SEISMIC RESPONSE OF BATTER PILE FOUNDATIONS: DYNAMIC STIFFNESSES AND KINEMATIC INTERACTION FACTORS

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ABSTRACT

This paper gathers a set of dynamic stiffnesses and kinematic interaction factors for square pile groups with inclined elements embedded in a viscoelastic homogeneous half-space, allowing to glimpse the main features of the problem. Horizontal and rocking kinematic interaction factors are provided for vertically-incident shear waves. The functions are computed in the frequency domain. Results are given in ready-to-use dimensionless graphs in order to facilitate its use in soil-structure interaction studies. The influence of angle of pile inclination, number and configuration of the elements, or group kinematics, on the dynamic stiffnesses and kinematic interaction factors is discussed, and their influence on the superstructure response is explored. The increasing ability of pile foundations to filter the horizontal seismic input and to dissipate energy, as the pile inclination increases, is highlighted.

INTRODUCTION

The specification of batter piles as foundation systems in seismically active regions is generally avoided due to several related reasons: the lack of understanding regarding the seismic response of batter piles and pile foundations with inclined elements; the poor performance of such systems in past earthquakes of the late eighties and early nineties (Gerolymos et al., 2008); and, in the European case, the note in part 5 of Eurocode 8 stating that the use of inclined piles for transmitting lateral loads to the soil is not recommended when designing for earthquake resistance. Indeed, the need for more research on all facets of this problem is apparent from the lack of information available in the literature and in the different building codes. This fact, together with the pieces of evidence suggesting a beneficial role of inclined piles (Gazetas and Mylonakis, 1998; Gerolymos et al., 2008), explains why the research on the topic has boosted in the last years (Juran et al., 2001; Sadek and Shahrour, 2004, Poulos, 2006, Gerolymos et al., 2008, Giannakou, 2010; Padrón et al., 2010; Padrón et al., 2012 Escoffier, 2012; Goit and Saitoh, 2013).

Furthermore, the seismic response of the foundation affects not only the foundation itself but also the superstructure as its dynamic properties will change, together with the experienced displacements and internal efforts, if soil-structure interaction is relevant. In order to study such effects, substructuring methodologies are of great interest due to its simplicity and reduced...
computational cost (Medina et al., 2013). For the problem at hand, applying such methodologies needs of the previous definition of impedance functions and kinematic interaction factors.

This paper gathers a set of dynamic stiffnesses and kinematic interaction factors for square pile groups with inclined elements (see Figure 1) embedded in a viscoelastic homogeneous half-space, allowing to glimpse the main features of the problem at hand. Horizontal and rocking kinematic interaction factors are provided for vertically-incident shear waves.

METHODOLOGY AND PROBLEM DEFINITION

The functions are computed in the frequency domain making use of a previously developed boundary element – finite element formulation where the soil is modelled as a homogeneous, viscoelastic half-space by the Boundary Element Method (BEM) and the piles rigidity is introduced in the model by means of monodimensional Euler-Bernouilli beam three-noded elements formulated by the Finite Element Method (FEM). The interested reader can find the details of the formulation in (Padrón et al., 2011).

Two kinds of problems are solved using this numerical approach: the computation of a) dynamic stiffness functions and b) kinematic interaction factors. The main parameters of these two problems are illustrated in the left and right parts of figure 1, being \( d, L \) and \( \theta \) the pile diameter, length and rake angle; \( s \) the separation between centres of adjacent piles; \( K_{hh} \) and \( K_{rr} \) the horizontal and rocking impedance functions; and \( u_g \) and \( \varphi_g \) the horizontal displacement and rocking produced at the pile cap by the vertically-incident shear waves.

For the first problem, pile heads are subjected to harmonic forced vibration in each of the oscillation modes (horizontal and rocking). Each component of the impedance matrix \( K_{ij} \) is computed as the relationship between the vector of forces (and moments) applied at the pile top and the corresponding term of the resulting vector of displacements (and rotations) at the same point. It is assumed that the pile heads are constrained by a rigid pile-cap, and the foundation stiffness can be computed as the addition of the contributions of each pile. The impedance matrix, as a function of the frequency of the excitation \( \omega \), is usually expressed as

\[
K_{ij} = k_{ij} + i \omega c_{ij}
\]  

(1)

Figure 1. Geometric definition of the square pile groups of 4 and 9 elements with inclined elements. Dynamic stiffnesses (left) and kinematic interaction problem (right) under vertically-incident shear waves.
where \( i = \sqrt{-1} \), \( k_{ij} \) and \( c_{ij} \) are the frequency-dependent dynamic stiffness and damping coefficients, \( a_o \) is the dimensionless frequency

\[
a_o = \frac{\omega d}{c_s}
\]

and \( c_s \) is the shear wave velocity.

On the other hand, kinematic interaction factors are computed by considering a vertically-incident shear wave field defined by the resulting horizontal free-field displacement \( u_{go} \) at ground surface. The translational and rotational kinematic interaction factors are then defined as

\[
I_u = \frac{u_g}{u_{go}}
\]

and

\[
I_\rho = \frac{\varphi_g b}{u_{go}}
\]

where \( b \) is the foundation half-width defined as \( b = s \) for 2×2 pile groups and \( b = 3s/2 \) for 3×3 pile groups.

The results presented in this paper correspond to the following dimensionless parameters: pile slenderness ratio \( L/d = 15 \), soil-pile density ratio \( \rho_s/\rho_p = 0.7 \), pile-to-pile separation ratio \( s/d = 5 \), soil Poisson’s ratio \( \nu_s = 0.4 \), soil internal hysteretic damping coefficient \( \beta_s = 0.05 \), and pile-soil modulus ratios \( E_p/E_s = 10^2 \) (soft soil) and \( 10^3 \) (stiff soil). Symmetric 2×2 and 3×3 pile groups will be analyzed with the external piles inclined parallel or perpendicularly to the direction of excitation with inclination angles \( \theta = 0^\circ, 10^\circ, 20^\circ \) and \( 30^\circ \).

**RESULTS**

This section presents impedance functions and kinematic interaction factors corresponding to the cases defined above. Results are given in ready-to-use dimensionless graphs in order to facilitate its use in soil-structure interaction studies. Figures 2 to 5 correspond to 2×2 pile groups, while figures 6 to 9 correspond to 3×3 pile groups. In each group of figures, the first two (one for \( E_p/E_s = 10^2 \) and the next one for \( E_p/E_s = 10^3 \)) present results for piles inclined in the direction of excitation, and the other two for piles inclined perpendicularly to the such direction. All plots are presented as a function of the dimensionless frequency. Every figure provides horizontal and rotational stiffness and damping coefficients followed by real part and magnitude of translational and rotational kinematic interaction factors.

Focusing firstly on the case where piles are inclined parallel to the direction of excitation, the following aspects are worthy of comment. As expected, the horizontal stiffness increases significantly. This is the case not only at low but also at mid and high frequencies. The group effect associated to this horizontal vibration is not significantly modified when inclining the piles, although the peak moves slightly to lower frequencies. Horizontal damping coefficients increase also with rake angle (nearly doubling for \( \theta = 30^\circ \) in some cases). This happens mainly for \( a_o < 0.6 \), where the magnitude of the increase is much more significant than that of the stiffness coefficients. The associated translational kinematic interaction factors show that batter piles allow the foundation to filter part of the seismic input motion even at very low frequencies. Such filtering can reach 20% for \( \theta = 20^\circ \) and \( E_p/E_s = 10^3 \), and more than 30% for \( \theta = 20^\circ \) and \( E_p/E_s = 10^2 \). On the contrary, pile inclination is not beneficial from the point of view of input filtering for dimensionless frequencies above 0.5. The rotational response is quite different. Due to the decrease in vertical stiffness of individual inclined piles with respect to the vertical case, rocking stiffness coefficients of pile groups decrease with rake angle for mid-to-low frequencies although the tendency tends to be the opposite for mid-to-high
frequencies in the 2×2 case, when pile group effects tend to be magnified (note that rocking stiffness of single piles is, on the contrary, independent of inclination angle). In the 3×3 case, on the contrary, the group effect tends to be less apparent when inclining the piles. As in the horizontal case, damping increases with rake angle for low-to-mid frequencies. The associated rotational kinematic interaction factors show a consistent and significant increase in the rotational input motion to the superstructure for increasing rake angles. This increase is really significant when comparing rake angles $\theta \geq 20^\circ$ with the vertical case, in which the magnitude of $I_\phi$ can be multiplied by a factor of 30 (low frequency of 2×2 and $E_p/E_s = 10^2$ case).

Figure 2. Horizontal and rocking stiffness and damping coefficients together with translational and rotational kinematic interaction factors (for vertically incident $S$ waves) of 2×2 pile group with inclined elements parallel to the direction of excitation. $E_p/E_s = 100$, s/d=5.
Figure 3. Horizontal and rocking stiffness and damping coefficients together with translational and rotational kinematic interaction factors (for vertically incident S waves) of 2×2 pile group with inclined elements parallel to the direction of excitation. $E_p/E_s = 1000$, $s/d = 5$. 
Figure 4. Horizontal and rocking stiffness and damping coefficients together with translational and rotational kinematic interaction factors (for vertically incident S waves) of 2×2 pile group with inclined elements perpendicular to the direction of excitation. $E_p/E_s = 100$, $s/d=5$. 
Figure 5. Horizontal and rocking stiffness and damping coefficients together with translational and rotational kinematic interaction factors (for vertically incident S waves) of 2×2 pile group with inclined elements perpendicular to the direction of excitation. $E_p/E_s = 1000, s/d = 5$. 
The response of the foundations changes when piles are inclined perpendicularly to the direction of excitation (figures 4, 5, 8 and 9). Horizontal static stiffnesses are not influenced significantly by rake angle. With frequency, horizontal stiffness coefficients can increase or decrease slightly with such an angle for frequencies below or above the one corresponding to the peak of the functions due to the group effect. Rocking stiffness coefficients tend to decrease with rake angle. At the same time, damping (both for horizontal and rocking) decreases, and the translational kinematic interaction...
Figure 7. Horizontal and rocking stiffness and damping coefficients together with translational and rotational kinematic interaction factors (for vertically incident S waves) of 3×3 pile group with inclined elements parallel to the direction of excitation. $E_p/E_s = 1000$, $s/d=5$.

Factors are closer to one, for increasing inclination angles. Finally, rotational kinematic interaction factors are much less influenced by rake angle, begin almost independent of it for mid-to-low frequencies, and showing a tendency to increase slightly for higher frequencies.

All in all, and as expected, the response of both configurations differs significantly, being the following the most important differences from the impedance and kinematic response points of view: a) damping coefficients increase significantly with rake angle when the excitation is parallel to the plane in which the piles are inclined, while such damping coefficients decrease in the other configuration.
tion; b) horizontal stiffness functions increase for all frequencies in the first configuration, while the variations in the second configuration do not follow this trend and are much less significant in magnitude; c) when the excitation is parallel to the plane in which the piles are inclined, the filtering capabilities of the pile foundations improve significantly when inclining the pile, while such tendency is exactly the opposite in the other configuration; and d) the increase in the rotational input motion to the superstructure is much less important in the second configuration than in the first one, where the high values of $I_\phi$ could be a cause of problems in slender constructions.
CONCLUSIONS

In order to look into some aspects related to the possible influence of inclination angles of piles on the dynamic response of pile foundations and piled structures, this paper provides a set of impedance functions and kinematic interaction factors of 2×2 and 3×3 pile groups founded on a homogeneous half-space. The main trends observed in the response functions are discussed.
In short, it is shown that, apart from the well known larger horizontal stiffness, inclined piles provide a significantly larger capacity to dissipate energy and to filter the horizontal input motion to the superstructure when the direction of the excitation coincides with that of the inclination of the piles, which might make them useful for non-sleender structures. On the other hand, the larger rotational input motions and rocking damping but smaller rocking stiffness makes it difficult to make a general prediction of the seismic response of slender structures, which will have to be studied in detail. When the direction of the excitation is such that is perpendicular to the planes in which the piles are inclined, the capacity of the foundations to filter out the input motions and dissipate energy decrease, which might partially overcome the advantages commented above. Configurations with piles inclined in different directions could provide solutions with beneficial responses in all directions, as suggested by the promising results obtained for diagonal configurations (not shown in this paper). In any case, future work is needed in this field involving not only different types of configurations and system parameters but also the presence of the superstructure and the influence of rake angle on internal moments and shear forces developed in the piles.

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