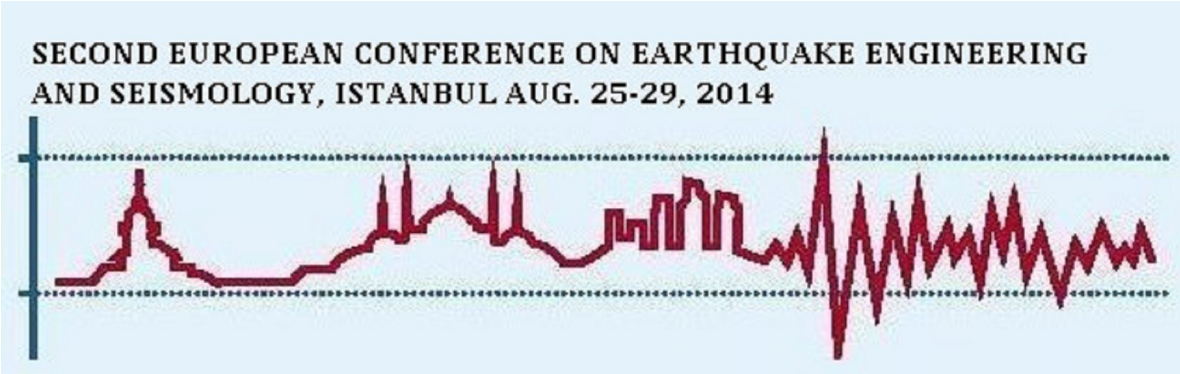

Sliding Mode Fault Tolerant Control for Actuator Failure in Civil Infrastructures under Seismic Action

Arash Yeganeh Fallah[1], Touraj Taghikhany[2]

ABSTRACT

In recent decades interest for utilizing active and semi-active control strategies for seismic protection of civil infrastructures have been widely increased. However, the reliability of these systems is still in doubt as there is a possibility that the critical components, such as actuators and sensors, malfunction during earthquake.

This paper deals with attenuation of actuator's fault effects on performance of the control system. In this regard, sliding mode concept is used due to its inherent robustness to matched uncertainties. The method is applied for designing fault detection observer and fault tolerant controller. The robust observer estimates the state of system and reconstructs the actuator fault. The fault tolerant sliding mode controller reconfigures itself by the fault distribution matrix and accommodates the fault effect on the system. Numerical simulation on a three-story structure demonstrates the effectiveness of the proposed fault tolerant system. It was shown that the fault tolerant control system maintains performance of the structure at an acceptable level in the post-fault case.

Keywords: *Fault detection, fault tolerant control, sliding-mode control, Actuator Fault, civil infrastructures*

Introduction:

In recent decades, the application of active and semi-active structural control strategies for protection of civil infrastructures against vibration has gained much attention. Never the less, smart actuators and sensors are not eternal in functionality during the control process and each components is vulnerable to partial or total malfunctioning (Zhang and Jiang, 2008; Zhang et al, 2011). Consequently, the control system should be designed in a way that it can recover itself in partial or total failure of these components. These self-recovery methods are known as Fault-Tolerant Control Systems (FTCS).

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FTCS are classified into passive and active strategy. In passive fault-tolerant control systems (PFTCS), controller is designed in a way to be robust respect to presumed faults. Active fault-tolerant control systems (AFTCS) can tolerate unpredictable faults through reconfiguring the control action. AFTCS comprises two important stages: first Fault Detection and Diagnosis mechanism (FDD) and second control reconfiguration, (Mahmoud et al, 2003; Zhang and Jiang, 2008). On-line FDD provides information about the system status and the occurred fault. The FDD unit generates a residual signal to detect the faults that occur during the control process. The residue signal is the difference between measured parameters of real systems and estimated parameters of healthy model, (Kinnaert, 1999; Besançon, 2003; Liberatore et al, 2006).

In model-based fault detection methods, the observers play an essential role in generating the residual signals, (Edwards and Spurgeon, 1998; Edwards et al, 2000). The basic idea behind the use of the observers for fault detection is to form residuals from the weighted difference between the actual system outputs and the estimated outputs using an observer. Instances of such observers include the unknown input observer, the H∞ method, the Kalman filter, the H2/LQG techniques and Sliding Mode observer, (Eryurek and Upadhyaya, 1995). Sliding mode observer has a distinct ability in reconstructing immeasurable signals (unknown inputs) by appropriate scaling and filtering, which is called 'equivalent output'. Sliding surface behavior in sliding mode observer creates this individual property, (Alwi et al, 2011). After reconstructing fault signals, the reconfiguration algorithm accommodates the fault effect on system.

As sliding mode controller robust to the matched uncertainty, it can be selected as a reconfigurable control, (Shtessel et al. 2002; Hess and Wells, 2003). Sliding mode employs nonlinear control and injection signals to force the system trajectories and attain the motion along the sliding surface in a finite time, (Utkin, 1992)

In the present study, an active fault tolerant controller is developed for a system with faulty actuators. The sliding mode observer is designed to estimate the state vector of the system and faults of the actuators. The variable structure systems along with adaptive sliding mode reconfigure control feedback through the matrix distribution for actuator fault. The effectiveness of the proposed method is validated by a numerical simulation on a three-story structure.

Problem Definition and Assumptions:

A class of uncertain systems is assumed by the following equations:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BW\,u(t) + M\xi(t) \\ y(t) &= Cx(t) + Qu(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $u(t) \in \mathbb{R}^m$ is the control input, and $y(t) \in \mathbb{R}^p$ is the measurement output. $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{p\times n}, M \in \mathbb{R}^{n\times r}$ $M \in \mathbb{R}^{n\times r}$ and $Q \in \mathbb{R}^{p\times m}$ are real constant matrices. $W(t) = diag\{w_1,...,w_m\}$ is a diagonal weighting matrix. The scalars $w_1,...,w_m$ model the effectiveness level of the actuators, for example if $w_i = 1$, the ith actuator is healthy, and if $w_i = 0$ the ith actuator has failed completely. The signal $\xi(t) : \mathbb{R}^+ \to \mathbb{R}^r$ stands for the uncertainty of the system or the external disturbance. It is assumed unknown but bounded subject to $\|\xi(t)\| \le \phi$, where ϕ is a known real constant.

If unknown function $f(t):\mathbb{R}^{+}\rightarrow\mathbb{R}^{q}$ represents the fault (or unknown input) and is defined as $f(t)=(W(t)-I)u(t)$ where $\|f(t)\|\leq\rho$, and ρ is a known real constant, then the system Eq. (1) can be presented as follows:

$$\dot{x}(t)=Ax(t)+Bu(t)+\overline{B}\eta(t) \tag{2}$$

where $\overline{B}=[B \quad M]$ and $\eta(t)=[f(t)^{T} \quad \xi(t)^{T}]^{T}$.

Before stating the main results, the following assumptions are made on system (1).

Assumption 1: Matrix $[CB]$ has full column rank, i.e., $rank(C\overline{B})=rank(\overline{B})=q$.

Assumption 2: Any invariant zero of triple $(A,C,\overline{B})$ lies in the left half plane, i.e., for every complex λ number with non-negative real part, $rank\begin{bmatrix} A-\lambda I & \overline{B} \\ C & 0 \end{bmatrix}=n+rank(\overline{B})$.

Sliding Mode Observer for Fault Detection and Diagnose:

A sliding mode observer is introduced in this part for estimation of actuator fault and states of the systems (1). So as to synthesize and analyze the observer, a Lyapunov second method is employed, (Hui and Zak, 2005; Kalsi et al, 2011).The estimated parameter of this observer is denoted as $\hat{x}$ and its difference with real parameter x is estimation error (e) which is defined as;

$$e(t)=\hat{x}(t)-x(t) \tag{3}$$

The observability of (A,C) implies the existence of a matrix $L\in\mathbb{R}^{n\times p}$ such that the matrix $(A-LC)$ has prescribed (symmetric with respect to the real axis), eigenvalues in the open left-half plane. Because $(A-LC)$ is asymptotically stable, there is a unique $P=P^{T}>0$ such that

$$(A-LC)^{T}P+P(A-LC)<0 \tag{4}$$

Therefore, by having P and L, the observer can be formed as below:

$$\dot{\hat{x}}=(A-LC)\hat{x}+Bu+Ly-LQu+\overline{B}v(e,k) \tag{5}$$

The discontinuous output error injection input $v(e,t)$ is defined as

$$v(e,k)=\begin{cases} k\dfrac{FCe}{\|FCe\|_{2}} & \text{if } e\neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where k is a design parameter and $k\geq\varphi$ where φ is a known real constant and $\varphi\geq\|\eta(t)\|$. Using the arguments by Walcott et al (1987), it can be shown that the state $\hat{x}$ of the dynamic system (5) estimates asymptotically the state x of system (2):

$$\lim_{t\rightarrow\infty}e(t)=\lim_{t\rightarrow\infty}(x(t)-\hat{x}(t))=0 \tag{7}$$

The differential equation describing the estimation error is

$$\dot{e} = \dot{x} - \dot{\hat{x}} = (A - LC)e - \bar{B}v(e,k) + \bar{B}\eta \tag{8}$$

Since $\bar{B}$ is of a full rank, the following approximation is achieved:

$$\eta = v(e,k) \tag{9}$$

Sliding Mode Fault Tolerant Controller

Considering system (1), the information about the actuator effectiveness level (i.e. the matrix W) is assumed to be available through the previous section. The system states can always be reordered, and the matrix B from (1) can be partitioned as:

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{10}$$

where $B_1 \in \mathbb{R}^{(n-l)\times m}$ and $B_2 \in \mathbb{R}^{l\times m}$ are of rank $l < m$. It is assumed as in Alwi and Edwards (2010) that B_2 is greater in norms in comparison with B_1, i.e. $\|B_2\| > \|B_1\|$, therefore, B_2 dominantly delivers the effect of the control action to the system dynamics. In the design, it is also assumed that the states are scaled to ensure that $B_2 B_2^T = I$.

The block diagram of the ISM FTC scheme with online CA is shown in Figure 1. In Online CA, it is assumed that there is an actuator effectiveness estimator, $W(t)$, available from the sliding mode fault observer. The control law $u(t)$ considered in this section has information about the actuator effectiveness level (matrix W) to distribute the control signals, (Hamayun et al, 2013; Hamayun et al, 2012)

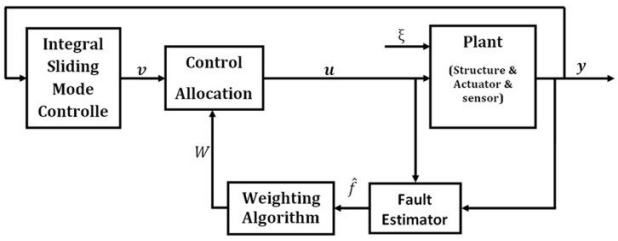

Figure 1. Block diagram of the ISM FTC scheme with online control allocation

For online CA, an expression for the physical control law $u(t)$ is defined as

$$u(t) = WB_2^T (B_2 W^2 B_2^T)^{-1} v(t) \tag{11}$$

where $v(t) \in \mathbb{R}^l$ is the virtual control input and providing that $\det(B_2 W^2 B_2^T) \neq 0$.

This way, the sliding mode control law maintains sliding during faults or failures and is responsible to keep the performance of system as close as possible to the nominal system, which is defined as

$$\bar{v}(t) = \bar{v}_l(t) + \bar{v}_n(t) \tag{12}$$

$$\bar{v} = (B_2 W^2 B_2^T)^{-1} v \tag{13}$$

where

$$\bar{v}_l(t) = -\left(GB_w\right)^{-1} Fx \tag{14}$$

$$\bar{v}_n(t) = -\rho\left(GB_w\right)^{-1} \frac{\sigma(x,t)}{\|\sigma(x,t)\|} \quad for \quad \sigma(x,t) \neq 0 \tag{15}$$

where $B_w = \begin{bmatrix} B_1 W^2 B_2^T \\ B_2 W^2 B_2^T \end{bmatrix}$ and with the current sliding surface and to simplify the following calculation, G is assumed as;

$$G := B_2 (B^T B)^{-1} B^T \tag{16}$$

and ρ is a gain to enforce the sliding. The design of the state feedback gain F in (14) is based on the augmented system (A, B_v), which is assumed controllable. The gain F is derived through Linear Matrix Inequality (LMI) optimization in H_∞ structure and plays an important role in guaranteeing the closed-loop stability of the integral sliding motion (ISM).
The regulated output of controlled system (1) is described as bellow:

$$Z(t) = C_z x(t) + D_z u(t) + M_z \xi(t) \tag{17}$$

To design feedback gain in H_∞ structure with the L_2 gain matrix from $\tilde{u}$ to $\tilde{y}$ should be extended into the L_2 gain from $[\tilde{u}\ \zeta]$ to $[\tilde{y}^T\ Z^T]^T$ which equals H_∞ norm of the new transfer function $\bar{G}(s)$:

$$\begin{bmatrix} \tilde{y} \\ Z \end{bmatrix} = \bar{G}(s) \begin{bmatrix} \tilde{u}^T \\ \xi^T \end{bmatrix}^T \tag{18}$$

and it satisfies $\|\bar{G}(s)\|_\infty \leq \gamma$ if and only if $X = X^T > 0$ and $\gamma \geq 0$ such that minimization of γ is subject to:

$$\begin{bmatrix} AX + XA^T - B_v Y - Y^T B_v & \tilde{B} & M & Y^T & XC_z^{\ T} + Y^T D_z^{\ T} \\ \tilde{B}^T & -\gamma^2 I & 0 & 0 & 0 \\ M^T & 0 & -\gamma^2 I & 0 & M_z^{\ T} \\ Y & 0 & 0 & -I & 0 \\ C_z X + D_z Y & 0 & M_z & 0 & -I \end{bmatrix} < 0 \tag{19}$$

The feedback gain F can be recovered as $F = YX^{-1}$

Numerical simulation of a three-story structure

A three-story shear–frame structure was simulated to demonstrate the effectiveness and feasibility of the proposed scheme. This model which has been previously presented by Wang et al (2009) and Larbah and Patton (2010) is an in-plane lumped-mass shear structure. Its control devices were allocated between every two neighboring floors, as shown in Figure 2. The mass, stiffness, and damping matrices are given as:

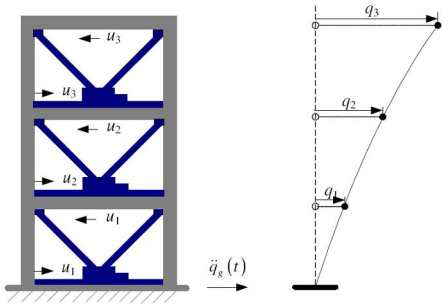

Figure 2. A three-story controlled structure excited by unidirectional ground motions

$$\bar{M}=\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}\times10^3\,kg,\quad \bar{K}=\begin{bmatrix}3.4&-1.8&0\\-1.8&3.4&-1.6\\0&-1.6&1.6\end{bmatrix}\times10^6\,N/m,\quad \bar{C}=\begin{bmatrix}12.4&-5.16&0\\-5.16&12.4&-4.59\\0&-4.59&7.20\end{bmatrix}\times10^3\,N/(m/s) \tag{20}$$

It is assumed that inter-story velocities and floor accelerations are measurable. For the purpose of simulation, the 1940 El Centro NS (Imperial Valley Irrigation District Station) ground motion record was considered as an input excitation with peak acceleration scaled to $1m/s^2$. The actuators generated the desired control force at each floor without any problems.

Actuator Fault Definition

Different faulty cases may happen for each actuator, which may be caused by mechanical imperfections or driver laws of actuator (Cai et al, 2013). Herein, dead zone fault considered as occurring mechanical imperfections. This fault generates zero output within a specified region which is called as dead zone. It has specify lower limit (LL) and upper limit (UL) as the start of dead zone and end of dead zone parameters, and the output of dead zone is defined as:

$$u_{dead\,zone}\begin{cases}u-UL & u>UL\\ 0 & LL\le u\le UL\\ u-LL & u<LL\end{cases} \tag{21}$$

It is assumed that the dead zone fault of the third floor actuator force is of a lower and upper limit of $LL=-5000N$ and $UL=4500N$. Figure 3 compares the generated force by the healthy actuator with that of the faulty one in third floor. Figure 4 shows the estimated and the actual fault that confirms the accuracy of the observer.

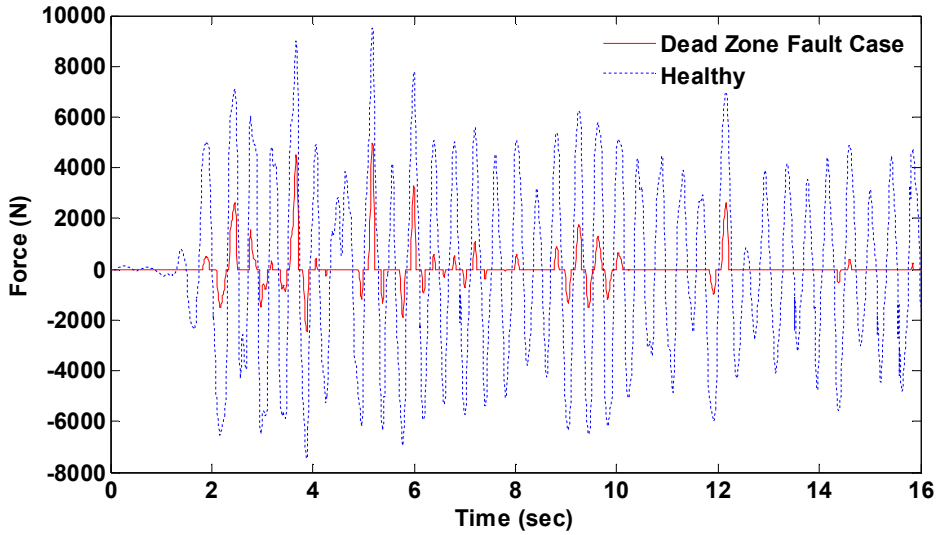

Figure 3. Actuator force in third floor for healthy and dead zone fault case

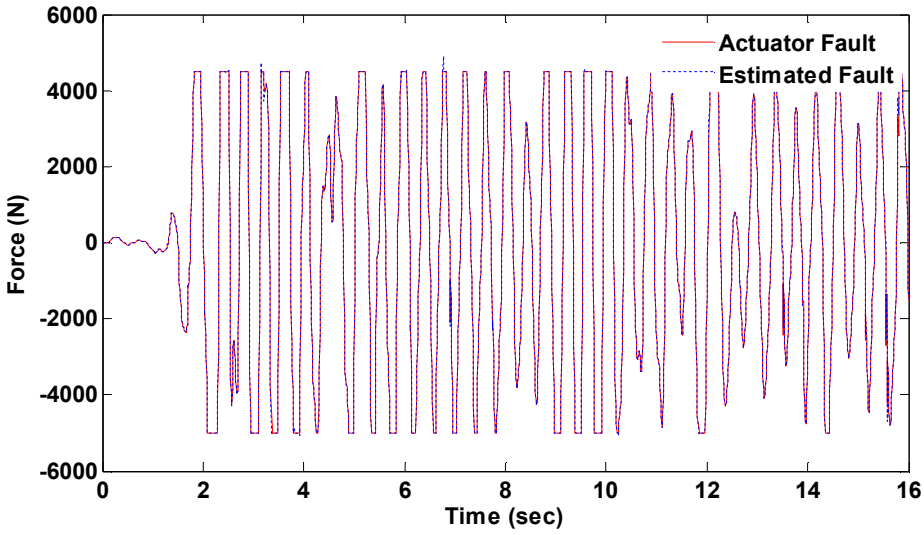

Figure 4. Actual and estimated dead zone fault occurred in actuator of the third floor

Online Control Allocation:

Online control allocation is the proccess of on-line redistribuation of control signal, in case of an actuator fault, in which other actuators are used to compensate lack of the faulty actuator and maintain the perfomance of system. The level of contribution of each actuator in the new control system is determined by an online control allocation, which is function of the actuators effectiveness level matrix. The actuators effectiveness level matrix is a diagonal matrix function of actuators faults in each step of time, and it is defined as the ratio of the estimated force to the desired force in each step of time. For each of its diagonal arrays, $W(t) = diag\{w_1,...,w_m\}$, a lower boundary and an upper boundaries is considered, $0 \le w_i \le 1$.

Since even a faulty actuator may work at 1% of its capacity, considering w_i as 0 may cause an error in numerical simulations. Accordingly, the lower boundary of actuator effectiveness should be limited to 0.01.

As shown in Figure 5, upon occurrence of fault in actuator on third floor, the first and second diagonal arrays of the $W(t)$ equal 1 and the third diagonal array varies by time as shown in Figure 5

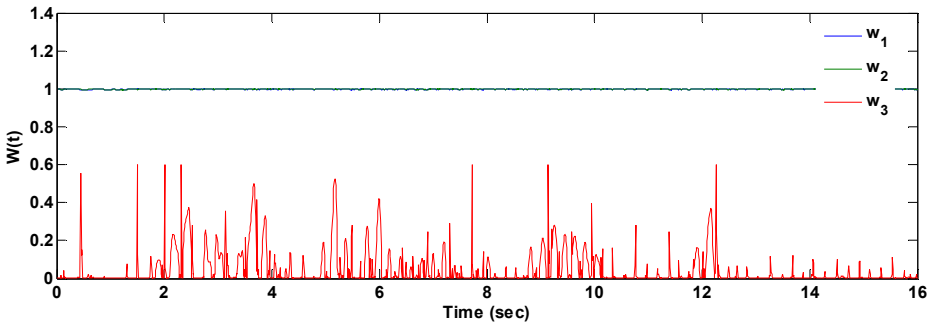

Figure 5. Diagonal arrays of the actuators effectiveness level matrix

Here, the simulation is carried out for the healthy and faulty actuators with and without considering online control allocations and then, the effectiveness of the assumed controller is evaluated by fault free simulation. Figure 6 depicts inter-story drift of the first floor by using three different control systems: with a healthy actuator, a faulty actuator with online control allocation and a faulty actuator without control allocation. As shown, the maximum drift of the first floor using online control allocation is effectively lower than that in the system without control allocation and in some peaks, their difference is smaller than 42%. Surprisingly, the maximum drift of system with online control allocation is also lower than control system with a healthy actuator. This deviation refers to redistribution of control actions in online control allocation.

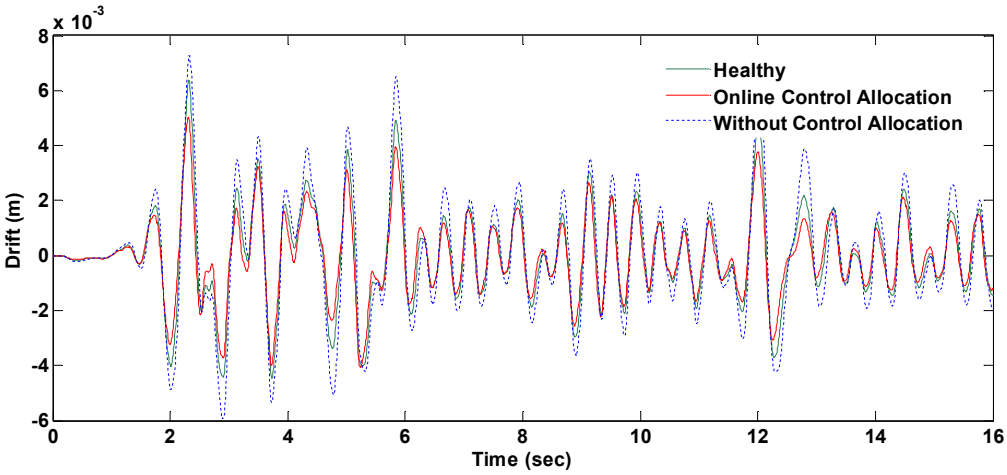

Figure 6. Drift response of the first floor for the healthy system and the dead zone fault case

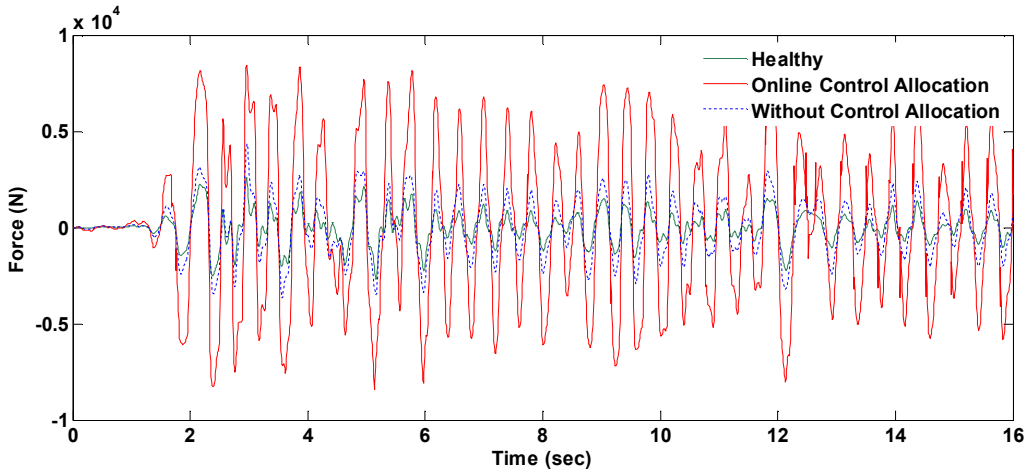

Figure 7Figure 7 to Figure 9 depict applied forces of actuators in first, second and third floor of the structure. As illustrated in these figures for faulty case, the online control allocation tries to compensate for the imperfections of the third floor actuator by increasing the generated force in first and second floors. In some cases, the applied forces in reconfigured control system by actuators are three times of the generated force of actuators in a healthy case.

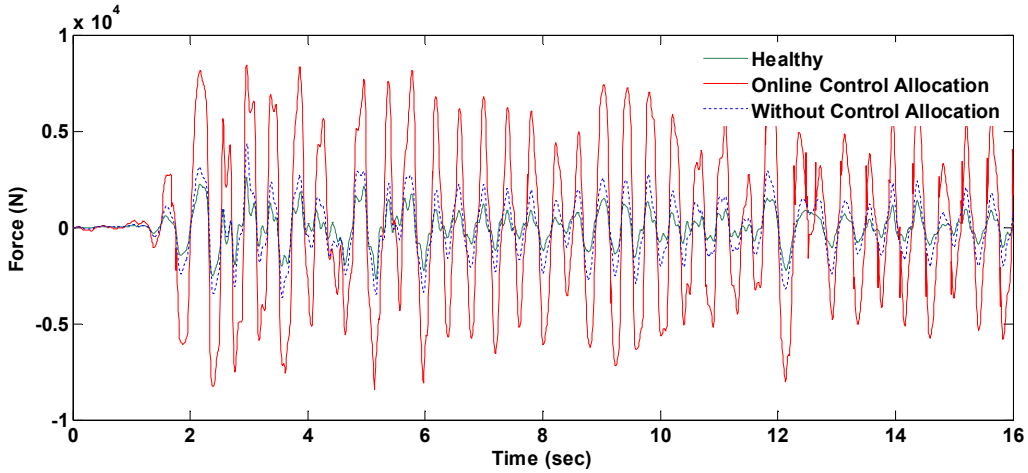

Figure 7. Applied force by the actuator of the first floor in the healthy system and for the dead zone fault case

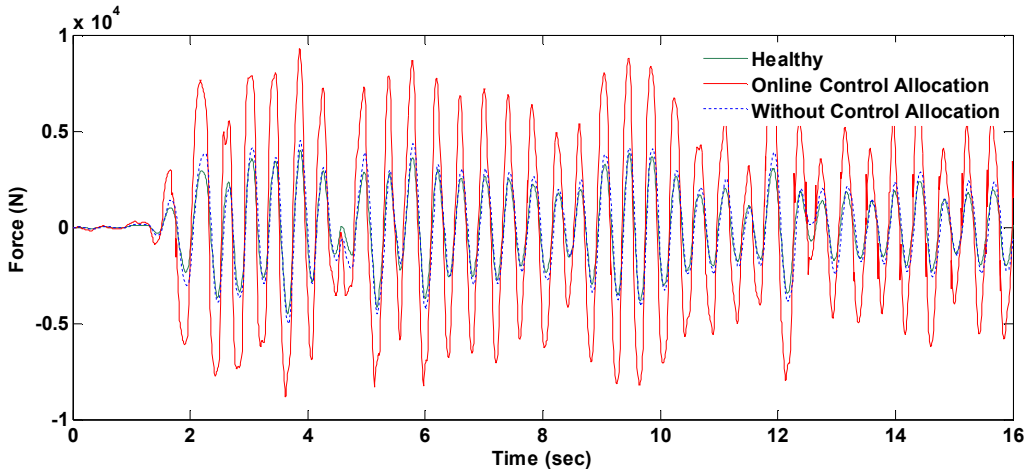

Figure 8. Applied force by the actuator of second floor in the healthy system and for the dead zone fault case

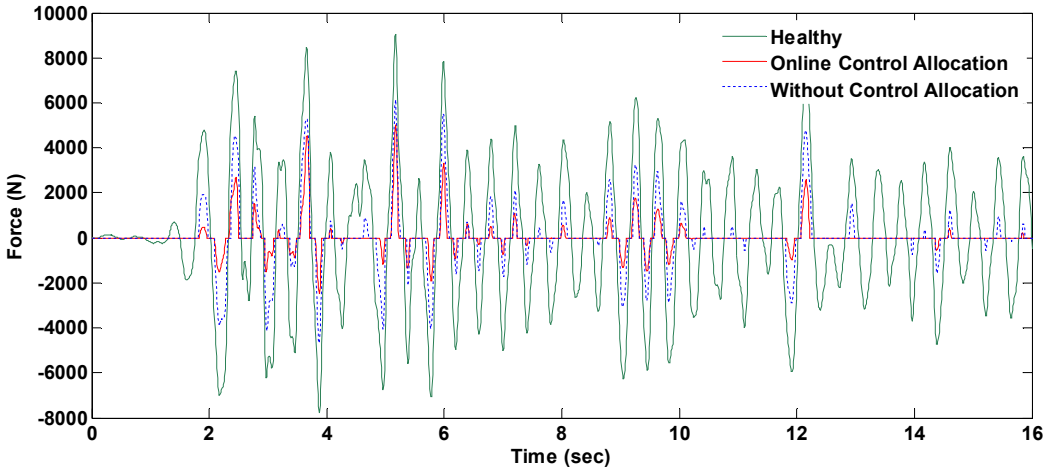

Figure 9. Applied force by the actuator of third floor in the healthy system and for the dead zone fault case

Conclusion:

As imperfection and fault of actuators in active and semi-active structural control system are inevitable, and actuators and sensors are vulnerable to faults, the fault tolerant control is vital for having a reliable and robust performance of structural control systems. In this paper, the Sliding modes were used to detect and diagnose faulty actuators. In this regard, a robust sliding mode observer was introduced to estimate the unknown input and detect faults in the control system.

After fault detection, the effectiveness matrix of the actuators was built up for the fault tolerant control. Effectiveness matrix is introduced to correct the sliding mode with redistribution of control signals to the healthy actuators. Next, the desire gain of the controller was designed in H_∞ framework to regulate robust outputs of the faulty actuators.

A three-story shear–frame structure was used to demonstrate the effectiveness and feasibility of the proposed active fault-tolerant control system (AFTCS). The obtained results show that if mechanical

imperfection of the actuator is assumed as dead zone fault type, the sliding mode with online control allocation perfectly reconfigures its control signals and maintains the system performance at the most available appropriate level.

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