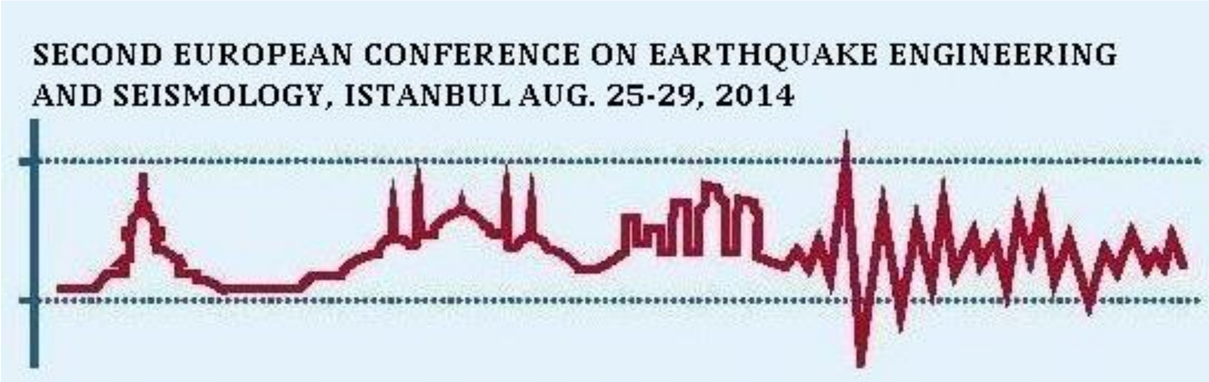

COMPARATIVE STUDY ON BEHAVIOR OF VARIABLE CURVATURE FRICTION PENDULUM ISOLATOR

Parisa SHAHBAZI[1] and Touraj TAGHIKHANY[2]

ABSTRACT

In recent years, there has been growing interest to improve the efficiency of the base isolators when the superstructure is subjected to near-fault ground motions. The seismic response of conventional isolated systems will be amplified in earthquakes with strong long-period wave components. It is found that Variable Curvature Friction Pendulum (VCFP) isolator can provide a promising solution to alleviate the mentioned problem. An VCFP isolator is similar to a friction pendulum system (FPS) isolator, except that its sliding surface has variable curvature rather being spherical. VCFP as an innovative isolation system, exhibits different hysteretic properties at different stages of displacement and effectively reduce the super structure responses in near-fault earthquakes. Herein, the behavior of a seismically isolated structure mounted on several VCFP type isolators is investigated under series of ground motions. The ground motions are scaled to SLE, DBE and MCE levels. Further, its performance is compared to Friction Pendulum System (FPS). The numerical results show that the isolator displacement and acceleration of the superstructure can be controlled within a desirable range with the installation of VCFP. Therefore, the VCFP can be adopted for upgrading the seismic resistance of the structures adjacent to an active fault.

INTRODUCTION

Seismic isolation is an approach to reduce transmitted earthquake forces to structure by shifting the fundamental period of structure away from the predominant frequencies of ground excitation and minimize the structural damage as a result (Soni et al. 2011). In this method which is used to seismic resistant design of low and middle rise structures with high natural frequencies, the super-structure remain almost elastic during excitation, due to concentrating the displacements in base level (Warn and Ryan 2012).

Different seismic isolation devices have been introduced and investigated thus far. Among these, sliding isolators having a simple mechanism are highly popular. This type of isolator is relatively insensitive to variations in the frequency content and amplitude of the input excitation (Mostaghel and Tanbakuchi 1983) and can control the maximum acceleration exerted on the structure by limiting the friction coefficient. FPS is one of most popular sliding isolator using gravity action to supply restoring force (Zayas, Low and Mahin 1990). This isolator contains a spherical sliding surface and a rigid slider. Effective stiffness of isolator and period of isolated structure are controlled by radius of sliding surface (Fenz and Constantinou 2008a). The spherical sliding surface of the FPS isolator provides a relatively constant time period of oscillation, resulting in several practical disadvantages. The FPS isolator is designed for specific ground motion amplitude and frequency characteristics, limits its efficiency under broad range of ground excitation (Sinha and Pranesh 1998).

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In order to improve seismic performance of FPS isolators and provide fully passive adaptive devices many researchers have introduced different innovative isolators. The first proposed isolator was Multi-Spherical Sliding Bearings, which consists of more than one concave surface; thereby it shows different hysteresis behavior as the slider slides on one or more than one of concaves. Variation of stiffness is discrete in this kind of isolator. Another imperfection of Multi-Spherical Sliding Bearings is the negative effects of impact when the slider contacts the displacement restrainer (Fenz 2008). A number of authors such as Fenz and Constantinou have investigated the mechanical behavior of these isolators (Fenz 2008, Fenz and Constantinou 2008a). Another method to provide an adaptive behavior is a kind of spherical sliding isolator which its coefficient of friction varies continuously with isolator displacement; therefore it possesses adaptive damping (Panchal and Jangid 2008a). Pranesh, Sinha, Punchal and Jungid and some other researchers have studied about such systems (Panchal and Jangid 2008a, Panchal and Jangid 2008b). What investigated in this paper as one of the best passive adaptive systems is an isolator called Variable Curvature Friction Pendulum (VCFP). VCFP isolator consists of a sliding surface and an articulated slider, resembling FPS isolator, except that the sliding surface of VCFP is non-spherical and has a variable curvature (Pranesh and Sinha 2000, Tsai, Chiang and Chen 2003, Lu, Wang and Hsu 2006, Lu, Lee and Yeh 2011). As a result, the period of the isolation system is not constant and varies along with the isolator displacement. Therefore, we determine the variation of the period of isolation over isolator displacement during designing process, rather than specifying a constant period of isolation. Pranesh and Sinha were the first researchers who evaluate the performance of VCFP isolators, as a new effective technology to overcome seismic forces, by introducing VFPI (Variable Frequency Pendulum Isolator) (Pranesh and Sinha 2000). The sliding surface of VFPI has an elliptical shape that its major axis extends as the slider takes away from the center point of sliding surface. So the period of oscillation increases with isolator displacement. Consequently it shows a behavior between FPS isolator and Pure Friction (PF) isolator. Accordingly, the possibility of low-frequency resonant will be attenuated but it leads to excessively large isolator displacement and residual displacement. The VFPI performance is found to be stable during low-intensity excitations, and fail-safe during high-intensity excitations (Pranesh and Sinha 2000). Gillich et al. and Lu et al. used polynomial functions in their studies (Gillich et al. 2012 and Lu et al. 2006).

What is recommended in this paper is to design the sliding surface in order to include a hardening part to control the displacement response during high intensity earthquakes. It means a part in which, period of isolation system decreases as the slider moves away from its neutral position. One mathematical function that can possess such property and is used in this study is order 4 polynomial function. The hardening part can emerge after a softening part too. The purpose of the softening part is to control the acceleration response during low and intermediate intensity earthquakes. This can be achieved by an order 6 polynomial function. In addition, A VCFP isolator with elliptical cross-sectional function is being investigated to highlight the advantages of hardening part in chosen polynomial functions.

Since the use of VCFP isolator is a new approach to seismic design of structures, the previous studies were less parametric and more introductive. This means that the role of selection of various functions for sliding surface and any changes in their design parameters, on the performance of isolation system has being discussed a little. Therefore, the effect of surface geometry and different design parameters of each chosen function on seismic behavior and structural responses of isolated structure under three seismic levels (SLE, DBE and MCE levels) is being evaluated and compared with that of FPS isolator in this paper. The selected seismic demand parameters in present study are maximum isolator displacement and peak structural acceleration.

MATHEMATICAL MODEL

Similar to a conventional friction pendulum system, a VCFP isolator primarily consists of a slider and a concave sliding surface to uncouple the super-structure motion from the ground excitation. However, unlike the FPS whose sliding surface is spherical with a constant radius, the sliding surface of the VCFP has variable curvature. As a result, the restoring stiffness and isolation frequency becomes adaptive to the isolator displacement (Lu, Shih and Wu 2004).

The free-body diagram of a variable curvature friction pendulum isolator shown in Figure 1 is used to simulate the behavior of VCFP isolator. The super-structure is assumed to be a rigid body. The radiant cross-section of the sliding surface is defined by a geometric function *y(x)* in the *x*-*y* coordinates.

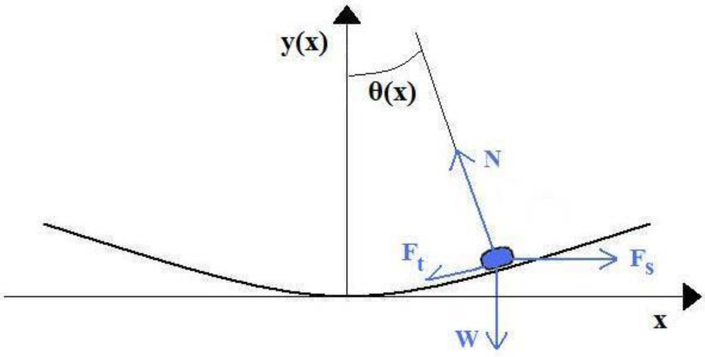


Figure 1. Free body diagram of VCFP isolator

Based on researches done by Lu et al. (Lu et al. 2004) the isolator shear force of the sliding surface with variable curvature can be written as:

$$F_s(x) = F_r(x) + F_f(x) \quad (1)$$

$$F_r(x) = Wy'(x) \quad (2)$$

$$F_f(x) \approx \mu W sign(\dot{x}) \quad (3)$$

Where $F_r(x)$ and $F_f(x)$ denote the restoring force and the friction force respectively. W is the total weight of the super-structure. y' is the first derivate of the geometric function *y(x)* .

The isolator stiffness $k_r(x)$, which is the rate of change of the restoring force $u_r(x)$, and the tangential isolation frequency $\omega(x)$ further computed by equations 4 and 5 (Lu et al. 2004).

$$k_r(x) = Wy''(x) \quad (4)$$

$$\omega(x) = \sqrt{gy''(x)} \quad (5)$$

As equations 2, 4 and 5 show, with designing geometric function *y(x)* of the sliding surface, the restoring force $F_r(x)$, the isolator stiffness $k_r(x)$ and isolation frequency $\omega(x)$ which are variable and implicit functions of the slider displacement x, can be determined immediately by these equations (Lu et al. 2011). This is the difference between a conventional sliding isolation system and a sliding isolator with variable curvature. By appropriately choosing the cross-sectional geometric function y(x) of the sliding surface, the VCFP may achieve the desired hysteretic property (force –displacement relation) and favourable dynamic characteristics.

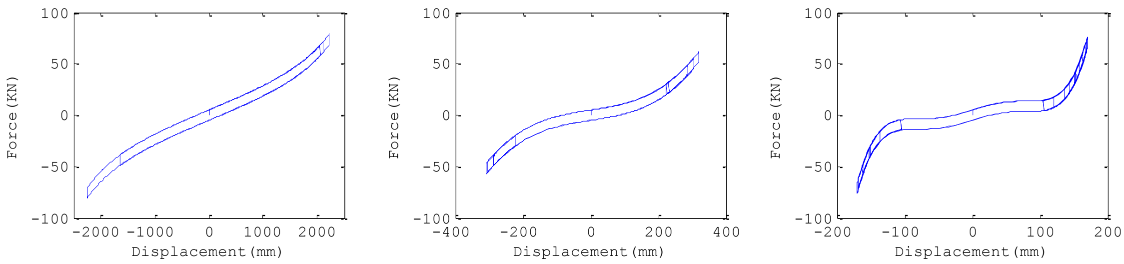
DESIGNING PROCESS

According to Eq.(2) restoring force $F_r(x)$ is an explicit function of *y'(x)*. So if an increasing restoring force is desired, we need to select a mathematical function which its first derivate increases with displacement and vice versa. Order 4 polynomial function and order 6 polynomial function can afford hardening behavior entitled "O4" and "O6" isolators, respectively. The later can show an "O6" isolator has softening behavior in displacements before the hardening part, because of its first derivate function that is an order 5 polynomial function displayed in Table 1. The VFPI isolator entitled "ELL", has an elliptical function with a seismic behavior softer than FPS isolator. It means that it possesses a restoring force lower than that of FPS for a certain distance from the isolator neutral point. This results a lower transmitted force but larger displacements that is not desirable during high intensity earthquakes (Shahbazi et al. 2013). Studying the behavior of such systems beside the hardening isolators and comparing the responses of all with conventional FPS isolator, helps us to have a better percipience of a decreasing period of isolations and its effects on seismic isolation performance. Table 1 shows the geometric functions of three chosen VCFP isolators and their first derivate to calculate the required responses.

Table 1. Geometric Functions Used for VCFP isolators, and Their First Derivate

Function	ELL	O4	O6
$y(x)$	$b(1-\sqrt{1-\frac{x^2}{a^2}})$	$\frac{1}{4}ax^4+\frac{1}{2}cx^2$	$\frac{1}{6}ax^6+\frac{1}{4}cx^4+\frac{1}{2}ex^2$
$y'(x)$	$bx/(a^2*\sqrt{1-\frac{x^2}{a^2}})$	ax^3+cx	ax^5+cx^3+ex

Figure 2 shows the hysteretic loop of different VCFP isolators chosen for this study under a sinusoidal loading. The hysteretic loop of the ELL isolator possesses only a softening behavior as the slider moves away from its neutral position. Unlike ELL the hysteretic loop of the O4 isolator only possesses a hardening behavior. On the other side O6 isolator possesses a softening-and-then-hardening behavior.



(a) (b) (c)

Figure 2. Schematic hysteretic loop of a) ELL isolator b) O4 isolator c) O6 isolator, under sinusoidal loading

As it's clear from Table 1, The primary design parameters which define the geometric function and consequently the exact shape of sliding surface, are "*a*" and "*b*" for ELL isolator, "*a*" and "*c*" for O4 isolator and "*a*", "*c*" and "*e*" for O6 isolator. To convert these parameters to more physical ones, we use equations listed in Table 2 for each VCFP isolator. In this table, k_0 represents the normalized initial stiffness at $x=0$ (x is the distance from neutral point) and can be computed by the Eq.(6) based on given initial period T_0. D is the isolator displacement that possesses the exact isolator stiffness of k_1. It is assumed that D be the displacement of the retro flexion point for O6 isolator too.

$$k_0 = \left(\frac{2\pi}{T_0}\right)^2 / g \qquad (6)$$

Table 2. Assumptions and Design Parameters for isolators

Function	Isolator	Assumptions	Design Parameters
Elliptical	ELL	$a = 3$	$b = \frac{a^2}{g}\left(\frac{2\pi}{T_0}\right)^2$
Order 4 polynomial function	O4-1	$k_1 = 4\ (1/m)\ \&\ D = 0.1\ (m)$	$a = \frac{k_1 - k_0}{3D^2}$ $c = k_0$
	O4-2	$k_1 = 4\ (1/m)\ \&\ D = 0.2\ (m)$	
Order 6 polynomial function	O6-1	$k_1 = 0\ (1/m)\ \&\ D = 0.1\ (m)$	$a = \frac{k_0 - k_1}{5D^4}$ $c = \frac{k_1 - k_0}{3D^2}$ $e = k_0$
	O6-2	$k_1 = 0\ (1/m)\ \&\ D = 0.2\ (m)$	

As showed in Table 2, parameter "*a*" of ELL isolator is assumed to be constant and equal to 3 meters. This value helps the isolator to keep servicing under severe earthquakes and do not fail when it possesses slight coefficient of friction. Parameter "*b*" is calculated directly by T_0.

For the two polynomial functions, *D* has two values of 0.1 and 0.2 meter. The effects of design parameter, *D* on seismic behavior of isolated structures can be observed by comparing the responses with these two different values of *D*. As mentioned before, k_1, the isolator stiffness at $x=D$, is assumed to be constant. The coefficients of polynomial functions as primary design parameters are calculated easily by equations shown in Table 2.

For better comparison and evaluating the seismic performance, the effects of each design parameter has been investigate in three different VCFP isolators. The effect of parameters as; friction coefficient and initial period are examined under different seismic levels of El Centro and Imperial Valley earthquakes. Accordingly, 20 cases of each VCFP isolator (ELL, O4-1, O4-2, O6-1 and O6-2) are chosen (a total of 100 types of VCFP) to be evaluated and compared with 20 cases of conventional FPS with the same properties. These cases properties are selected in a way that includes different combinations of friction coefficient and initial period. The chosen properties of VCFP cases are listed in Table 3. The range of minimum friction coefficient is from 0.02 to 0.065 and the initial period varies between 1 to 5 seconds. These values cover a wide range of different properties to place the performance of VCFP isolator under scrutiny.

In all models the maximum friction coefficient were also assumed as:

$$f_{max} = 2 \times f_{min} \tag{7}$$

The velocity dependence of the coefficient of friction is described by Eq.(8) (Constantinou, Mokha and Reinhorn 1990):

$$\mu = f_{max} - (f_{max} - f_{min})exp(-a|\dot{u}|) \tag{8}$$

where $|\dot{u}|$ is the sliding velocity, f_{max} and f_{min} are the sliding coefficients of friction at large velocity and nearly zero sliding velocity, respectively and *a* is a rate parameter that controls the transition from f_{min} to f_{max}. *a* is considered to be 100 s/m for single concave bearings (Constantinou et al. 1990).

Table 2. Chosen Properties for VCFP Cases

Case No.	Minimum friction coefficient	Initial period
1	0.02	1
2	0.02	2
3	0.02	3
4	0.02	4
5	0.02	5
6	0.035	1
7	0.035	2
8	0.035	3
9	0.035	4
10	0.035	5
11	0.05	1
12	0.05	2
13	0.05	3
14	0.05	4
15	0.05	5
16	0.065	1
17	0.065	2
18	0.065	3
19	0.065	4
20	0.065	5

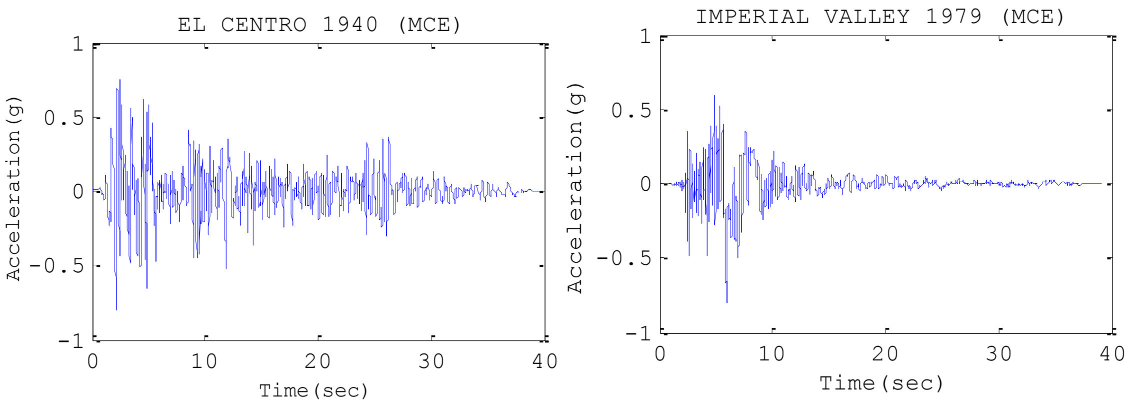
Evaluation of VCFP isolation performance

1- Superstructure properties

The SDOF superstructure chosen to be isolated has the same mass, stiffness, and damping properties as the superstructure in the Fenz et al. research (Fenz and Constantinou 2008b). So the total weight of structure is W_s= 133.33 kN, the structural period is T_s= 0.20 second and damping ratio is ζ_s = 0.025. This superstructure is assumed to be mounted on four similar bearings.

2- Records

Two different types of unidirectional ground motions were used for nonlinear time history analysis of the isolated structure. These records are; El Centro (1940) as a far-field ground motion and Imperial Valley (1979), which is a near-field ground motion with pulse period of 3.8 seconds. Both ground motions were scaled to 0.3g, 0.5g and 0.8g, as SLE, DBE and MCE levels, respectively. The waveforms of these ground accelerations are shown in Figure 3.



(a) (b)

Figure 3. Waveforms of the Ground Accelerations Used in the Numerical Study: (a) El Centro earthquake (far field) (b) Imperial Valley earthquake (near field)

3- Results

To investigate the isolation performance of the variable curvature friction pendulum system, the measured peak responses of structure isolated by each VCFP isolator has been compared with the simulated responses of the FPS isolated structure and un-isolated structure. The peak acceleration of super-structure and maximum displacement of isolator are computed by state space formulation of the equations of motion in MATLAB.

The peak responses of the un-isolated structure under two types of ground motions have been listed in Table 4.

Table 4. Un-isolated Structure Peak Responses under El Centro and Imperial Valley ground motions

Earthquake	El Centro			Imperial Valley		
	SLE	DBE	MCE	SLE	DBE	MCE
Roof Displacement (mm)	7.883	13.137	21.019	6.300	10.501	16.802
Structural Acceleration (g)	0.798	1.331	2.129	0.634	1.0572	1.692

Figure 4 shows the peak isolator displacement and the peak structural acceleration of various VCFP and FPS isolators under different seismic levels of El Centro and Imperial Valley ground motions. These diagrams compare VCFP isolators with different geometrical cross section and design properties. A perfect VCFP isolator is the one which reduce peak acceleration as well as controlling isolator displacement in least value.

The peak responses of FPS and ELL isolators are very similar in both earthquakes and all seismic levels. For Imperial Valley ground motion which is a near-field record, large displacement in isolator level indicated on poor performance of FPS and ELL isolators. On the other hand the VCFP isolators with order 4 and order 6 polynomial functions limit the isolator displacement, especially when structure is subjected to near-field earthquake, because of proportional relation of curvature with displacement. O6 and O4 isolators result low level of spectral acceleration under El Centro ground motion however this parameter is increased almost linearly during Imperial Valley due to hardening of stiffness in large displacement.

The adaptability of VCFP isolators under different seismic levels of earthquakes with different characteristics is observable in Figure 4. The equality of responses of VCFP cases with that of FPS in SLE level and the dramatic difference between responses during higher seismic levels is an example of such behavior. The difference between various VCFP cases responses and FPS responses intensify as earthquake intensity grows higher, because as the earthquake intensify grows higher the isolator displacement increase and consequently the VCFP demonstrates its performance more complete, so the difference will be noticeable. This difference is more when a near field earthquake is applied on isolated structure.

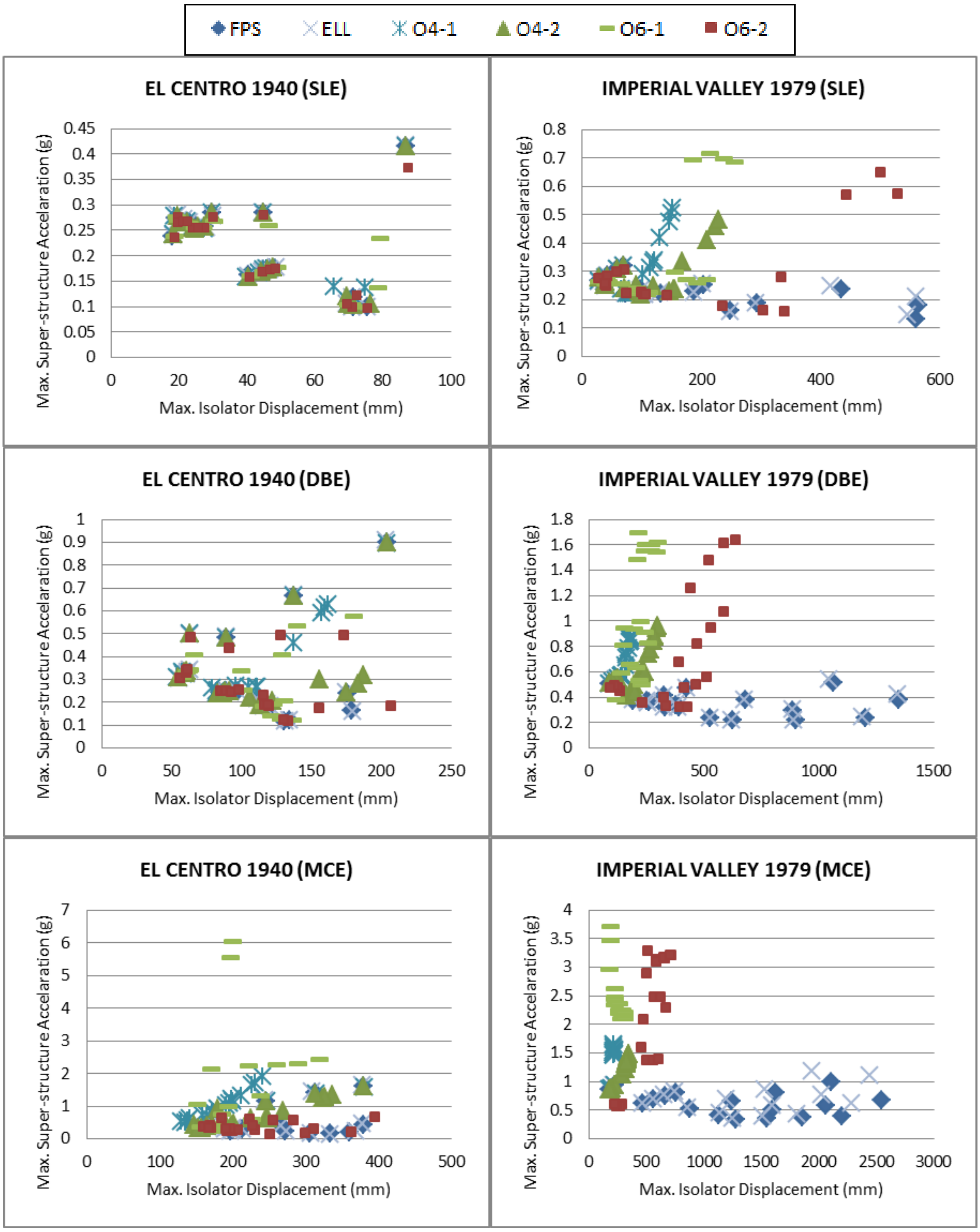


Figure 4. Comparison of Peak Responses of Various VCFP and FPS Cases, under El Centro and Imperial Valley Ground Motions at SLE, DBE and MCE Level

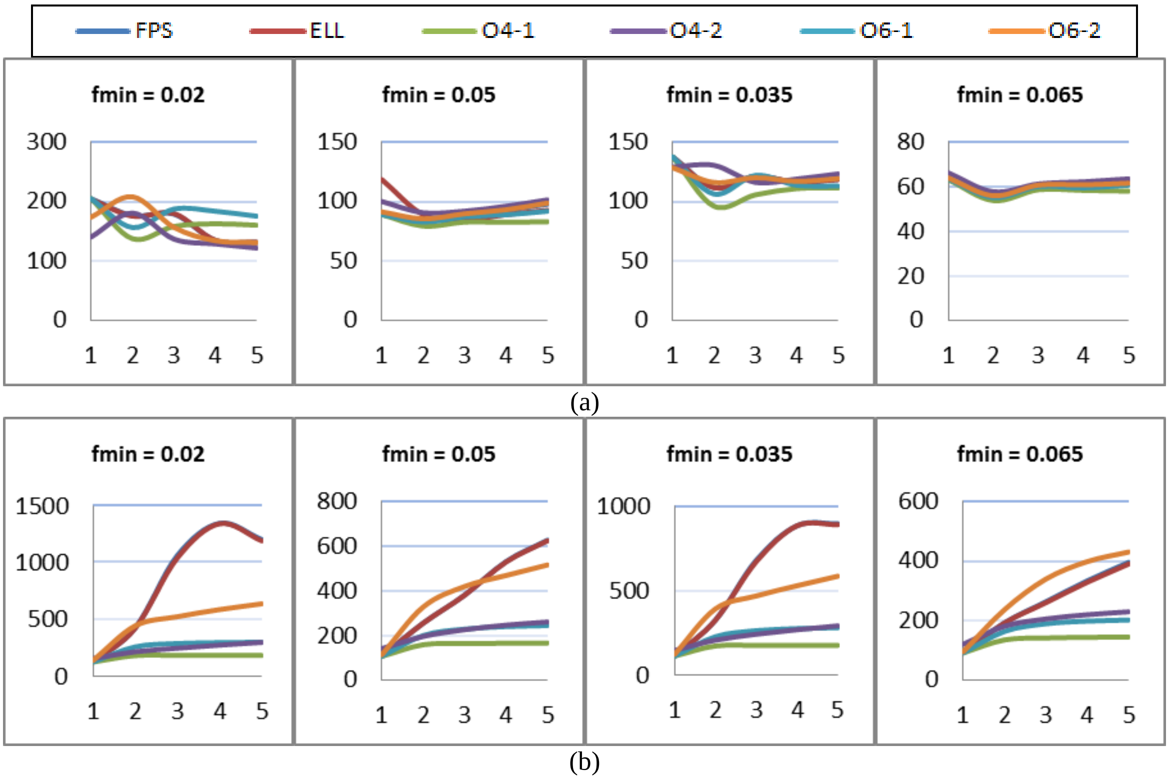


Figure 5. The Effect of Initial Period and Friction Coefficient on Maximum Isolator Displacement in Different VCFP Isolators under DBE level of (a) El Centro and (b) Imperial Valley Ground Motions
Horizontal Axis: Initial Period (sec), Vertical Axis: Max. Isolator Displacement (mm)

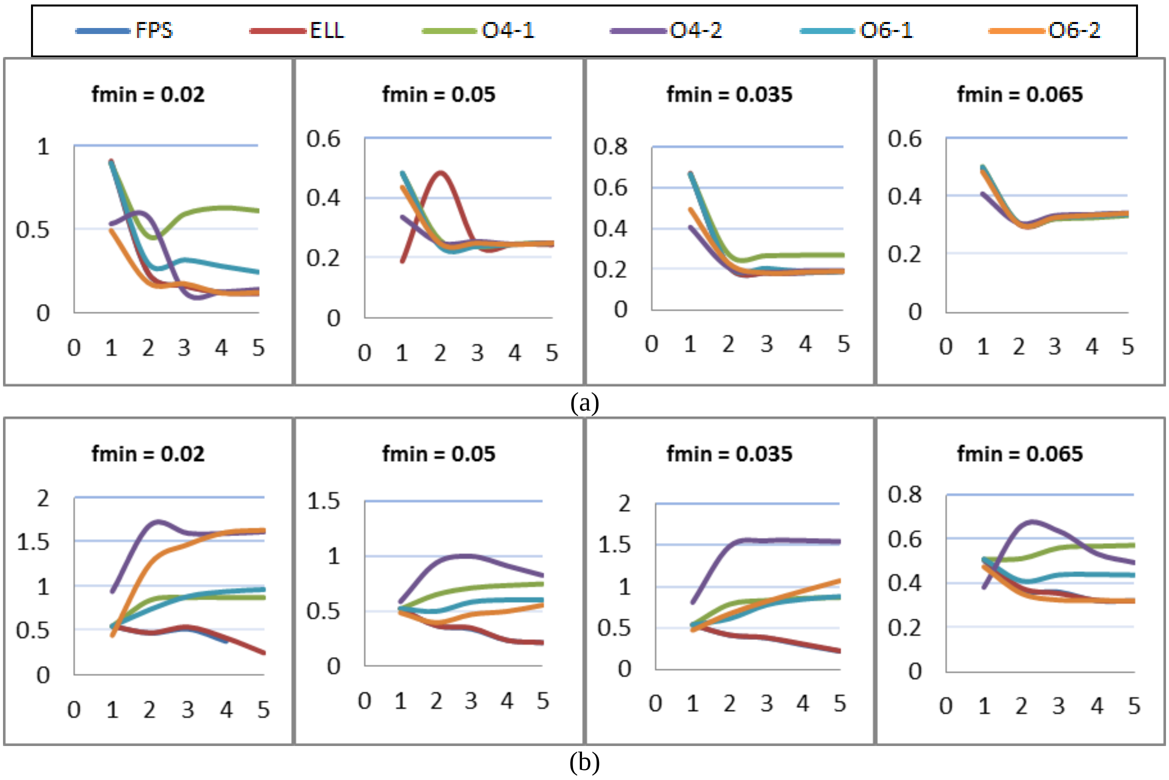


Figure 6. The Effect of Initial Period and Friction Coefficient on Peak structural Acceleration in Different VCFP Isolators under DBE level of (a) El Centro and (b) Imperial Valley Ground Motions
Horizontal Axis: Initial Period (sec), Vertical Axis: Peak Structural Acceleration (g)

Figures 5 and 6 display the effect of initial period and friction coefficient on maximum isolator displacement and peak structural acceleration in different VCFP isolators under DBE level of El Centro and Imperial Valley ground motions. These figures show that choosing different initial period for VCFP isolators under far field earthquakes, doesn't effect on structural responses in a same way under near field earthquakes. In general, under El Centro ground motion that cause low displacement, increasing the initial period of isolation lead to decrease in structural responses, and under near-field Imperial Valley ground motion, lead to decrease in structural acceleration but increase in isolator displacement.

Increasing the amount of friction coefficient leads to increase structural acceleration and decrease in isolator displacement in general. However in cases of O4 and O6, the story acceleration will decrease as well. Because by decreasing displacement due to rougher surface, the slider slides less in hardening part of O4 or O6 isolators, so the restoring force and consequently the structural acceleration will decrease. As a result, higher friction coefficient is recommended for O4 and O6 isolators.

The structural responses revealed that the seismic behavior depends on design parameters severely, but the sensitivity to all parameters is not equal. For example, design parameters such as the retro flexion point position, initial period and friction coefficient have a determinative role on seismic behavior of O6 isolator. On the other hand, O4 isolator behavior is less impressible by design parameters especially in higher seismic levels. The most effective parameter in designing O4 isolator is critical displacement *D*. O4-2 isolator with $D = 0.2\ (m)$ behaves better than O4-1 with $D = 0.1\ (m)$ generally, because it causes lower structural acceleration. O4-2 isolator has the best performance among chosen VCFP isolators during EL Centro and Imperial Valley ground motions, because of its ability to control isolator displacement and structural acceleration both.

CONCLOUSION

It is concluded that variable curvature friction pendulum isolators as a kind of passive adaptive system behave differently based on the chosen function of sliding surface and satisfy the engineer purpose of design. This purpose can be controlling of structural acceleration or isolator displacement or both of them simultaneously, in a way that minimize the structural damage during high intensity earthquakes and represent a good performance during less severe earthquakes. In VCFP isolator designer has more options to choose according to what they expect from isolator.

Considering the problem of excessive isolator displacement, that may be induced by a near-field earthquake with strong long-period components, the possibility of using VCFP isolator, was investigated numerically in this paper. The fundamental period of the isolated structure with VCFP isolator is variable with increase of the isolator displacement; accordingly, the amplification caused by a long-period pulse-like ground excitation, when structure mounted on a conventional isolation system, will not occur here, while still maintaining a good isolation performance. The hysteresis loop of VCFP is different with that of FPS and is based on its restoring force defining by sliding surface geometric function.

After evaluating 100 cases of different VCFP isolators and comparing the results with corresponding FPS cases responses, it is concluded that VCFP isolator can be a promising isolation technology to upgrade effectively seismic resistibility. The best seismic performance during VCFP cases studied in this paper belongs to the VCFP type with order 4 polynomial function, entitled O4.

One challenging issue in the subject of application of VCFP isolators is the design process, that takes more time and energy than a conventional isolator, because it is a new technology and has more parameters to be designated. Therefore, research works and experimental investigation needs to be carried out in order to arrive at complete understanding of the behavior of the VCFP isolators and finding the optimum parameters for them in future.

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