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PROBABILISTIC ANALYSIS OF FLEXURAL OVERSTRENGTH FOR NEW DESIGNED RC BEAMS

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ABSTRACT

To ensure an overall ductile structural behavior, the reinforcing steel used in the seismic design of reinforced concrete (RC) structures is governed by certain specific requirements given in many international codes, such as the Eurocodes and the current Italian Building Code (IBC). İn particular, recent studies confirms an improvement in the quality of reinforcing steel used in RC structures, reflected in reduced variability and increased bias factors for yield and ultimate strenghts.

This study investigates the flexural overstrength of RC beams designed according to the current IBC - consistent with Eurocodes - and the accuracy of code requirements in light of realistic material models both for concrete and reinforcing steel and of uncertainties associated with mechanical models, structural members geometry and material properties. In particular, reinforcing steel properties, and their statistical characterization, are derived based on data from over 600 material tests, including a wide range of reinforcing steel bars (from 12 to 26 mm) provided by different Italian industries and used for a large structure built in Naples (southern Italy).

The obtained results show that code provisions do not seems conservative and provide a basis for an improved calibration of future editions of seismic design codes for buildings.

INTRODUCTION

As ductility is an essential property of structures responding inelastically during severe shaking, the design of reinforced concrete (RC) buildings to resist seismic action should provide the structure with an adequate capacity to dissipate energy and an overall ductile behavior.

The seismic design of structures aims to ensure ductility by appropriately dimensioning and detailing regions intended for energy dissipation (i.e., plastic hinges or “critical regions”) in response to seismic action. Moreover, a controlled inelastic response must be achieved by preventing brittle failure modes (with a certain safety margin) through the *capacity design* of structural members: ductile modes of failure (e.g., flexure) must precede brittle failure modes (e.g., shear) with sufficient reliability since brittle failure implies near-complete loss of resistance and the absence of adequate warning. In other words, capacity design rules must be used effectively to obtain the *hierarchy of resistance* of the various structural components and failure modes necessary for ensuring a suitable plastic mechanism and for avoiding brittle failure modes or other undesirable failure mechanisms: e.g., concentration of plastic hinges in columns of a single story of a multistory building, shear failure of structural elements, or failure of beam-column joints.

Ductility in structural members can be developed only if the constituent materials themselves are ductile. To achieve overall ductility in a structure, appropriate concrete and steel qualities must be adopted to ensure local ductility first of cross-sections and then of elements. In particular, the reinforcement properties required for use with Eurocode are given in Annex C of Eurocode 2 (or EC2,

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CEN, 2004a); further requirements for use in seismic design are given in Eurocode 8 (or EC8, CEN, 2004b), e.g. in *Sec. 5.4.1.1* and in *Sec. 5.5.1.1*. Similarly, IBC08 (*Sec. 11.3.2*) gives the reinforcement properties appropriate for use with the Italian code.

Specifically, the ability of reinforcing steel to sustain repeated load cycles to withstand high levels of plastic strain without significant reduction in strength is the prime source of ductility for RC structural members. In particular, the steel used in critical regions of primary seismic elements must have high uniform ultimate elongation (i.e., the strain at the peak stress, ε_{su}) to ensure a minimum curvature ductility and flexural deformation capacity; the ultimate (i.e., the maximum stress, f_t) strength-to-yield strength ratio (i.e., the hardening ratio, f_t / f_y) must be significantly higher than unity, i.e., greater than 1.15. However, the steel ultimate strength must not exceed its yield strength by more than a certain percentage (i.e., 35%); a low (i.e., less than 25%) variability of actual yield strength from the specified nominal value, $f_{y,\mathrm{nom}}$, is also a desirable characteristic of reinforcing steel. The codes refer to characteristic values of each material property, noting the maximum percentage of tests results falling below (or above) the characteristic value.

The importance of these properties stems from the requirements of capacity design. For example, since beams under inelastic shear deformation do not exhibit characteristics of energy dissipation, shear strength at all sections in critical regions is designed to be higher than the shear corresponding to the flexural strength at the chosen plastic hinge locations (Paulay and Priestley, 1992). If the reinforcing steel exhibits early and rapid strain hardening, the steel stress at a section with high ductility may exceed the yield stress (typically used as design value) by an excessive margin. Similarly, if a specific grade of reinforcing steel is subjected to a considerable variation in yield strength, the actual flexural strength of a plastic hinge may greatly exceed the nominal specified value or the steel may not yield before the concrete crushes. All these cases result in a need to adopt high overstrength factors to protect against shear failures or unexpected flexural hinging: i.e., to prevent loss of control in the hierarchy of resistance. In other words, the indiscriminate provision of excess strength, which standard design procedures usually consider to be positive, may adversely affect the nonlinear seismic behavior of a structural system (e.g., Magliulo et al., 2007), especially in the case of irregular buildings (e.g., Magliulo et al., 2012).

Table 1. Outcome of surveys of steel used in seismic regions of Europe (adapted from Fardis, 2009)

	Spain, Portugal	Italy	Belgium. France, Germany, Italy, Luxembourg, Netherlands, Portugal, Spain	UK	**Galasso et al., 2014**
$f_{y,nom}$ [MPa]	400	430	500	500	450
Mean yield strength [MPa]	496	478	571	552	549
$\left(\frac{f_y}{f_{y,nom}}\right)_{0.95}$	*1.335*	1.19	1.23	1.165	*1.36**
Mean ultimate strength [MPa]	598	733	663	653	659
$\left(\frac{f_t}{f_y}\right)_{0.10}$	1.15	1.44	*1.10*	*1.13*	1.16
$\left(\frac{f_t}{f_y}\right)_{0.90}$	1.27	1.62	1.23	1.23	1.24
$(\varepsilon_{su})_{0.10}$	9.6	9.7	8.6	9.7	21.9

*90%-fractile according to IBC08.

On the basis of these preliminary remarks, this study's main objective is the probabilistic characterization of flexural strength of RC beams designed according to the current Italian Building Code (IBC08, CS.LL.PP., 2008). To this aim, realistic material models for both concrete and reinforcing steel and uncertainties in mechanical models, structural members geometry and material properties are properly considered. More specifically, reinforcing steel properties, and their statistical characterization, are derived based on data from over 600 material tests, including a wide range of reinforcing steel bars (from 12 to 26 mm) provided by different Italian industries and used for a large structure built in Naples (southern Italy); see Galasso et al. (2014) for details. In particular, Table 1 reports the statistical outcome of the widest survey of ductile steels of the type used in European seismic regions (Fardis, 2009). That survey, carried out in the early 1990s, was the basis for EC8 current provisions on steel reinforcing properties. For comparison, results from Galasso et al. (2014), as used in the current study, are also reported in the last column of Table 1 (the subscripts indicate the assumed percentile for each quantity according to the considered codes). In Table 1, the values in italics violate the corresponding limit for the steel to be used in High Ductility Class (DCH) buildings. As a general comment, the values in Table 1 confirm a remarkable improvement in quality of materials and reduced variability in strength. As a consequence, current reinforcing steel does not usually satisfy the codes requirements regarding steel overstrength; see Galasso et al. (2014) for a discussion.

FLEXURAL OVERSTRENGTH OF EUROCODE-DESIGNED RC BEAMS

The proportioning and detailing requirements for buildings in seismic zones are intended to ensure that inelastic response is ductile. As discussed above, current seismic design pursues the control of inelastic seismic response through capacity design to achieve a strong-column/ weak-beam design that spreads inelastic response over several stories and to avoid relatively brittle shear failure, in both beams and columns. To achieve this aim, international building codes require that the sum of the columns strengths exceed the sum of beams strengths at all joints between a frame's primary or secondary seismic beams and primary seismic columns and in the two orthogonal directions, with an amplification (overstrength) factor, γ_{Rd}, equal to 1.3 for DCH (for both EC8 and IBC08), applied to the design values of the moments of resistance of the beams framing the joints. Similarly, shear failure in beams is avoided by calculating the design shear forces based on the beam equilibrium under the transverse load acting on it (in a seismic design situation) and the moments of resistance at the beam ends, again with an overstrength factor, γ_{Rd}, equal to 1.2 for DCH. In practice, due to inherent uncertainties in material properties, geometrical dimensions, and the equations used to compute member strengths, the actual moment of resistance of a RC structural member, M_R, differs from its design (i.e., nominal) flexural capacity, M_{Rd}, which is calculated based on nominal values. For that reason, the basis for designing structural members following the capacity design procedure is to accurately assess the beams flexural overstrength in terms of M_R/M_{Rd} ratio at an appropriate upper fractile. The estimation of RC beams flexural overstrength (and the calibration of overstrength factors) must necessarily be expressed in probabilistic terms because most, if not all, the factors possibly affecting the moment of resistance are uncertain despite the values assumed in design, as widely discussed in the previous sections in the case of reinforcing steel. Currently, calibration of the overstrength factors used in codes seems to be based mainly on engineering judgment rather than scientifically sound assessments (Nofal et al., 2012).

To assess the probability distribution of flexural overstrength for IBC08-designed RC beams, this study analyzes 4,320 cross-sections, representing combinations of variations in:

1) concrete geometry: this study considers rectangular cross-sections obtained by varying the concrete width between 20 cm and 40 cm and the concrete depth between 50 cm and 85 cm; width and depth are samples with a step of 5 cm.
2) Geometric reinforcement ratio in tension (ρ): this parameter is varied between 0.3% and 2% (with a 0.1% step).

3) Geometric reinforcement ratio in compression, ρ': three different values are considered for this parameter, i.e., equal to 50% and 75% of the reinforcement ratio in tension and in the case of symmetric reinforcement (ρ = ρ').
4) Design stress-strain diagrams for reinforcing steel in computing M_{Rd}: two bilinear stress-strain relationships for reinforcing steel (according to EC8 and IBC08) are used: *i*) with a horizontal top branch without a strain limit (elasto-ideal plastic); and *ii*) with an inclined, linear, top branch with a strain limit, ε_{ud}, of 6.75% (recommended value in EC8 and IBC08) and hardening ratio, *k*, equal to 1.35 (maximum allowable value based on the codes requirement, as widely discussed in this paper); see Figure 1 (left panel).

In computing M_R, concrete and steel are characterized by a characteristic compressive cylinder strength, f_{ck}, of 25 MPa and a characteristic yield strength, f_{yk}, of 450 MPa (i.e., B450C type according to the IBC08) respectively; following IBC08, partial safety factors of γ_c = 1.5 and γ_s = 1.15 for concrete and steel respectively are used. In this way, the assessment is general and covers a large number of realistic design conditions that reflect IBC08 (and EC8) provisions (e.g., Kappos, 1997; Magliulo et al., 2007).

UNCERTAINTY CHARACTERIZATION

A Monte Carlo sampling procedure is applied to accomplish the overstrength assessment. For steel properties, the results of the statistical analysis discussed in Galasso et al. (2014) are used in the Monte Carlo simulation. Representative statistics and appropriate probability distributions for the other basic resistance variables are selected from previous related studies. In particular, a literature review (Ellingwood et al., 1980; Galambos et al., 1982; Nowak and Szerszen, 2003) was carried out to select the statistical characterization for each random variable referring to materials (i.e., concrete strength), geometry (i.e., cross-section dimensions and reinforcement area), and models. The resulting assumptions, corresponding to average-quality construction, are summarized in Table 2 and described in the following sub-sections; the parameters given in Table 2 are the bias (i.e., the ratio between the mean of the sample to the reported nominal value) and the coefficients of variation (or CoV: i.e., the ratio of the standard deviation to the mean). All random variables considered were treated as stochastically independent, except for the reinforcing steel properties, for which the correlation structure described in Galasso et al. (2014) is used in the simulation. In particular, f_y and f_t are assumed to be fully correlated, as in Kappos et al., 1999, assuming an intercorrelated multivariate lognormal distribution.

Table 2. Summary of resistance statistics and distributions

Category	*Variable*	*Bias*	*CoV [%]*	*Distribution*
Material	Concrete compressive cylinder strength	1.35	18	Normal
	Steel yielding strength	1.22	5	Lognormal
	Steel ultimate strength	1.22	5	Lognormal
	Steel ultimate deformation	3.45	13	Lognormal
Geometry	Width of beam	1.10	4	Normal
	Effective depth of beam	0.99	4	Normal
	Reinforcement area	1.00	1	Normal
Model	Experimental/Theoretical flexural capacity	1.02	6	Normal

Materials

The uncertainty involving concrete properties is modeled by assuming a normal distribution for the ultimate compressive cylinder strength; the bias factor is assumed to be equal to 1.35 with a CoV of 18%. The ultimate strain of concrete is assumed to be deterministic and equal to 0.0035. Concrete tensile strength is not considered, as its effect is negligible. The uncertainty involving reinforcing steel properties is modeled by assuming a lognormal distribution for f_y, f_t and ε_{su} (Galasso et al., 2014). A

CoV value of 5% with a bias factor equal to 1.22 is assumed for both f_y and f_t while a CoV of 13% is assumed for ε_{su} with a bias factor of 3.45.

Sectional geometry

Uncertainties in geometry (or *fabrication*, Nowak and Szerszen, 2003) account for the heterogeneity in the dimensions of the considered structural element due to construction quality. The considered statistical parameters are based on Ellingwood et al., (1980). In particular, for concrete beams dimensions in bending, the bias factor is assumed to be equal to 1.01 for the width with a CoV of 4%, and equal to 0.99 for the effective depth with a CoV of 4%. The area of reinforcing steel is treated as a practically determinist value (bias factor equal to 1 with a CoV of 1%). All cases assume a normal model. The thickness of concrete cover is assumed to be deterministic and equal to 4 cm.

Mechanical models

Model (or *professional*, Nowak and Szerszen, 2003) uncertainties characterize the heterogeneity in sectional capacity estimation which is caused by design equations. In fact, such uncertainties are generally measured by comparing the flexural capacity obtained in experimental tests with the corresponding values obtained via analytical formulations. Models statistical properties are comprehensively documented in (Ellingwood et al., 1980). The normal distribution is typically used to represent these modeling factors, whose mean and CoV depend on the limit-state considered. In particular, the mean value for the ratio of the test-to-predicted flexural strength for RC beams is 1.02 with a 6% CoV.

METHODOLOGY

The probability distributions and statistics for M_R are determined using a Monte Carlo sampling procedure, employing the uncertainty characterization discussed above. To achieve this aim, the authors developed an *ad hoc* MATHWORKS-MATLAB® script.

In particular, for each case-study cross-section, defined by a set of nominal material strengths and nominal dimensions (and the selected steel constitutive model), the following steps are carried out:

1) M_{Rd} is computed based on the design material strengths, nominal dimensions, and chosen design stress-strain diagrams for reinforcing steel.
2) Given the cross-section nominal characteristics, a set of material strengths and dimensions is generated randomly from the statistical distributions of each variable that affects M_R and discussed above. This set of strength, etc., plus a randomly-generated value of the model error, is used to estimate the cross-section theoretical capacity, M_R. M_R is computed based on strain compatibility, equilibrium among internal forces, and the controlling mode of failure (i.e., concrete crushing). A bilinear stress-strain relationship with an inclined, linear top branch with a strain limit ε_{su}, at the ultimate strength f_t, is used for reinforcing steel (for both tension and compression); a parabola-rectangle diagram is employed for concrete under compression (see IBC08 and EC2 for details).
3) The overstrength ratio, M_R/M_{Rd} is finally calculated. This procedure is repeated 5,000 times, enabling the probability distribution of M_R/M_{Rd} to be determined numerically. Then the mean and the 10% and 90% fractiles from this distribution are evaluated.

Although the selected steel constitutive model used in Step 2 may look simplistic, it is fully based on the steel properties available from the test data; a more sophisticated model would require additional experimental parameters not available here and would not significantly improve computational accuracy.

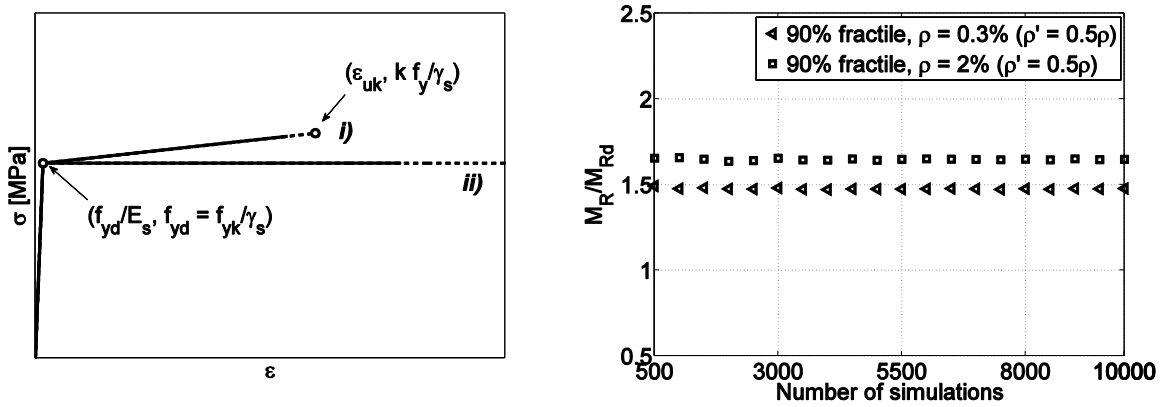

Figure 1. Design stress-strain diagrams for reinforcing steel used in computing M_{Rd} (left), and effect of sample size on the beams overstrength ratios at the upper 10% fractile (right).

As in Aydemir and Zorbozan (2012), Monte Carlo simulation is used to conduct sensitivity analyses in order to investigate the effect of the sample size on the probability distribution of M_R/M_{Rd} (particularly the 90% fractile). This part of the study used different sample sizes ranging between 500 and 10,000 for randomly-generated values of the considered random variables. As an example, Figure 1 (right panel) shows the overstrength ratios (90% fractile) for two selected cross-sections (40 cm x 60 cm with ρ = 0.3% and 2% and ρ' = 0.5ρ). In particular, Figure 1 (right) shows that the overstrength ratio (90% fractile) is stable and does not change significantly for larger sample sizes. Thus, a statistical assessment was performed for an arbitrary sample size of 5,000 for the remaining part of the study, since no differences in computational effort are observed for smaller sample sizes.

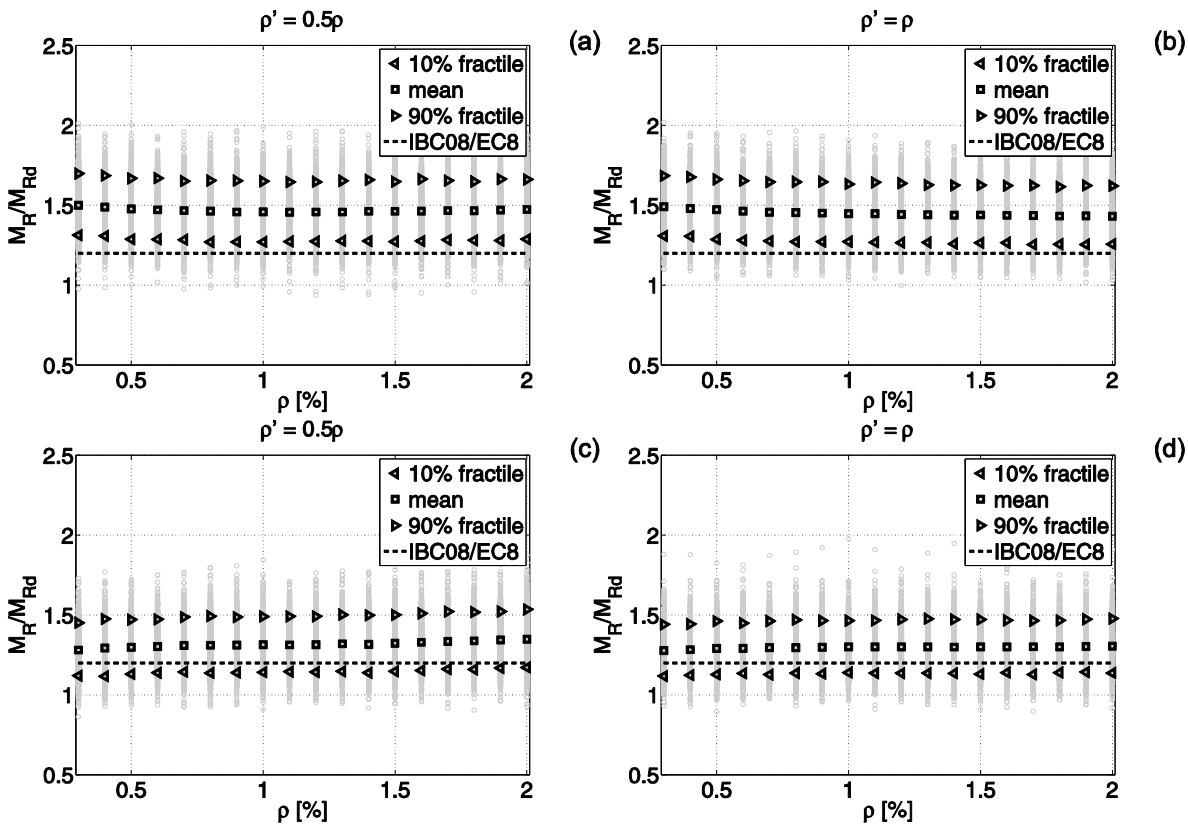

Figure 2. Flexural overstrength in terms of M_R/M_{Rd} ratios for sample cross-sections (a) elasto-ideal plastic diagram for steel (in M_{Rd} computation) and ρ' = 0.50ρ; (b) elasto-ideal plastic diagram for steel (in M_{Rd} computation) and ρ' = ρ; (c) bilinear with hardening diagram for steel (in M_{Rd} computation) and ρ' = 0.50ρ; (d) bilinear with hardening diagram for steel (in M_{Rd} computation) and ρ' = ρ.

RESULTS AND DISCUSSIONS

As an example, Figure 2 shows the M_R/M_{Rd} distributions as a function of the level of tensile reinforcement (ρ) for a selected geometry (40 cm x 60 cm) with $\rho' = 0.5\rho$ and $\rho' = \rho$; all the randomly-generated samples are plotted (grey circles). The lower 10% fractile, mean, and upper 10% fractile of M_R/M_{Rd} ratios, obtained from all generated samples, are shown in the same figure (see legend) together with the IBC08 overstrength factor value (= 1.2, black dashed line). The upper panels refer to the elasto-ideal plastic stress-strain curve used in computing M_{Rd}, while the lower panels refer to the bilinear stress-strain relationship with hardening used in computing M_{Rd}. As expected, the use of a more realistic stress-strain relationship results in lower values of the flexural overstrength.
Figure 2 demonstrates that randomness in materials, section properties, and model significantly affect the M_R/M_{Rd} ratio. The overstrength factor suggested by IBC08 (and EC8), equal to 1.2, is always lower than mean M_R/M_{Rd} ratio of the sample beams for each tensile reinforcement ratio and for the two considered values of the compression reinforcement ratio.
Therefore, the code provisions do not seem to be very accurate and conservative, although a constant increase in the design flexural strength for the calculation of probable flexural strength of RC beams is a simple and practical way to account for the flexural overstrength. This finding is consistent with the results of similar studies: e.g., Aydemir and Zorbozan (2012) and Somja et al. (2013).
To summarize the results obtained for all the considered case study cross-sections and to suggest an improvement to the code provisions, Figure 3 shows the obtained CDFs for the 90% fractile of the M_R/M_{Rd} ratio (for all cross-sections) considering the actual nominal yield strength prescribed by IBC08 (i.e., 450 MPa) and the value suggested in Galasso et al. (2014), i.e., 500 MPa. The median overstrength ratios extracted from these curves (i.e., at a cumulative probability of 50%) are equal to 1.65 and 1.50, respectively; these values could be used as overstrength factors in future editions of the code.

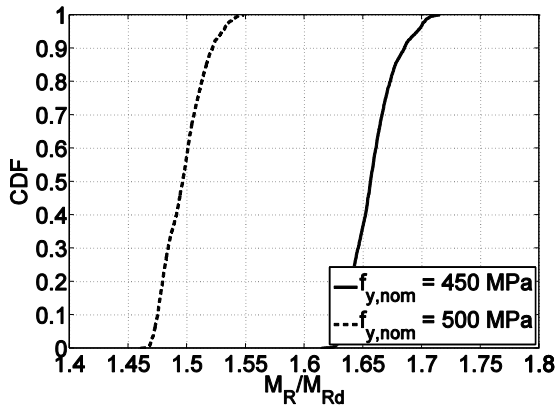

Figure 3. Cumulative probability of M_R/M_{Rd} ratio (90% fractile) for all the considered case study cross-sections.

CONCLUSIONS

The ability of ductile structures to dissipate energy by post-elastic deformations may be the most important factor in avoiding collapse during major earthquakes. The fundamental source of ductility is the constituent materials ability to sustain plastic strains without significant reduction of strength. Both the strength and ductility of RC structures depend to a large extent on certain proprieties of reinforcing bars, properties controlled in practice by code specifications.
This paper investigated the probable flexural strength of IBC08-designed RC beams by using Monte Carlo simulation and considering randomness in material, cross-section geometry, and mechanical models. It is concluded that uncertainties in material properties and cross-sectional behavior significantly affect M_R/M_{Rd} ratio and that code provisions do not seem accurate enough to address these effects.

The results obtained provide a basis for the reliability analysis of the RC components of building structures and for an improved calibration of future editions of seismic design codes for buildings.

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