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RESPONSE VALUES IN HYSTERESIS PROPERTIES BY ACCELERATION RECORDS

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ABSTRACT

This paper describes the response values of acceleration records by earthquakes. Recent years, many agency or institutes disseminated the acceleration records on ground. In order to know generally how much displacement took place in buildings by earthquakes, the response values, calculated by non-linear dynamic analysis, are helpful for seismic structural design.

Non-linear dynamic analysis has many options in Bi-linear model, Tri-linear model, the base shear coefficient, the reduction ratio of stiffness, the period of the storey and others.

Therefore this paper shows the spectral results of non-linear dynamic analysis in the Single - Degree-of-Freedom (SDOF). The period is from 0.1 (sec) to 5.0 (sec), and the shear coefficient of the yielding point is equal to 0.2 or 0.1. The hysteresis property is Bi-linear model, Peak Oriented Tri-linear model and others.

And the predominant periods in the analysis of Multi-Degree-of-Freedom-System (MDOF) are compared with the periods by the eigenvalues.

INTRODUCTION

The following is the overview of the inelastic dynamic analysis method. The equation of motion about SDOF is the following Eq. (1).

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{y} \tag{1}$$

where
m : mass (N/(cm/sec^2))
c : damping factor (N/(cm/sec))
k : stiffness (N/cm)
x : relative displacement (cm)
$\dot{x}$: relative velocity (cm/sec)
$\ddot{x}$: relative acceleration (cm/sec^2)
$\ddot{y}$: horizontal acceleration record on the ground surface (cm/sec^2)

Fig. 1 shows the hysteresis property of the Peak Oriented Bi-linear Model which is simplest analysis models for building structures and similar to these structural property. The figure of the hysteresis property shows the relationship between the shear coefficient and the displacement. The

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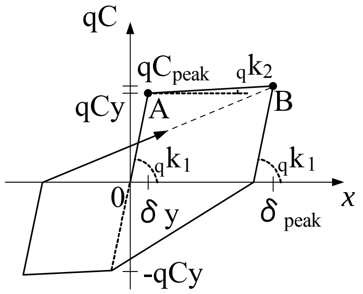
shear coefficient is 0.1, or 0.2, the stiffness degrading ratio is 1/1000, and the damping ratio is 0.05. The initial stiffness and the weight are used when the period T is calculated (Eq. (2)). In this analysis, each case has the period from 0.1 to 5.0 (sec) in about 30 cases.

$$T = 2\pi\sqrt{\frac{m}{k}} = \frac{2\pi}{\sqrt{{}_{q}k_{1} \cdot g}} \tag{2}$$



Notes)
(1) qC : Shear Coefficient
$\left(qC = \frac{\text{Storey Shear Force (Q)}}{\text{Weight } (m \cdot g)} \right)$
(2) qCy : Shear Coefficient at the yielding point of A
(qCy = 0.2, or 0.1)
(3) qC_{max} : Shear Coefficient at the peak of B
(4) Q : Storey Shear Force (N) ($= k \cdot x$)
(5) ${}_{q}k_{1}$, ${}_{q}k_{2}$: Stiffness Coefficient (1/cm)
$\left({}_{q}k_{1} = \frac{\text{Initial Stiffness } k_{1}}{\text{Weight } (m \cdot g)} \right)$
(6) Stiffness Degrading Ratio : 1/1000
$\left({}_{q}k_{2} = \frac{1}{1000} \times {}_{q}k_{1} \right)$
(7) h : Damping Coefficient (= 0.05)
(8) g : Gravity Acceleration (=980 (cm/sec^{2}))

Figure 1. Hysteresis property of Peak Oriented Bi-linear Model

ACCELERATION RECORDS

The acceleration records for the analysis of qCy=0.1, are recorded in ;

2011 New Zealand Christchurch Earthquake, 2010 Chile Earthquake,
2009 Italy L'Aquila Earthquake, 2005 Pakistan Earthquake,
2003 Algeria Earthquake, 2001 El Salvador Earthquake,
1995 Hyogo-ken Nanbu Earthquake, 1994 Northridge Earthquake,
1985 Michoacan Mexico Earthquake, and 1940 Imperial Valley Earthquake.

They are 10 earthquakes and 13 acceleration records. These are the highest peak acceleration records in large earthquakes and others.

In 2011 New Zealand Christchurch Earthquake, the acceleration record is located in building. In 2010 Chile earthquake, the 4 acceleration records which calculate larger response displacement are also included.

The acceleration records for the analysis of qCy=0.2, are recorded in ;

2011 Great East Japan Earthquake, 2007 Niigata-ken Chuetsu-Oki Earthquake,
2004 Niigata-ken Chuetsu Earthquake, 1995 Hyogo-ken Nanbu Earthquake, and
BCJ Level2(Artificial Earthquake Acceleration Data on Bedrock).

They are 4 earthquakes, and 10 acceleration records. In 2011 Great East Japan Earthquake, the acceleration records, which are located in buildings and measured by BRI, are also included.

This value of qCy is nearly equal to the values for static analysis of the seismic design codes in Japan, Mexico, USA and other countries.

RESPONSE VALUES BY INELASTIC DYNAMIC ANALYSIS

By the inelastic dynamic analysis in the section "Introduction", the response values are calculated. Fig. 2-1 and Fig. 2-2 show the analysis results in response displacements spectra and ductility spectra. The response ductility μ peak is calculated, by the peak displacement δ peak divided by the yielding displacement (Eq. (3)). The period of structures is considered to be about 0.5 - 1.0 (sec) for low-rise buildings. When the height of a storey of building structure is 3 meters and the storey drift by earthquake loads is 1/30 or 1/60 (rad.), the horizontal displacement in a storey is 10 or 5 (cm).

$$\mu \text{ peak} = \frac{\delta_{peak}}{\delta y} \qquad (3)$$

The response displacements of SDOF can help us to understand how much displacement took place in storeys of structures during earthquake. According to Fig. 2-1, the response displacement more than 5 cm in period from 0.5 to 1.0 are analysed in the 9 acceleration records of ;

[2011 HVSC S26W, GeoNet], [2010 CCSP NS, UCS], [2010 Cons EW, UCS],
[2010 Conc NS, UCS], [2009 AQV EW, Itaca], [2001 La Libertad NS, UCA],
[2005 Abbottabad EW, MSSP], [1995 Kobe NS, JMA], and [1994 Tarzana EW, COSMOS].

The displacement more than 10 cm in period from 0.5 to 1.0 are analysed in the 4 acceleration records of ;

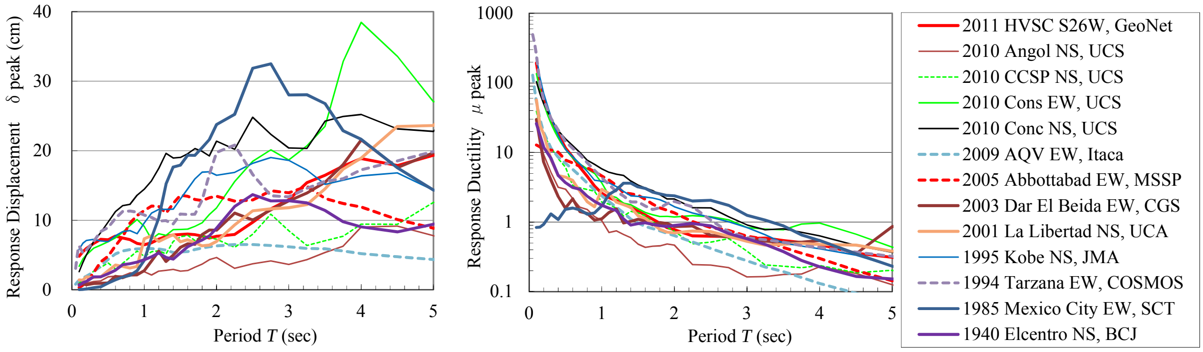
[2010 Cons EW, UCS], [2010 Conc NS, UCS], [2005 Abbottabad EW, MSSP], and
[1994 Tarzana EW, COSMOS].

And the response ductility also can help us consider how much ductility is required. At 1985 Mexico City, the response ductility are at most 3 in the period of 1.5 (sec) while the middle - rise buildings were collapsed. At the other acceleration records, the response ductility are higher than 3 in the period of less than 1.5 (sec).

According to Fig. 2-2, the response displacement more than 5 cm in period from 0.5 to 1.0 are analysed in the 9 acceleration records of ;

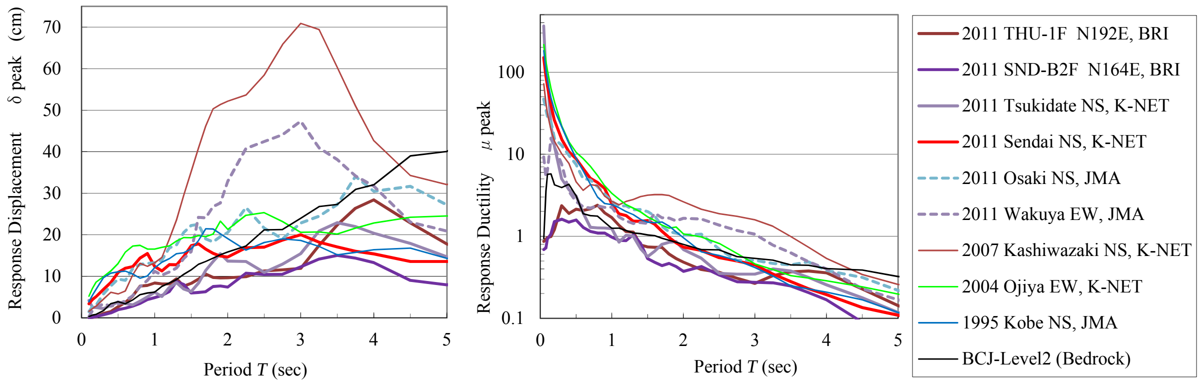
[2011 THU-1F N192E, BRI], [2011 Tsukidate NS, K-NET], [2011 Sendai NS, K-NET],
[2011 Osaki NS, JMA], [2011 Wakuya EW, JMA], [2007 Kashiwazaki NS, K-NET],
[2004 Ojiya EW, K-NET], [1995 Kobe NS, JMA], and [BCJ Level2 (Bedrock)].

The displacement more than 10 cm in period from 0.5 to 1.0 are analysed in the 6 acceleration records of ;
[2011 Sendai NS, K-NET], [2011 Osaki NS, JMA], [2011 Wakuya EW, JMA],
[2007 Kashiwazaki NS, K-NET], [2004 Ojiya EW, K-NET], and [1995 Kobe NS, JMA].



(1) Response Displacement Spectra (2) Response Ductility Spectra

Figure 2-1. Inelastic Response Spectra (qCy=0.1, *h*=0.05)



(1) Response Displacement Spectra (2) Response Ductility Spectra

Figure 2-2. Inelastic Response Spectra (qCy=0.2, *h*=0.05)

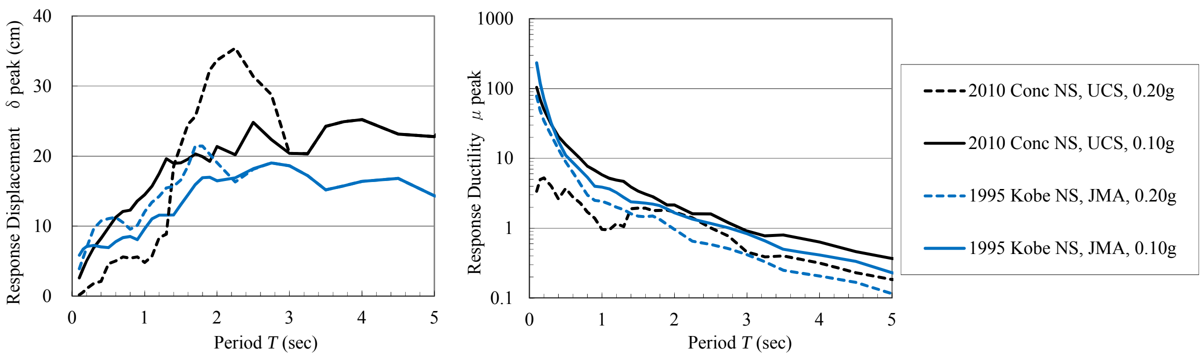
And the response ductility is required to be less than 2 by the time history analysis in the building code for high rise buildings in Japan. The response ductility more than 2 in periods from 0.1 to 5.0 are analysed in the 9 acceleration records without [2011 SND-B2F N164E, BRI] among the 10 acceleration records.

COMPARISON IN SHEAR COEFFICIENTS

In the previous section, the response values of [1995 Kobe NS, JMA] are analysed when qCy is 0.1 and 0.2 in order to know the vulnerability in the low seismic buildings. In this section, the response values when qCy is 0.1 or 0.2 are compared by [1995 Kobe NS, JMA] and [2010 Conc NS, UCS]. The response values of [2010 Conc NS, UCS] are analysed when qCy is 0.1 or 0.2 because the static analysis of the seismic design codes in Chile describes clearly the specifications of more than 0.2 for qCy, and in Fig. 2-1(2), the response ductility of [2010 Conc NS, UCS] is higher than the others around the period 1 (sec).

The comparison of the response displacements or the response ductility between the shear coefficient (qCy) of 0.1 or 0.2 is executed. Any ductility when qCy is 0.2, is less than those when qCy is 0.1, even if some of the displacement for qCy=0.2 is larger than those for qCy=0.1.

It means that the higher base shear coefficient almost gives lower response ductility.



(1) Response Displacement Spectra (2) Response Ductility Spectra

Figure 3. Inelastic Response Spectra (qCy=0.1, or 0.2, and *h*=0.05)

COMPARISON IN HYSTERESIS PROPERTIES

The building structures have their hysteresis properties during earthquakes loads.

In the above sections, the hysteresis property is Peak Oriented Bi-linear Model. In this section, the 4 hysteresis properties are used in inelastic dynamic analysis. They are shown in Fig. 4-1 ~ Fig. 4-

4. The hysteresis properties should be modified by some experimental results, but Fig. 4-1 ~ Fig. 4-4 show the simple hysteresis properties of Bi-linear model and Tri-linear model for SDOF in many periods defined by the secant stiffness at the yielding point and the weight. To compare the results of Bi-linear model and Tri-linear model in this analysis, the secant stiffness, and the stiffness after the yielding point, is same in these 4 hysteresis properties. Therefore, the initial stiffness of Bi-linear model ${}_q k_1$ is equal to the secant stiffness of Tri-linear model ${}_q ky$. And the stiffness after the yielding point of Bi-linear model ${}_q k_2$ is equal to the stiffness after the yielding point of Tri-linear model ${}_q k_3$.

In Fig. 4-1, Hisada Model (1962) is Peak oriented Tri-linear model. The stiffness of unloading is same as the initial stiffness. It has options of the shear coefficient in the crack point qCc and the stiffness degrading ratio before the yielding point. These options are described in this analysis according to the experimental results of reinforced concrete beams which usually show the Eq. (4) and Eq. (5).

$$\mathrm{qCc} = 0.2\ \mathrm{qCy} \tag{4}$$

$${}_q k_2 = \frac{{}_q k_1}{6} \tag{5}$$

The Eq. (4) and the Eq. (5) make the relationship between ${}_q k_1$, which is the initial stiffness divided by the weight, and the secant stiffness at the yielding point $\frac{\mathrm{qCy}}{\delta y}$ by Eq. (6).

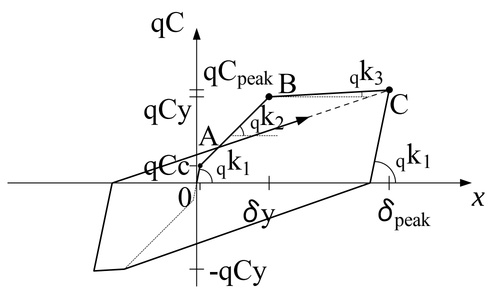
$${}_q k_1 = 5 \cdot \frac{\mathrm{qCy}}{\delta y} \tag{6}$$

The stiffness degrading ratio after the yielding point is defined as Eq. (7).

$${}_q k_3 = \frac{1}{1000} \frac{\mathrm{qCy}}{\delta y} \tag{7}$$

The secant stiffness at the yielding point $\frac{\mathrm{qCy}}{\delta y}$ of Tri-linear model in Fig. 4-1 gives the period *Ty* of this Hisada Model is defined as the Eq. (8), same as Eq. (2).

$$Ty = 2\pi\sqrt{\frac{m}{ky}} = \frac{2\pi}{\sqrt{{}_q ky \cdot g}} = 2\pi\sqrt{\frac{\delta y}{\mathrm{qCy} \cdot g}} \tag{8}$$



Hisada Model(1962) (Peak oriented Tri-linear model. The stiffness of unloading is same as the initial stiffness)

qCc=0.2·qCy, $\frac{\mathrm{qCy}}{\delta y} = \frac{ky}{mg}$ (same in 4 cases)

${}_q k_1 = 5 \cdot \frac{\mathrm{qCy}}{\delta y}$ ${}_q k_2 = \frac{{}_q k_1}{6}$

${}_q k_3 = \frac{1}{1000} \frac{\mathrm{qCy}}{\delta y}$

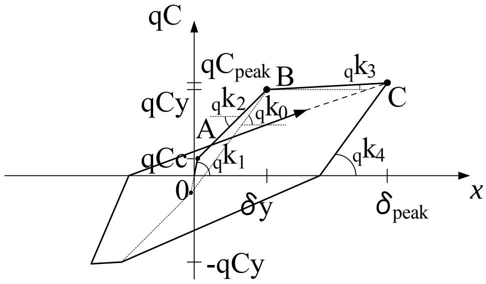
Figure 4-1. Hysteresis property of Tri-linear Model 1

In Fig. 4-2, Takeda Model (1970) has the oriented point before the yielding point is the crack point in the opposite. The stiffness ${}_{q}k_4$ of unloading after the yielding point has some relationship with the stiffness ${}_{q}k_0$ between the yielding point and the crack point in the opposite side by Eq. (9).

$$ {}_{q}k_4 = {}_{q}k_0 \left(\frac{\delta y}{\delta \text{ peak}} \right)^{0.4} \tag{9} $$

The other symbols of qCc, ${}_{q}k_1$, ${}_{q}k_2$, ${}_{q}k_3$, and *Ty* are calculated by Eq. (4) ~ Eq. (8).

İn Fig. 4-3, Jenkin Model (1922) is Bi-linear model. The initial stiffness divided by the weight ${}_{q}k_1$, and the secant stiffness at the yielding point $\frac{\text{qCy}}{\delta y}$ is defined by Eq. (10).



Takeda Model(1970) (The oriented point before the yielding point is the crack point in the opposite. The stiffness of unloading after the yielding point ; ${}_{q}k_4 = {}_{q}k_0 \left(\frac{\delta y}{\delta \text{ peak}} \right)^{0.4}$)

$$ {}_{q}k_1 = 5 \cdot \frac{\text{qCy}}{\delta y} \qquad {}_{q}k_2 = \frac{{}_{q}k_1}{6} $$

$$ {}_{q}k_3 = \frac{1}{1000} \frac{\text{qCy}}{\delta y} $$

Figure 4-2. Hysteresis property of Tri-linear Model 2

In Fig. 4-3, Jenkin Model (1922) is Bi-linear model. The initial stiffness divided by the weight is calculated by the yielding point.

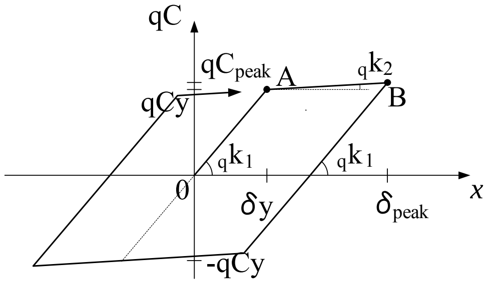
$$ {}_{q}k_1 = \frac{\text{qCy}}{\delta y} \tag{10} $$

The stiffness degrading ratio after the yielding point is defined as Eq. (11).

$$ {}_{q}k_2 = \frac{{}_{q}k_1}{1000} \tag{11} $$

The period *T* of this Jenkin Model is defined as the Eq. (12), same as Eq. (2).

$$ T = 2\pi \sqrt{\frac{m}{k}} = \frac{2\pi}{\sqrt{{}_{q}k \cdot g}} = 2\pi \sqrt{\frac{\delta y}{\text{qCy} \cdot g}} \tag{12} $$

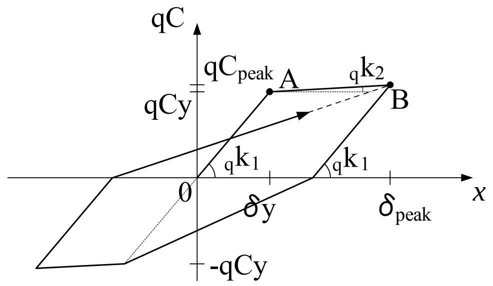


Jenkin Model(1922) (Bi-linear model)

$$ {}_{q}k_1 = \frac{\text{qCy}}{\delta y} \left(= \frac{k}{mg} \right) \qquad {}_{q}k_2 = \frac{{}_{q}k_1}{1000} $$

Figure 4-3. Hysteresis property of Bi-linear Model 1

In Fig. 4-4, Clough Model (1966) is the Peak oriented Bi-linear model which is same as Fig. 1. The stiffness of unloading is same as the initial stiffness. The symbols of ${}_q k_1$, ${}_q k_2$,and T is calculated by Eq. (9) ~ Eq. (11).



Clough Model(1966) (Peak oriented Bi-linear model.)

$$ {}_q k_1 = \frac{qCy}{\delta y} \left(= \frac{k}{mg} \right) \qquad {}_q k_2 = \frac{{}_q k_1}{1000} $$

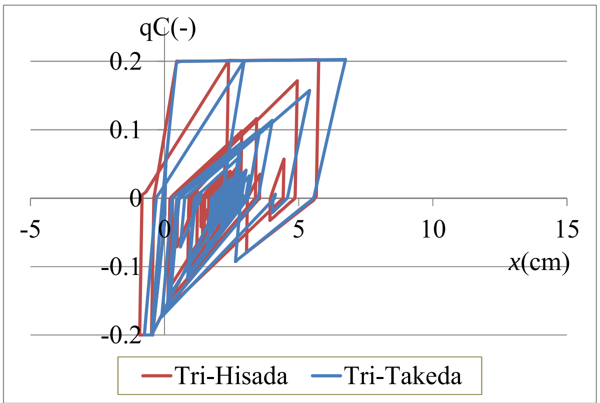
Figure 4-4. Hysteresis property of Bi-linear Model 2

RESULTS OF ANALYSIS FOR COMPARISON IN HYSTERESIS PROPERTIES

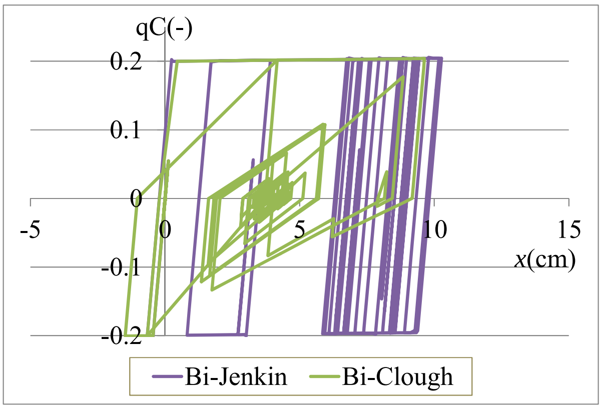
The results of inelastic dynamic analysis for the comparison in the 4 hysteresis properties when the period is 0.3 (sec) is show in Fig. 5. The shear coefficient at the yielding point is 0.2.

All results of 4 hysteresis properties show the large displacement to the North side 2 times.

Among the results of the 4 hysteresis properties, the minimum peak of the response displacement is the result of Tri-Hisada Model. The next larger peak is one of Tri-Takeda Model. The third one is Bi-Clough Model, and the fourth is Bi-Jenkin Model.



(1)Results of Tri-linear models



(2)Results of Bi-linear models

Notes)

Shear Coefficient at the yielding point :
$qCy = 0.2$

Period by the secant stiffness at the yielding point :

$$ Ty = 2\pi\sqrt{\frac{m}{k}} = \frac{2\pi}{\sqrt{{}_q k \cdot g}} = 2\pi\sqrt{\frac{\delta y}{qCy \cdot g}} = 0.3 \text{ (sec)} $$

Peak values of the response displacement :
(Tri-Hisada) < (Tri-Takeda) < (Bi-Clough) < (Bi-Jenkin)

Figure 5. Comparison of the relationship between the response shear coefficient and the response displacement in Hysteresis Properties (qCy= 0.2, *h*=0.05)

These differences in the results of the peak response displacement are considered to be made by the resonant shaking between the period by the stiffness on the hysteresis loop of the models and the predominant period of the acceleration record. The peak of Tri-Hisada Model is the minimum because the stiffness of unloading is larger than the stiffness of Tri-Takeda Model, and the period by the stiffness is far from the predominant period of the acceleration record. Moreover, the periods by the stiffness of loading and unloading of Tri-linear models are considered to be different from those of Bi-linear models, or the predominant period of the acceleration record.

The peak of Bi-Jenkin Model is the maximum because the response displacement is changed largely only when the response shear coefficient is more than the positive qCy or less than the negative qCy. On the other hand, the peak of Bi-Clough Model is less than one of Bi-Jenkin Model because Bi-Clough Model has the property of the peak oriented which makes the response displacement smaller.

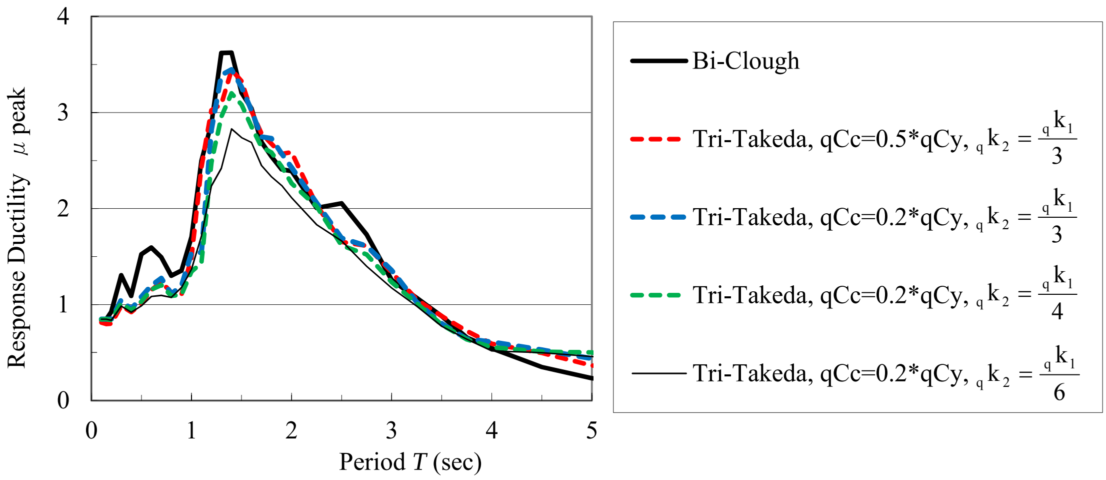
COMPARISON IN TRI-TAKEDA MODEL

According to the previous section, each hysteresis property has its response values. In the structural analysis, Bi-linear models are used for steel structures and Tri-linear models are used for RC structures, usually.

Among the acceleration records which earthquakes brought severe damages, the smaller peak acceleration record should analyse the larger results, when the larger peak acceleration record usually analyse the large ones.

In 1985 Michoacan Mexico Earthquake, some middle rise reinforced concrete buildings in Mexico City were collapsed severely and many storeys were collapsed completely even if the peak acceleration was 167.9 (cm/sec^2) which was 0.17 in shear coefficient. It is considered that a specified period waves were predominant on these RC structures

So, in this section, the acceleration record of [1985 Mexico City EW, SCT] is analyzed in Tri-Takeda Model. The response ductility spectra by the same hysteresis poperty in Eqs (4) ~ (7) and (9) gives the smaller response ductility spectra than those of Bi-Clough Model in Fig. 2-1. İt is the same results in the previous section for the specific period. Fig. 6 shows the comparison of these response dutility spectra.



Notes) (1) Acceleration Record : [1985 Mexico City EW, SCT]

(2) qCy=0.1, h=0.05

(3) $_qk_2 = \frac{_qk_1}{1000}$ in Bi-Clough Model, $_qk_3 = \frac{1}{1000}\frac{qCy}{\delta y}$ in Tri-Takeda Model

(4) The shear coefficients at the crack point are 2 cases of (qCc=0.2*qCy, and 0.5*qCy)

(5) The second stiffness degrading ratios of Tri-linear model are 3 cases of ($_qk_2 = \frac{_qk_1}{6}$, $_qk_2 = \frac{_qk_1}{4}$ and $_qk_2 = \frac{_qk_1}{3}$)

Figure 6. Comparison of Response Ductility Spectra in Bi-Clough Model and Tri-Takeda Model

In Fig. 6, the range of the period in Bi-Clough Model, when the response ductility is more than 2, is from 1.1(sec) to 2.5(sec), and the maximum response ductility is 3.6 when the period is 1.4(sec).

The range of the period in [Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/6], when the response ductility is more than 2, is from 1.2(sec) to 2.0(sec), and the maximum response ductility is 2.8 when the period is 1.4(sec).

In the section "Comparison In Hysteresis Properties", the hysteresis property of [Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/6] is based on the experimental results of reinforced concrete beams, but this property maybe not be matching to the property of storeys in RC structures which collapsed completely in 1985 Michoacan Mexico Earthquake.

In order to know what is the hysteresis property of storeys in RC structures, other analysis are carried out when the shear coefficient at the crack point qCc is (0.5*qCy), and the second stiffness degrading ratios of Tri-linear model ${}_q k_2$ are $\frac{{}_q k_1}{4}$ and $\frac{{}_q k_1}{3}$.

These response ductility spectra are also shown in Fig. 6. Totaly, it shows the 5 response ductility spectra of ;

[Bi-Clough],
[Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/3],
[Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/6].
[Tri-Takeda, qCc=0.5*qCy, (qk2)=(qk1)/3],
[Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/4], and

The range of the period in [Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/4], when the response ductility is more than 2, is from 1.2(sec) to 2.25(sec), and the maximum response ductility is 3.2 when the period is 1.4(sec).

The range of the period in [Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/3], when the response ductility is more than 2, is from 1.2(sec) to 2.25(sec), and the maximum response ductility is 3.4 when the period is 1.4(sec).

The range of the period in [Tri-Takeda, qCc=0.5*qCy, (qk2)=(qk1)/3], when the response ductility is more than 2, is from 1.1(sec) to 2.25(sec), and the maximum response ductility is 3.4 when the period is 1.4(sec).

Therefore, the results of [Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/4] are larger than those of [Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/6] and less than those of [Bi-Clough Model] in the almost periods.

The larger results are [Tri-Takeda, qCc=0.5*qCy, (qk2)=(qk1)/3] or [Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/3], and they are near to the results of [Bi-Clough]. Among these 2 cases of the hysteresis properties, qCc are (0.5*qCy) or (0.2*qCy), and the second stiffness degrading ratios of Tri-linear model ${}_q k_2$ is $\frac{{}_q k_1}{3}$ commonly. İt shows that ${}_q k_2$ has much effect to the results than qCc.

By the way, when qCy is 0.2 and the hysteresis property is [Tri-Takeda, qCc=0.2*qCy, (qk2)=(qk1)/6], the response displacement is less than those of qCy=0.1. The range of the period, when the response ductility is more than 1, is from 1.9(sec) to 2.25(sec), and the maximum response ductility is 1.13 when the period is 2.25(sec). The qCy also has much effect to the results.

PREDOMINANT PERIOD OF MDOF ANALYSIS

In the above sections, the SDOF analysis is discussed. In this section, the Multi-Degree-of-Freedom-System (MDOF) analysis is discussed.

In the SDOF analysis in Eq. (1), the mass, the damping factor, and the stiffness can give the predominant period, and the response values. In the MDOF analysis, after the mass matrix $[M]$, the damping matrix $[C]$, and the stiffness matrix $[K]$ can give the eigenvalues, the modes and others, the MDOF is translated to a series of the SDOF equations, and the response values are analyzed. Therefore, in the MDOF analysis, the predominant periods of the global MDOF system are not discussed.

So, the predominant periods of the MDOF analysis are also analyzed and the dynamic amplification ratios are shown. The analysis method is the assumption that, as well as the periodic motion $\ddot{y}$ ($= p\sin\omega t$) in the external force of SDOF analysis in Eq. (13), the system of the periodic motion in the external force of MDOF analysis is applied in Eq. (14) and the system of the response values is also applied to be the periodic motion in Eq. (15).

$$m\ddot{x} + c\dot{x} + kx = -mp\sin\omega t \tag{13}$$

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{\ddot{y}\} \tag{14}$$

where

$[M]$, $[C]$, $[K]$: n-by-n square matrix

$\{\ddot{x}\}$, $\{\dot{x}\}$, $\{x\}$: n-by-1 column matrix

$\{\ddot{y}\}$: n-by-1 column matrix for external force ($\ddot{y} = p\sin\omega t$) (cm/sec^2)

p : constant value (cm/sec^2)

ω : angular frequency of the periodic motion (radian/sec)

Eq. (15) is the particular solution of Eq. (14) by the method of undetermined coefficient which has the assumption that $\{x\}$ on i-th storey is linear combinations of shaking functions < $\{x_i\}$ = $\{c_{1i}\sin\omega t + c_{2i}\cos\omega t\}$ >, and the each element of $\{x_i\}$ includes the periodic motion in $\{\ddot{y}_{\phi\, i}\}$ calculated by the shaking functions with the angular frequency ω and the phase lag ϕ_i on i-th storey. The dynamic amplification matrix $[A]$, which doesn't include any shaking function, is calculated by $[M]$,$[C]$,$[K]$, and ω. The multiple of $[A]$ and $[M]$ can be amplitudes of the periodic motion $\{\ddot{y}_{\phi\, i}\}$. Therefore, Eq. (15) gives the predominant periods of the global MDOF by the spectral analysis of ω.

$$\{x_i\} = [A][M]\{\ddot{y}_{\phi\, i}\} \tag{15}$$

where

$[A]$: n-by-n square matrix. $[A]$ is a part of the particular solution and also the dynamic amplification matrix, which is calculated by $[M]$, $[C]$, $[K]$, and ω

$\{\ddot{y}_{\phi\, i}\}$: n-by-1 column matrix for a periodic motion part of the particular solution with the phase lag ϕ_i on i-th storey, { $\ddot{y}_{\phi\, i} = p\sin(\omega t + \phi_i)$ } (cm/sec^2)

In the analysis of SDOF, the complementary solution of Eq. (13), by the assumption that the external force is zero, can give the predominant period $\omega_{sgl} = \sqrt{k/m}$, and the particular solution by the method of undetermined coefficient of Eq. (13) can give the dynamic amplification ratio $\{\mu_S\}$ explicitly by the ratio of the dynamic amplification to the static one in Eq. (16).

$$\{\mu_S\} = \frac{k}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \tag{16}$$

However, in the analysis of MDOF, the dynamic amplification matrix $[A]$ of Eq. (15) doesn't explicitly show the dynamic amplification ratio, likely Eq. (16) of SDOF. Moreover, the complementary solution of Eq. (14), can give the eigenvalues and the eigenvectors. These eigenvalues are considered to be the square of the angular frequency in the solutions and usually the high level modes are negleted in the analysis.

In order to know what the eigenvalues are in the analysis of MDOF, one example of a stucture data is as follows. The structure is the 2 storeys wooden house with the roof truss and the bricks footing. İt is assumed to be the shear system of 4 mass. The matrix equation of motion in forced shaking with external damping is shown in Eq. (17). The external forces are the periodic motions.

Fig. 7 and Table 1 show the analysis model, and the stiffness k_i and the period Ti of each 4 storeys. These values can give the dynamic amplification $\{x_{DA}\}$ and the static one $\{x_{SA}\}$ in Eq. (18-1) and Eq. (18-2). The ratio $\{\mu_D\}$ of the dynamic amplification to the static one is shown in Eq. (19).

$$\begin{bmatrix} m_3 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_0 \end{bmatrix} \begin{Bmatrix} \ddot{x}_3 \\ \ddot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_0 \end{Bmatrix} + \begin{bmatrix} c_3 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & c_1 & 0 \\ 0 & 0 & 0 & c_0 \end{bmatrix} \begin{Bmatrix} \dot{x}_3 \\ \dot{x}_2 \\ \dot{x}_1 \\ \dot{x}_0 \end{Bmatrix} + \begin{bmatrix} k_3 & -k_3 & 0 & 0 \\ -k_3 & k_3+k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2+k_1 & -k_1 \\ 0 & 0 & -k_1 & k_1+k_0 \end{bmatrix} \begin{Bmatrix} x_3 \\ x_2 \\ x_1 \\ x_0 \end{Bmatrix} = -\sin\omega t \begin{bmatrix} m_3 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_0 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix} \quad (17)$$

where

Period of i-th Storey : $T_i = 2\pi\sqrt{\dfrac{\sum_{j=i}^{n} m_j}{k_i}}$ [sec]

Damping Factor of i-th Storey : $c_i = 2h\omega_i m_i = 2hm_i\sqrt{\dfrac{k_i}{\sum_{j=i}^{n} m_j}}$ $\left[\mathrm{N}\dfrac{\mathrm{sec}}{\mathrm{cm}}\right]$

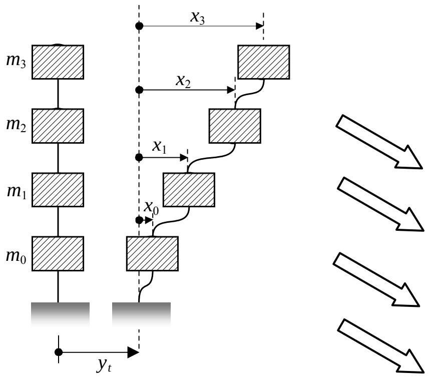


Figure 7. Analysis Model for Shear System of 4 Mass

Table 1. Example Values of Stiffness and Period in each Storey for Shear System of 4 Mass

Storey	Stiffness k_i (kN/cm)	Period T_i (sec)
3 (roof truss)	69.3	0.56
2	17.3	1.70
1	21.3	2.07
0 (footing)	262	0.70

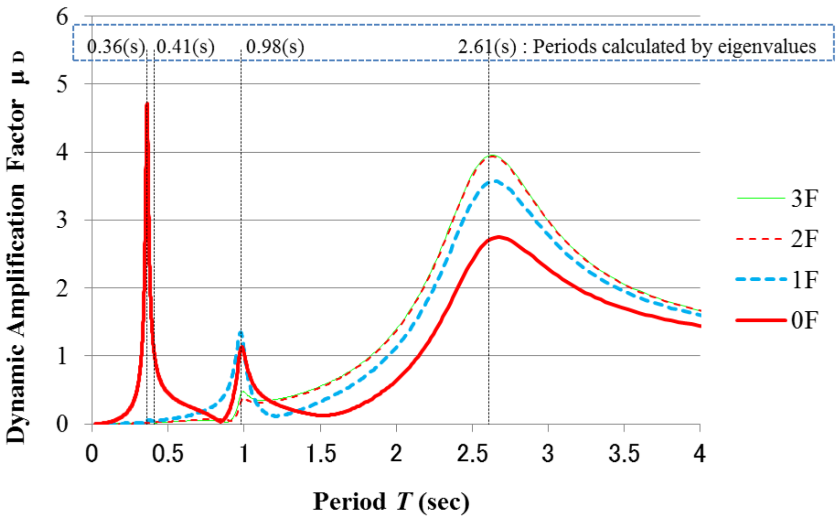


Figure 8. Spectra of Dynamic Amplification Ratio μ_D

$$\{x_{DA}\} = [A][M]\{1\} = \begin{Bmatrix} x_{DA3} \\ x_{DA2} \\ x_{DA1} \\ x_{DA0} \end{Bmatrix} \quad (18\text{-}1),$$

$$\{x_{SA}\} = [K]^{-1}[M]\{1\} = \begin{Bmatrix} x_{SA3} \\ x_{SA2} \\ x_{SA1} \\ x_{SA0} \end{Bmatrix} \quad (18\text{-}2)$$

$$\{\mu_D\} = \begin{Bmatrix} x_{DA3}/x_{SA3} \\ x_{DA2}/x_{SA2} \\ x_{DA1}/x_{SA1} \\ x_{DA0}/x_{SA0} \end{Bmatrix} \tag{19}$$

Fig. 8 shows the spectra of μ_D on each floor in each period T ($= 2\pi/\omega$). The values "0.36(s), 0.41(s), 0.98(s) and 2.61(s)" at the upper level in Fig. 8 are the periods calculated by the 4 eigenvalues. At the results, it is cleared that the predominant periods of the global MDOF are close to be the periods by the eigenvalues, and in the short period, or in the high level modes, the dynamic amplification ratios are sometimes high which means that the higher level modes should not be neglected. The dynamic amplification $\{x_{DA}\}$ at the predominant periods are usually different from the eigenvectors.

CONCLUSIONS

In this paper, the response values of acceleration records by earthquakes are shown. The analysis method is inelastic dynamic analysis, using Bi-linear models or Tri-linear models.
The main points of these results are as follows;

(1) The 13 acceleration records by the 10 larger earthquakes are analysed when the shear coefficient at the yielding point of the Peak Oriented Bi-linear Model is 0.1.
(2) The 10 acceleration records by the 4 larger earthquakes and an artificial acceleration data on bedrock are analysed when the shear coefficient at the yielding point of the Peak Oriented Bi-linear Model is 0.2.
(3) In the results, the response displacement spectra and the response ductility spectra are shown.
(4) The results of many acceleration records are larger than the typical values of the response displacement 5 or 10 (cm), and the response ductility which is equal to 2 in some periods.
(5) According to the comparison when the shear coefficient at the yielding point of the Peak Oriented Bi-linear Model is 0.1 or 0.2 for the 2 acceleration records, the response ductility spectra when the shear coefficient is 0.2 is always less than those of 0.1 in all periods.
(6) According to the comparison of the 4 hysteresis properties, each hysteresis property has the peak response displacement. When the period by the initial stiffness is 0.3 (sec), the minimum peak is analysed by Tri-linear (Hisada Model), the next one is Tri-linear (Takeda Model), the third one is Bi-linear (Clough Model) and the largest one is Bi-linear (Jenkin Model).
(7) The smaller peak acceleration record of [1985 Mexico City EW, SCT] could show an assumed hysteresis property for a storey of reinforced concrete buildings.
(8) According to the dynamic amplification ratio μ_D in the sample data of MDOF, the predominant periods of the global MDOF are close to be the periods by the eigenvalues.

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