SECOND EUROPEAN CONFERENCE ON EARTHQUAKE ENGINEERING AND SEISMOLOGY, ISTANBUL AUG. 25-29, 2014

ROTATION-INVARIANT FORMULATION OF STRONG GROUND-MOTION PARAMETERS

Rajesh RUPAKHETY[1] and Ragnar SIGBJÖRNSSON[2]

ABSTRACT

Ground motion components along orthogonal axes are dependent on the orientation of the axes. In a three-dimensional space, the motion recorded by a tri-axial instrument represents only three samples from an ensemble of infinite number of time series possible at the site of measurement. This means that ground-motion parameters derived from the as-recorded motion may not be necessarily representative of a range of values possible at the site. The arbitrariness of instrument orientation then implies that ground-motion parameters derived from as-recorded motion may not be suitable for engineering applications. This contribution presents the conceptual and mathematical formulation of ground-motion measures that are independent of sensor orientation. The proposed measures are conceptually straightforward. The recorded motion components are treated as realizations of a sequence (in time) of random variables. Alternatively, they can be thought of as random variables (at a given time) with the randomness governed by the orientation of the sensor. A fractile (for example, the 50 percentile) level of the sequence (in time) of random variables can then be considered as a statistical representative of ground motion. Such a fractile measure, by definition, is invariant to sensor orientation. In this work, 50 percentile values are considered as statistically representative ground motion on the horizontal plane. These concepts are mathematically formulated to obtain rotation-invariant ground-motion parameters from the as-recorded motion. Various ground-motion parameters are considered, such as, peak amplitude values, power spectral densities, response spectral amplitudes, strong-motion duration, and different intensity measures derived from elastic response spectra. The formulation is described in detail with both graphical and physical interpretation of the proposed rotation-invariant measures. A large set of ground-motion data from Europe and the Middle East is used to verify the applicability of the proposed measures.

INTRODUCTION

Strong ground motion parameters are commonly estimated from empirically calibrated regression equations. Regression equations, as known as Ground Motion Prediction Equations (GMPEs), are calibrated and used to predict a variety of ground motion parameters. These parameters include peak ground acceleration (PGA), peak ground velocity (PGV), peak ground displacement (PGD), elastic response spectra, inelastic response spectra, duration, Arias intensity, etc. A detailed review of different GMPEs can be found in Douglas (2003). Most commonly, GMPEs are calibrated for the horizontal ground shaking records. A ground motion record consists of ground acceleration time series in three orthogonal directions, two of which are on the horizontal plane. Each record of horizontal ground acceleration data thus consists of two time series at a recording station. Ground motion parameters derived from these two time series are, in general, significantly different. On the other

[1] Associate Professor, Earthquake Engineering Research Center, University of Iceland, rajesh@hi.is
[2] Professor, Earthquake Engineering Research Center, University of Iceland, ragnar.sigbjornsson@hi.is

hand, the independent variables (such as earthquake magnitude, source-site distance, etc.) commonly used in GMPEs are the same for the time series of a record. It appears then that the direction of the instrument sensor has to be considered as an additional independent regression variable. However, sensor orientation in installed accelerographs has limited engineering significance as such since the orientation is arbitrary on the horizontal plane relative to the seismic source. It is therefore neither practical nor meaningful in the engineering sense to use sensor orientation as a predictor variable. To account for the difference in recorded ground motion in two orthogonal horizontal directions, ground motion parameters in these two directions are combined into a single measure which is then used as the dependent variable in regression analysis. This is achieved in different ways. For example, the larger value of the two horizontal components is considered. Review of other combination methods can be found in Douglas (2003). The most common approach seems to be in taking the geometric mean of ground motion parameters in the two horizontal directions. In this approach, the standard deviation of predicted motion is corrected to approximate a randomly chosen direction of ground motion (Boore, 2005). The use of geometric mean of the as recorded horizontal components is promoted, apparently, by the observation that this measure results in smaller aleatory uncertainty of the predicted motion. This statistical observation aside, it is obvious that the geometric mean of the two horizontal components is not invariant to the sensor orientation. When the ground motion is linearly polarized in the direction of one of the horizontal axes of the sensor, and in absence of noise, the motion in the orthogonal sensor is zero, and so is the geometric mean. Although such an extreme condition is not to be expected in reality, it helps to conceptualize the fact that the geometric mean is not a reliable engineering measure when the motion is highly anisotropic or polarized. In other developments, Boore et al. (2006) considered a certain fractile of geometric means of two orthogonal components obtained by rotating, in the horizontal plane, the as-recorded vector time series by all non-redundant angles. Consideration of the fractile motion makes this measure of Boore et al. (2006) independent of the sensor orientation. It is also true that when a vector is rotated in the plane, the components of the vector replace each other, periodically, for rotation angles in the multiple of $\pi / 2$. When rotation in a full circle is considered, the 'statistical' distribution of one of the orthogonal components of the vector is exactly the same as that of the other. This renders the method of Boore et al. (2006) in considering the fractile of geometric means of the rotated components unnecessary. Boore (2010) recognized this and proposed to use a certain fractile of ground motion parameter obtained from the distribution of the parameter obtained from one of the components of motion corresponding to different orientations of the sensor. In this contribution, we present a measure of rotation-invariant ground motion parameter which is comparable to the measure proposed by Boore (2010), but is easier to compute and have a clearer physical meaning. Boore (2010) focussed on elastic response spectra, while we provide a consistent framework for rotation-invariant measure for the most commonly used ground motion parameters.

The proposed measure is conceptually straightforward. Owing to the dependence of recorded motion on the orientation of the sensor, which is arbitrary at least in the horizontal plane, the as-recorded motion represents only three samples from an ensemble of infinite number of time series possible at the site. These samples are treated as realizations of a sequence (in time) of random variables. Alternatively, they can be thought of as random variables (at a given time) with the randomness being governed by the orientation of the sensor. A fractile (for example, the 50 percentile) of the sequence (in time) of random variables can then be considered as a statistical representative of ground motion. Such a fractile measure, by definition, is invariant to sensor orientation. The basic formulation of such rotation-invariant measures for response spectral ordinates and strong-motion duration is described in detail in Rupakhety and Sigbjörnsson (2013a; and b). This contribution extends this formulation to other commonly used measures of ground motion. For illustration of the methodology and verification of results, several recorded ground acceleration data from Europe and the Middle East are used. These data were obtained from the Internet Site for European Strong-Motion Data (ISESD, Ambraseys et al., 2004).

PEAK GROUND MOTION

Peak ground motion, such as peak ground acceleration (PGA), peak ground velocity (PGV), and peak ground displacement (PGD), are commonly used to describe seismic effects on structures. These parameters, collectively referred to here as PGX, of as-recorded components of motion are dependent on sensor orientation. Let $a_x(t)$ and $a_y(t)$ denote the as-recorded acceleration in the two horizontal directions. By applying a counter-clockwise rotation of an angle θ to the sensor axes in the horizontal plane, one of the rotated components of motion is obtained as

$$a(\theta,t)=\sqrt{a_x^2(t)+a_y^2(t)}\cos(\theta+\phi)=\rho(t)\cos(\theta+\phi) \tag{1}$$

where ϕ is the phase lag between the recorded motions and $\rho(t)$ is the resultant horizontal acceleration which is invariant to coordinate transformation. The largest ground acceleration for all sensor orientations can then be obtained as

$$a_{\max}(t)=\max_{\theta} a(\theta,t)=\rho(t) \tag{2}$$

which is the resultant horizontal acceleration. The largest PGA is then defined as

$$PGA_{\max}=\max_{t}\rho(t) \tag{3}$$

which is simply the peak value of resultant horizontal acceleration. This measure of PGA is invariant to sensor orientation, but might be too conservative considering the small probability of having a structural orientation exactly in the direction where this acceleration is observed. A more representative engineering measure is the statistical expectation of PGA for all possible sensor orientations. Since the maximization operator is nonlinear, an exact expression for this estimate is not available. However, an approximate estimate which is close to the statistical expectation of PGA for all sensor orientation can be obtained. Such a measure can be formulated projecting the as-recorded motion in a new coordinate system. The new coordinate system is defined by a linear orthogonal transformation that diagonalizes the cross correlation matrix of the recorded time series. Such a coordinate system defines the principal directions of motion. Principal directions consist of a major axis along which the variance of motion is the maximum and an orthogonal minor axis along which the variance is the minimum. The components of motion along these directions are called as the major and minor components, respectively. Let $a_1(t)$ and $a_2(t)$ represent the major and minor components of motion. The corresponding peak values are denoted as PGA_1 and PGA_2. Along an axis oriented by an angle θ counter-clockwise from the major axis, the motion is described by

$$a(\theta,t)=a_1(t)\cos\theta+a_2(t)\sin\theta \tag{4}$$

If σ_1^2 and σ_2^2 are the variances of the principal components, and because the principal components are uncorrelated, the variance of $a(\theta,t)$ is $\sigma_\theta^2=\sigma_1^2\cos^2\theta+\sigma_2^2\sin^2\theta$. The peak ground accelerations in the three directions are related to their corresponding variances by peak factors (see, for example, Vanmarcke, 1975). The peak factors are, in general, expected to vary with θ. However, the formulation can be considerably simplified by assuming a peak factor which is independent of θ. It will be shown subsequently that this simplification is, for practical purposes, justifiable. Denoting peak factor by η, the following equality can be presented.

$$PGA_\theta=\max_{t} a_\theta(\theta,t)=\eta\sigma_\theta=\sqrt{\eta^2\sigma_1^2\cos^2\theta+\eta^2\sigma_2^2\sin^2\theta}=\sqrt{PGA_1^2\cos^2\theta+PGA_2^2\sin^2\theta} \tag{5}$$

By assuming that θ is uniformly distributed in the interval 0 to π , the statistical expectation of peak ground acceleration can be obtained as

$$PGA_m=\sqrt{\frac{PGA_1^2+PGA_2^2}{2}} \tag{6}$$

An alternative, but lengthy, method to obtain the statistical expectation is to rotate the as-recorded data through numerous angles obtaining a distribution of PGA from which the mean value can be obtained numerically. Such an estimate is denoted as $\overline{PGA}$ here. The proposed estimate of Eq. 6 is much easier to compute and is, for practical purposes, approximately equal to $\overline{PGA}$.

An example comparing the two measures is presented first in Figure.1. Horizontal ground acceleration obtained at the Kaldarhlot station during the 17 June 2000 Mw 6.57 South Iceland Earthquake is used in the illustration. The variation of PGA at this station with the orientation of sensor is shown in Figure.1 with the blue trace. It can be seen that for different orientations of the sensor, PGA varies significantly in the range about 5-8 m/s^2. The mean PGA corresponding to the proposed measure is shown in green while the actual mean is shown in red. The proposed measure is very close to the actual mean. The small difference between the two measures is due to the variation of peak factor with sensor orientation. To study if such differences may be significant, we compared PGA_m to $\overline{PGA}$ for 462 ground motion records obtained from the ISESD. It can be seen that the proposed measure matches the statistical average well. The average ratio between the two is 0.99, indicating a small bias. The standard deviation of the ratio is 0.05, which is small for all practical purpose. It can thus be concluded that the proposed measure of PGA is a good representative of the statistical average of PGA corresponding to all possible orientations of sensors on the horizontal plane.

In the isotropic case, i.e., when the peak ground accelerations along the major and the minor axes are identical, and according to Eq. 6, the proposed estimate is equal to the PGA along the major or the minor direction. In other situations, the proposed measure can be interpreted as the resultant of PGA in the major and minor directions, with the factor of $\sqrt{2}$ in the denominator of Eq. (6) acting as a partitioning factor for unidirectional motion, which might be relevant in some simplified analysis such as in analysis of structures considering seismic excitation in one direction only. The proposed measure is thus not only simple to estimate but also has a sound physical interpretation. It is clear that similar measures can also be defined for PGV and PGD.

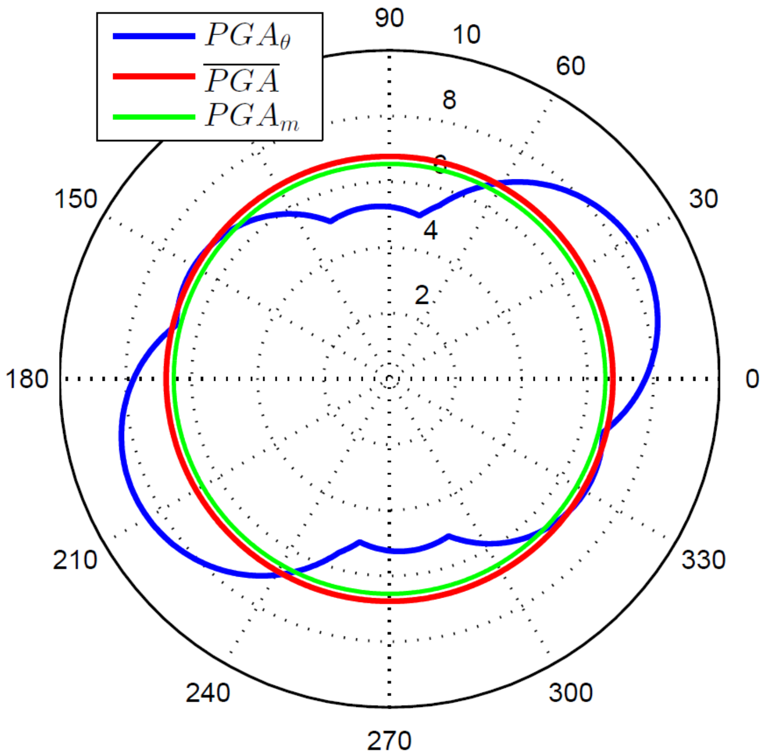

Figure 1. Variation of PGA with sensor orientation. The PGA values and rotation angles are shown in a polar representation.

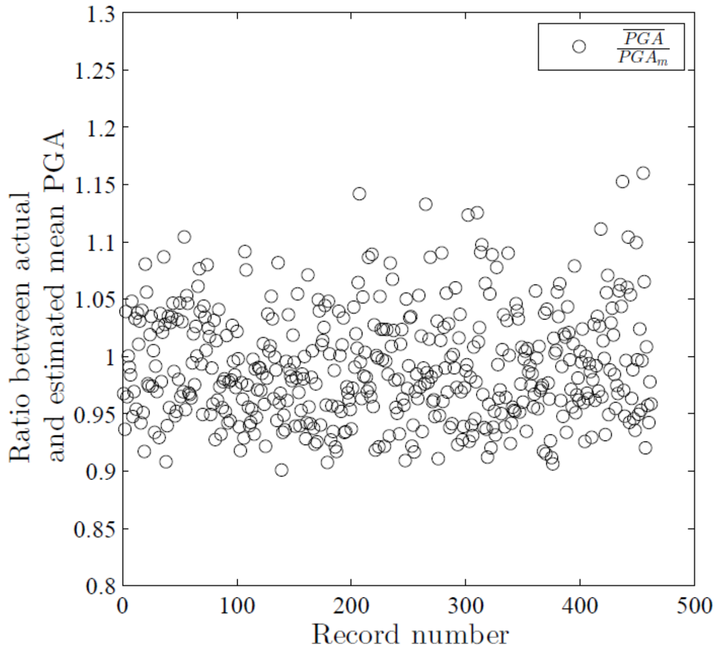

Figure 2. Comparison between the proposed measure of rotation-invariant mean PGA and the actual statistical average PGA for all possible sensor orientations.

ARIAS INTENSITY

Arias intensity (Arias, 1970) is an important ground motion parameter. It incorporates the amplitude, frequency content, and duration characteristics of ground motion into a single measure. It has been found to be well correlated to the damage of stiff structures (see, Travasarou et al., 2003). This parameter has also been found to be useful in geotechnical applications, including liquefaction potential (see, for example, Kramer and Mitchell, 2006). Arias Intensity tensor is defined as

$$I_{ij} = \frac{\pi}{2g}\int_0^T a_i(t)a_j(t)dt \tag{7}$$

where $a_i(t)$ represents the i^{th} component of acceleration (i, j = 1,2,3), and T is the duration of motion. The second order tensor is symmetric by definition and its first invariant, using the summnation convention is denoted as I_{ii}, which is invariant to coordinate transformation. Considering only the two horizontal components of motion, the first invariant is given by

$$I_I = I_{xx} + I_{yy} \tag{8}$$

It is then obvious from equations 7 and 8 that the first invariant is the arias intensity of the resultant motion, which will, henceforth be denoted as I_R. The first invariant of this tensor is a rotation-invariant measure of Arias Intensity of the recorded motion representing total energy in the motion contained in both of its components. For structures with circular symmetry, this measure can be used as representative ground motion intensity applied in any single direction. However, if intensity of motion in only one direction is needed, for example, in unidirectional analysis of structures or soil systems, this measure is not appropriate. In such situations, mean Arias Intensity in one of the components of motion for all orientations of sensor need to be applied. Consider the motion at an angle of θ from the x axis. The Arias Intensity in this direction is

$$I_\theta = \frac{\pi}{2g}\int_0^T \left(a_x(t)\cos\theta + a_y(t)\sin\theta\right)^2 dt \tag{9}$$

which can be simplified as

$$I_\theta = I_{xx}\cos^2\theta + I_{yy}\sin^2\theta + 2I_{xx}I_{yy}\cos\theta\sin\theta \tag{10}$$

Considering θ to be a random variable uniformly distributed in the interval 0 to 2π, the expected value of I_θ can be obtained as

$$I_m = E[I_\theta] = \frac{I_{xx} + I_{yy}}{2} = \frac{I_r}{2} \tag{11}$$

The mean Arias Intensity is thus found to be half the Arias Intensity of the resultant horizontal motion. This implies that the total energy is equally divided among the two orthogonal components in the average sense. When three dimensional motion is considered, the mean Arias Intensity in one direction can be obtained similarly, and is given by

$$I_m = \frac{I_r}{3} \tag{13}$$

where I_r now is the Arias Intensity of the resultant of three orthogonal components of motion. The formulation presented here for Arias Intensity is exact, and involve no approximations. It is, therefore, not necessary to verify it with data. An example is, nevertheless, presented for illustrating the main idea. In Figure 3, the variation of Arias Intensity with the direction of sensor is presented in a polar representation. The blue and the red traces represent the Arias Intensity of two orthogonal horizontal components. It is evident that one of these traces are identical expect for a rotation of 90°. The green trace is the Arias Intensity of resultant horizontal acceleration.

The invariant nature of this intensity is evident from the circular shape of the trace. This trace also represents the first invariant of the two-dimensional Arias Intensity tensor, and is the sum of the red and the blue traces. The mean Arias Intensity for all sensor orientation is represented by the dashed black circle. The mean level is equal to half of the Arias Intensity of the resultant motion (green circle). The red, blue, and the black traces intersect at common points, which indicate that the total energy of motion is equally divided into two components in the average sense.

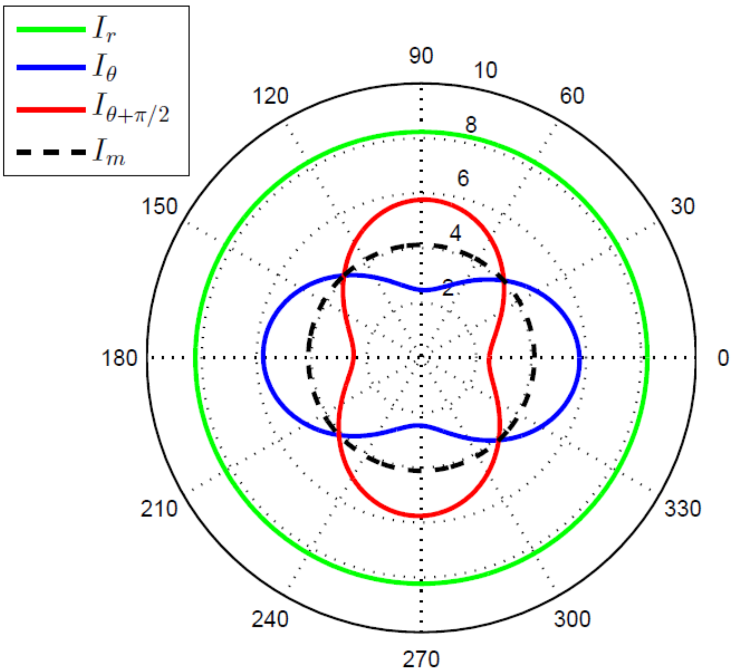

Figure 3. Variation of Arias Intensity with orientation of sensor. The intensity values are shown in a polar representation in units of m/s. The blue and the red traces are the Arias Intensities of two orthogonal components of motion recorded at the Kaldarholt station during the 17 June 2000 South Iceland Earthquake. The green circle is the Arias Intensity of the resultant horizontal motion. The dashed black line is the mean intensity for all orientations of the sensor, which can be obtained as the mean of either the blue or the red trace.

POWER SPECTRAL DENSITY

Power spectrum or power spectral density function (PSDF) is an important ground motion parameter. It describes the frequency content of ground motion. It is also used to model earthquake ground motion as a stationary stochastic process. This has direct application in simulating artificial ground motion using the spectral representation method (see, for example, Shinozuka and Deodatis, 1988). PSDF is also used in stochastic response analysis of structures (see, for example, Yang, 1986). It is also closely related to the Fourier Amplitude Spectra (FAS) of ground motion, which are often used to calibrate parameters of theoretical source spectra (for example, Brune, 1970).

Power spectral density function (PSDF) is defined as the Fourier transform of autocorrelation function (ACF) of ground motion modelled as a stationary process. Unbiased and consistent estimates of PSDF can be obtained when an ensemble of ground motions is available. This requires a set of time series recorded at the same point in space due to the same earthquake, which is, for obvious reasons, not available. In lack of ensemble averages, ergodic assumptions are invoked, and ACF is estimated from a single time series by averaging over time. A common method in this regard is the use of so-called periodogram estimate. If $A(f)$ is the Fourier transform of a component of recorded ground motion, $a(t)$, the periodogram estimate is given as

$$S(f) = \frac{T}{2\pi}\left|A(f)\right|^2 \tag{14}$$

where T is the duration of ground motion and f represents frequency in Hz. The periodogram is an inconsistent estimate of PSDF which is associated with large variance. To reduce the variance, smoothing windows are commonly applied in frequency domain. This may, however, introduce a bias error, especially for narrow-banded spectral densities. The smoothed periodogram is a consistent estimate of the PSDF, denoted here as $\bar{S}_{jj}(f)$. In formulating a rotation-invariant measure of PSDF, we first derive a rotation-invariant measure of the periodogram, which can then be smoothed using an appropriate procedure to obtain consistent PSDF.

The Fourier transforms of $a_x(t)$ and $a_y(t)$ are denoted in complex notation as $A_x(f) = R_x(f) + iC_x(f)$ and $A_y(f) = R_y(f) + iC_y(f)$. The corresponding periodograms are denoted as $S_{xx}(f)$ and $S_{yy}(f)$. The Fourier transform of ground motion at an angle θ from the x direction is

$$A_\theta(f) = R_\theta(f) + iC_\theta(f) \tag{15}$$

where the real and complex parts of the transform are given by

$$\begin{aligned} R_\theta(f) &= R_x(f)\cos\theta + R_y(f)\sin\theta \\ C_\theta(f) &= C_x(f)\cos\theta + C_y(f)\sin\theta \end{aligned} \tag{16}$$

The periodogram at this direction is obtained as

$$S_{\theta\theta}(f) = \frac{T}{2\pi}\left|A_\theta(f)\right|^2 = S_{xx}(f)\cos^2\theta + S_{yy}(f)\sin^2\theta + 2\cos\theta\sin\theta\left[R_x(f)R_y(f) + C_x(f)C_y(f)\right] \tag{17}$$

Treating the rotation angle θ as a random variable uniformly distributed in the interval 0 to 2π, the expected value of periodogram for all directions of the measurement sensor, can be obtained as

$$S(f) = E\left[S_{\theta\theta}(f)\right] = \int_2^{2\pi} \frac{S_{\theta\theta}(f)}{2\pi} d\theta = \frac{S_{xx}(f) + S_{ff}(f)}{2} \tag{18}$$

This shows that the expected periodogram for all sensor orientation is the mean periodogram of any two orthogonal components. This also indicates that the sum of the periodogram of two orthogonal components is invariant to rotation in the plane of the two components. If the spectral matrix defined by the auto and cross spectral densities of the two components of motion is thought of as a second order tensor, the expected PSDF given by Eq. 18 represents the isotropic part of the tensor. This is analogous to saying that the total power in the ground motion is, at each frequency, distributed equally in orthogonal directions, in the average sense.

An example is presented for illustrating the main idea. In Figure 4, the variation of periodogram (at a frequency of 1 Hz) with the direction of sensor is presented in a polar representation. The blue and the red traces represent the periodogram of two orthogonal horizontal components. It is evident that one of these traces are identical expect for a rotation of 90°. The green trace is the expected periodogram as given by Eq. 18. The green trace is, therefore, the average of the red and the blue traces. The red, blue, and the black traces intersect at common points, which indicate that the total power of motion is, in the average sense, equally divided into two components. It is evident, graphically from Figure.4 that the proposed rotation-invariant represents an isotropic equivalent power spectral density. It is noted that this measure is conceptually similar to rotary spectra (see, for example, Gonella, 1972).

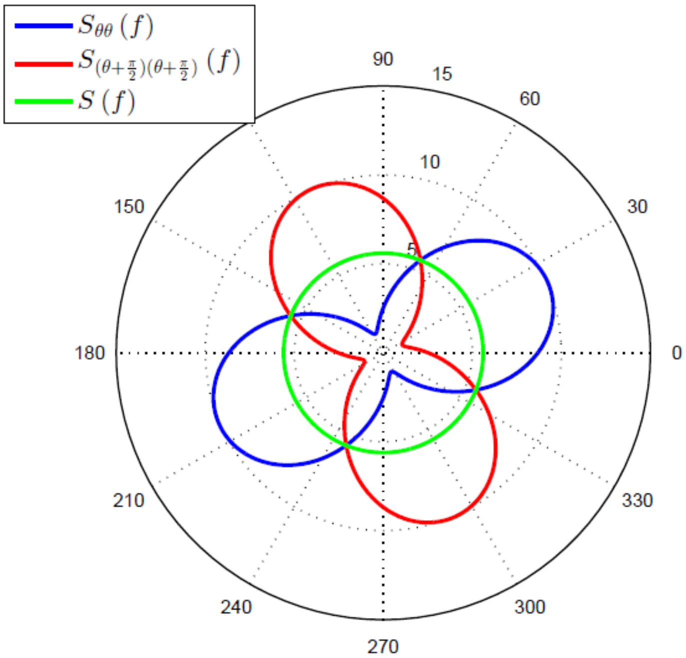

Figure 4. Variation of periodogram at 1 Hz with orientation of sensor. The intensity values are shown in a polar representation in units of m^2/s^3. Horizontal ground acceleration recorded at Kaldarholt station during the 17 June 2000 South Iceland Earthquake is used in this illustration.

ELASTIC RESPONSE SPECTRA

Rotation-invariant elastic response spectra of horizontal ground motion can be formulated along similar lines as the formulation presented here for PGA. More details of this formulation and verification of the proposed measure using a large set of data is given in Rupakhety and Sigbjörnsson (2013a). In this formulation, the displacement response histories of a single degree of freedom system (SDOF) with undamped natural period T and critical damping ratio ζ when excited by two horizontal components of motion are projected in principal directions. The peak displacements in the major and minor principal directions are denoted as $D_1(T,\zeta)$ and $D_2(T,\zeta)$, respectively. The expected value of spectral displacement for all orientations of the sensor is then obtained as (see, Rupakhety and Sigbjörnsson, 2013a)

$$D(T,\zeta)=\sqrt{\frac{D_1^2(T,\zeta)+D_2^2(T,\zeta)}{2}} \tag{19}$$

Rotation-invariant pseudo-spectral velocity (PSV) and pseudo-spectral acceleration (PSA) can then be derived from the rotation-invariant spectral displacement of Eq.19 by successive multiplication with the angular frequency of the SDOF.

DURATION

The conceptual and mathematical formulation of rotation-invariant measure of strong-motion duration is given in Rupakhety and Sigbjörnsson (2013b). It has been shown that relative significant duration obtained from the resultant horizontal motion is the expected duration for all sensor orientations. This is also clear from Equation.11 above, which formulates the expected Arias Intensity for all sensor orientations in the horizontal plane. For further details and verification of the proposed measure using a large set of recorded data, the readers are referred to Rupakhety and Sigbjörnsson (2013b).

RESPONSE SPECTRUM INTENSITY

Response spectrum intensity (SI) also known as Housner Intensity (named after Housner, 1959), is defined as the area under the pseudo-velocity spectrum between periods 0.1s and 2.5s:

$$SI(T,\zeta) = \int_{0.1}^{2.5} PSV(T,\zeta)dT \tag{20}$$

Rotation-invariant measure of this parameter can be obtained from the rotation-invariant spectral displacement (Eq. 19) and the corresponding pseudo-spectral velocity spectrum. Some approximations are involved in formulating Eq. 19, which result in minor difference between the actual mean response spectrum (for all sensor orientations) and the proposed measure. We verify using the 462 ground-motion records from the ISESD that such small discrepancies in the response spectral ordinates are, for all practical purposes, negligible in computing response spectrum intensity. The mean value of SI computed for different orientations of the sensor is denoted as $\overline{SI}(T,\zeta)$. The value of SI corresponding to the response spectra obtained from Eq. 18 is denoted as $SI_m(T,\zeta)$. In Figure 5 the ratio between the two measures is shown for 462 horizontal ground-motion records obtained from the ISESD. It is observed that on the average, the ratio between the two is 1.02 with a standard deviation of 0.02. These differences are small and negligible for all practical engineering purposes.

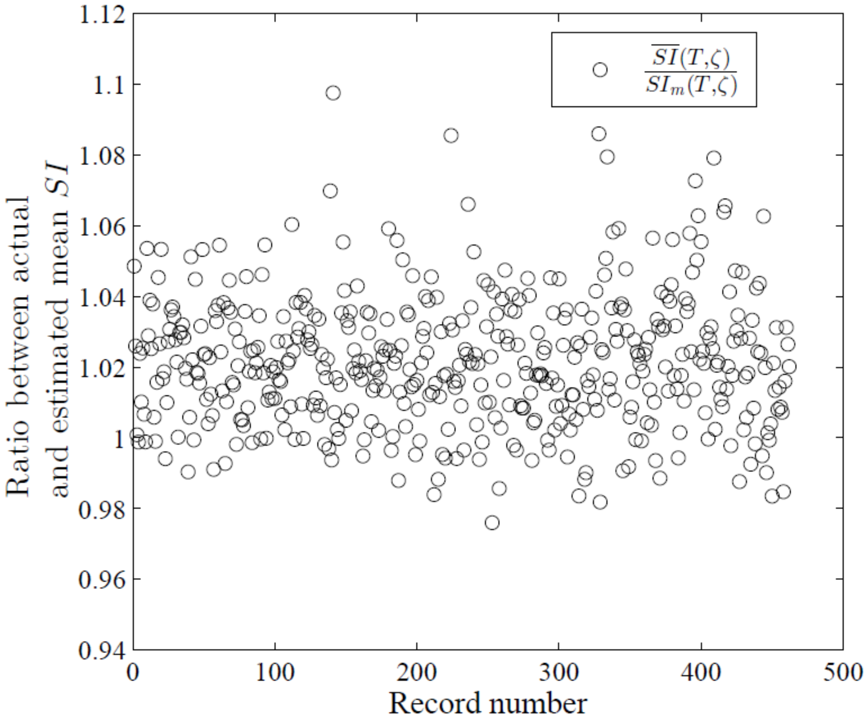

Figure 5. Comparison between the proposed measure of rotation-invariant response spectrum intensity (SI) and the actual statistical average SI for all possible sensor orientations. Damping ratio is taken as

5% of critical damping. The ratio between the two is shown for 462 horizontal ground motion records obtained from ISESD.

OTHER INTENSITY MEASURES

Other measures of ground motion intensity derived from elastic response spectra are velocity spectrum intensity and acceleration spectrum intensity. Velocity spectrum intensity, as defined by Von Thun et al. (1988) is the response spectrum intensity defined in Equation.20 at 5% of critical damping. Similarly, acceleration spectrum intensity is defined as (Von Thun et al., 1988)

$$ASI = \int_{0.1}^{0.5} \left(\frac{2\pi}{T}\right) D^2\left(T, \zeta = 0.05\right) dT \tag{21}$$

Our analysis of ground-motion records from ISESD shows that the rotation-invariant response spectral measures (see Rupakhety et al., 2000b) can be used for estimating rotational-invariant measures of velocity spectrum intensity and acceleration spectrum intensity without significant loss of accuracy.

CONCLUSIONS

Ground-motion time series recorded by a triaxial instrument describes motion along three mutually orthogonal directions. The recorded time series depend on the orientation of the measuring instrument. In most situations, one of the axes of the instrument is kept vertical, whereas the two horizontal sensors are often arbitrarily oriented. This implies that ground-motion parameters derived from the as-recorded data represent only two samples of an infinite number of possible values in the horizontal plane. The as-recorded motion is not necessarily representative of all possible motion at the site of measurement. The dependence of ground-motion parameters on the direction of measurement is significant and should be considered in engineering applications. Conceptually, the recorded motion can be rotated by various angles on the horizontal plane, generating a set of ground motion time series. Each of those time series can be used to derive a set of ground-motion parameters from which, certain representative values, such as the statistical average can be used as a rotation-invariant ground-motion parameter. In this study, rotation-invariant measures of various ground-motion parameters are presented. The present formulation avoids rotation of recorded motion, and is therefore, easier to compute. The proposed formulation is based on modelling the orientation of the sensor as a random variable with uniform probability distribution on the horizontal plane. Based on this assumption, statistical expectation of many of the most commonly used ground motion parameters are expressed in terms of the corresponding parameters of the as-recorded components. The proposed measures are shown to be simple to compute, and have clear physical interpretation. An analysis of a large number of recorded ground-motion data from the ISESD shows that the proposed measures are accurate for practical applications.

ACKNOWLEDGEMENTS

We acknowledge the support from University of Iceland research fund.

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