE OF A RIGID FOUNDATION EMBEDED IN A HETEROGENEOUS SOIL

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### ABSTRACT

This study analyzes the influence of soil-structure interaction on the seismic response of a three dimensional (3-D) rigid foundation embedded in a viscoelastic heterogeneous soil limited by bedrock. Vibrations result from P, SV and SH harmonic seismic waves. The key step is the characterization of the soil-foundation interaction with the impedance matrix on one hand and the input motion matrix on the other hand. For each frequency the impedance matrix connects the applied forces to the resulting displacement, and the input motion matrix connects the displacement vector of the foundation to amplitudes of the free field motion. The mathematical approach is based on the method of integral equations in the frequency domain using the formalism of Green's functions for layered soil. Appropriate discretization of the soil foundation interfaces in constant quadrilateral elements is used to obtain algebraic equations and transform the force-displacement relationship into a system of linear equations. This approach has been applied to analyze the effect of soil-structure interaction on the seismic response of the foundation.

### INTRODUCTION

The analysis of the behavior of foundations under dynamic loads has grown considerably over the past four decades. Stringent security requirements imposed on design of certain types of structures have played a particularly important role in the development of analytical methods. The key step in studying the dynamic response of foundation is the determination of the relationship between forces. This relationship which results in displacement is expressed using impedance functions (dynamic stiffness) or the compliance functions (dynamic flexibility). Consideration of the soil-structure interaction in the analysis of the dynamic behaviour of foundation allows taking realistically into account the influence of soil on its vibration.

The response of foundation under dynamic loading is greatly influenced by the foundation geometry, rigidity, embedment and characteristics of soil underneath the foundation. A myriad of methods has been proposed to solve the soil-structure interaction problem. To simplify the problem linear-analysis techniques have been developed. One of the most commonly used approaches is the sub-structuring method that allows the problem to be analyzed in two parts (Kausel et al (1978), Aubry et al (1992) and Pecker (1984)). In this approach the dynamic response of superstructure elements and substructure are examined separately. The analysis of foundation systems can be reduced to the study of the dynamic stiffness at the soil-foundation interface (known as impedance function) and driving forces from incident waves. The kinematic interaction of the foundation with incident waves is implemented in the form of a driving-force vector.

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Determining the foundation response thereby becomes a wave propagation problem. Due to the mixed-boundary conditions of the problem (displacement compatibility with stress distribution underneath the foundation and zero tension outside) solutions are complex. The determination of impedance functions and forces of movement related to the incident waves is a complex process. Several studies have been conducted on the dynamic response of foundation using the finite-element and boundary-element methods. Wong and Luco (1978) have shown the importance of the effect of non-verticality of SV, SH harmonics on the response of a foundation.

Apsel and Luco (1987) used an integral-equation approach based on Green's functions for multilayered soils determined to calculate the impedance functions of foundation. Using this approach, Wong and Luco (1986) studied the dynamic interaction between rigid foundations resting on a half-space. Boumekik (1985) studied the problem of 3-D foundations embedded in soil limited by a rigid substratum. The finite-element method was applied by Kausel et al (1978), Kausel and Roesset (1981) and Lin and Tassoulas (1987) to determine the behavior of rigid foundations placed on or embedded in soil layer limited by a rigid substratum. A formulation of the boundary-element method in the frequency domain has been developed to address wave-propagation problems of soil-structure interaction and structure-soil-structure which limits the discretization at the interface soil-foundation. In this approach, the field of displacement is formulated as integrals equation in terms of Green's functions Beskos (1987), Aubry et al (1992), Qian and Beskos (1987), Karabalis & Mohammadi (1991) and Mohammadi (1992). Celebi et al (2006) used the boundary-element method with integral formulation (BIEM) to compute the dynamic impedance of foundations. In this context, the analytical solutions of 3-D wave equations in cylindrical coordinates in layered medium with satisfying the necessary boundary conditions are employed by Liou (1992) and Liou and Chung (2009). Sbartai and Boumekik (2008) used the BEM-TLM method to calculate the one hand, the dynamic impedance of rectangular foundations placed or embedded in the soil layered limited by a substratum and also the propagation of vibrations in the vicinity of a vibrating foundation. In this study, the method has been applied to analyze the effect of some parameters on the dynamic response of the foundations (depth of the substratum, embedding, masses and shape of the foundation, soil heterogeneity and frequency). However, Sbartai and Boumekik (2006, 2007) have studied the dynamic response of two square foundations placed or embedded in soil layered limited by a substratum. Spryakos and Xu (2004) have developed a hybrid BEM-FEM and have conducted several studies for parametric analysis of soil-structure interaction.

Recently, McKay(2009) used the reciprocity theorem based on the BIEM to analyze the influence of soil-structure interaction on the seismic response of foundations. However, Suarez et al (2002) applied the BIEM to determine the seismic response of an L-shaped foundation. In addition, experimental work has been carried out by researchers in Japan to determine the effect of soil-structure interaction on the response of real structure Fujimori et al (1992), Akino et al (1996), Mizuhata et al (1988), Watakabe et al (1992) and Imamura et al (1992).

However the effects of no-homogeneity in the soil medium on the dynamic response of the foundations have not been addressed adequately in the past because of the obvious difficulty in analyzing this case. Gazetas (1980) is developed an analytical-numerical formulation for study the foundations on an elastic isotropic medium consisting of heterogeneous layers.

In our study, the solution is derived from the BEM in the frequency domain with constant quadrilateral elements and the thin-layer method is used to analyze the influence of heterogeneous layer soil on the response of seismic foundation. The results are presented as of displacements as a function of dimensionless frequency, angle of incidence (vertically and horizontally) and the effects of rigidity soil layer.

## **PHYSICAL MODEL AND BASIC EQUATIONS**

The geometry of the calculation model is shown in Fig 1. Consider 3-D rigid massless surface foundation of arbitrary shape  $S$  in full contact with a homogeneous, isotropic, and linearly-elastic soil that is limited by bedrock. The soil is characterized by its density  $\rho$ , shear modulus  $G$ , damping coefficient  $\beta$  and Poisson's ratio  $\nu$ . The foundation is subject to harmonic oblique-incident waves that are time-dependent: P, SV, SH and R.

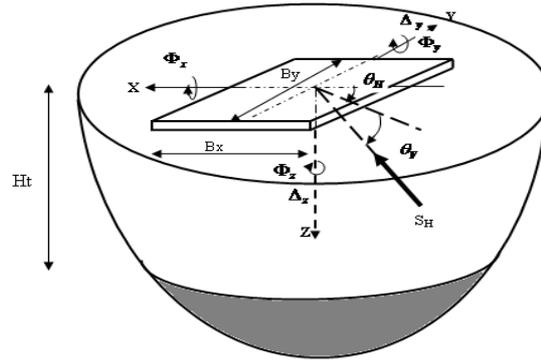


Figure 1. Geometry of a foundation subjected to harmonic seismic waves

The movement of a not-specified-point "  $\zeta$  " can be obtained from solving the wave equation:

$$(C_p^2 - C_s^2) u_{j,ij} + C_s^2 u_{i,ij} - \omega^2 u_i = 0, \quad (1)$$

Where  $C_s$ , and  $C_p$  are the velocities of shear and compression waves and  $\omega$  the angular frequency of excitation.

$u_i$  is the component of the harmonic displacement-vector in the x-direction;  $u_{j,ij}$  is the partial derivative of the displacement field with respect to x and y;  $u_{i,ij}$  is the second partial derivative of the displacement field with respect to y.

The solution of equation (1) may be expressed by the following integral equation:

$$u_j(x, \omega) = \int_S G_{ij}(x, \xi, \omega) T_i(\xi, \omega) ds(\xi), \quad (2)$$

with  $G_{ij}$  denoting the Green's functions at point i due to unit-harmonic load (vertical and horizontal) of the ground at point j and  $T_i$  being a load (traction) distributed over an area of soil.

The medium is continuous so this relationship is very difficult to assess. However, if the soil mass is discretized appropriately, this relationship can be made algebraic and displacement can be calculated. The key step of this study is to determine the impedance matrix linking the harmonic forces applied to the resulting harmonic displacement. Even with a continuous medium the determination of the impedance matrix is still very difficult, if not impossible, due to the propagation problem and its mixed-boundary conditions. However, if the medium is discretized vertically and horizontally then it is possible to making the problem algebraic by considering that the variation of interface displacement is a linear function.

We limit ourselves to the general representation of the problem. If a source acts on (or in) a massif multilayered, a multitude of reflection and refraction takes birth to the level of the interfaces. Even if the nature of the primary incidental wave is known, it would be very difficult to know the nature of the resulting waves in the massif. The quantity of energy transmitted of a layer to the other by the refracted waves would be therefore difficult to determine.

Considering the pressure of confinement, the rigidity of the layers generally increases with the depth. The energy transmitted of a layer to the other decreases therefore with the depth. Besides, the Bedrock is (of very elevated rigidity) sufficiently deep, the reflection of the waves would be total to this level. The incidental and reflexive waves form this fact, in the massif, a system of stationary waves, whose energy would propagate itself mainly in the parallel direction to the interfaces (horizontally). it would be therefore reasonable, to simplify the survey of problem, to make the hypothesis of a horizontal propagation of the waves thought in the multilayered massif

The principle of horizontal and vertical discretization of soil mass is shown in Fig 2. The principle of vertical discretization based on the division of every soil layer into a number of sub-layers of height  $h_j$  with similar physical characteristics. Each sublayer is assumed to be horizontal,

viscoelastic, and isotropic, and characterized by constant Lamé  $\lambda_j$ , a shear modulus  $\mu_i$  and a density  $\rho_j$ . The bedrock at depth  $H_t$  is considered infinitely rigid and is not discretized. The reflection wave is assumed to be total and the displacements null.

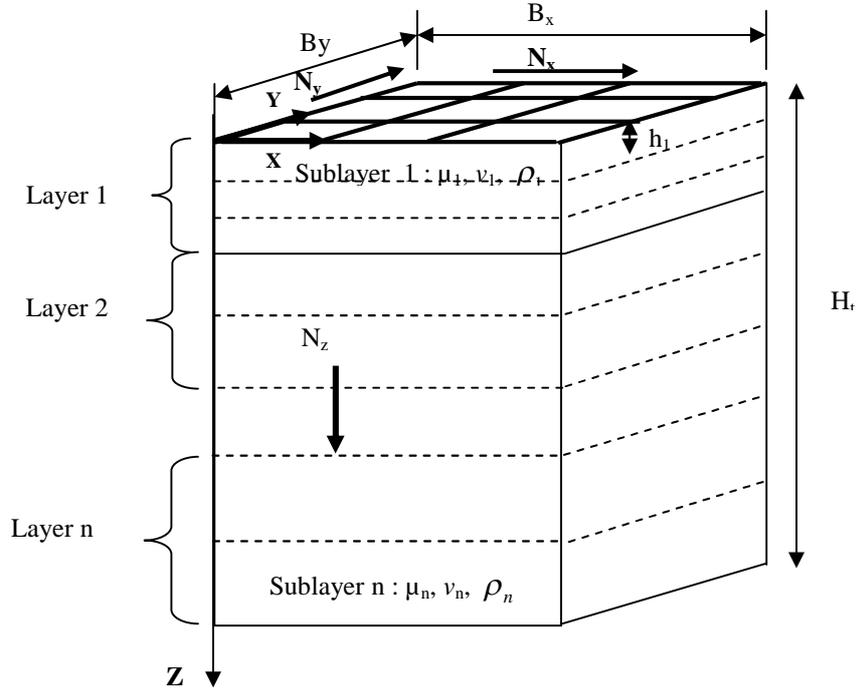


Figure 2. The model of calculation

$H_t$ : height of soil mass;  $h_1$ : height of the sublayer 1;  $N_x$ : number of elements in the x-direction for a horizontal plane;  $N_y$ : number of elements in the y-direction for a horizontal plane;  $N_z$ : number of soil layers;  $n$ : number of soil layer;  $N$ : is the total number of elements in soil-foundation interface and  $B_x$  and  $B_y$  are the dimensions of the foundation.

Within a given sublayer, the displacement is assumed to be a linear function of interface displacement above and below. This is true when the height of the sublayer is small in relation to the wavelength considered (in the order of  $\lambda/10$ ). This method is comparable to the FEM in the sense that the movements within each sublayer are completely defined from the displacements in the middle of the interfaces. The interaction between the elements is done only through the nodes. The degrees of freedom of the soil mass are reduced to the degrees of freedom of the nodes. The stiffness matrix of soil mass is obtained in a similar manner to how it is determined in the FEM.

This technique has been developed by Lysmer and Waas [17] and is known as the thin-layer method (TLM) and is used mainly for horizontal soil layers. This method has the advantage to making the problem algebraic and thus obtains the Green's functions by applying the boundary elements method in the soil-foundation interface. For this reason a horizontal discretization of the interface soil-foundation is established.

The horizontal discretization permits to subdivide any horizontal interface of soil-foundation by elements of square-sections  $S_k$ . These elements where the constant-moving average is replaced by the movement of the center, assumes that stress distribution is uniform and is shown in Fig 2. Seeking simplicity of the integration calculation and economy of computing time the square elements are approximated disc elements. If the units loads (along the direction x, y, z) are applied to disc j, the Green's functions at the center of disc i can be determined. By successively applying these loads on all discs, the flexibility matrix of soil complex at a frequency of a given  $\omega$  can be formed. The discretized model to calculate the impedance functions of the foundation is also presented in Figure (2). In the discrete model, Equation (2) is expressed in algebraic form as follows:

$$u_j = \sum_{i=1}^N \int_S G_{ij} T_i ds \quad (3)$$

The Green's function for a layered stratum is obtained by an inversion of the thin-layer stiffness matrix using a spectral-decomposition procedure of Kausel and Peek [10]. The advantage of the thin-layer-stiffness matrix technique over the classical transfer-matrix technique for finite layers and the finite-layer-stiffness matrix technique Kausel and Roesset [11] is that the transcendental functions in the layered-stiffness matrix are linearized. The obtained Green's functions are complex and constitute the starting point for the determination of the flexibility matrix of an arbitrary soil volume. The determination of this flexibility matrix gives the impedance function of one or several foundations and the amplitudes of vibrations in the neighborhood of a foundation Sbartai and Boumekik (2006, 2008).

## CALCULATION MODEL

The total displacement of the soil matrix is obtained by successive application of unit loads on the constituents of the discretized solid ground. The displacements in the soil are then expressed by

$$\{u\} = [G] \{t\} \quad (4)$$

The vectors  $\{u\}$  and  $\{t\}$  are the nodal values of the amplitudes of displacements and tractions respectively at the interface soil-foundation.  $[G]$  is the flexibility matrix of the soil.

When the foundation is in place, it requires different components of soil displacement consistent with rigid body motions. Compatibility of displacements at the contact area  $S$  between the soil and the rigid foundation leads to the matrix equation

$$\{u\} = [R] \{\Delta\} \quad (5)$$

$[R]$  is the transformation matrix;  $\{\Delta\} = \{\Delta_x, \Delta_y, \Delta_z, \Phi_x, \Phi_y, \Phi_z\}^t$  is the displacement vector;  $\Delta_i (i = x, y, z)$  represents translations and  $\Phi_i (i = x, y, z)$  rotations, (Fig 1).

If we denote  $\{P\}$  the vector of load applied to the foundation, the equilibrium between the vector of loads applied and the forces (tractions) distributed over the elements discretizing the volume of the foundation is expressed by the following equation:

$$\{P\} = [R]^t \{t\} \quad (6)$$

Combining equations (4), (5) and (6) we obtain the following equation:

$$\{P\} = \left( [R]^t [G]^{-1} [R] \right) \{\Delta\} = [K(\omega)] \{\Delta\} \quad (7)$$

with  $\omega$  is the circular frequency of vibration and  $[K(\omega)]$  the impedance or dynamic-stiffness matrix of the rigid foundation.

Considering an incident plane, SH, P, SV and R harmonic waves are characterized by the vertical and horizontal angles of incidence  $\theta_v$  and  $\theta_H$  respectively, as shown in Fig 1. The motion of the half-space due to these seismic waves can be expressed by the following equation:

$$\{u^f\} = \{U^f\} e^{-i\omega (x \cos \theta_H + y \sin \theta_H) / c} \quad (8)$$

$\{U^f\} = \{U_x^f, U_y^f, U_z^f\}^t$ , is known as the vector of amplitudes of the soil, that depends on the  $z$  coordinate if we want to study the embedded foundations case. However, in the case of surface foundations ( $z = 0$ ), it is known as the vector of amplitudes of the free field.  $c$  is the apparent velocity of the incident waves having the form  $c = \frac{c_1}{\cos \theta_v}$  or  $c = \frac{c_2}{\cos \theta_v}$  for P or S waves, respectively, and being equal to the R-wave. The explicit expressions of the vector  $\{U^f\}$  of waves SH, P, SV and R may be found in Wong and Luco (1978).

The presence of a rigid foundation on the surface of the half-space results in diffraction of the above waves so that the total displacement field  $\{u\}$  is expressed by the following equation:

$$\{u\} = \{u^f\} + \{u^s\} \quad (9)$$

where  $\{u^s\}$  represents the scattered wave field that satisfies the equation of motion (4). Also, the total displacement field in the contact region between the foundation and the half space must be equal to the rigid body motion of the foundation. Substituting equation (5) into equation (9), written in terms of the scattered field leads to the force-displacement relation:

$$\{u^s\} = [R]\{\Delta\} - \{u^f\} \quad (10)$$

Substituting equation (4) into equation (10), written in terms of the traction forces:

$$[G]\{t\} = [R]\{\Delta\} - \{u^f\} \quad (11)$$

hence

$$\{t\} = [G]^{-1} [R]\{\Delta\} - [G]^{-1} \{u^f\} \quad (12)$$

Multiplying both sides of the equation (12) by the transpose of the transformation matrix and combining with equations (6) and (7) yields the external forces:

$$[R]^t \{t\} = [R]^t [G]^{-1} [R]\{\Delta\} - [R]^t [G]^{-1} \{u^f\} \quad (13)$$

The equilibrium between external forces and seismic forces can be as follows:

$$\{P\} = [K]\{\Delta\} - [K^*]\{U^f\}, \quad (14)$$

with  $[K^*]$  is the driving force matrix given by the following formula:

$$[K^*] = [R]^t [G^{-1}] e^{[-i\omega (x \cdot \cos \theta_H + y \cdot \sin \theta_H) / c]}. \quad (15)$$

Equation (14) can be replaced by the alternative form:

$$\{\Delta\} = [C]\{P\} + [S^*]\{U^f\} \quad (16)$$

where

$[C] = [K]^{-1}$  is the dynamic compliance matrix and  $[S^*]$  is the input motion matrix given by the following formula:

$$[S^*] = [C][K^*] \quad (17)$$

When the rigid foundation is acted upon by seismic waves only, the external forces are zero ( $P = 0$ ), and the seismic response of the foundation is obtained from equation (13) or (15) by the following expression:

$$\{\Delta\} = [S^*]\{U^f\} \quad (18)$$

When the mass of the foundation is not zero, one simply has to replace  $[K]$  by  $[K] - \omega^2 [M]$  in the above equations, where  $[M]$  is mass matrix of the foundation.

## CALCULATION MODEL

The accuracy of the BEM-TLM method used in this paper to study the 3D-response of foundations subject to plane-harmonic waves with variable angles of incidence and dimensionless frequency  $a_0$  is validated through comparisons with results obtained by Luco and Wong (1977) and Qian and Beskos (1996) for a semi-infinite ground. A parametric study was conducted to define the parameters of the calculation model. The influence of the discretization of the soil-foundation interface was studied. The thickness of a sub-layer “h” must be small enough that the discrete model can transmit waves in an appropriate manner and without numerical distortion. This size depends on the frequencies involved and the velocity of wave propagation. The frequency of loading and velocity of wave propagation affect the precision of the numerical solution. Kausel and Peek (1982) showed that the thickness of sub-layer must be smaller than a quarter of the wavelength  $\lambda$ . Consequently, the maximum dimensionless frequency must not exceed the number of sub-layer N divided by four.

Consider a rigid massless square foundation of side  $B_x = 2a$  on the surface of the half-space with a Poisson's ratio of  $\nu = 1/3$  and subjected to plane P, SV, or SH harmonic waves ( $\theta_H = 90^\circ$  and  $\theta_V = 45^\circ$ ). Fig 4 shows the variation of the real and the imaginary part of coefficient  $S_{xx}$  movement based on

the dimensionless frequency  $a_0 = \frac{\omega}{C_s} \frac{B}{2}$ . The results obtained by the proposed method are in agreement with those obtained by the method used by Qian and Beskos (1996). Considering the same foundation subjected to a Rayleigh wave where the angle of horizontal incidence is  $(\theta_H = 0)$  and the corresponding velocity taken is equal to  $c_R = 0.9325.c$  for a Poisson's ratio  $\nu = 1/3$ .

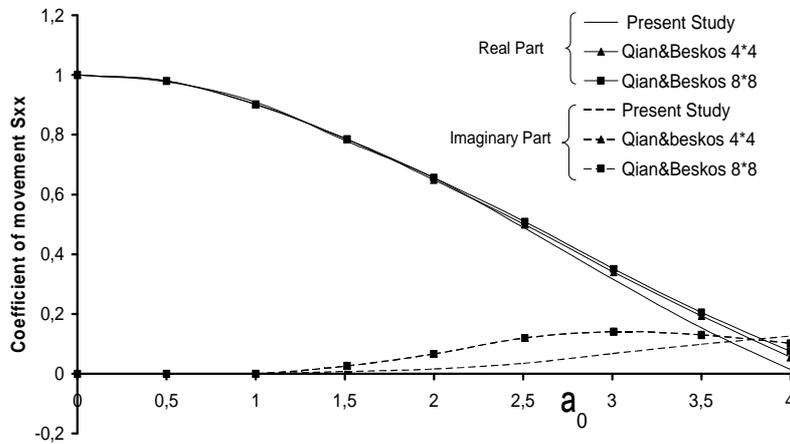


Figure 4. The coefficient of movement  $S_{xx}$  a square foundation ( $\theta_H=0^\circ$ ,  $\theta_V=45^\circ$  and  $c_r/c=0.70711$ )

Fig 5 shows the real and the imaginary part of dimensionless displacement  $\Delta x/H_R$  function of frequency  $a_0$ . The results of this study using the BEM-TLM method were compared with those of Qian and Beskos (1996) and those of Luco (1977).

The results obtained are in agreement with those of Beskos and Qian (1996) and those of Luco and Wong (1977). However, a difference is present for dimensionless frequencies  $a_0$  higher than 2.5. This difference can be explained in two ways:

- Qian and Beskos isoparametric elements were used to determine the soil-foundation interface. These types of elements are more accurate than constant elements used by the method developed in this article.
- Quian and Beskos and Luco and Wong use the Green's functions of a semi-infinite soil whereas our method used the Green's functions of soil bounded by bedrock

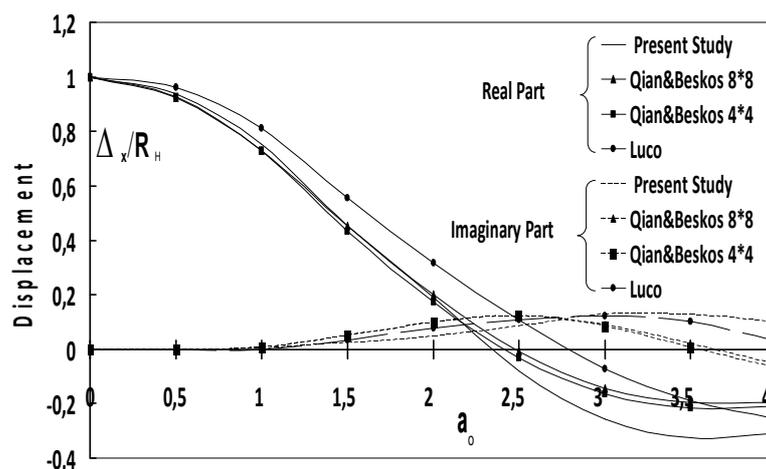


Figure 5. The response a square foundation under the Rayleigh wave  $c_R/c=0.9325$

## RESULTS

A square rigid surface dimension  $B_x = 2a = 1\text{m}$  posed in a heterogeneous soil and subject to P, SV and SH waves (Fig 6) is considered. The medium is laminated and consists by two layers soil. Characteristics are the following: The layers soil is characterized by, their Poisson's ratio is  $\nu = 1/3$ , the coefficient of the hysteretic damping  $\beta = 0.05$ , the shear modulus  $\mu = 1$ , and its density  $\rho = 1$ . The height of the first layer soil is 1m and the height of the second layer soil is greater than 10 m. By varying the stiffness ratio of the first layer on second layer ( $G_1/G_2 = 0, 0.25, 0.5, 1$ ). The wave velocity is calculated by the following relationship:  $C_s = \sqrt{\frac{\mu}{\rho}}$ ; where  $\mu$  and  $\rho$  are the characteristics of the first layer.

To present results that are easier to understand visually, the driving-force vectors are converted to input-motion vectors by multiplying by the inverse of the impedance matrix. The response of the massless foundation to incident body waves of type P, SV SH traveled the heterogeneous soil is considered. The nature of the P, SV and SH waves traveled the medium is determined depending on the characteristics of the first layer. It is assumed that the reflected energy transmitted by the waves propagating is parallel to the layers soil interfaces. The results are presented in terms of displacement and torsion as functions of the dimensionless frequency  $a_0 = \frac{\omega B}{C_s 2}$ .

In this section of results, the apparent velocity of the P, SV and SH waves considered is constant. The Flexibility matrix of soil layer is determined by the characteristics of the heterogeneous medium. The dynamic response of the foundation has been obtained for incident P and SH waves through heterogeneous soil (Fig. 6) for a vertical angle of incidence equal to  $45^\circ$  the amplitudes of motion in free field wave SV equal to zero.

Figures 7 to 8 show the response of a massless foundation to an incident P-wave. The wave travels in the  $x$ -direction with its particle motion in the  $z$  and  $x$ -directions. Except for the vertical incidence  $\theta_v = 90^\circ$ , the incident P-wave causes a conversion mode and reflected SV-wave results. One considered an angle of incidence of  $45^\circ$  measured with respect to the  $x$ -axis. The wavelength of the incident P-wave is twice as long as that of the incident S-waves; therefore, the kinematic interaction is less prominent. In general, the P-wave induces displacement along the  $x$  and  $z$ -axes and rotation around the  $y$ -axis.

Figures 7-8 show the variation of displacement and rotation as a function of dimensionless frequency and the influence of the  $G_1/G_2$  ratio on the motion of foundation.

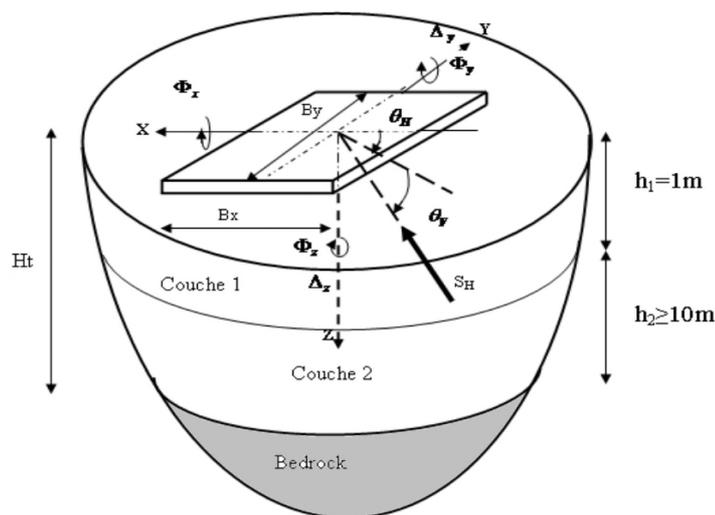


Figure 6. Geometry of a rigid surface foundation posed in the heterogeneous subjected to seismic harmonic waves.

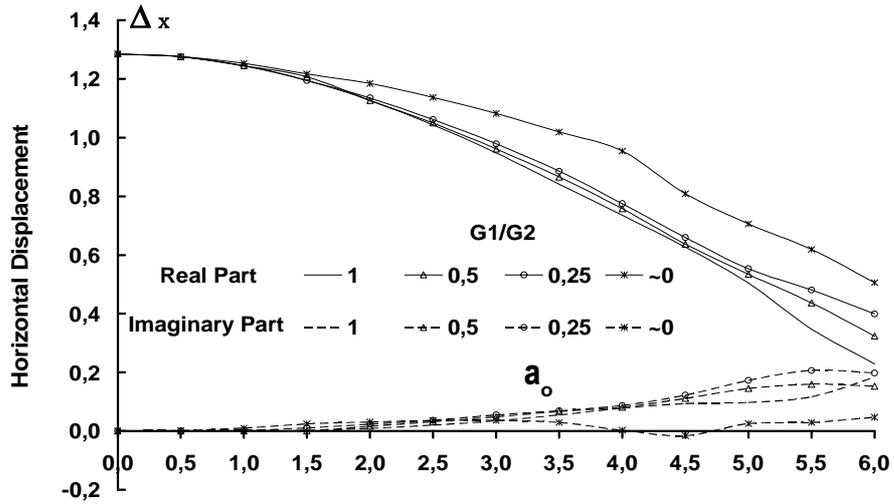


Figure 7. Horizontal input motion  $\Delta_x$  due to incident P-waves ( $\theta_v=45^\circ$ ,  $\theta_H=0^\circ$ )

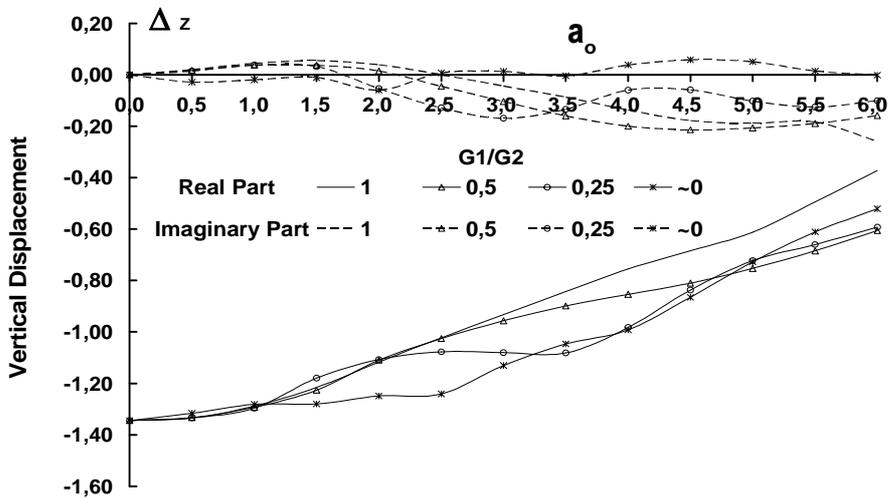


Figure 8. Horizontal input motion  $\Delta_z$  due to incident P-waves ( $\theta_v=45^\circ$ ,  $\theta_H=0^\circ$ )

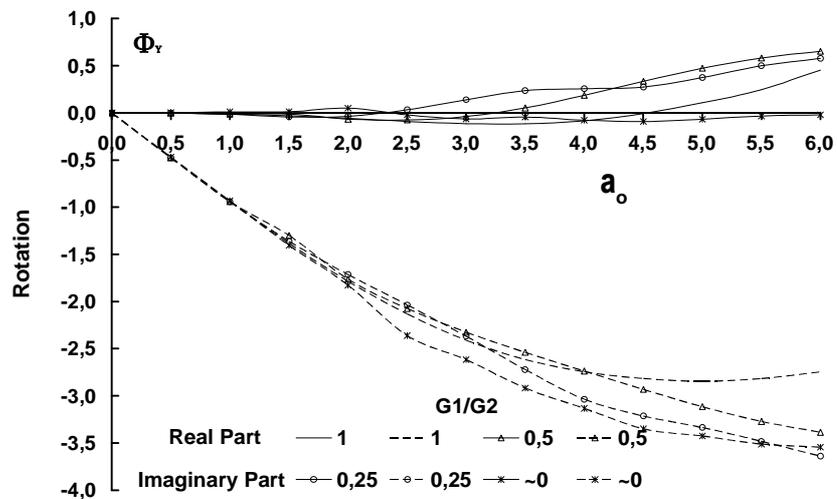


Figure 9. Rocking input motion  $\phi_y$  due to incident P-waves ( $\theta_v=45^\circ$ ,  $\theta_H=0^\circ$ ).

The displacements ( $\Delta_x, \Delta_z$ ) and rotation ( $\phi_y$ ) are affected due to the diminution of shear ratio (rigidity) of two soil layers, especially for the high frequencies. In addition, the imaginary part of the vertical displacement  $\Delta_z$  is strongly affected by the rigidity of first soil layer that the horizontal displacement  $\Delta_x$  and rotation.

The response of the square, massless foundation to a SH-wave is presented in Figures 10 and 11, with a horizontal angle of incidence  $\theta_H = 90^\circ$ . The incident wave travels in the  $y$ -direction; therefore, the particle motion of the wave is in the  $x$ -direction. The shear wave causes displacement and torsion. Figures 10 and 11 show the influence of  $G1/G2$  ratio on displacement  $\Delta_x$  and torsion  $\phi_z$ .

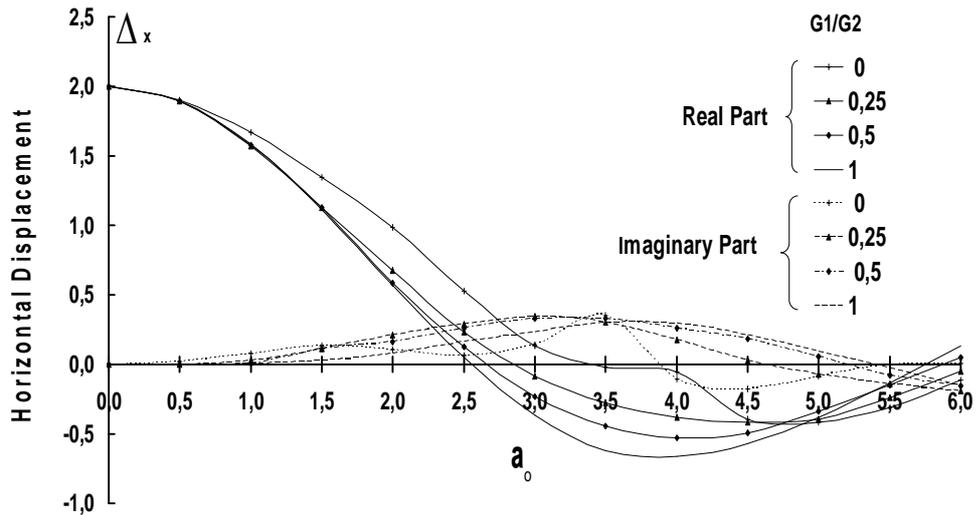


Figure 10. Horizontal input motion  $\Delta_x$  due to incident SH-waves ( $\theta_v=45^\circ, \theta_H=90^\circ$ )

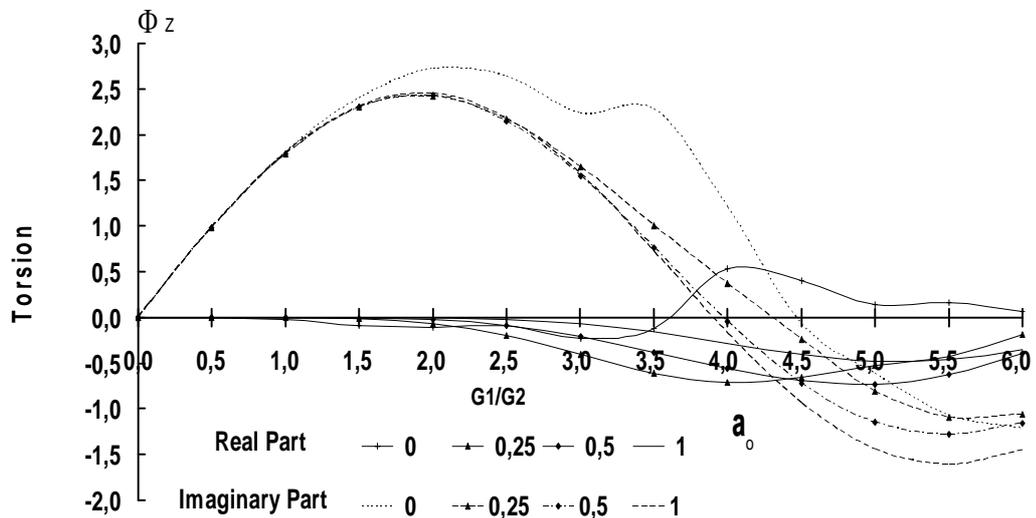


Figure 11. Torsion-input motion  $\phi_z$  due to incident SH-waves ( $\theta_v=0^\circ, \theta_H=90^\circ$ )

Figure 10 shows the influence of shear ratio  $G1/G2$  on the seismic response of a square foundation resting on a layered medium. The real part of Fig.10 shows, for the ratio  $G1/G2 = 1$  (layered medium is homogeneous) the displacement of the foundation is strongly attenuated compared to displacement given by the low ratios of  $G1/G2$ . For the ratio  $G1/G2 = 0$ , the second layer is relatively rigid to the first layer, the figure shows that the displacement is strongly affected by the effect of environment.

Figure 11 shows the influence of shear ratio  $G1/G2$  on the variation of the torsion function of frequency. The imaginary part of the figure shows that for lower  $G1/G2$  ratios, the torsion is strongly affected by the effect of medium and the frequency becomes high.

Figures 10 and 11 show that horizontal displacement and torsion are strongly affected by the rigidity of layers soil. For dimensionless frequencies lower than 3, the torsion  $\phi_z$  is not affected by increasing the relative rigidity especially for the reports  $G1/G2 = 0.25, 0.5, 1$ . However, the horizontal displacement  $\Delta_x$  is strongly affected by an increase of the rigidity.

## CONCLUSIONS

The interaction of a seismic square-rigid foundation placed in a heterogeneous viscoelastic soil and subjected to obliquely incident harmonic P, SV and SH waves were implemented. A simplified BEM-TLM was developed and used to calculate the foundation-input motion under different travelling seismic waves. The solution was formulated by the boundary-element method in the frequency domain using the formalism of Green's functions. Constant quadrilateral elements were used to study the seismic response of a foundation. The efficiency of this technique was confirmed by comparison with results rigorously obtained by the use of the relationship between the solution of the radiation problem associated with the soil-impedance functions, and the solution of the scattering problem associated with the foundation-input motion. This remarkably simple technique was concluded to be both highly effective and economical to determine input motions for rigid foundations of arbitrary geometry. The originality of the method lies first in the insignificance of the number of elements used in the discretization of the model, and second, in the ability to simulate a limited height of bedrock. We notice that heterogeneous soil affects the displacement, rotation and torsion in significative way. The movement of the foundation is strongly affected by the rigidity of second layer especially at high frequencies.

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