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ACCOUNTING FOR THE AFTERSHOCK EFFECT IN THE LIFE-CYCLE ASSESSMENT OF STRUCTURES

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ABSTRACT

Most of life-cycle models for earthquake resistant structures consider that damage accumulation and failure are possibly due to subsequent mainshocks. Because there is a chance that also aftershocks worsen structural conditions, it may be appropriate to include this effect in the life-cycle assessment. Recently, stochastic processes for occurrence of aftershocks and their effect on cumulative structural damage have been formalized. These can be employed to develop stochastic damage accumulation models for earthquake resistant structures, accounting for the effect of the whole cluster (i.e., a single mainshock-aftershock sequence). In the paper, a model of such kind is formulated with reference to simple elastic-perfectly-plastic single degree of freedom systems. Temporal distribution of mainshocks is modeled via a homogeneous Poisson process; temporal distribution of aftershocks in a cluster, is modeled by means of a conditional non-homogeneous Poisson process. An approximated closed-form solution is derived for the reliability assessment under the two hypotheses that total damages produced by events pertaining to different clusters are independent and identically distributed; the gamma distribution is adopted to represent the distribution of damage in one cluster. An application illustrates the implications of the model on the life-cycle assessment, also in comparison with the case in which the effect of damaging aftershocks is ignored.

INTRODUCTION

Stochastic modeling of structures cumulating damage due to mainshock-aftershock seismic sequences, or clusters, is the issue addressed in the presented study. The work builds on recent results of the authors about stochastic modeling of degradation in earthquake resistant structures for life-cycle assessment (i.e., Iervolino et al., 2013a), short-term structural risk assessment based on aftershock probabilistic seismic hazard analysis (or APSHA; Yeo and Cornell 2009), and damage accumulation in aftershock sequences (i.e., Iervolino et al. 2013b). In the study, earthquake clusters are considered point events, because their duration is negligible with respect to structural life. Therefore, seismic clusters are described by a marked point process, where each event is represented by its occurrence time (i.e., the occurrence time of the triggering mainshock) and damage that it produces. The occurrence of earthquake clusters is modelled via the same homogeneous Poisson process (HPP) considered for the mainshocks (Boyd, 2012), while in each cluster aftershocks are assumed to occur according to a (conditional) non-homogeneous Poisson process (NHPP), the intensity of which depends on the characteristics of the sequence-triggering mainshock (Yeo and Cornell, 2009). On the

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structural vulnerability side, it is considered that the structure may suffer damage both in the mainshock and in the following aftershocks, and that performance degradation due to these seismic damages can eventually lead to failure.

The main assumptions of the model are that increments of damage accumulated over different seismic sequences are independent and identically distributed (i.i.d.) random variables (RVs), which are also independent of the process regulating occurrence of clusters. It has been shown in Iervolino et al. (2013a-b) that these assumptions are applicable to simple elastic-perfectly-plastic (EPP) single degree of freedom (SDOF) systems, at least if energy-based damage indices are adopted. The model also explicitly accounts for the fact that not all earthquakes are strong enough to damage the structure.

The paper is structured such that the compound Poisson process modeling structural damage accumulation is described first. Then, the variable selected to define the damage of the structure at risk is briefly discussed. Subsequently, starting from the hypotheses taken for hazard and vulnerability, the distribution of damage in a single cluster is derived. Hence, the problem of formulating the reliability of the considered structure is addressed. Finally, the gamma distribution is adopted to represent the damage in a single cluster. Main motivation for this is that the reproductive property of this RV enables closed-form solution, or at least closed-form approximation, for the reliability problem. An illustrative application of the proposed methodology, to an EPP-SDOF structure located in an ideal seismic source zone, closes the work. For this simple structure, the model is calibrated and the probability of failure is obtained. Results of the life-cycle assessment are also compared with those in the case aftershock effect is ignored.

FORMULATION

In the study, the source of deterioration is related to damaging events in seismic sequences comprised of a mainshock and following aftershocks (Yeo and Cornell, 2009). The effect of the (whole) sequence on the structure is evaluated considering the effective occurrence time and location of the triggering mainshock and all the aftershocks in the cluster. This modeling approach allows describing the sequence effect as that of a single event, as schematically illustrated in Figure 1. Clearly, this approach works satisfactorily in the case that no maintenance activity is performed within a sequence.

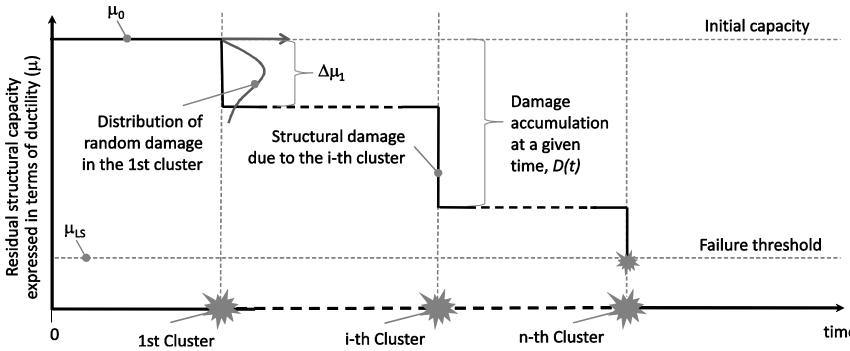

Figure 1. Seismic cycle representation for a structure subject to cumulative earthquake damages.

Given a metric of the damage effect on the structural performance, for example the residual ductility to collapse, $\mu(t)$, the degradation process may be expressed as in Equation (1), where μ_0 is the initial capacity in the cycle and $D(t)$ is the cumulated damage due to all clusters, $N(t)$, occurring within t.

$$\mu(t) = \mu_0 - D(t) = \mu_0 - \sum_{i=1}^{N(t)} \Delta\mu_i \tag{1}$$

It follows from Equation (1) that the probability the structure fails within time *t*, $P_f(t)$, that is the cumulative probability function (CDF) of structural lifetime, $F_T(t)$, complement to one of reliability, $R(t)$, is the probability that the structure passes the limit-state (LS) threshold, μ_{LS}. It can also be expressed as the probability the cumulated damage is larger than the difference between the initial capacity and the threshold, $\bar{\mu} = \mu_0 - \mu_{LS}$, as in Equation (2).

$$P_f(t) = F_T(t) = 1 - R(t) = P\left[\mu(t) \le \mu_{LS}\right] = P\left[D(t) \ge \mu_0 - \mu_{LS}\right] = P\left[D(t) \ge \bar{\mu}\right] \quad (2)$$

Because in this approach the damage in the single cluster, $\Delta\mu_i$, and $N(t)$ are both RVs, the structural reliability problem may be computed by means of the total probability theorem as in Equation (3), where the probability of occurrence of *k* clusters and the probability of failure given *k* clusters, appear.

$$\begin{aligned} P_f(t) &= \sum_{k=0}^{+\infty} P\left[D(t) \ge \bar{\mu} \middle| N(t) = k\right] \cdot P\left[N(t) = k\right] = \\ &= \sum_{k=1}^{+\infty} P\left[\sum_{i=1}^{k} \Delta\mu_i \ge \bar{\mu} \middle| N(t) = k\right] \cdot \frac{\left(E\left[N(t)\right]\right)^k}{k!} \cdot e^{-E\left[N(t)\right]} = \\ &= \sum_{k=1}^{+\infty} P\left[\sum_{i=1}^{k} \Delta\mu_i \ge \bar{\mu} \middle| N(t) = k\right] \cdot \frac{(\lambda \cdot t)^k}{k!} \cdot e^{-\lambda \cdot t} \end{aligned} \quad (3)$$

The equation assumes that the process regulating the occurrence of clusters is a HPP. Indeed, if mainshock occurrence is stochastically modeled by a HPP with rate equal to λ (a common assumption in the classical probabilistic seismic hazard analysis; PSHA) then, the cluster initiation may be seen as described by the same process (Boyd, 2012; Iervolino et al., 2013d; Iervolino et al., 2014). Thus, $E\left[N(t)\right] = \lambda \cdot t$ is the expected number of clusters in $(0,t)$.

The last issue to solve, is to evaluate the probability of exceedance of a threshold for any given number of clusters. Such a probability may be easily computed if $\Delta\mu_i$, the damage in a single sequence, is modeled via a random variable that enjoys additive *reproductive* property.[4] A RV, featuring the needed property is the gamma (G). Because reproducibility requires that effects of clusters are independent, this hypothesis will be discussed in the next section along with the assumption that cluster damages are identically distributed.

DAMAGE MEASURES AND I.I.D. DAMAGE INCREMENTS HYPOTHESIS

This section focuses on the properties of damage measures that may characterize the dynamic performance of common structures in literature. According to Cosenza and Manfredi (2000) damage indices are usually comprised between two extremes: (i) *displacement-related* and (ii) *energy-related*. Measures in the former class assume that collapse is related to attainment or exceedance of some maximum strain limit. Those in the latter postulate that damage is related to the amount of energy dissipated by hysteretic loops. In fact, the most representative damage index of (i) is the maximum displacement demand, while hysteretic energy is a key member of (ii).

If the simplest non-linear inelastic structure is considered, that is an EPP-SDOF, according to a displacement-based damage criterion, the accumulation of degradation occurs in the second shock, that is part of a sequence of two, only if the maximum displacement reached (in a certain direction) in the second one is larger than the maximum in the first one. This makes the damage increment dependent at least on the residual displacement of the structure at the time of the shock, and violates the hypothesis that the cumulative damage process has independent increments. In this case, state-dependent approaches (e.g., Yeo and Cornell, 2005; Luco et al., 2011; Giorgio et al., 2010) may be required to stochastically model degradation.

On the other hand, due to the non-evolutionary (Cosenza and Manfredi, 2000) features of the EPP-SDOF system response, the area of hysteretic loops during the shaking from the second shock is measured regardless of the previous shaking demand. If a damage index measuring dissipated

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[4] The sum of i.i.d. RVs pertaining to a family that enjoys the reproductive property also belongs to the considered family.

hysteretic energy is chosen, the response of the structure to a specific shock is independent of its status prior to the shock, so it may be assumed that damage increments in subsequent events are i.i.d. RVs.

In this work the kinematic ductility, μ, is considered as a simplistic proxy for dissipated hysteretic energy. It is the maximum displacement demand when the yielding displacement is the unit. To capture energy dissipation in a single shock only, ductility is computed as if the residual displacement at beginning of each ground motion is zero. Note that this implies that only events with intensity larger than that required to yield the structure may produce increment of damage. The collapse is assumed to occur when kinematic ductility, conservatively accumulated independently on the sign of maximum displacement, reaches some capacity value.

CLUSTER DAMAGE DISTRIBUTION

This section targets the formulation of the distribution of damage increment in a single seismic cluster, $\Delta\mu_i$. It is the fundamental component to obtain the distribution of the sum of damage in *k* clusters as per Equation (3). Under the hypotheses discussed in the preceding sections, $\Delta\mu_i$ may be seen as the damage in the mainshock, $\Delta\mu_{E,i}$, plus that accumulated in the aftershock sequence, $\Delta\mu_{A,i}$, pertaining to the same cluster, Equation (4).

$$\Delta\mu_i = \Delta\mu_{E,i} + \Delta\mu_{A,i} = \Delta\mu_{E,i} + \sum_{j=1}^{N_{A,i}(\Delta T_A)} \Delta\mu_{A,ij} \tag{4}$$

In the equation, $N_{A,i}$ is the number of aftershocks in the sequence following the *i*-th mainshock and $\Delta\mu_{A,ij}$ is the damage in the *j*-th aftershock (the duration of the sequence is indicated as ΔT_A). The developed model considers all the terms of Equation (4) as random. Therefore, in the following it will be discussed first how $N_{A,i}$ is stochastically modelled, then the distribution of $\Delta\mu_{E,i}$ is addressed, and $\Delta\mu_{A,ij}$ is discussed. Finally, the strategy adopted to get $\Delta\mu_i$ is illustrated.

Conditional aftershock occurrence process and APSHA

Given the occurrence of the mainshock, aftershocks are modeled herein following the aftershock probabilistic seismic hazard analysis approach of Yeo and Cornell (2009). In APSHA, assuming that the mainshock occurred at $t=0$, the occurrence of aftershocks is described by a NHPP the daily rate of which, $\lambda_{A|M_E}(t)$, is provided by Equation (5). It refers to the aftershocks with magnitude bounded between a minimum value of interest, $m_{\min}$, and that of the mainshock, coefficients *a* and *b* are from a suitable Gutenberg and Richter (GR) relationship (Gutenberg and Richter, 1944), while *c* and *p* are those of the *modified Omori law* (Utsu, 1961) for the considered sequence. Therefore, the process regulating the aftershocks' occurrence may be considered conditional to the mainshock characteristics. Moreover, it follows from Equation (5) that the expected number of aftershocks in $(0,\Delta T_A)$ is given by Equation (6).

$$\lambda_{A|M_E}(t) = \left(10^{a+b\cdot(m_E - m_{\min})} - 10^a\right) \Big/ (t+c)^p \tag{5}$$

$$E\left[N_A(\Delta T_A)\middle|M_E = m_E\right] = \int_0^{\Delta T_A} \lambda_{A|M_E}(\tau)\cdot d\tau = \frac{10^{a+b\cdot(m_E - m_{\min})} - 10^a}{p-1}\cdot\left[c^{1-p} - (\Delta T_A + c)^{1-p}\right] \tag{6}$$

APSHA, provides the rate of exceedance of a ground motion intensity measure (IM) at a site of interest, $\lambda_{IM_A|M_E,R_E}(t)$, during the aftershock sequence, via Equation (7).

$$\lambda_{IM_A|M_E,R_E}(t) = \lambda_{A|M_E}(t)\cdot\int_{r_A}\int_{m_A} P\left[IM > im\middle|M_A = w, R_A = z\right]\cdot f_{M_A,R_A|M_E,R_E}\left(w,z\middle|x,y\right)\cdot dw\cdot dz \tag{7}$$

In the equation, $f_{M_A,R_A|M_E,R_E}$ is the distribution of the aftershock magnitude and distance, $\{M_A,R_A\}$, conditional to those of the mainshock, $\{M_E=x,R_E=y\}$,[5] while $P\left[IM>im|M_A=w,R_A=z\right]$ is the probability of exceedance of IM conditional to magnitude and distance from a ground motion prediction equation (GMPE). It is worth to note that, to obtain Equation (7), APSHA also assumes that IMs in different aftershocks are i.i.d., given $\{M_E,R_E\}$.

Mainshock damage

The probability density function (PDF) of the first term at the right hand side of Equation (4) that is the damage in the mainshock, $\Delta\mu_{E,i}$, is computed consistently with the performance-based earthquake engineering framework (PBEE; e.g., Cornell and Krawinkler, 2000). Indeed, the distribution of $\Delta\mu_{E,i}$, $f_{\Delta\mu_{E,i}}(\delta\mu)$, is calculated as in Equation (8).

$$\begin{aligned} f_{\Delta\mu_{E,i}}(\delta\mu) &= \int_{im} f_{\Delta\mu_{E,i}|IM}(\delta\mu|u)\cdot f_{IM_E}(u)\cdot du = \\ &= \int_{im} f_{\Delta\mu_{E,i}|IM}(\delta\mu|u)\cdot \int_{r_E}\int_{m_E} f_{IM|M_E,R_E}(u|x,y)\cdot f_{M_E,R_E}(x,y)\cdot dx\cdot dy\cdot du \end{aligned} \tag{8}$$

In the equation $f_{\Delta\mu_{E,i}|IM}$ is the distribution of damage given an IM value (e.g., from incremental dynamic analysis or IDA; Vamvatsikos and Cornell, 2002), while f_{IM_E} is the PDF of the chosen IM given the occurrence of a mainshock. Indeed, as per the right hand side of Equation (8), the latter can be computed as in PSHA, via the joint PDF of mainshock magnitude and distance RVs for the site of the construction, f_{M_E,R_E}, and the distribution of IM given the mainshock parameters, $f_{IM|M_E,R_E}$, provided by a GMPE. In the case $\{M_E,R_E\}$ may be considered stochastically independent, the joint PDF is just the product of the marginal distribution of magnitude, often described by a GR relationship, and that of source-to-site distance that depends on the source-site configuration. In fact, it will be shown in the following that to compute the distribution in the cluster, the PDF of damage in the mainshock, conditional to $\{M_E,R_E\}$ is of interest. It follows from Equation (8) and it is given in Equation (9), in the case the structural response is independent of $\{M_E,R_E\}$, given IM.

$$f_{\Delta\mu_{E,i}|M_E,R_E}(\delta\mu|x,y) = \int_{im} f_{\Delta\mu_{E,i}|IM}(\delta\mu|u)\cdot f_{IM|M_E,R_E}(u|x,y)\cdot du \tag{9}$$

Damage in the generic aftershock

To compute the distribution of damage in the single aftershock of a certain mainshock, a similar approach can be used, as depicted in Equation (10). In the equation, $f_{IM_A|M_E,R_E}$ is the distribution of the ground motion intensity given the occurrence of a mainshock of magnitude $M_E=x$ and separated by a distance $R_E=y$ from the site.[6] In fact, $f_{IM_A|M_E,R_E}$ is the PDF corresponding to the integral term of Equation (7).

[5] This factually makes the aftershock rate to be dependent also on location of the mainshock.
[6] Models used in this study consider that the aftershock source zone depends on the magnitude and location of the mainshock. Considering magnitude and distance is equivalent herein. It is also to note that both $f_{IM_A|M_E,R_E}$ and $f_{IM_E|M_E,R_E}$ should be indicated as $f_{IM_{A,ij}|M_E,R_E}$ and $f_{IM_{E,i}|M_E,R_E}$, yet the notation is simplified due to the i.i.d. feature of these RVs. Actually, while also damages are i.i.d., subscript are kept there to avoid confusion, as it will be clarified in the following.

$$
\begin{aligned}
& f_{\Delta\mu_{A,ij}|M_E,R_E}\left(\delta\mu|x,y\right)=\int_{im} f_{\Delta\mu_{A,ij}|IM}\left(\delta\mu|u\right)\cdot f_{IM_A|M_E,R_E}\left(u|x,y\right)\cdot du= \\
& =\int_{im} f_{\Delta\mu_{A,ij}|IM}\left(\delta\mu|u\right)\cdot\int_{r_A}\int_{m_A} f_{IM|M_A,R_A}\left(u|w,z\right)\cdot f_{M_A,R_A|M_E,R_E}\left(w,z|x,y\right)\cdot dw\cdot dz\cdot du
\end{aligned} \qquad (10)
$$

Note that the $f_{\Delta\mu_{A,ij}|IM}$ term is the same as $f_{\Delta\mu_{E,i}|IM}$ in Equation (9). Indeed, in both equations it is assumed that the response of the structure is, given the IM, the same in mainshock and one aftershock, $f_{\Delta\mu_{E,i}|IM}=f_{\Delta\mu_{A,ij}|IM}=f_{\Delta\mu|IM}$, and independent of the specific features of the earthquake (see the application). In this case, the IM is said to be a sufficient one (Luco and Cornell, 2007). Moreover, it is also assumed that the same GMPE can be used for both mainshock and aftershocks, so also the $f_{IM|M_A,R_A}$ term is the same as $f_{IM|M_E,R_E}$.

Cluster damage

On the basis of the above equations, it is possible to approach the distribution of damage in the whole cluster. Recalling Equation (4), the probability of exceedance of any damage level can be computed as in Equation (11).

$$
\begin{aligned}
& P\left[\Delta\mu_i>\delta\mu\right]=P\left[\Delta\mu_{E,i}+\Delta\mu_{A,i}>\delta\mu\right]= \\
& =1-P\left[\Delta\mu_{E,i}+\Delta\mu_{A,i}\le\delta\mu\right]=1-P\left[\Delta\mu_{E,i}+\sum_{j=1}^{N_{A,i}(\Delta T_A)}\Delta\mu_{A,ij}\le\delta\mu\right]
\end{aligned} \qquad (11)
$$

Because of the features of the EPP-SDOF response introduced in previous sections, it may be argued that, conditional to $\{M_E,R_E\}$, the increment damage in the mainshock and in the aftershock sequence are independent random variables. Hence, applying the total probability theorem, the $P\left[\Delta\mu_{E,i}+\Delta\mu_{A,i}\le\delta\mu\right]$ term of Equation (11) can be rewritten as in Equation (12).

$$
\begin{aligned}
& P\left[\Delta\mu_{E,i}+\Delta\mu_{A,i}\le\delta\mu\right]=\int_{r_E}\int_{m_E} P\left[\Delta\mu_i\le\delta\mu|M_E=x,R_E=y\right]\cdot f_{M_E,R_E}\left(x,y\right)\cdot dx\cdot dy= \\
& =\int_{r_E}\int_{m_E}\int_0^{\delta\mu} P\left[\Delta\mu_{E,i}\le\delta\mu-l|M_E=x,R_E=y\right]\cdot f_{\Delta\mu_{A,i}|M_E,R_E}\left(l|x,y\right)\cdot f_{M_E,R_E}\left(x,y\right)\cdot dl\cdot dx\cdot dy
\end{aligned} \qquad (12)
$$

In the above equation, the term $P\left[\Delta\mu_{E,i}\le\delta\mu-l\,|\,M_E=x,R_E=y\right]$ is obtained from Equation (9), while $f_{\Delta\mu_{A,i}|M_E,R_E}$ represents the PDF of damage cumulated during the aftershock sequence, given the features of the mainshock. Because the aftershock sequence is comprised by a random number of events, $f_{\Delta\mu_{A,i}|M_E,R_E}$ can be evaluated applying the total probability theorem again; Equation (13).

$$
\begin{aligned}
& f_{\Delta\mu_{A,i}|M_E,R_E}\left(l|x,y\right)=\sum_{j=0}^{+\infty} f_{\Delta\mu_{A,i}|M_E,R_E,N_{A,i}}\left(l|x,y,j\right)\cdot P\left[N_{A,i}\left(\Delta T_A\right)=j|M_E=x\right]= \\
& =\sum_{j=0}^{+\infty} f_{\Delta\mu_{A,i}|M_E,R_E,N_{A,i}}\left(l|x,y,j\right)\cdot\frac{\left(E\left[N_{A,i}\left(\Delta T_A\right)|M_E=x\right]\right)^j}{j!}\cdot e^{-E\left[N_{A,i}\left(\Delta T_A\right)|M_E=x\right]}
\end{aligned} \qquad (13)
$$

Note that, following the APSHA approach, the probability of having *j* aftershocks in the cluster is provided by a Poisson distribution with mean in Equation (6). In the equation it is assumed that $f_{\Delta\mu_{A,i}|M_E,R_E,N_{A,i}}$ degenerates in a unitary mass at zero when *j* equals zero.

Under the assumption that damages produced in different aftershock events are i.i.d. RVs, given $\{M_E,R_E\}$, the distribution of the sum of damages in a given number of aftershocks, conditional to magnitude and distance of the mainshock, $f_{\Delta\mu_{A,i}|M_E,R_E,N_{A,i}}$, is just the *j*-th order convolution of $f_{\Delta\mu_{A,ij}|M_E,R_E}$ from Equation (10), with itself, and it will be indicated as $f^{(j)}_{\Delta\mu_{A,i}|M_E,R_E}$ in the following. Applying a simplification of the delta method (e.g., Oehlert, 1992) to Equation (13), the infinite-terms summation may be approximated by calculating $f^{(j)}_{\Delta\mu_{A,ij}|M_E,R_E}(l|x,y)$ with j equal to the expected number of aftershocks in the time interval of interest, Equation (14).

$$\begin{cases} f_{\Delta\mu_{A,i}|M_E,R_E}(l|x,y)=\sum_{j=0}^{+\infty} f^{(j)}_{\Delta\mu_{A,ij}|M_E,R_E}(l|x,y)\cdot\dfrac{\left(E\left[N_{A,i}(\Delta T_A)|M_E=x\right]\right)^j}{j!}\cdot e^{-E\left[N_{A,i}(\Delta T_A)|M_E=x\right]}\approx \\ \approx f^{(\tilde{N}_A)}_{\Delta\mu_{A,ij}|M_E,R_E}(l\,|\,x,y) \\ \tilde{N}_A=\mathrm{int}\left\{E\left[N_{A,i}(\Delta T_A)|M_E=x\right]\right\}=\mathrm{int}\left\{\int_0^{\Delta T_A}\lambda_{A|m_E=x}(\tau)\cdot d\tau\right\} \end{cases} \tag{14}$$

At this point, combining Equation (14) with Equation (12), the probability of exceedance of an increment damage value in the single cluster results, and it is given in Equation (15), where it is assumed that $f^{(\tilde{N}_A)}_{\Delta\mu_{A,ij}|M_E,R_E}$ degenerates in a unitary probability mass at $\delta\mu_{A,ij}=0$ when $\tilde{N}_A=0$.[7]

$$\begin{aligned} &P\left[\Delta\mu_i>\delta\mu\right]= \\ &=1-\int_{r_E}\int_{m_E}\int_0^{\delta\mu} P\left[\Delta\mu_{E,i}\le\delta\mu-l|M_E=x,R_E=y\right]\cdot f^{(\tilde{N}_A)}_{\Delta\mu_{A,ij}|M_E,R_E}(l|x,y)\cdot f_{M_E,R_E}(x,y)\cdot dl\cdot dx\cdot dy \end{aligned} \tag{15}$$

The strategy to compute the integral in Equation (15) will be discussed in the application, while the next section discusses the advantage of assuming that $\Delta\mu_i$ follows a gamma distribution.

RELIABILITY FOR GAMMA-DISTRIBUTED CLUSTER DAMAGE

Because the EPP-SDOF assures the RVs adopted to model damages, $\Delta\mu_i$, accumulated over different clusters are i.i.d., a closed-form solution of the reliability problem may be obtained if the sum of the damages in multiple mainshock-aftershock sequences may be expressed using a (non-negative) RV, which possesses the reproductive property. An option discussed in Iervolino et al. (2013a) is given in Equation (16), in which it is considered that the damage increment is a gamma-distributed RV (Γ is the gamma function). The PDF of this RV is indexed by two parameters, γ_D and α_D, the scale and shape parameters, respectively. The mean is α_D/γ_D while the variance is α_D/γ_D^2.

$$f_{\Delta\mu_i}(\delta\mu)=\frac{\gamma_D\cdot(\gamma_D\cdot\delta\mu)^{\alpha_D-1}}{\Gamma(\alpha_D)}\cdot e^{-\gamma_D\cdot\delta\mu} \tag{16}$$

The main advantage in using the gamma model in the context of this study is that the sum of k_D i.i.d. gamma RVs, with scale and shape parameters γ_D and α_D, is still gamma with parameters γ_D and $k_D\cdot\alpha_D$. Therefore, the probability of cumulative damage exceeding the threshold, conditional to k_D shocks, is given by Equation (17) where $\Gamma(k_D\cdot\alpha_D)$ and $\Gamma_U(k_D\cdot\alpha_D,\gamma_D\cdot\bar{\mu})$ are referred to as the incomplete and the upper-incomplete gamma functions, respectively.

[7] In Equation (15), and in others above, the distribution of damage is always indicated as a PDF, for simplicity of notation. However, it is not perfectly appropriate because the damage in a single event is not a continuous RV.

$$P\left[D(t) \geq \bar{\mu} \middle| N_D(t) = k_D\right] = \int_{\bar{\mu}}^{+\infty} \frac{\gamma_D (\gamma_D \cdot x)^{k_D \cdot \alpha_D - 1}}{\Gamma(k_D \cdot \alpha_D)} \cdot e^{-\gamma_D \cdot x} \cdot dx = \frac{\Gamma_U(k_D \cdot \alpha_D, \gamma_D \cdot \bar{\mu})}{\Gamma(k_D \cdot \alpha_D)} \tag{17}$$

Equation (17) allows a closed-form solution of the reliability problem given in Equation (3). However, because the gamma is a continuous RV, it can be adopted to account only for the effects of damaging clusters (this justifies the subscript *D*). This is the reason why the rate in Equation (3) has to be the one referring to damaging sequences, which can be obtained as the total cluster rate, λ, times the probability that a cluster is damaging, that is $\lambda_D = \lambda \cdot P[\Delta\mu_i > 0]$.

It might be worth to introduce an approximation enabling closed-form for the reliability assessment. This is given in Equation (18) where $P_f(t)$ is replaced by the probability conditional to the expected number of damaging clusters until t. Tolerability of this approximation will be discussed in the application section.

$$P_f(t) = \sum_{k_D=1}^{+\infty} P\left[\sum_{i=1}^{k_D} \Delta\mu_i \geq \bar{\mu} \middle| N_D(t) = k_D\right] \cdot \frac{(\lambda_D \cdot t)^{k_D}}{k_D!} \cdot e^{-\lambda_D \cdot t} \approx P\left[D(t) \geq \bar{\mu} \middle| N_D(t) = E\left[N_D(t)\right]\right] =$$

$$= P\left[D(t) \geq \bar{\mu} \middle| N_D(t) = \lambda_D \cdot t\right] = \frac{\Gamma_U(\lambda_D \cdot t \cdot \alpha_D, \gamma_D \cdot \bar{\mu})}{\Gamma(\lambda_D \cdot t \cdot \alpha_D)} \tag{18}$$

ILLUSTRATIVE APPLICATION

To evaluate the developed models, an ideal application is performed. To this aim a simple EPP-SDOF system with unloading/reloading stiffness always equal to initial one, is considered. The period of the SDOF system is assumed to be 0.5 s, its weight is 100 kN and the yielding force is equal to 10 kN, viscous damping is set at 5%. The following sub-sections first illustrate the calibration of the cluster damage model. Then, the results of the reliability assessment are discussed. Finally, a comparison with the case the effects of aftershocks are neglected is carried out.

Mainshock and aftershock intensity distributions

The structure was assumed to be within a generic seismogenic source zone, the size of which is $20 \times 80\ km^2$. Mainshock epicenters were assumed as uniformly distributed in the source zone discretized by means of the lattice depicted in Figure 2.

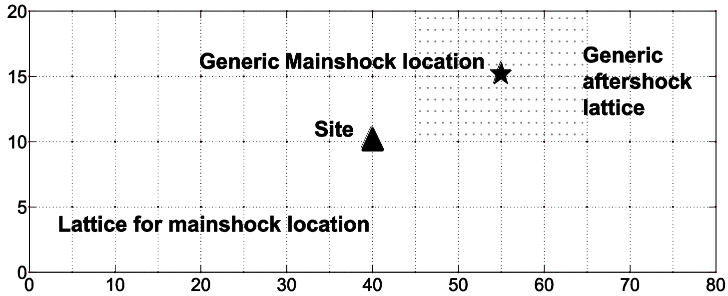

Figure 2. Seismogenic source lattice for mainshocks, generic aftershock lattice around a mainshock epicenter, and site.

The event rate of mainshocks, and then of clusters, was arbitrarily, assumed to be $\lambda = 0.013\,[events/yr]$. The magnitude distribution of mainshocks was taken as a truncated exponential defined in the $[5, 6.5]$ range. The *b*-value of the GR relationship was set to 1.056; $\{M_E, R_E\}$ were considered independent RVs. It was assumed that each mainshock has its aftershocks constrained in an area around the epicenter. The size of the aftershock seismogenic zone in squared kilometers, S_A, depends on the triggering event's magnitude according to Equation (19), from Utsu (1970). Within this area, arbitrarily assumed to be a square, epicenters are uniformly distributed on a lattice with 0.5 km spacing (Iervolino et al., 2014).

$$S_A = 10^{(m_E - 4.1)} \quad (19)$$

The length of aftershock sequences is set to 90 days after the mainshock (following Yeo and Cornell, 2009). The parameters appearing in Equation (5), were: $a = -1.66$, $b = 0.96$, $c = 0.03$, $p = 0.93$, and $m_{\min} = 4.5$; i.e., those of generic aftershock sequences in Italy according to Lolli and Gasperini (2003).

Given this set of parameters and source models, the distributions of IM in the mainshock and in the generic aftershock, given magnitude and location of the mainshock, were computed via the integrals over magnitude and distance appearing at the right hand sides of Equation (8) and Equation (10).[8] The required $f_{IM|M_E,R_E}$ and $f_{IM|M_A,R_A}$ terms for these calculations were taken considering the Ambraseys et al. (1996) GMPE, on rock sites, converting the epicentral distance, to R_{jb} distance (Joyner and Boore 1981) used by this GMPE, via a semi-empirical relationship (Gruppo di Lavoro, 2004).

Distribution of damage given IM of a single earthquake

As discussed in previous sections, the parameter chosen as a proxy for dissipating hysteretic energy in a single earthquake is the kinematic ductility computed as if the residual displacement of the structure before the earthquake was zero. Hence, the damage increment, $\Delta\mu$, in each earthquake event, may be evaluated via Equation (20).

$$\Delta\mu = \frac{\delta_{\max} - \delta_y}{\bar{\delta} - \delta_y} = \frac{\mu}{\mu_0} \quad (20)$$

In the equation $\delta_{\max}$ is the maximum absolute value of plastic displacement demand and $\bar{\delta}$ is the displacement associated to the ductility capacity; recalling that μ_0 is the initial capacity, values of $\Delta\mu$ larger than one imply failure. Moreover, as discussed, damage is zero in those shocks not able to push the structure beyond yielding, which means ground motions with 5% damped spectral acceleration at 0.5 s lower than 0.10 g.

Because the response of the considered structure in terms of hysteretic energy in a generic earthquake shock should have always the same distribution given a sufficient IM – e.g., first mode spectral acceleration at the elastic period of the SDOF, or $Sa(T)$ – and it is independent on the shaking history, then a single set of IDAs is sufficient to calibrate the damage distribution conditional to earthquake intensity, $f_{\Delta\mu|IM}$. In particular, it is sufficient to analyze the response of the as-new structure (see also Iervolino et al., 2013a-c). To this aim, IDAs have been performed using 30 records selected via REXEL (Iervolino et al., 2010), with moment magnitude between 5 and 7, epicentral distances lower than 30 km and stiff site class.[9] For $f_{\Delta\mu|IM}$ a lognormal distribution was assumed, as well-established hypothesis in the PBEE context.

Damage in the mainshock, in the aftershocks, and in the cluster

According to Equation (9) and Equation (10), the PDFs of damage in the mainshocks and in a generic aftershock conditional to any value of the magnitude and distance of the mainshock are derived first. Then, $f^{(\tilde{N}_A)}_{\Delta\mu_{A,ij}|M_E,R_E}$, the PDF of the sum of damage in the aftershocks sequence conditional to a

[8] In fact, they are hazard integrals where the rate is not considered as these PDF are *given* the occurrence of the event.
[9] The same records and analyses have been used to calculate the response of the structure to aftershocks.

$\{M_E, R_E\}$ mainshock, when the expected number $(\tilde{N}_A)$ of aftershocks in ΔT_A occurs is obtained; Equation (14). In fact, it is convenient here to refer to the process counting the number damaging aftershocks, the only contributing to damage accumulation. Because of the properties of Poisson processes, the rate of damaging aftershocks is simply that in Equation (5) times the probability that an aftershock is damaging, Equation (21), where $P^0_{A,ij|M_E,R_E}$ is the probability that the generic aftershock is undamaging. The integer approximation of the expected number of damaging aftershocks is termed $\tilde{N}_{A,D}$.

$$\lambda_{A,D|M_E,R_E} = \lambda_{A|M_E}(t)\cdot P\left[\Delta\mu_{A,ij|M_E,R_E} > 0\right] = \lambda_{A|M_E}(t)\cdot\left(1 - P^0_{A,ij|M_E,R_E}\right) \tag{21}$$

Because, given $\{M_E, R_E\}$, damage in different aftershocks are i.i.d., $f^{(\tilde{N}_{A,D})}_{\Delta\mu_{A,ij}|M_E,R_E}$ is just the convolution of $f_{\Delta\mu_{A,ij}|M_E,R_E}$ with itself of order $\tilde{N}_{A,D}$. Finally, $P[\Delta\mu_i > \delta\mu]$, the distribution of damage in the generic cluster, is obtained as per Equation (15). It is compared in Figure 3a with the distribution obtained when the contribution of aftershocks is neglected, that is with the results of Equation (8) in terms of complementary cumulative distribution function (CCDF). Changes in probability, in the case the aftershock sequences are accounted for, are depicted in Figure 3b. Note that the distribution of damage in the cluster is characterized by a probability mass in zero, as not all clusters are damaging, this accounts for the chance that the mainshock and all aftershocks are undamaging. For the considered example, the probability that the cluster is undamaging is $P_i^0 = 1 - P[\Delta\mu_i > 0] = 0.62$. For comparison it may be worth to report also about the probability that the mainshock only is undamaging, which is $P^0_{E,i} = 0.65$, marginally with respect to $\{M_E, R_E\}$.

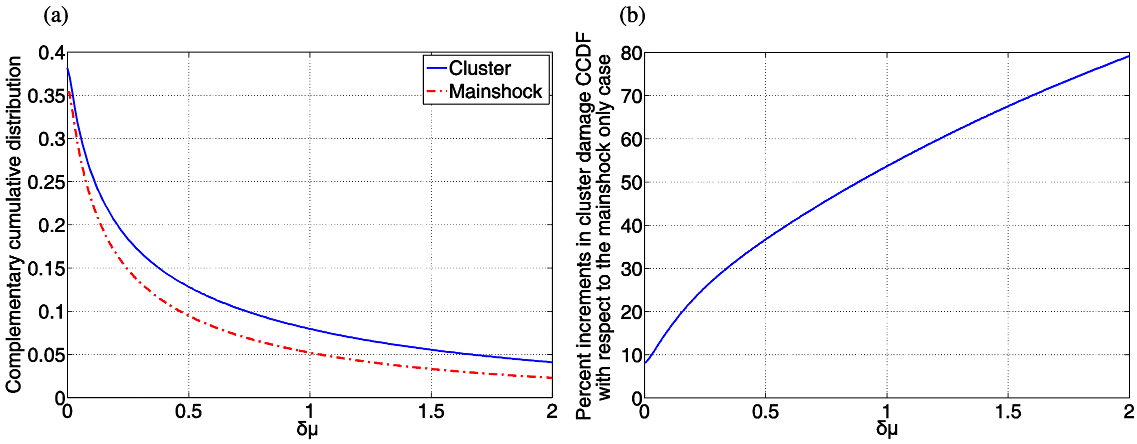

Figure 3. CCDFs of damage increment in the cluster and in the mainshock only (a); percent probability increments if the aftershock sequence effect is not neglected (b).

Results

The distribution of damage in the cluster, $P[\Delta\mu_i > \delta\mu]$, given that the cluster is damaging, was then represented by means of a gamma distribution. This continuous RV has been adopted to approximate the damage increment larger than zero (whose area is normalized to one). The criterion to calibrate the parameters of this distribution was to set its mean and variance the same as that of the damage conditional to the occurrence of a damaging cluster. The mean and variance are equal to 0.77 and 2.18, respectively; the corresponding parameters are $\gamma_D = 0.3556$ and $\alpha_D = 0.2762$.

At this point it is possible to compute the CDF of the lifetime of the structure, $F_T(t)$. In fact, Figure 4a shows such distribution computed in different cases: (i) according to Equation (18) that is when the expected number of damaging clusters is considered and the damage in the cluster is modelled via a Gamma RV; (ii) when the approximation of the expected number of damaging clusters relaxed, that is reliability is computed by means of Equation (3), yet still approximating the damage in the cluster via a Gamma RV; (iii) directly using the distributions of damage in Figure 3 for both the cluster and the mainshock (neglecting aftershocks) obtained by means of structural simulation, that is without using the gamma distribution to represent damage in the cluster; also in this case the approximation of the number of occurring damaging clusters via its expected values is removed.

The cluster simulation curve is a reference case; therefore, Figure 4b reports on the ratios of the failure probabilities reported in Figure 4a as a function of time and with respect to this case. Obtained results show that considering the mainshock only leads to an appreciable un-conservative estimate of failure probability.

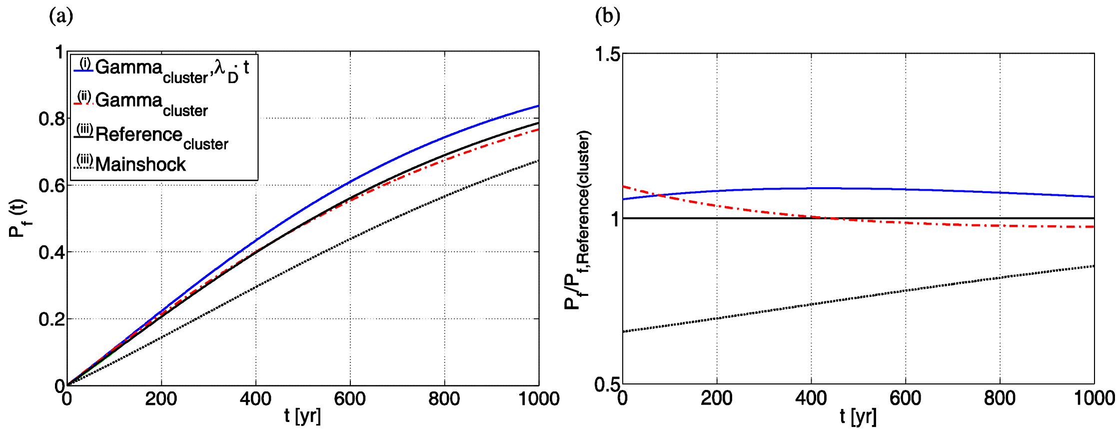

Figure 4. Lifetime distributions accounting for the cluster effect with different degrees of approximation along with that when only mainshocks are considered (a); ratio of failure probabilities from curves in the left panel to the reference case (b).

CONCLUSIONS

Starting from: classical stochastic modeling of mainshock occurrence, conditional process modeling of aftershock sequences, and a probabilistic structural damage accumulation model, life-cycle reliability of constructions subject to seismic clusters was addressed. The developed model, consistent with the classical framework of performance-based earthquake engineering, assumes that the occurrence of seismic clusters may be described by the same homogeneous Poisson process characterizing mainshock occurrence, while aftershocks' occurrence follows a non-homogeneous Poisson process based on the modified Omori-law, and therefore it is conditional on mainshock magnitude. The structural damage model postulated leads damage increments in different mainshocks to be independent and identically distributed; damage increments in aftershocks pertaining to a specific mainshock are also independent and identically distributed random variables, given the mainshock's features. This allowed to formulate the distribution of damage in a generic cluster, which is also i.i.d. with respect to other clusters. These characteristics of the cluster-damage distribution enable to formulate the non-negative damage accumulation process, which in turn, under the additional hypothesis that damage is a gamma RV, allowed closed-form solution, even if approximate, for the life-cycle reliability assessment. An elastic perfectly plastic single-degree-of-freedom system located in a generic seismogenic areal source was considered: (i) to appreciate the effect of changes in reliability assessment when aftershock contribution is neglected, and (ii) to evaluate the tolerability of the approximated closed-form. Therefore, the distribution of damage in the single cluster was computed and fitted by the mentioned reproductive model calibrated to retain mean and variance of damage computed via structural analysis. Results show that, at least in the examined case, the contribution of aftershocks to the life-cycle assessment of earthquake-resistant structures may be non-negligible, yet the problem may be addressed via stochastic modeling consistent with PBEE, which may lead to convenient closed-form approximations.

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REFERENCES

Ambraseys NN, Simpson KA, Bommer JJ (1996) "Prediction of horizontal response spectra in Europe", *Earthquake Eng. Struct. Dyn.* 25: 371-400.

Boyd OS (2012) "Including foreshocks and aftershocks in time-independent probabilistic seismic hazard Analyses", *B. Seism. Soc. Am.* 102(3): 909-917.

Cornell CA, Krawinkler H (2000) "Progress and challenges in seismic performance assessment", Peer Center Newletter 3(2).

Cosenza E, Manfredi G (2000) "Damage indices and damage measures", *Prog. Struct. Eng. Mat.* 2(1): 50-59.

Giorgio M, Guida M, Pulcini G. (2010) "A state-dependent wear model with an application to marine engine cylinder liners", *Technometrics* 52(2): 172-187.

Gruppo di Lavoro (2004) Redazione della Mappa di Pericolosità Simica Prevista dall'Ordinanza PCM 3274 del 20 Marzo 2003, Rapporto conclusivo per il Dipartimento della Protezione Civile. Istituto Nazionale di Geofisica e Vulcanologia, Rome, Italy. (In Italian)

Gutenberg R, Richter CF (1944) "Frequency of earthquakes in California", *B. Seism. Soc. Am.* 34(4): 185-188.

Iervolino I, Galasso C, Cosenza E (2010) "REXEL: computer aided record selection for code-based seismic structural analysis", *B. Earthq. Eng.* 8(2): 339-362.

Iervolino I, Giorgio M, Chioccarelli E (2013a) "Gamma degradation models for earthquake resistant structures", *Struct. Saf.* 45: 48–58.

Iervolino I, Giorgio M, Chioccarelli E (2013b) "Closed-form aftershock reliability of damage-cumulating elastic-perfectly-plastics- ystems", *Earthquake Eng. Struct. Dyn.* http://dx.doi.org/10.1002/eqe.2363 (in press)

Iervolino I, Giorgio M, Chioccarelli E (2013c) "Gamma modelling of aftershock reliability of elastic-perfectly-plastic systems", *Proc. of Vienna Congress on Recent Advances in Earthquake Engineering and Structural Dynamics (VEESD)* – Vienna, Austria.

Iervolino I, Giorgio M, Polidoro B (2013d) "Probabilistic seismic hazard analysis for seismic sequences", *Proc. of Vienna Congress on Recent Advances in Earthquake Engineering and Structural Dynamics (VEESD) –* Vienna, Austria.

Iervolino I, Giorgio M, Polidoro B (2014) "Sequence-based probabilistic seismic hazard analysis" *B. Seism. Soc. Am. doi: 10.1785/0120130207 (in press)*

Joyner WB, Boore D.M. (1981) "Peak horizontal acceleration and velocity from strong motion records including records from the 1979 Imperial Valley, California, earthquake", *B. Seism. Soc. Am.* 71(6): 2011-38.

Lolli B, Gasperini P (2003) "Aftershocks hazard in Italy Part I: Estimation of time-magnitude distribution model parameters and computation of probabilities of occurrence", *J. Seismol.* 7(2): 235-257.

Luco N, Cornell CA (2007) "Structure-specific scalar intensity measures for near-source and ordinary earthquake ground motions", *Earthq. Spectra* 23(2): 357-392.

Luco N, Gerstenberger M, Uma S, Ryu H, Liel A, Raghunandan M (2011) "A methodology for post-mainshock probabilistic assessment of building collapse risk", Proc. of the 9th Pacific Conference on Earthquake Engineering, Auckland, New Zealand.

Oehlert GW (1992) "A note on the delta method", *The American Statistician* 46(1): 27-29.

Utsu T (1961) "A statistical study on the occurrence of aftershocks", *Geophys. Mag.* 30: 521–605.

Utsu T (1970) "Aftershocks and earthquake statistics (1): Some parameters which characterize an aftershock sequence and their interrelations", *Journal of the Faculty of Science*, Hokkaido University, Series 7, Geophysics 3(3): 129-195.

Vamvatsikos D, Cornell CA. (2002) "Incremental dynamic analysis", *Earthquake Eng. Struct. Dyn.* 31(3): 491-514.

Yeo GL, Cornell CA (2005) "Stochastic characterization and decision bases under time-dependent aftershock risk in performance-based earthquake engineering", PEER Report 2005/13, Pacific Earthquake Engineering Research Center, Berkeley, CA, USA.

Yeo GL, Cornell CA (2009) "A probabilistic framework for quantification of aftershock ground-motion hazard in California: Methodology and parametric study", *Earthquake Eng. Struct. Dyn.* 38: 45-60.