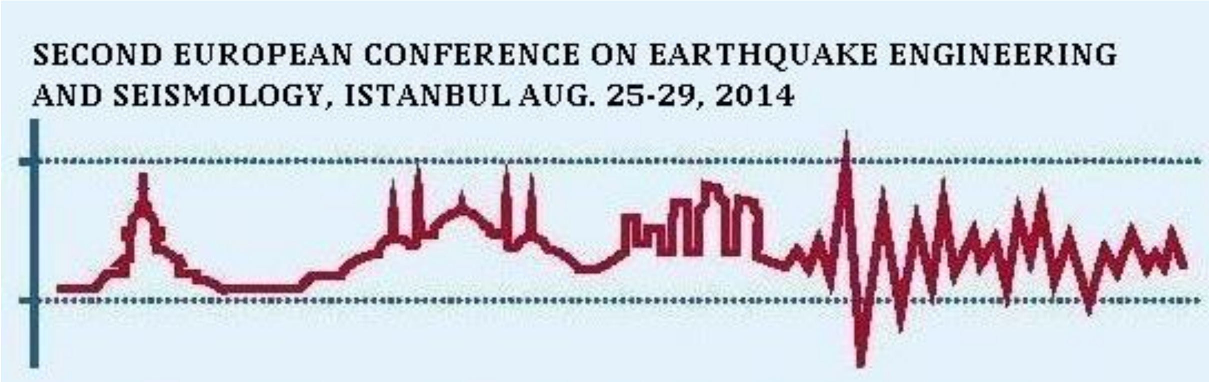

RESPONSE OF MASONRY COLUMNS AND ARCHES SUBJECTED TO BASE EXCITATION

Tamás THER[1] and László P. KOLLÁR[2]

ABSTRACT

A new numerical model is introduced to determine the planar motion of a dry-stone masonry column subjected to base excitation. The model and the numerical calculation were verified by experiments, where the multi-block motion had been recorded by a camera, and captured by image procession. The results of the numerical calculation and the tests are close to each other. So far we investigated columns consisting one, two or three blocks. Housner's classical model was also analyzed and generalized for the case when there are aggregates between the interfaces.

INTRODUCTION

Historic masonry buildings are vulnerable to earthquakes. In Hungary several churches were built in the XII – XIX centuries with vaults supported by arches, many of them were severely damaged by moderate ground motions. For example, in 1956 the vaults of a baroque church in Taksony was collapsed by the Dunaharaszti earthquake, M5,6 (Szeidovitz, 1984). In the archive photos (Fig.1) it is clearly visible (*Historia Domus*, 1956), that the motion of the arches were so big that the vaults collapsed, while the arches themselves became seriously damaged but were not destroyed. This is the reason that in the investigation of arches both the stability of the structure and the motions during the excitation must be determined. It is also important to note that these structures were not designed for earthquakes, however today they must be investigated for the expected seismic event.

It is well known, that the classical analysis used for the design of regular buildings, such as the Response Modal Analysis or even the time history analysis of elasto-plastic structures are not directly applicable for masonries, where the "rocking" of the blocks (opening and closing with impact) plays an important role in the nonlinear response of masonry structures (Makris and Konstantinidis, 2003). The investigated structural elements are usually the masonry columns and the masonry arches.

This is the reason why several researchers were investigating this problem. Based on the well-known work of Housner (Housner, 1963), the dynamic behavior of the single rocking block was investigated by Shi and Anooshehpoor (1996), and Anooshehpoor and Brune (2002); the motion of the rocking two-block assemblies was determined by Psycharis, (1990); Spanos et al. (2001) and the analytical solution of the multi-block structures was presented by Sinopoli (1991), Augusti and Sinopoli (1992) and Castrillo (2007).

The discrete element method is also widely used (Psycharis et al., 2000; DeJong, 2009; Dimitri et al., 2011; Smoljanović et al., 2013). Analytical solutions were presented by (Oppenheim, 1992; Castrillo, 2007; DeJong, 2009; Theodossopoulos and Sinha, 2012) taking into account the impact between the elements of the masonry column or the masonry arch. There are several quasy-dynamic

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solutions to find the limit load of the structure (Livesley, 1992; De Luca et al., 2004; Romano, 2005; Drosopoulos, 2008; De Santis and de Felice, 2012).

Figure 1. The ruined *Szent Anna* parish church after the earthquake in Dunaharaszti, 12th Jan. 1956 (*Historia Domus,* 1956)

In theory, both the solutions given by DEM and by some of the simplified analytical solutions can be feasible, however the first one requires the proper choice of some non-physical parameters to ensure the stability of the calculation and it is computationally very demanding; while the second option may results oversimplification, for example by considering the arch as a single degree of freedom structure. Moreover, the effect of the impact described by Housner found to be too optimistic: the energy dissipation of the rocking structure can be less than in Housners's model (see the *Discussion on Housner's model* section).

In addition, there are only very few well-documented experiments, which verify the above calculations and the new solutions. The experimental results mostly include the final result of the motion, telling just that the investigated structure collapsed or not (Anooshehpoor and Brune, 2002; Romano, 2005; Castrillo, 2007; DeJong, 2009).

PROBLEM STATEMENT

In this paper a new model is developed which is taking into account both the elastic deformations of the elements and the concentrated motions at the cracked interfaces. When the interfaces are closing an impact model is introduced to calculate the motions after the impact. This model can be considered as the generalization of Housner's model. To verify the analysis several tests were carried out.

NUMERICAL MODEL

A model and a numerical procedure is presented which is capable to calculate the rocking motion of multi-block structures (Fig.2). Between the deformable beam-elements we define eccentric hinges that represent the opening and closing cracks in the structure. During the motion, the geometry and the stiffness of the structure are updated, hence the second order effects are taken into account. The only considered damping effect is the one which occurs during the impact of the closing element.

It is assumed that the motion is 2D, there is no sliding between the elements, and the strength of the masonry is not investigated.

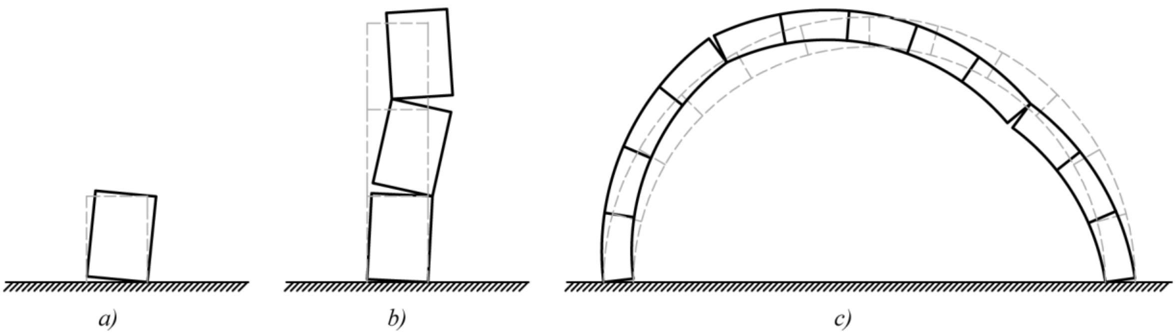

Figure 2. Rocking motion of a single (a) and a multibody column (b), and the motion of a circular arch (c).

To describe the motion of the rocking mechanism we consider n blocks, which can be elastic. Assuming n nodes with 2 degrees of freedom the total number of unknowns is m=2×n+n_h, where n_h is the number of interfaces, where rocking occurs (Fig.3).

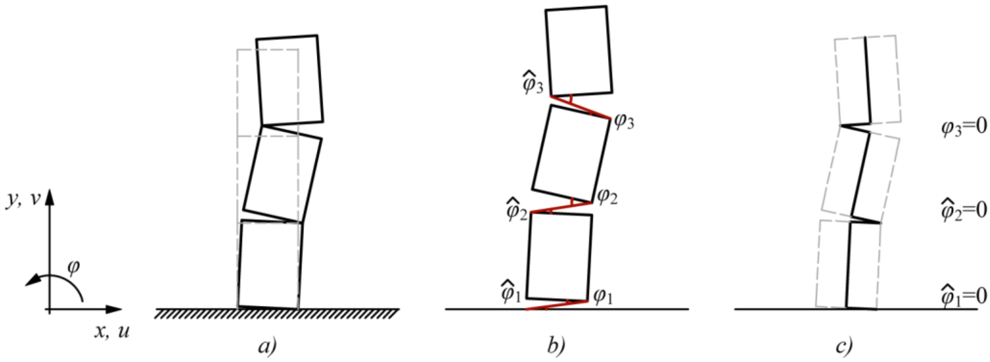

Figure 3. The degrees of freedom of the model. One possible shape of a three-block system (a), the theoretically possible motions (b) and the choice of the zero and non-zero rotations (c).

The equation of motion (neglecting damping) is:

$$\mathbf{Ku}+\mathbf{M\ddot{u}}=\mathbf{p}, \tag{1}$$

where **K** is the stiffness matrix, **M** is the mass matrix, **p** is the vector of excitation forces – due to base excitation and gravity – and $\mathbf{u}^T=[u_{1x}, u_{2x}, \ldots, u_{1y}, u_{2y}, \ldots, \varphi_1, \varphi_2, \ldots]$ is the transpose of the displacement vector.

When the blocks are assumed to be rigid, Eq.(1) simplifies to:

$$\mathbf{K}_{red}\varphi+\mathbf{M}_{red}\ddot{\varphi}=\mathbf{p}_{red}, \tag{2}$$

where $\boldsymbol{\varphi}$ contains only the relative rotations of the rocking interfaces.

The above equations can be solved by Newmark's method, we applied the Wilson algorithm (Chopra, 1995).

In theory, at every interface, when rocking may occur there are three options (Fig.2b)

- there is no rocking ($\varphi_i=\hat{\varphi}_i=0$)
- the interface is opening to the right ($\hat{\varphi}_i=0, \varphi_i<0$)
- the interface is opening to the left ($\varphi_i=0, \hat{\varphi}_i>0$)

If the eccentricity of the forces are between ± b_i (the half width of the block), there is no rocking. When the eccentricity reaches ± b_i the rocking is initiated while the eccentricity remains constant.

In the solution algorithm there are two important steps, which are discussed below: a) calculation of the velocities during impact, b) choice of the mechanism after impact.

The impact

We denote the displacement vector before and after the impact by $\mathbf{u}^{\text{before}}$ and $\mathbf{u}^{\text{after}}$, respectively. Let the duration of impact be a very short time step: Δt, and the equation of motion be multiplied by Δt:

$$\underbrace{\mathbf{Ku}\Delta t}_{\approx 0} + \mathbf{M}\underbrace{\ddot{\mathbf{u}}\Delta t}_{\Delta\mathbf{v}} = \underbrace{\mathbf{p}\Delta t}_{\Delta\mathbf{I}}, \tag{3}$$

where $\Delta\mathbf{v}$ is the change in velocity, while $\Delta\mathbf{I}$ is the impulse. (If the blocks are rigid, $\mathbf{K}$, $\mathbf{u}$, $\mathbf{M}$, and $\mathbf{p}$ must be replaced by $\mathbf{K}_{\text{red}}$, $\boldsymbol{\varphi}$, $\mathbf{M}_{\text{red}}$, $\mathbf{p}_{\text{red}}$.)

It is assumed that the impact occurs at the i^{th} displacement component (which must be one of the φ-s), hence $\dot{u}_{\text{i}}^{\text{after}}$ becomes zero:

$$\Delta v_i = -\dot{u}_{\text{i}}^{\text{before}}, \tag{4}$$

and the only nonzero element of $\Delta\mathbf{I}$ is the i^{th} one:

$$\begin{bmatrix} m_{11} & m_{12} & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{21} & m_{22} & & & & & \\ \vdots & & \ddots & & & & \\ \vdots & & & \ddots & & & \\ \vdots & & & & \ddots & & \\ \vdots & & & & & \ddots & \\ \vdots & & & & & & \ddots \end{bmatrix} \begin{Bmatrix} \Delta v_1 \\ \Delta v_2 \\ \vdots \\ \Delta v_{\text{i}} \\ \vdots \\ \vdots \\ \Delta v_{\text{m}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ \Delta I_{\text{i}} \\ 0 \\ \vdots \\ 0 \end{Bmatrix}. \tag{5}$$

Disregarding the i^{th} row of the above equation, and solving Eq.(5) we obtain

$$\Delta\hat{\mathbf{v}} = \begin{Bmatrix} \Delta v_1 \\ \cdots \\ \Delta v_{\text{i}-1} \\ \Delta v_{\text{i}+1} \\ \cdots \\ \Delta v_{\text{m}} \end{Bmatrix} = \hat{\mathbf{M}}^{-1}\mathbf{M}^{\text{i}}\dot{u}_{\text{i}}^{\text{before}}, \tag{6}$$

where $\mathbf{M}^{\text{i}}$ is the i^{th} column of $\mathbf{M}$ (without the i^{th} element) while $\hat{\mathbf{M}}$ is the sub-matrix of $\mathbf{M}$ without the i^{th} row and column.

Hence the velocities after the impact are

$$\dot{u}_j^{after} = \dot{u}_j^{before} + \Delta v_j, \ (j \neq i) \tag{7}$$
$$\dot{u}_i^{after} = 0.$$

Choice of mechanism

In the above calculation it was assumed that the rocking mechanism is known. In reality, this is not the case, and we have to follow the steps below.

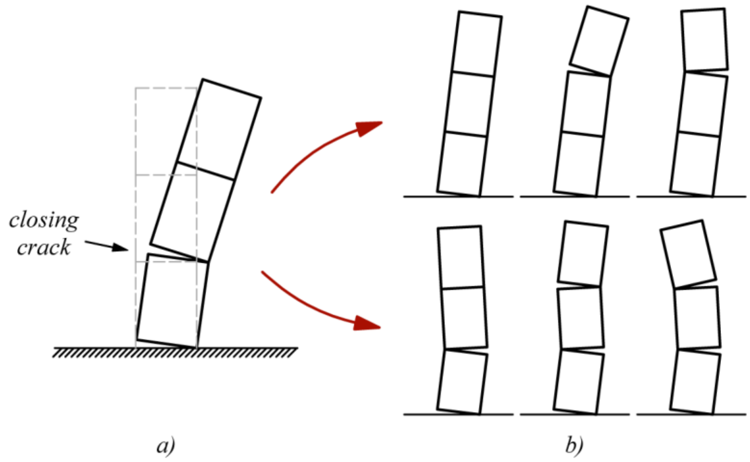

Figure 4. The possible rocking motions after impact (n_{closed}=1)

If there are n_{closed} interfaces, which were closed before the impact, they can be closed or open to the right or to the left after impact. It means that the total number of cases is

$$2 \times 3^{n_{\text{closed}}}, \tag{8}$$

where 2 is due to the fact that the closing interfaces (where the impact occurs) may either open up (opposite to the closing direction), or remains closed.

All these cases must be calculated, together with the kinetic energy of the structure:

$$U = \frac{1}{2}\left(\dot{\mathbf{u}}^{\text{after}}\right)^{\mathrm{T}} \mathbf{M}\dot{\mathbf{u}}^{\text{after}}. \tag{9}$$

There are impossible cases ($\varphi_i > 0$, $\hat{\varphi}_i < 0$), these must be thrown out. It is important to note that usually there are more than one possible cases (Fig.4b). Here we choose the motion for which the kinetic energy is the highest, i.e. where the dissipated energy is the smallest.

DISCUSSION OF HOUSNER'S MODEL

When the numerical model presented in the previous section is applied for one rigid element, it becomes identical to the well-known Housner-model, which was verified also numerically.

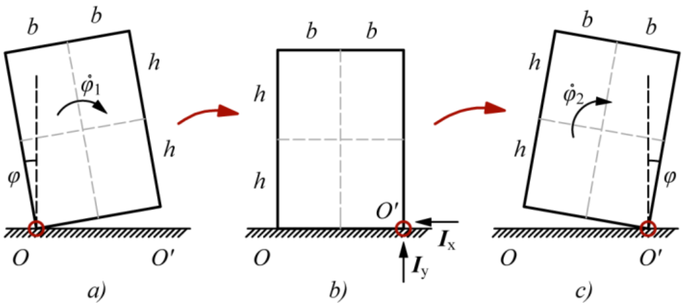

Figure 5. Housner's model.

In Housner's model it is assumed that during impact there are concentrated forces (or impulses) at the corner of the rocking block (Fig.5b). Due to the perfectly plastic impact there is a loss in kinetic energy, which is written in the following equations (Eq.10) and shown as a function of the slenderness of the element in Fig.8 (uppermost line).

$$r = \frac{\frac{1}{2}\Theta_0\dot{\varphi}_2^2}{\frac{1}{2}\Theta_0\dot{\varphi}_1^2} = \frac{\dot{\varphi}_2^2}{\dot{\varphi}_1^2}, \tag{10}$$

where the angular velocity after impact is $\dot{\varphi}_2$ can be calculated as (Housner, 1963)

$$\dot{\varphi}_2 = \dot{\varphi}_1\left(1 - \frac{2mb^2}{\Theta_0}\right), \tag{11}$$

where Θ_0 is the moment of inertia of the block around the corner point (see Fig.5a).

Now, we assume that the face of the block is not plane, but there is an infinitesimally small "aggregate" ($\Delta \ll b$) in the middle (Fig.6b). In this case the rocking will occur in more steps (there are two consecutive impacts), as it is illustrated in Fig.7b. As a consequence, even if $\Delta \rightarrow 0$, the loss in energy (and velocity) will be significantly different as it is shown in Fig.8.

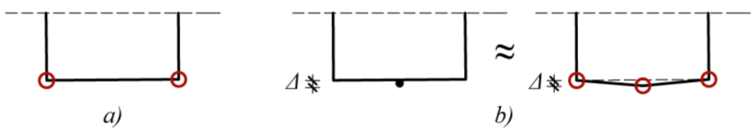

Figure 6. The impact points of the plane interface (a) and impact points with an aggregate (b).

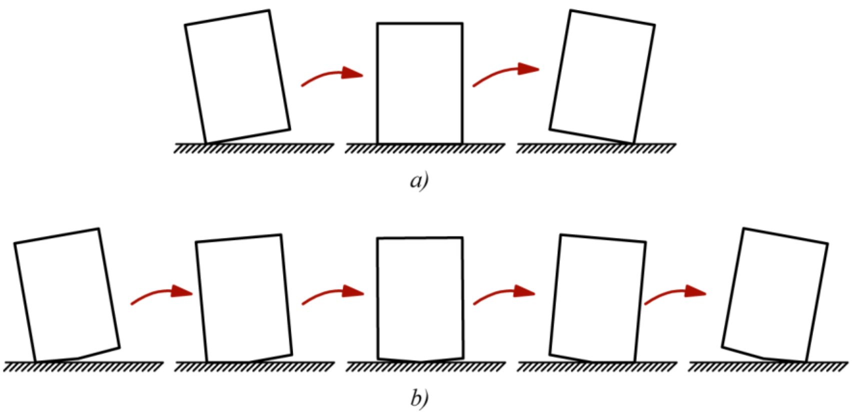

Figure 7. The impact at the Housner model (a) and impact with an aggregate between the closing faces (b).

The more "aggregates" are assumed, the smaller the loss in energy. In the limit, when the number of aggregates is infinite, the behavior is close to a smooth "rolling" and there is no loss in the kinetic energy.

It is important to emphasize that Housner's model gives an upper bound for the impact, when the loss in energy is maximum. Even the presence of a very small aggregate can modify the response of the blocks, hence we introduced the effects of the "aggregate" also on our numerical model.

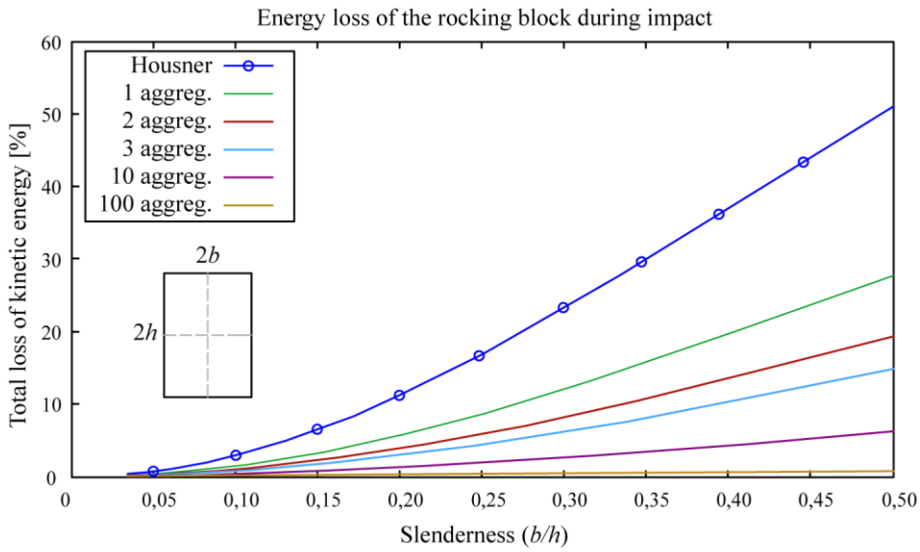

Figure 8. The amount of energy dissipation based on the number of aggregates between the closing surface, and the slenderness of the rocking block

EXPERIMENTAL RESULTS AND VERIFICATION

During several laboratory experiments we investigated the motion of one to three bricks placed on the top of each other (see Fig.9). The base excitation was a hand-shake oscillation to carry out the higher vibration modes of the columns. The motions of the bricks have been recorded by a Full HD camcorder, with 50 frames per sec. accuracy. The rotations of the elements have been identified by an image-procession algorithm, written by the authors. As a result, we obtained the exact motion of all the elements, and the base excitation.

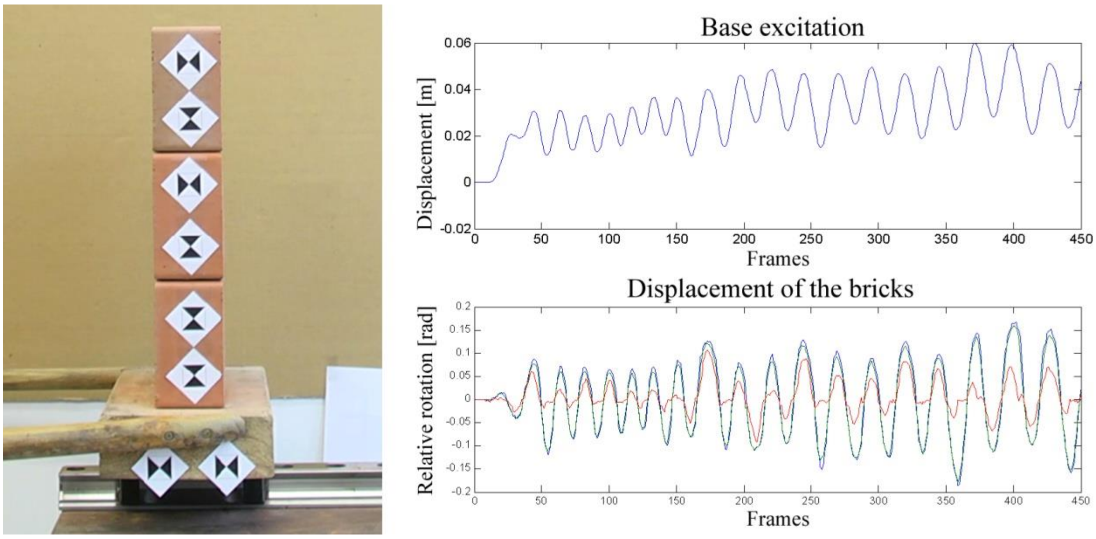

Figure 8. The experimental set-up and the results of the image-processing

Based on the experimental data the presented numerical model was verified. An illustrative example is shown in Fig.9.

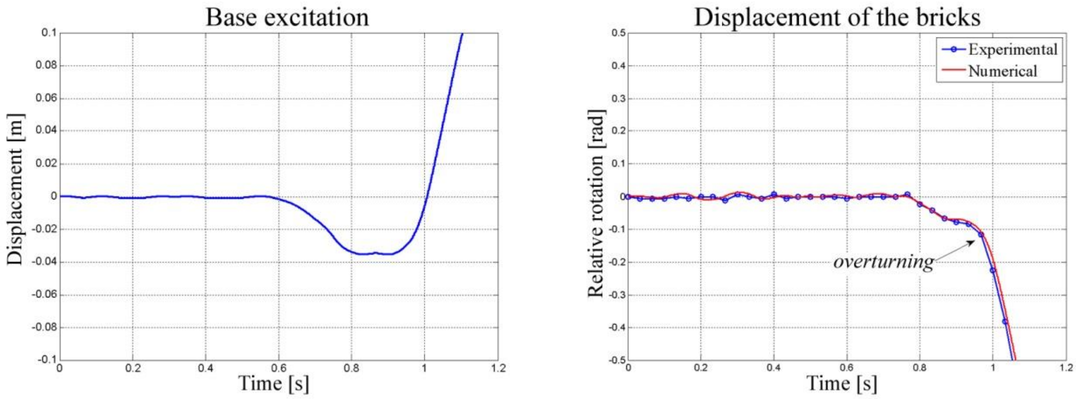

Figure 9. Experimental data of the base excitation and the verification of the numerical model for a single element

CONCLUSION

A new model and a computer code were developed to calculate the response of columns and arches having opening and closing cracks. It was shown that Housner's model can be unconservative and may underpredict the velocities of the elements after impact.

The model and the code can serve as a tool in the analysis and design of arches and vaults, which will be presented elsewhere.

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