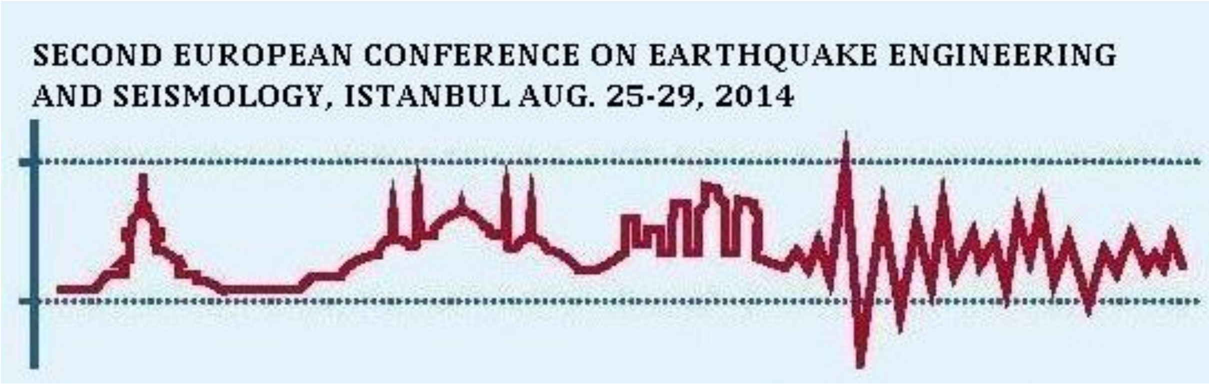


STRUCTURAL DAMAGE DETECTION USING DAMAGE LOCATING VECTOR WITH WIRELESS SMART SENSORS

Mohsen Hashemi[1], Nastaran Dabiran[2] and Touraj Taghikhany[3]

ABSTRACT

Most existing health monitoring methodologies require direct measurement of input excitation for implementation, however, there is no easy way to measure these inputs. Therefore, SHM methods based on ambient vibration have become important in civil engineering. In this paper, a structural damage detection method integrating the damage locating vector (DLV) method and ARMAV model for system identification of frame structures has been explored. A four-storey steel frame with diagonal bracings is considered as the objective building. The damage condition of the structure is simulated by removing some of the diagonals. With the flexibility matrices of both the intact and damaged structure identified from seismic structural responses, results indicate that the damaged locations can be successfully identified by the DLV method if sufficient modes of vibration are taken into account in the realization of the flexibility matrices. In the study, the feasibility of using DLV method for damage detection of frame structures is confirmed and appropriate results supported the employed method in localization of the damages.

INTRODUCTION

Structural health monitoring (SHM) has received significant amount of interests from researchers over the last two decades, due to progress in the development of smart sensing and system identification techniques (Doebling et al., 1996).

Damage in structures is an intrinsically local phenomenon, but damage to a structure also alters its global modal parameters, such as the modal frequencies, damping ratios and mode shapes, as well as its physical parameters, such as the damping, stiffness and flexibility matrices. Despite that the damage conditions might be revealed from changes in the modal parameters, it is difficult to locate the damages by using such information. The physical parameters are indeed more useful as far as damage localization is concerned. Stiffness matrix is intuitively a possible candidate for damage localization. However, the sensitivity analysis developed for damage detection based on the stiffness matrix requires an accurate analytical model of the intact structure, which is itself a difficult task to obtain. Moreover, the stiffness matrix is sensitive to higher mode responses which in general are difficult to be identified from the dynamic response data. Construction of the flexibility matrix, on the other hand,

[1] Graduate student, Amirkabir University of Technology, Tehran, mohsen.hashemi@aut.ac.ir
[2] Graduate student, Amirkabir University of Technology, Tehran, nastaran.dabiran@gmail.com
[3] Assistant Professor, Amirkabir University of Technology, Tehran, ttaghikhany@aut.ac.ir

requires only the first few vibration modes of the structure as the modal contribution on the flexibility matrix decreases with the square of the corresponding modal frequency.

The pioneering work of Pandey and Biswas (1994) demonstrated that, via exploring the change in the flexibility matrix derived from the measurement data, the damage locations in a wide-flange steel beam can be identified. The flexibility-based technique has been considered of great potential in damage localization of structures from global vibration response measurements. The flexibility-based damage localization technique has been advanced further by Bernal (2002) with the method of damage locating vectors (DLVs). This methodology has also been verified experimentally by Gao and Spencer et al. (2007). The concept of the DLV method is to identify the members with zero stress under some specific loading patterns, namely the DLVs, derived from the changes in flexibility matrix of the structure before and after the damage state. The loading patterns can be obtained by simply performing the singular value decomposition (SVD) of the flexibility differential matrix. The DLVs, regarded as the sets of external static forces, are then applied to the structure. Those elements resulting with zero stresses (internal forces) under the DLV loads are considered potentially damaged (Bernal et al., 2004). The DLV technique is capable of identifying multiple damages in the structure via a truncated modal basis constructed from information of limited sensor locations without using a reference analytical model. Nevertheless, the success of the DLV method depends still on how well the realization of the flexibility matrix of the target structure is identified.

A structural damage detection method integrating the DLV method and ARMAV model for system identification of frame structures has been explored in this paper. The auto-regressive moving average vector (ARMAV) technique is one of the most promising techniques to make use of ambient vibration data. By means of ARMAV technique, modal analysis can be conducted for structures under unknown excitation forces, presumed to be random, such as wind gusts and traffic loads, which allow the fully automated real-time monitoring of the structure under in-service damage assessment. A Four-Storey Benchmark Structure, which instrumented by Wireless Smart Sensor Network and excited by ambient excitation is considered as the objective building. The damage condition of the Benchmark Structure is simulated by partially removing some of the diagonals. With the flexibility matrices of both the intact and damaged structure identified from acceleration responses of the structure, results indicate that the damaged locations can be successfully identified by the DLV method if sufficient modes of vibration are taken into account in the realization of the flexibility matrices.

SYSTEM IDENTIFICATION IN TIME-DOMAIN

To provide an accurate estimate of structural damage, the reliable identification of modal properties is a prerequisite. Although forced vibrations provide accurate quantitative modal information, the use of ambient loading constitutes an attractive alternative in terms of cost and simplicity. The auto-regressive moving average vector (ARMAV) technique is one of the most promising techniques to make use of ambient vibration data (Giraldo et al., 2009). By means of ARMAV technique, modal analysis can be conducted for structures under unknown excitation forces, presumed to be random, such as wind gusts and traffic loads, which allow the fully automated real-time monitoring of the structure under in-service damage assessment (Song et al., 2006).

Although several algorithms, such as ARX, ARMAX, ARMAV and … have been proposed to implement modal identification, in current studies ARMAV model has become a modulus apparatus in both system description and control design. After extensive evaluation, Auto-regressive moving average vector (ARMAV) model was applied for analysis of ambient excitation of multi-DOF's systems. This model only uses time series obtained from output signals of the system. It can be shown that the ARMAV model allows us to describe dynamic of structure subjected to filtered white noise. The parametric ARMAV (p,q) model is described by the matrix Eq.1 for a m-dimensional time series output y[n] and the time sampling interval of t (Piombo et al., 1993).

$$y[n] = \sum_{k=1}^{p} a_k y[n-k] + u[n] + \sum_{k=1}^{q} b_k u[n-k] \qquad (1)$$

Where, u[n] is a stationary zero-mean Gaussian white noise process, a_k and b_k are matrices of AR (auto-regressive) and MA (moving-average) coefficients, respectively. The AR part of order, p, describes the system dynamics while the MA part of order, q, is related to the external noise as well as to the white noise excitation. In this linear parametric model, the system output y[n] is supposed to be produced by a stationary Gaussian white noise input u[n]. In the state space, the ARMAV model can be demonstrated through Eq.2 and Eq.3. Where C is the observation matrix, A is a matrix containing the different coefficients of the auto-regressive part while $\underline{u}$[n] includes the moving-average terms of the ARMAV model (Piombo et al., 1993).

$$\underline{y}[n] = A\underline{y}[n-1] + \underline{u}[n] \tag{2}$$

$$y[n] = C\underline{y}[n] \tag{3}$$

Parameters of the model are estimated by the prediction error method. The vector θ is defined as Eq.4:

$$\theta = [a_1 a_2 \dots a_p b_1 b_2 \dots b_q]^T \tag{4}$$

As the systems are stochastic the output y[n] at time t_n cannot be determined exactly from data available at time t_{n-1}. Therefore ŷ[n|n-1,θ]is defined, the one-step ahead predicted system response at time t_n based on parameter θ and on the available data for t_{n-1} (Ljung L, 1987).

$$\hat{y}[n|n-1,\theta] = \sum_{k=1}^{p} a_k y[n-k] + \sum_{k=1}^{q} b_k \varepsilon[n-k|\theta] \tag{5}$$

The variable ε[n|θ] is the prediction error and is defined as:

$$\varepsilon[n|\theta] = y[n] - \hat{y}[n|n-1,\theta] \tag{6}$$

The variable ε[n|θ] thus represents the part of the output y[n] that cannot be predicted from the past data.

$$C = [I \;\; 0 \dots 0 \;\; 0] \tag{7}$$

$$A = \begin{bmatrix} 0 & I & \cdots & 0 & 0 \\ \vdots & & \ddots & \vdots & \\ 0 & 0 & \cdots & 0 & I \\ a_p & a_{p-1} & & a_2 & a_1 \end{bmatrix} \tag{8}$$

Let us define L, the matrix formed with the eigenvectors of A positioned as columns. The complex mode shapes stocked in matrix Φ are extracted from the matrix L as:

$$\Phi = CL \tag{9}$$

FLEXIBILITY BASED DAMAGE LOCALIZATION

The procedures of the DLV method proposed by Bernal (2002) are briefly summarized as the following:

(1) Determining the change in flexibility matrix (F_{Δ}) between the intact and damaged structures.

(2) Performing the singular value decomposition (SVD) of the flexibility differential matrix, F_{Δ}, to obtain the singular values and eigenvectors.

(3) Calculating the singular value normalized, svn_i, corresponding to each eigenvalue and screening out the damage locating vectors, L_i, whichever satisfy the empirical rule $svn_i \leq 0.20$. An index *svn* was proposed by Bernal (2002) and defined as:

$$svn_i = \sqrt{\frac{s_i c_i^2}{\max_k (s_k c_k^2)}} \qquad (10)$$

In which s_i = *i*th singular value of the matrix F_{Δ} , c_i = constant that is used to normalize the maximum stress in the structural element and $s_k c_k^2$ is the maximum value of all $s_i c_i^2$ $\forall$ *i*.

(4) Calculating the normalized cumulative stress index, nsi_i. Each of the DLVs is then applied to an undamaged analytical model of the structure. The stress in each structural element is calculated and a normalized cumulative stress is obtained. If an element has zero normalized cumulative stress, then this element is a possible candidate of damage. The normalized stress index for the *j*th element is defined as:

$$nsi_i = \frac{\sigma_j}{\max_k (\sigma_k)} \quad \text{where} \quad \sigma_j = \sum_{i=1}^{m} \mathrm{abs}\left(\frac{\sigma_{ij}}{\max_k (\sigma_{ik})}\right) \qquad (11)$$

In Eq. (11), σ_j = cumulative stress in the *j*th element; σ_{ij} = stress in the *j*th element induced by the *i*th DLV; and *m* = number of DLVs. In practice, the normalized cumulative stresses induced by the DLVs in the damaged elements may not be exactly zero due to noise and uncertainties. (The independent internal stresses in every element are reduced to a single value denoted as characterizing stress, σ to discriminate between large and small stresses. The characterizing stress is defined in such a way that the strain energy per unit length (or unit area or volume, in 2-D or 3-D elements, respectively) is proportional to σ^2. For a truss bar, for example, σ can be taken as the absolute value of the bar force, whereas for a planar prismatic beam element for which two end moments (M_i and M_j) exist, σ can be taken as $(M_i^2 + M_j^2 + M_i M_j)^{0.5}$, for beams and columns of frame structures.)

(5) Determining the vector of WSI_j (weighted stress index). The smaller value of WSI_j is a sign of the higher possibility of damage in the *j*th element of the structure. The weighted stress index for the *j*th element is defined as:

$$WSI = \frac{\sum_{i=1}^{ndlv} \frac{\overline{\overline{nsi_i}}}{svn_i}}{ndlv} \qquad (12)$$

Where $\overline{svn}$ = max(*svn*,0.015) and ndlv is the number of DLVs. The potentially damaged elements are those having WSI <1.

CONSTRUCTION OF STRUCTURAL FLEXIBILITY MATRIX

The flexibility matrix of a structure can be obtained by using the modal frequencies and mode shapes as (Pandey and Biswas, 1994):

$$\mathbf{F} = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \boldsymbol{\varphi}_i \boldsymbol{\varphi}_i^{\mathrm{T}} \tag{13}$$

Where φ_i is the *i*-th mode shape, ω_i is the *i*-th modal frequency; *n* is the number of the identified vibration modes of the structure.

Note that the modal matrix Φ =[φ_1 φ_2 …. φ_n] is normalized in accordance with $\Phi^T M \Phi = I$, in which *M* is the mass matrix of the structure. In this study, the modal frequencies and mode shapes are obtained by a time-domain ARMAV system identification technique as will be introduced in the next section.

DESCRIPTION OF THE BENCHMARK AND INSTRUMENTATION

A four-storey two-bay by two-bay shear building under ambient excitation is considered to demonstrate the application of the proposed algorithm. The Benchmark designed by ASCE Task Group at the University of British Columbia, Canada (UBC). The structure has dimension of 2.5X2.5 m in plan and height of 3.6 m. Wireless Smart Sensor Network (WSSN), has four wireless sensors in each floor that recorded out-put data as shown in Fig.1 (Johnson et al., 2004). The excitation is wind which loading at each floor in the y-direction as shown in Fig.2. The sections are designed for a scale model, with properties as given in Table.1 (Johnson et al., 2004). The MATLAB program was provided by the ASCE Task Group (Johnson et al., 2004) generates the input and output sampling data with time interval equal to 0.001s.

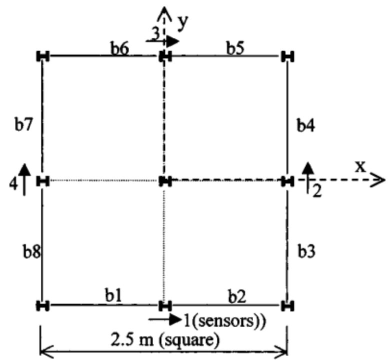


Figure 1. Floor plan (numbers shown are for level 1 and continue in same pattern in subsequent floors) (Johnson et al., 2004).

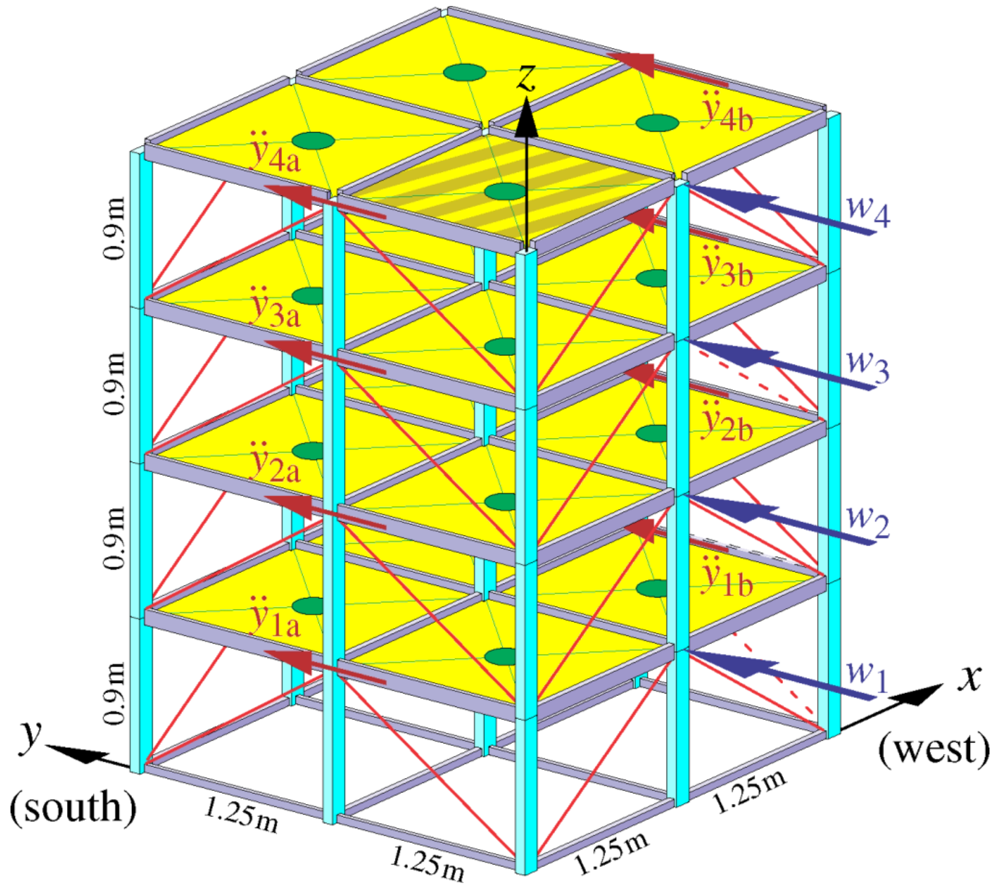


Figure 2. Diagram of model (The w_i are excitations and the $\ddot{y}_{ij}$ are accelerometer measurements) (Johnson et al., 2004).

Table 1. Properties of Structural Members

Property		*Columns*	*Floor Beams*	*Braces*
section type		B100x9	S75x11	L25x25x3
cross-sectional area	A [m^2]	$1.133\text{x}10^{-3}$	$1.43\text{x}10^{-3}$	$0.141\text{x}10^{-3}$
moment of inertia (strong direction)	I_y [m^4]	$1.97\text{x}10^{-6}$	$1.22\text{x}10^{-6}$	0
moment of inertia (weak direction)	I_z [m^4]	$.664\text{x}10^{-6}$	$.249\text{x}10^{-6}$	0
St. Venant torsion constant	J [m^4]	$8.01\text{x}10^{-9}$	$38.2\text{x}10^{-9}$	0
Young's Modulus	E [Pa]	$2\text{x}10^{11}$	$2\text{x}10^{11}$	$2\text{x}10^{11}$
Shear Modulus	G [Pa]	E / 2.6	E / 2.6	E / 2.6
Mass per unit volume	p [kg/m^3]	7800	7800	7800

NUMERICAL RESULTS

The identified modal parameters of the Benchmark discussed in this section. The first step of the identification procedure is the selection of the ARMAV model order. In order to validate the model order ARMAV(2,2), ARMAV(4,3), ARMAV(4,4), ARMAV(5,4) and ARMAV(6,6) were selected. As the results show, in compare with reported true mode shape in benchmark, the obtained results from model order ARMAV(6,6) gave the most accurate mode shapes. The four first mode shapes have been compared as follow:

$$\varphi_{ARMAV(6,6)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.911 & 0.292 & -0.585 & -1.464 \\ 0.694 & -0.708 & -0.813 & 1.222 \\ 0.383 & -0.984 & 0.864 & -0.449 \end{bmatrix}$$

$$\varphi_{ARMAV(5,4)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.926 & 0.310 & -0.607 & -1.486 \\ 0.718 & -0.693 & -0.740 & 1.343 \\ 0.403 & -1.007 & 0.797 & -0.529 \end{bmatrix}$$

$$\varphi_{true} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.907 & 0.313 & -0.573 & -1.425 \\ 0.690 & -0.689 & -0.825 & 1.215 \\ 0.379 & -0.998 & 0.825 & -0.463 \end{bmatrix}$$

Fig.3, Fig.4 and Fig.5 show the precision of ARMAV(4,4), ARMAV(5,4) and ARMAV(6,6) relative to the true mode shapes of the benchmark. Comparison of the true and identified values of mode shapes shows that the identified mode shapes using proposed algorithm are in a good compliance with true structural data. The accuracy can be also observed in all three modes of frequency.

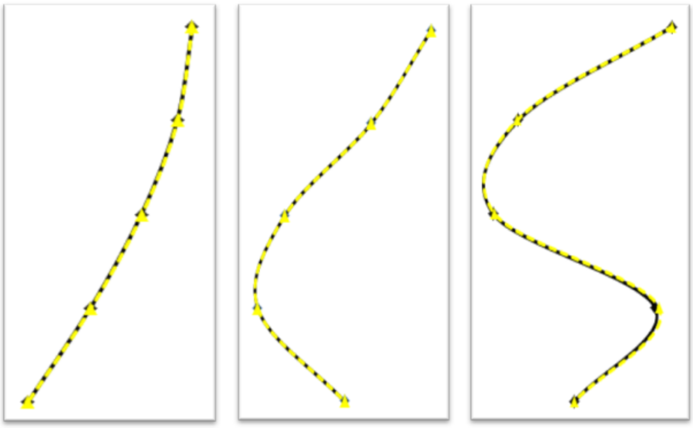


Figure 3. Comparing the first three mode shapes obtained from ARMAV(4,4) and true mode shapes of the structure (ARMAV(4,4):_ _ _ , true mode shapes:____)

Figure 4. Comparing the first three mode shapes obtained from ARMAV(5,4) and true mode shapes of the structure (ARMAV(5,4):_ _ _ , true mode shapes:____)

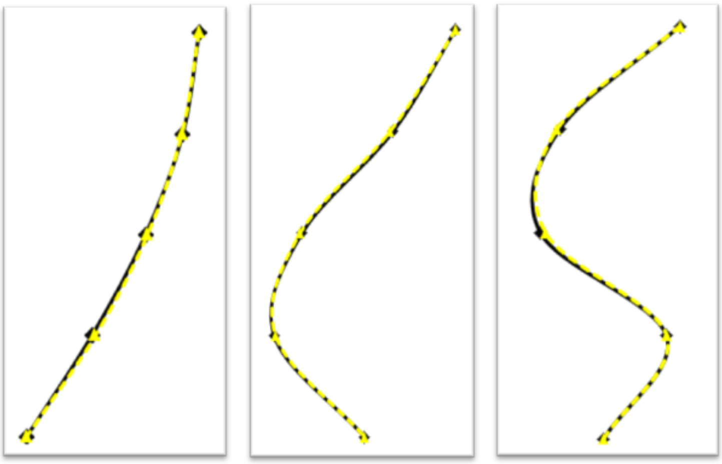


Figure 5. Comparing the first three mode shapes obtained from ARMAV(6,6) and true mode shapes of the structure (ARMAV(6,6):_ _ _ , true mode shapes:____)

In this section we present two cases with sufficient detail; these are fully braced structure (case 1, no damaged) and removed all braces on 1st floor (case 2, damaged). The identified modal frequencies and their corresponding mode shapes of the intact and damaged structures are summarized in Table. 2. It is seen that first four vibration modes are identified by using ARMAV model for the intact structure and damaged structure (in x and y directions). Note that the mode shapes have been normalized in accordance with $\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi} = \mathbf{I}$ where the mass matrix is (Johnson et al., 2004):

$$M = \begin{pmatrix} 3430 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3430 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2630 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2630 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2630 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2630 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1798 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1798 \end{pmatrix} (Kgf.s^2/m)$$

Table 2. Identified Modal Parameters of the Intact and Damaged Structure

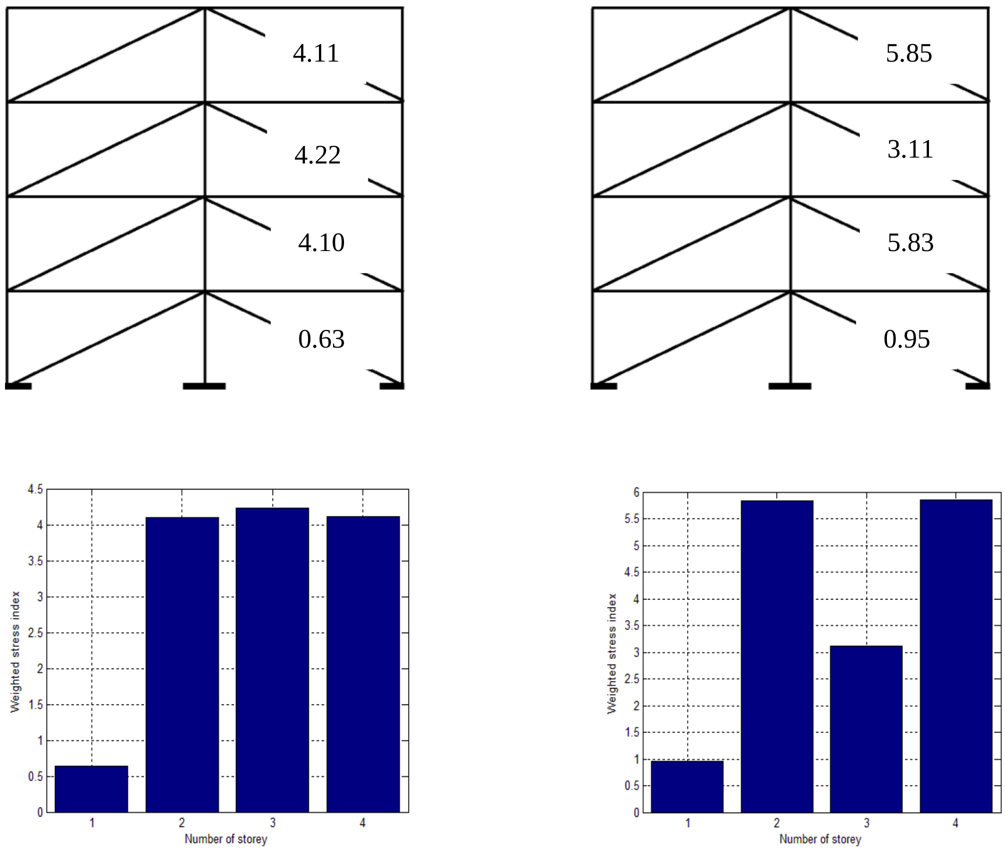
		Intact Structure				Damaged Structure			
Direction	Mode	1st	2nd	3rd	4th	1st	2nd	3rd	4th
x	Frequency(Hz)	11.75	29.89	48.46	59.89	9.89	28.78	47.35	59.73
	Floor 1	0.39	-1.04	1.58	-0.88	0.69	-1.92	1.33	-0.73
	Floor 2	0.73	-0.54	-1.20	1.79	0.76	-0.71	-1.34	1.72
	Floor 3	0.88	0.50	-0.84	-2.08	0.90	0.63	-0.06	-2.09
	Floor 4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
y	Frequency(Hz)	9.34	25.56	38.57	48.02	6.26	21.53	37.46	47.84
	Floor 1	0.43	-0.97	1.38	-0.91	0.71	-1.88	1.16	-0.86
	Floor 2	0.77	-0.44	-1.20	1.77	0.77	-0.50	-1.34	2.17
	Floor 3	0.91	0.54	-0.66	-2.11	0.90	0.75	-0.61	-2.56
	Floor 4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Since the damage is symmetrical we proceeded to combine the sensors in the x-x direction and discarded the response in y-y. The next step is to normalize the modes so that the flexibility proportional matrices can be computed. The flexibility proportional matrices are readily assembled using the normalized modes and the identified eigenvalues. The application of the DLV technique leads to the conclusion that there are 2 vectors and is;

$$F_\Delta = F_D - F_U = \begin{bmatrix} 1.0000 & 1.1753 & 1.1676 & 1.1783 \\ 1.1753 & 1.4662 & 1.8355 & 2.0510 \\ 1.1776 & 1.8355 & 2.3510 & 2.3036 \\ 1.1783 & 2.0510 & 2.3036 & 2.8888 \end{bmatrix}$$

$$\mathrm{DLV} = \begin{Bmatrix} -0.1027 & -0.0772 \\ 0.1007 & -0.2926 \\ -0.9303 & -0.5924 \\ 0.3374 & 0.7466 \end{Bmatrix}$$

A 3-D 8-DOF shear building model is used to represent the undamaged model of the structure and the characterizing stress is selected as the storey shears and the average of the end moments of the beams for the braced and the unbraced structure, respectively. Treating these vectors as loads on the system (order corresponds to the sensor numbering) and combining the results into the ‘weighted stress index’ WSI (which is a weighted combination of the stresses induced by the loads on the elements) one gets the results in Fig.6. As explained before, the WSI is defined in such a way that the potentially damaged elements are those for which WSI<1. As can be seen from Fig.6, the damage is correctly localized as being in level 1.



x-x *y-y*

Figure 6. WSI index for the stories in the x-x and y-y frames.

CONCLUSIONS

As the structure is excited by ambient excitation, where the excitation cannot be measured, auto-regressive moving average vector (ARMAV) algorithm was selected in this study. First of all, the ARMAV(6,6) was determined as a most accurate order to estimate mode shapes. In order to verifying this statement, the mode shapes obtained from different order of ARMAV were compared with true mode shapes. After identification of system parameters by using ARMAV algorithm, a flexibility-based technique was described to locate linear damage. The proposed method integrated the DLV method and the time domain ARMAV model identification technique for damage detection of frame structures. The theory of DLV method is itself elegant and sound without doubts, nevertheless, a sufficient realization of the flexibility matrix is the key to make it a perfect tool in damage localization. The feasibility of proposed method has been verified by application of this algorithm in the case of ASCE SHM task group benchmark structure.

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