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IMPROVEMENT OF THE MODEL PROPOSED BY MENEGOTTO AND PINTO FOR STEEL

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ABSTRACT

The model proposed by Menegotto and Pinto is widely used to simulate the cyclic response of steel structures and steel bars of reinforced concrete structures. In the early 1980s, Filippou et al. improved the original model by introducing isotropic hardening and highlighted a flaw in the original formulation. Specifically, these researchers observed that if partial unloading takes place at strains lower than the maximum value the reloading path provides forces that are higher than those expected. Notwithstanding, Filippou et al. deemed that these errors were not particularly significant and did not propose any modification to the formulation of the model. In the opinion of the writers, this conclusion may be questionable and thus an improved version of the Menegotto and Pinto model is proposed here. In particular, the abovementioned flaw is eliminated and the improvement of this model over the original Menegotto and Pinto model is shown.

INTRODUCTION

The model originally proposed by Menegotto and Pinto is used to simulate the cyclic response of steel structures (e.g. [1-3]) and steel bars of reinforced concrete structures. Because of the simplicity of its formulation, it is implemented in numerous programs intended to the response of structures, e.g. OpenSees [4]. It is a uniaxial material model and is used for the description of the response of steel in cross-sections discretized by means of fibres and subjected to normal stresses. The first formulation of the model was published in 1970 by Giuffré and Pinto [5]. The analytical relations drew their origin from a previous research study by Goldberg and Richard (1963) [6] and considered the possibility of unloading and reloading paths. Shortly later, the model proposed by Giuffré and Pinto was enhanced by Menegotto and Pinto [7, 8] to simulate kinematic hardening. Then, in the early 1980s, Filippou et al. (1983) [9] proposed a further improvement of this model by introducing the possibility that also the isotropic hardening could be considered. These researchers also noticed a flaw in the formulation of the model examined. Specifically, they observed that the response was higher than the previous loading curve if partial unloading and reloading paths took place at deformations lower than the maximum values. However, they deemed that the errors were not particularly significant and did not propose any modification to the analytical formulation of the model. In the opinion of the writers, this observation is shareable when dealing with lightly reinforced concrete members but is questionable when referred to steel members. This paper intends to eliminate the abovementioned flaw from the

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model described by Menegotto and Pinto. The modified model retains the simplicity of the original formulation and, therefore, is adequate for use in structural analysis. The validity of the new formulation is shown by comparison of the response of the current and new models.

THE MODEL MENEGOTTO-PINTO (1973)

The model proposed by Giuffré and Pinto (1970) [5] modifies the relations previously considered by Goldberg and Richard (1963) [6] so that they were apt to describe the response of steel to reversed cyclic loading. The uniaxial stress σ is calculated from a normalized stress σ^*, in a similar manner to Ramberg and Osgood (1943) [10]. The relation that provides the normalized stress is

$$\sigma^* = \frac{\varepsilon^*}{\left(1+\varepsilon^{*R}\right)^{\frac{1}{R}}} \tag{1}$$

and represents the equation of a curved transition from one straight line asymptote with slope E_0 equal to the initial Young's modulus to another line asymptote with slope E_∞ equal to zero (Fig. 1). In Equation (1), ε^* is the normalized strain and R is the parameter that influences the shape of the transition curve and takes account of the Bauschinger effect. To describe this effect accurately, the parameter R is considered to vary as a function of the plastic excursion ξ of the previous loading path

$$R = R_0 - \frac{a_1\xi}{a_2+\xi} \tag{2}$$

In this equation, R_0 is the value of the parameter R during first loading and a_1 and a_2 are experimentally determined parameters.
For the curve of first loading, the relations which provide the normalized stress and strain are

$$\sigma^* = \frac{\sigma}{\sigma_0}\,; \qquad \varepsilon^* = \frac{\varepsilon}{\varepsilon_0} \tag{3}$$

where σ_0 is the yield strength and ε_0 is the yield strain.
For the curves subsequent to the first load reversal, these relations are

$$\sigma^* = \frac{\sigma-\sigma_r}{2\sigma_0}\,; \qquad \varepsilon^* = \frac{\varepsilon-\varepsilon_r}{2\varepsilon_0} \tag{4}$$

where σ_r and ε_r are the stress and strain, respectively, at the last load reversal.
The model proposed by Giuffré and Pinto is subsequently enriched by Menegotto and Pinto [8] to take

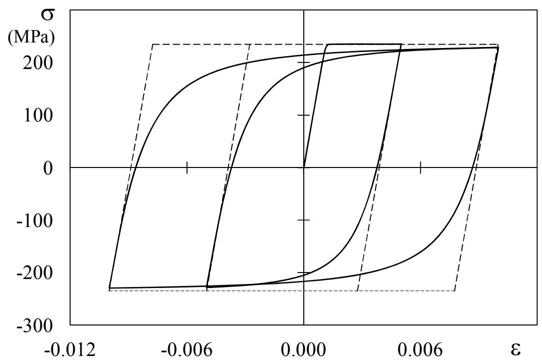

Figure 1. Model proposed by Giuffré and Pinto (1970)

account of the kinematic hardening. The relation that provides the normalized stresses is

$$\sigma^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{\left(1+\varepsilon^{*R}\right)^{\frac{1}{R}}} \tag{5}$$

where b is the strain hardening ratio, i.e. the ratio of the elasticity modulus E_∞ of the second asymptote to the initial Young's modulus. In this version of the model, Equation (4) is substituted for

$$\sigma^* = \frac{\sigma - \sigma_r}{\sigma_0 - \sigma_r}; \qquad \varepsilon^* = \frac{\varepsilon - \varepsilon_r}{\varepsilon_0 - \varepsilon_r} \tag{6}$$

where σ_0 and ε_0 are stress and strain at the point where the two asymptotes of the branch under consideration meet and σ_r and ε_r are stress and strain at the point where the last strain reversal with stress of equal sign takes place. As an example, Fig. 2a shows the points P_0 (ε_0, σ_0) and P_r (ε_r, σ_r) corresponding to the first load reversal of steel. R is considered dependent on the strain difference between the current asymptote intersection point and the previous load reversal point with maximum or minimum strain depending on whether the corresponding steel stress is positive or negative. The parameters ε_0, σ_0, ε_r, σ_r, and R are updated after each strain reversal. As an example, Fig. 2b illustrates the change in P_0 (ε_0, σ_0) and P_r (ε_r, σ_r) on the occurrence of four load reversals. The loading branches subsequent to the load reversals are labelled by means of roman numbers and similarly are points P_0 and P_r.
This formulation has been object of several studies intended to highlight qualities and deficiencies of the model [9, 11, 12, 13]. Some research studies, e.g. [11], have showed that the explicit formulation proposed by Menegotto and Pinto [8] offers numerical efficiency, and that the results provided by this model agrees well with experimental results from cyclic tests on reinforcing steel bars. Some researchers, e.g. [9, 11], have also proposed modifications to Equation (5) in order to improve the accuracy of the model. In particular, in the early 1980s Filippou et al. [9] included the possibility that the model considered isotropic hardening, as noted in laboratory tests on reinforcing steel bars. The

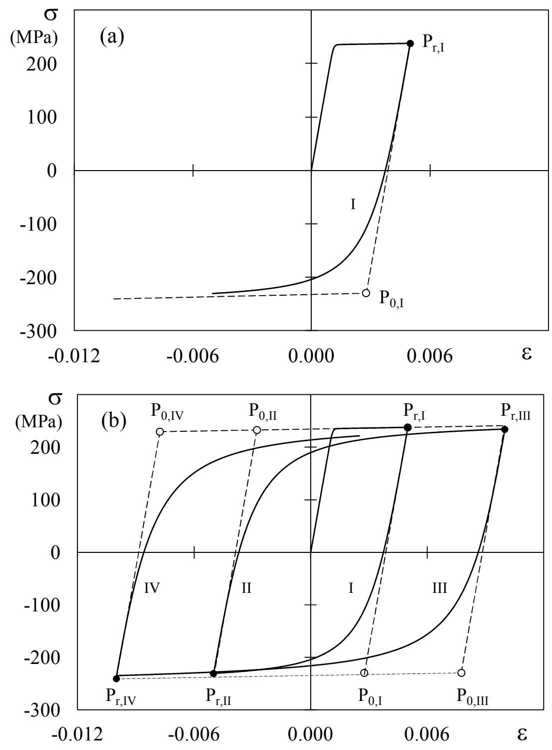

Figure 2. Model proposed by Menegotto and Pinto (1973): (a) first cycle and (b) subsequent cycles.

model proposed by Filippou et al. [9] accounts for isotropic strain hardening by shifting the position of the yield asymptote before computing the new asymptote intersection point following a strain reversal. The shift is effected by moving the initial yield asymptote through a stress shift parallel to its direction. This idea was introduced some years before by Stanton and McNiven [11], but the latter researchers considered both a strain and stress shift on the monotonic envelope curve to arrive at an accurate representation of the hysteretic behaviour of steel under generalized strain histories. The yield asymptote shift suggested by Filippou et al. [9] was dependent on the maximum plastic strain, as reported in the following relation

$$\frac{\sigma_{st}}{\sigma_0} = a_3\left(\frac{\varepsilon_{max}}{\varepsilon_0} - a_4\right) \quad (7)$$

where ε_{max} is the absolute maximum strain at the instant of strain reversal and a_3 and a_4 are experimentally determined parameters.

Filippou et al. [9] also added some clarification in regard to the set of rules for unloading and reloading. In particular, Filippou et al. [9] stated that if the analytical model had a memory extending over all previous branches of the stress-strain history, it would follow the previous loading branch, as soon as the new reloading curve reached it. This would require that the model store all necessary information to retrace all previous reloading curves that were left incomplete. According to Filippou et al. [9] this was impractical from a computational standpoint. Memory of the past stress-strain history was, therefore, limited to a predefined small number of controlling curves. Due to these restrictions, reloading after partial unloading connects with curves that are different from the expected envelope curve. According to Filippou et al. [9], the discrepancy between the adopted analytical model and the actual behaviour was, however, acceptable and no remedy was offered to this shortcoming. In the opinion of the writers, this statement is questionable. To explain this misgiving qualm, some cyclic strain histories are devised which consider reloading branches after partial unloading. The response of

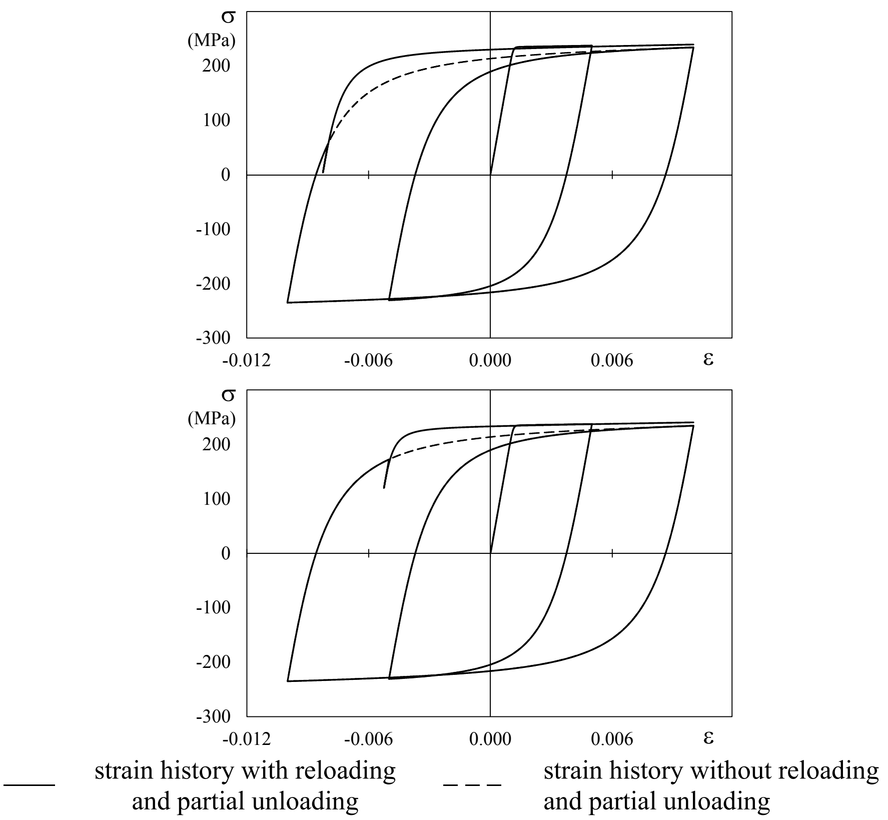

Figure 3. Error made by the model proposed by Menegotto and Pinto (1973) in case reloading occurs after partial unloading

the model proposed by Menegotto and Pinto [5] to these strain histories is shown in Fig. 3 together with the response to strain histories that do not consider partial unloading branches. The differences between the two responses are evident and not negligible in the light of current scientific expectations and computational capabilities. However, it is undoubted that when the model proposed by Menegotto and Pinto is used to contribute to the simulation of the structural behaviour of reinforced concrete members the inaccuracy in the prediction of the response of reinforcing steel bars is less important, i.e. the response of the cross-section is only partly influenced by that of steel. This last consideration may not be applied when dealing with all steel members for which the model proposed by Menegotto and Pinto is also used. In these cases, the inaccuracy of the original model proposed by Menegotto and Pinto may be evident if members are substantially deformed in flexure and greatly striking if members are subjected to axial forces, as in buckling restrained braces.

THE PROPOSED MODEL

As reported in the previous section, when reloading occurs after partial unloading the stress of the reloading path tends to be higher than that of the previous loading branch. The modification introduced by the writers is intended to eliminate this shortcoming, i.e. the proposed model is formulated so as to tend to the curve that would have developed if no unloading and reloading would have occurred (this curve is later named *basic loading curve*).
To introduce the proposed modification to the analytical formulation of the model, it is important to note that the uniaxial material response described by Menegotto and Pinto consists of two contributions. Fig. 4a shows the two contributions in terms of the normalized strain and stress. The first contribution to the normalized stress is given by a transition curve between two straight-line asymptotes; the first asymptote is characterised by a slope equal to 1-*b*, where *b* is the strain hardening ratio and the second is characterised by a null slope. The second contribution to the normalized stress is linear with the normalized strain and characterised by a slope equal to *b*. In particular, note that the total normalized stress σ^* is equal to unity at a strain ε^* equal to unity and that the second

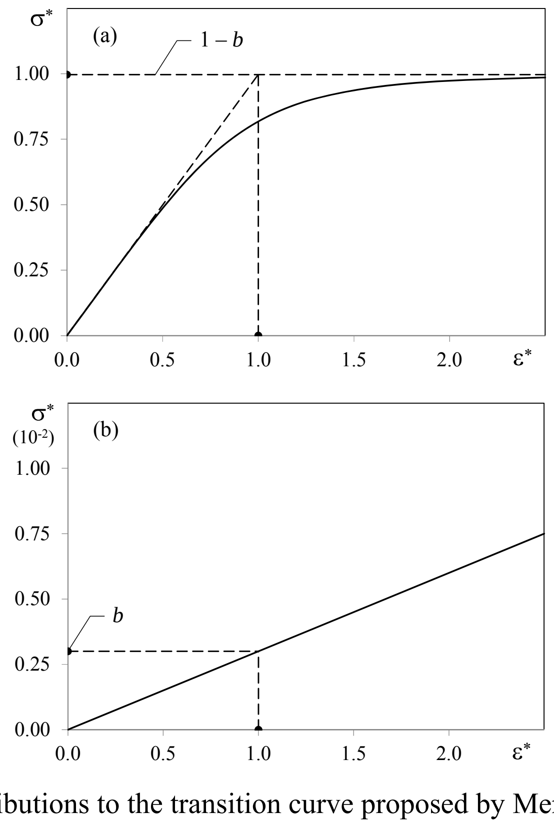

Figure 4. Contributions to the transition curve proposed by Menegotto and Pinto in terms of normalized strain and stress

contribution (Fig. 4b) is much smaller than the first (Fig. 4a) because of the common values of the strain hardening of steel.
Unlike the curve proposed by Menegotto and Pinto, the one suggested by the writers is a transition curve from a first straight line asymptote to the basic loading curve. The proposed curve in terms of normalized strains and stresses may still be thought as the sum of two functions. The points selected for the normalization of strains and stresses are however different from those suggested by Menegotto and Pinto [8], as well as the analytical relations of the abovementioned stress contributions. In particular, the determination of the point $P_0(\varepsilon_0, \sigma_0)$ is changed with respect to Menegotto and Pinto. To construct a transition curve able to tend to the basic loading curve it is opinion of the writers that point P_0 should be defined as the intersection of the first asymptote to the basic loading curve. The new point P_0 is named here $P_0^{'}$ so as to differentiate it from the point P_0 considered by Menegotto and Pinto (see Fig. 5). The analytical evaluation of the coordinates of $P_0^{'}$ is not straightforward because of the nonlinearity of Equation (5). Because of this, they are derived by means of a trial-and-error method, verifying that the inclination of the line from the beginning of the reloading path to the new point P_0 is equal to the Young's modulus of steel.
The proposed first contribution to the total normalized stress is given by a transition curve between two straight line asymptotes; again, the first asymptote is characterised by a slope equal to $1\text{-}e_{\mathrm{p}}^{*}$ and the second by a null slope (Fig. 6a). In this contribution, e_{p}^{*} is the slope of the basic loading curve in the reference system (O, ε^{*}, σ^{*}) at a unity normalized strain, i.e. in $P_0^{'}$. The transition curve proposed by the writers is

$$\sigma^{*}=\frac{\left(1-e_{\mathrm{p}}^{*}\right)\varepsilon^{*}}{\left(1+\varepsilon^{*R}\right)^{\frac{1}{R}}} \tag{8}$$

This relation is similar to that suggested by Menegotto and Pinto [see Eq. (5)] save that the strain hardening ratio *b* is substituted for the parameter e_{p}^{*}.
The second contribution to the normalized stress is globally nonlinear with the normalized strain (Fig. 6b). Specifically, the part of the curve with normalized strains not higher than unity is linear and characterised by a slope equal to e_{p}^{*}. The remaining part of the curve ($\varepsilon^{*}>1$), instead, is defined by a nonlinear function and takes account of the nonlinearity of the basic loading curve. The analytical relations of the two parts of this second contribution are

$$\sigma^{*}=e_{\mathrm{p}}^{*}\,\varepsilon^{*} \qquad \varepsilon^{*}\leq 1 \tag{9}$$

$$\sigma^{*}=f_{\mathrm{b}}\left(\varepsilon^{*}\right)-\left(1-e_{\mathrm{p}}^{*}\right) \qquad \varepsilon^{*}>1 \tag{10}$$

where $f_{\mathrm{b}}\left(\varepsilon^{*}\right)$ is the normalized stress of the basic loading curve in terms of the normalized strain.

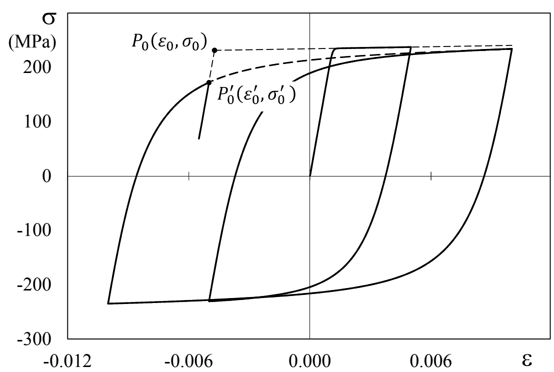

Figure 5. Definition of the new point $P_0^{'}$

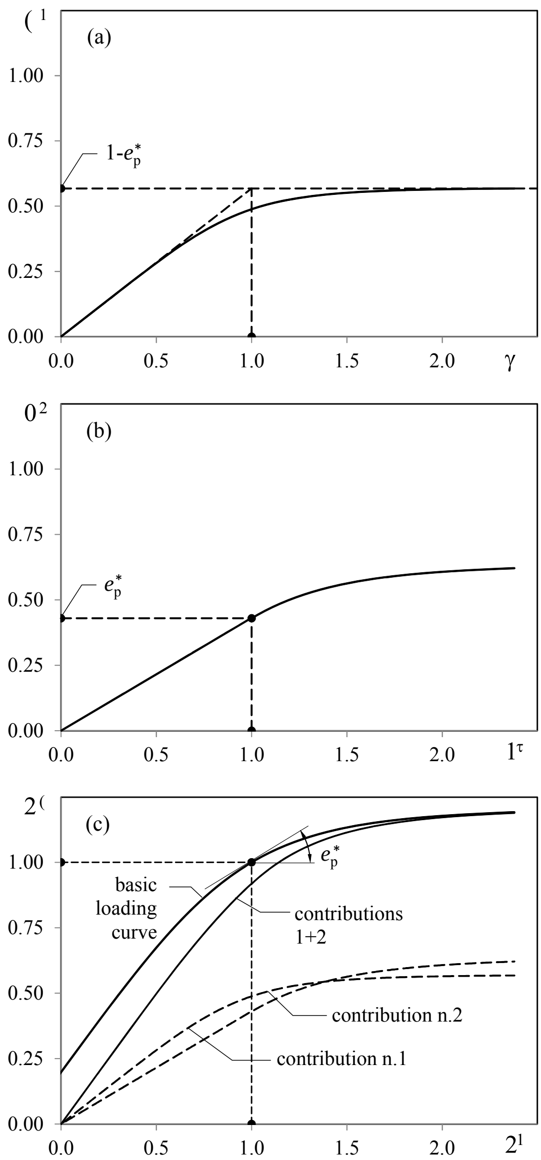

Figure 6. Contributions to the proposed transition curve in terms of normalized strain and stress

Thus, the proposal for the transition curve in case of reloading after partial unloading is

$$\sigma^* = e^*_p\,\varepsilon^* + \frac{\left(1-e^*_p\right)\varepsilon^*}{\left(1+\varepsilon^{*R}\right)^{\frac{1}{R}}} \qquad \varepsilon^* \le 1 \tag{11}$$

$$\sigma^* = f_b\left(\varepsilon^*\right) - \left(1-e^*_p\right) + \frac{\left(1-e^*_p\right)\varepsilon^*}{\left(1+\varepsilon^{*R}\right)^{\frac{1}{R}}} \qquad \varepsilon^* > 1 \tag{12}$$

The true (non-normalized) stress is obtained by means of the relation

$$\sigma = \sigma^*\left(\sigma_0 - \sigma_r\right) + \sigma_r \tag{13}$$

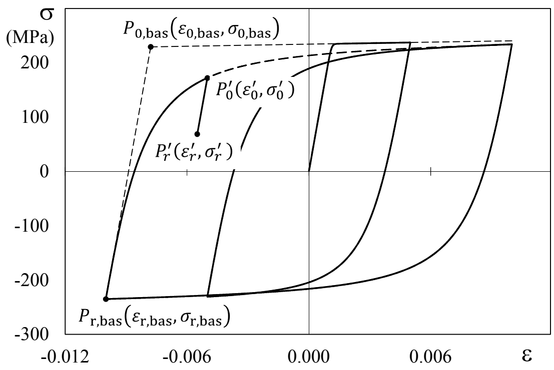

Figure 7. Definition of points $P_{0,bas}$ and $P_{r,bas}$

IMPLEMENTATION

The model has been implemented into the program OpenSees [4, 14] by changing the uniaxial material model "Steel02" of the program above. It is characterised by assigned values of the yield stress σ_0, yield strain ε_0, Young's modulus E, kinematic hardening ratio b, coefficients R_0, a_1, a_2, a_3 and a_4. Note that at the beginning of the analysis the plastic excursion is null and then the parameter R that characterise the roundness of the loading curve is equal to R_0. The curve proposed by Menegotto and Pinto, already implemented in the program OpesSees, is utilized in case loading occurs from the maximum or minimum strain. In all the other cases, the proposed relation in Equation (8) is used. To apply the proposed loading curve, the values of the parameters that characterise the basic loading curve must be recorded during the analysis. This holds for the basic curve corresponding to either increments or decrements of strain.

Note that the parameter e_p^* in Equations (8-12) is the slope of the basic loading curve and is calculated with respect to normalized values of stresses and strains of the reloading curve, i.e. these stresses and strains are normalized to the differences between the stresses and strains of the points $P_0^{'}$ and P_r of the reloading curve. To obtain the value of the parameter e_p^* three subsequent operations are required. The slope of the basic loading curve is first determined as normalized to the stresses and strains of points $P_{0,bas}$ and $P_{r,bas}$ of the basic loading curve (Fig. 7).

$$e_\mathrm{p}^{*,\mathrm{bas}} = b + \frac{(1-b)}{\left(1+\varepsilon^{*R}\right)^{\frac{1}{R}+1}} \tag{14}$$

Then, the slope is denormalized, i.e. expressed in terms of non-normalized stresses and strains

$$e_\mathrm{p} = e_\mathrm{p}^{*,\,\mathrm{bas}} \frac{\left(\sigma_{0,\mathrm{bas}} - \sigma_{\mathrm{r,bas}}\right)}{\left(\varepsilon_{0,\mathrm{bas}} - \varepsilon_{\mathrm{r,bas}}\right)} \tag{15}$$

Finally, the slope is calculated as a function of stresses and strains that are normalized to those of points $P_0^{'}$ and P_r of the reloading curve

$$e_\mathrm{p}^* = e_\mathrm{p} \frac{\left(\varepsilon_0^{'} - \varepsilon_r\right)}{\left(\sigma_0^{'} - \sigma_r\right)} \tag{16}$$

SIMPLE NUMERICAL APPLICATIONS

The proposed model is subjected to axial stress according to assigned load histories in order to show the results of the effects of the corrections applied (Fig. 8). In particular, in this figure the basic loading curve is plotted by dashed line while that corresponding to the new model is plotted by thick line. The parameters of the model are R_0=20, a_1=18.5, a_2=0.15, a_3=0 and a_4=1.

CONCLUSIONS

The writers show an improved version of the model proposed by Menegotto and Pinto in which the flaw noted by Filippou et al. in the early 1980s is eliminated. Specifically, the latter researchers observed that, if partial unloading takes place at strains lower than the maximum value, the reloading path of the model proposed by Menegotto and Pinto provides forces that are higher than those expected. The modified model retains the simplicity of the original formulation and, therefore, is adequate for use in structural analysis. The validity of the new formulation is proved by comparison of the response of the current and new uniaxial material models. However, the writers recognize that in order to quantify the significance of the improvement further studies should be made on multistorey structures.

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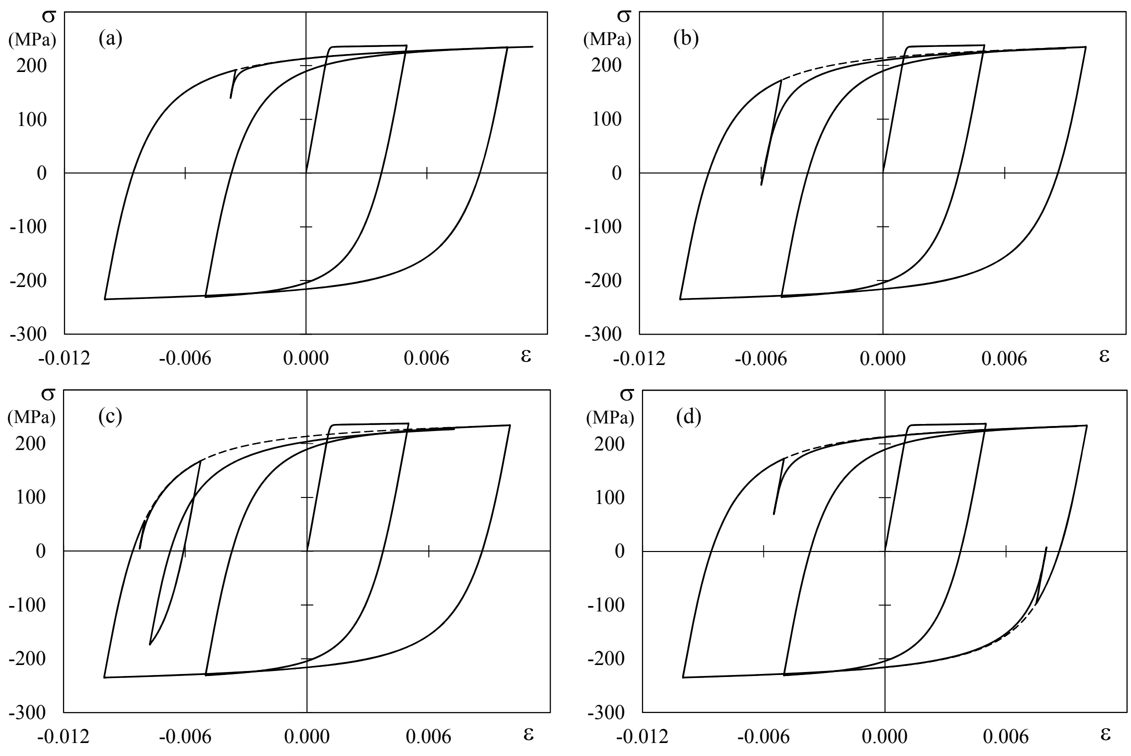

Figure 8. The improved model
in case reloading occurs after partial unloading

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