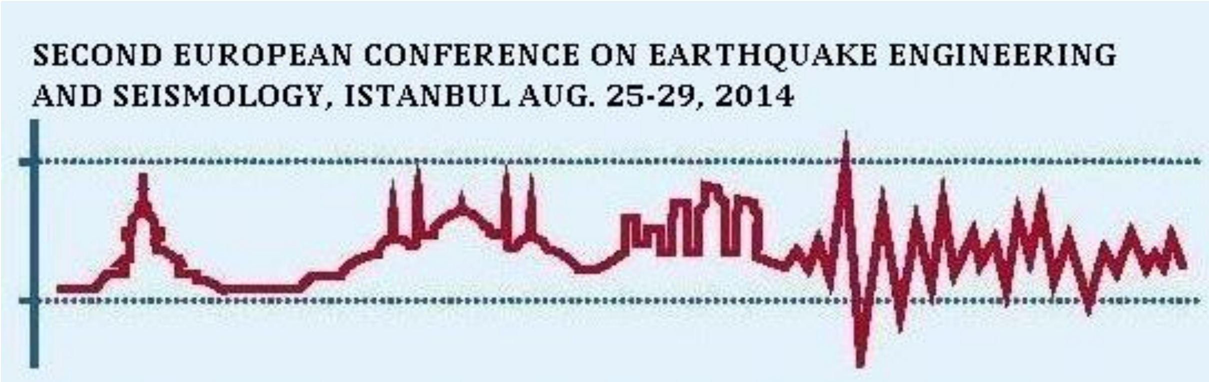

STUDY OF DYNAMIC SOIL-STRUCTURE INTERACTION OF CONCRETE GRAVITY DAMS

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ABSTRACT

The safety evaluation of the dams subjected to seismic loads is really very complex. One of the most important problems in evaluation of seismic behavior of concrete gravity dams is dam-foundation interaction. In this paper, we study the effect of soil-structure interaction (SSI) on seismic response of concrete gravity dams. For this purpose, two 2D finite element models using ANSYS software are created. The first model represents the dam alone without SSI. The second model represents the dam with ISS. Oued Fodda concrete gravity dam, located in the north west of Algeria, is chosen in the present study. Reservoir water is modeled using Westergaard approach. Drucker-prager model is used in nonlinear analysis for concrete dam. According to finite element analysis, numerical results show that considering interaction soil-structure in the model develops more stresses and displacements in the dam body. The dynamic soil-structure interaction phenomenon plays an important role in accurately estimating the concrete gravity dams response.

INTRODUCTION

Dams have contributed to the development of civilization for a long time. They will continue to keep their importance in satisfying the ever increasing demand for power, irrigation and drinking water, the protection of man, property and environment from catastrophic floods, and in regulating the flow of rivers.

There are several factors affecting the dynamic response of concrete gravity dams to earthquake ground motions. Some of them are the interaction of the dam with the foundation rock and water in reservoir (Alemdar et al., 2005). Hence it becomes imperative to consider the effect of soil-structure interaction for heavy structures such as concrete gravity dams.

Wolf (1985) first presented the direct method of soil-structure interaction analysis. Using this method the soil region near the structure along with the structure is modeled directly and the idealized soil-structure system was analyzed in a single step. An excellent amount of work on dam-reservoir-foundation interaction in the frequency domain has been carried out by Chopra and his colleagues (Chopra and Chakrabatry, 1981. Fenves and Chopra, 1985). However, frequency domain based methods are difficult to understand compared to the time domain based methods. Also, incorporation of nonlinear material behavior will be a prohibitive task for frequency domain based methods. The time domain based methods do not suffer from such limitations. Rizos and Wang (2002) developed a partitioned method for soil-structure interaction analysis in the time domain through a staggered solution method using both FEM and BEM.

Tahar Berrabah and Belharizi (2012) studied the effect of the soil-structure interaction on the dynamic behavior of Brezina concrete gravity arch dam, located in El-Beyadh at the west of Algeria. Dynamic analyses of Brezina dam showed that the presence of the soil-structure interaction in the

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model develops more stresses in the dam body. A time domain transient analysis of a concrete gravity dam and its foundation has been carried out in a coupled manner using finite element technique and the effect of soil-structure Interaction (SSI) has been incorporated using a simplified direct method (Burman and Nayak, 2011). The results illustrated that the displacements and stresses have increased for the elastic as compared to the rigid base.

This study investigates the effect of dynamic soil-structure interaction on the earthquake response of concrete gravity dams. Oued Fodda concrete gravity dam, located in Chlef, Algeria, is chosen in this work. For this purpose, two studied finite element models: dam alone without ISS and dam with ISS were employed in analyses using the ANSYS software (ANSYS, 2009). The effect of hydrodynamic pressure is considered according to added mass technique originally proposed by Westergaard (1933). The Drucker-Prager (1952) model is considered in nonlinear analyses for concrete dam.

METHODOLOGY

Formulation of soil-structure interaction system

In soil-structure interaction problems, the foundation soil and the structure do not vibrate as separate systems under external excitations, rather they act together in a coupled way. Therefore, these problems have to be dealt in a coupled way. The most common soil-structure interaction SSI approach used is based on the ‘‘added motion’’ formulation. In order to develop the fundamental SSI dynamic equilibrium equations, we consider the soil-structure system shown in Figure 1.

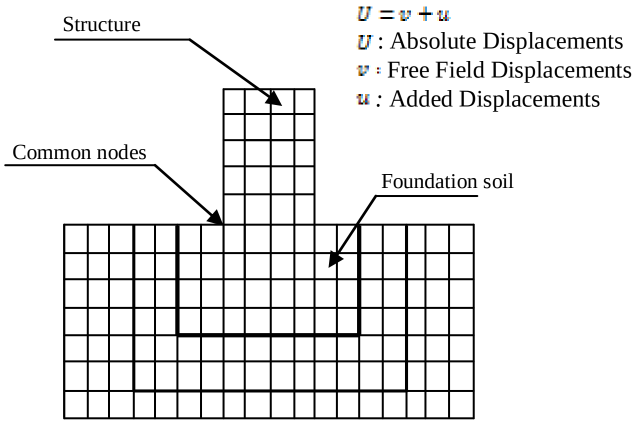


Figure 1. Soil-structure interaction model

The SSI model here is divided into three sets of node points (Wilson, 2002). The common nodes at the interface of the structure and soil are identified with the subscript ‘‘c’’; the nodes within the structure are with ‘‘s’’ and the nodes within the foundation soil are with ‘‘f’’. In this figure, the absolute displacement (U) is estimated out of the sum of the free field displacement (v) and the added displacement (u).

From the direct stiffness approach in structural analysis, the dynamic force equilibrium of the system is given in terms of the absolute displacements, U, by the following sub-matrix equation:

$$\begin{bmatrix} M_{ss} & M_{sc} & 0 \\ M_{cs} & M_{cc} & M_{cf} \\ 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{U}_s \\ \ddot{U}_c \\ \ddot{U}_f \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sc} & 0 \\ C_{cs} & C_{cc} & C_{cf} \\ 0 & C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{U}_s \\ \dot{U}_c \\ \dot{U}_f \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sc} & 0 \\ K_{cs} & K_{cc} & K_{cf} \\ 0 & K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} U_s \\ U_c \\ U_f \end{Bmatrix} = -\begin{bmatrix} M_{ss} & M_{sc} & 0 \\ M_{cs} & M_{cc} & M_{cf} \\ 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{U}_s^g \\ \ddot{U}_c^g \\ \ddot{U}_f^g \end{Bmatrix} \quad (1)$$

where the mass and stiffness at the contact nodes are the sum of the contributions from the structure (s) and foundation soil (f), and are given by:

$$M_{cc} = M_{cc}^{(s)} + M_{cc}^{(f)}, C_{cc} = C_{cc}^{(s)} + C_{cc}^{(f)}, K_{cc} = K_{cc}^{(s)} + K_{cc}^{(f)} \tag{2}$$

In order to solve the coupled soil-structure interaction problem, we would require to solve Eq. (1). Having solved Eq. (1) using Newmark's integration method, one would obtain the absolute displacements, velocities and accelerations of the coupled SSI problem. To avoid solving the SSI problem directly, the dynamic response of the foundation without the structure is calculated. The free-field solution is designated by the free-field displacements v , velocities $\dot{v}$ and accelerations $\ddot{v}$. Here, $\ddot{U}^g$ is the ground acceleration vector. By a simple change of variables, it becomes possible to express the absolute displacements U, velocities $\dot{U}$ and accelerations $\ddot{U}$ in terms of displacements u, relative to the free-field displacements v. Or,

$$\begin{Bmatrix} \ddot{U}_s \\ \ddot{U}_c \\ \ddot{U}_f \end{Bmatrix} = \begin{Bmatrix} \ddot{v}_s \\ \ddot{v}_c \\ \ddot{v}_f \end{Bmatrix} + \begin{Bmatrix} \ddot{u}_s \\ \ddot{u}_c \\ \ddot{u}_f \end{Bmatrix} \quad \begin{Bmatrix} \dot{U}_s \\ \dot{U}_c \\ \dot{U}_f \end{Bmatrix} = \begin{Bmatrix} \dot{v}_s \\ \dot{v}_c \\ \dot{v}_f \end{Bmatrix} + \begin{Bmatrix} \dot{u}_s \\ \dot{u}_c \\ \dot{u}_f \end{Bmatrix} \quad \begin{Bmatrix} U_s \\ U_c \\ U_f \end{Bmatrix} = \begin{Bmatrix} v_s \\ v_c \\ v_f \end{Bmatrix} + \begin{Bmatrix} u_s \\ u_c \\ u_f \end{Bmatrix} \tag{3}$$

After replacing the values of $\ddot{U}$, $\dot{U}$ and U from Eq. (3), Eq. (1) is expressed as

$$\begin{bmatrix} M_{ss} & M_{sc} & 0 \\ M_{cs} & M_{cc} & M_{cf} \\ 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{u}_s \\ \ddot{u}_c \\ \ddot{u}_f \end{Bmatrix} \begin{bmatrix} C_{ss} & C_{sc} & 0 \\ C_{cs} & C_{cc} & C_{cf} \\ 0 & C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{u}_s \\ \dot{u}_c \\ \dot{u}_f \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sc} & 0 \\ K_{cs} & K_{cc} & K_{cf} \\ 0 & K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} u_s \\ u_c \\ u_f \end{Bmatrix} = R + F \tag{4}$$

where

$$R = -\begin{bmatrix} M_{ss} & M_{sc} & 0 \\ M_{cs} & M_{cc} & M_{cf} \\ 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{v}_s \\ \ddot{v}_c \\ \ddot{v}_f \end{Bmatrix} - \begin{bmatrix} C_{ss} & C_{sc} & 0 \\ C_{cs} & C_{cc} & C_{cf} \\ 0 & C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{v}_s \\ \dot{v}_c \\ \dot{v}_f \end{Bmatrix} - \begin{bmatrix} K_{ss} & K_{sc} & 0 \\ K_{cs} & K_{cc} & K_{cf} \\ 0 & K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} v_s \\ v_c \\ v_f \end{Bmatrix} \tag{5}$$

$$\text{and, } F = -\begin{bmatrix} M_{ss} & M_{sc} & 0 \\ M_{cs} & M_{cc} & M_{cf} \\ 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{U}_s^g \\ \ddot{U}_c^g \\ \ddot{U}_f^g \end{Bmatrix} \tag{6}$$

This is a numerically cumbersome approach; hence, an alternative approach is necessary to formulate the solution directly in terms of the absolute displacements of the structure. Since the analysis is now for the foundation part only (free field analysis), hence the corresponding values of the displacement, velocity and acceleration for the structural part is taken as zero. This involves the introduction of the following change of variables:

$$\begin{Bmatrix} \ddot{U}_s \\ \ddot{U}_c \\ \ddot{U}_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ \ddot{v}_c \\ \ddot{v}_f \end{Bmatrix} + \begin{Bmatrix} \ddot{u}_s \\ \ddot{u}_c \\ \ddot{u}_f \end{Bmatrix} \quad \begin{Bmatrix} \dot{U}_s \\ \dot{U}_c \\ \dot{U}_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ \dot{v}_c \\ \dot{v}_f \end{Bmatrix} + \begin{Bmatrix} \dot{u}_s \\ \dot{u}_c \\ \dot{u}_f \end{Bmatrix} \quad \begin{Bmatrix} U_s \\ U_c \\ U_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ v_c \\ v_f \end{Bmatrix} + \begin{Bmatrix} u_s \\ u_c \\ u_f \end{Bmatrix} \tag{7}$$

In order to calculate the free field displacements v, only foundation domain is solved by considering no structure is present on it. The foundation domain is subjected to earthquake motion and the free-field displacement for the common and other foundation nodes are obtained.

$$\begin{bmatrix} M_{cc} & M_{cf} \\ M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{v}_c \\ \ddot{v}_f \end{Bmatrix} + \begin{bmatrix} C_{cc} & C_{cf} \\ C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{v}_c \\ \dot{v}_f \end{Bmatrix} + \begin{bmatrix} K_{cc} & K_{cf} \\ K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} v_c \\ v_f \end{Bmatrix} = - \begin{bmatrix} M_{cc} & M_{cf} \\ M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{U}_c^g \\ \ddot{U}_f^g \end{Bmatrix} \tag{8}$$

After obtaining the free field response (i.e. $v, \dot{v}$ and $\ddot{v}$) the interaction force R is calculated using Eq. (9) in the following simplified manner:

$$R = - \begin{bmatrix} M_{ss} & M_{sc} & 0 \\ M_{cs} & M_{cc} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ \ddot{v}_c \\ 0 \end{Bmatrix} - \begin{bmatrix} C_{ss} & C_{sc} & 0 \\ C_{cs} & C_{cc} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ \dot{v}_c \\ 0 \end{Bmatrix} - \begin{bmatrix} K_{ss} & K_{sc} & 0 \\ K_{cs} & K_{cc} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ v_c \\ 0 \end{Bmatrix} \tag{9}$$

After obtaining the interaction forces R, the added responses of the structure and foundation soil domain are calculated using Eq. (10). And then the added responses (i.e. $u, \dot{u}$ and $\ddot{u}$) are added to the free field responses to get the absolute responses of the coupled soil and structure domain, following Eq. (7):

$$\begin{bmatrix} M_{ss} & M_{sc} & 0 \\ M_{cs} & M_{cc} & M_{cf} \\ 0 & M_{fc} & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{u}_s \\ \ddot{u}_c \\ \ddot{u}_f \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sc} & 0 \\ C_{cs} & C_{cc} & C_{cf} \\ 0 & C_{fc} & C_{ff} \end{bmatrix} \begin{Bmatrix} \dot{u}_s \\ \dot{u}_c \\ \dot{u}_f \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sc} & 0 \\ K_{cs} & K_{cc} & K_{cf} \\ 0 & K_{fc} & K_{ff} \end{bmatrix} \begin{Bmatrix} u_s \\ u_c \\ u_f \end{Bmatrix} = R + F \tag{10}$$

The main assumptions used in this model are that the input motions at the level of the base rock are not considered to be affected by the presence of the structure and that all interface nodes will be subjected to the same free-field accelerogram (Leger and Boughoufalah, 1989). In theory any desired spatial variation of the free-field components could be considered at the interface, but there is seldom sufficient information to specify such variation. In this case, the mass of the foundation is taken into account in the analysis such that it will represent the soil-structure interaction in a relatively more realistic manner.

Drucker-prager model

There are many criteria for determination of yield surface or yield function of materials. Drucker-Prager criterion is widely used for frictional materials such as rock and concrete. Drucker and Prager (1952) obtained a convenient yield function to determine elasto-plastic behavior of concrete smoothing Mohr-Coulomb criterion (Figure 2) (Chen and Mizuno, 1990). This function is defined as

$$f = \alpha I_1 + \sqrt{J_2} - k \tag{11}$$

where α and k are constants which depend on cohesion (c) and angle of internal friction (φ) of the material given by :

$$\alpha = \frac{2 \sin\varphi}{\sqrt{3}\,(3 - \sin\varphi)} \tag{12}$$

$$k = \frac{6c \cos\varphi}{\sqrt{3}\,(3 - \sin\varphi)} \tag{13}$$

In Eq.(11), $I_1$is the first invariant of stress tensor (σ_{ij}) formulated as follows :

$$I_1 = \sigma_{12} + \sigma_{22} + \sigma_{33} \tag{14}$$

and J_2 is the second invariant of deviatoric stress tensor (s_{ij}) given by :

$$J_2 = \frac{1}{2} s_{ij} s_{ij} \tag{15}$$

where δ_{ij} is the deviatoric stresses as given below :

$$s_{ij} = \sigma_{ij} - \delta_{ij}\,\sigma_m \qquad (ij = 1,2,3) \tag{16}$$

In Eq. (16), δ_{ij} is the Kronecker delta, which is equal to 1 for $i=j$; 0 for $i\neq j$, and σ_m is the mean stress and obtained as follows:

$$\sigma_m = \frac{I_1}{3} = \frac{\sigma_{ij}}{3} \tag{17}$$

If the terms in Eq. (16) are obtained by the Eq. (17) and replaced in Eq. (15), the second invariant of the deviatoric stress tensor can be obtained as follows:

$$J_2 = \frac{1}{6}[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2] + {\sigma_{12}}^2 + {\sigma_{13}}^2 + {\sigma_{23}}^2 \tag{18}$$

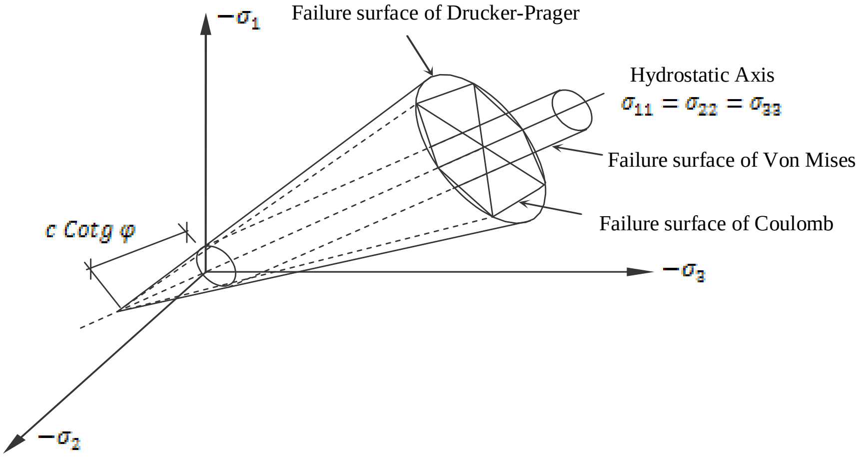


Figure 2. Failure criteria for Coulomb, Drucker-Prager and Von Mises

FINITE ELEMENT MODEL OF THE SYSTEM DAM-FOUNDATION SOIL

The concrete gravity dam Oued Fodda is located in the north west of Algeria. The geometry of the Oued Fodda dam-foundation is illustrated in Figure 3. The dam-foundation soil system is investigated using two 2D finite element models. The first model represents the dam alone neglecting the soil-structure interaction effect and assuming that the structure is fixed at its base on the soil (Figure 4). The second model represents the dam-foundation interaction system (Figure 5). The effect of hydrodynamic pressure is considered according to added mass technique originally proposed by Westergaard (1933). These finite elements models are created using software ANSYS (ANSYS, 2009).

A two-dimensional (2D) finite element model with 1521 nodes and 1394 plane solid elements (PLANE 82) is used to model Oued Fodda dam alone (Figure 4). A two-dimensional (2D) finite element model with 1521 nodes and 1394 plane solid elements (PLANE 82) is used to model dam-foundation soil system (Figure 5).

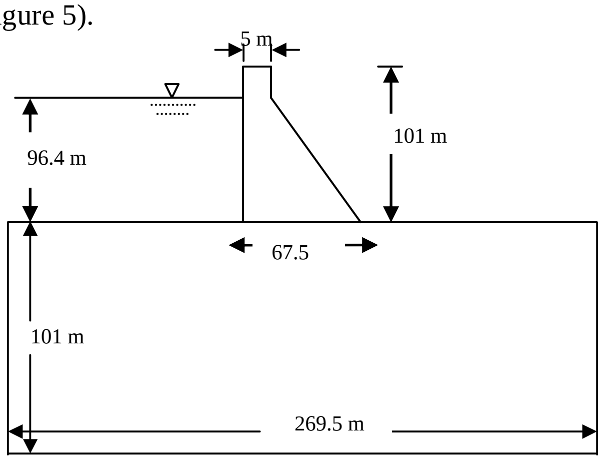


Figure 3. Dam-foundation system

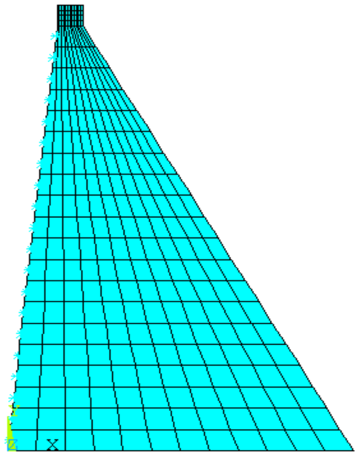


Figure 4. Finite element discretization of dam alone

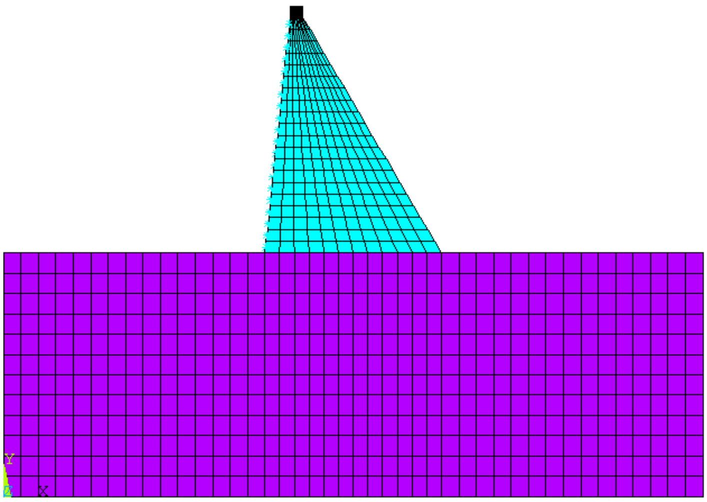


Figure 5. Finite element discretization of dam-foundation soil system

The material properties of Oued Fodda dam including its foundation are reported in Table 1 below.

Table 1. Material properties of the dam and foundation rock.

Material	Elasticity Modulus (Mpa)	Poisson's ratio	Mass density (Kg/m^3)	Cohesion (Mpa)	Angle of internal friction
Dam concrete	24600	0.2	2640	3	45
Foundation rock	20000	0.33	2000	-	-

RESULTS AND DISCUSSION

Earthquake response of Oued Fodda dam

The earthquake response of Oued Fodda concrete gravity dam was evaluated for horizontal component of 2003 boumerdes earthquake during 20 s. The peak ground acceleration (pga) of the horizontal component of the earthquake acceleration is 0.34 g (Figure 6). Linear and non linear time history analyses were performed using ANSYS software. Newmark algorithm is employed in numerical solutions. Horizontal displacements at the crest of dam and principal stress components at heel of dam for two models without and with ISS are presented.

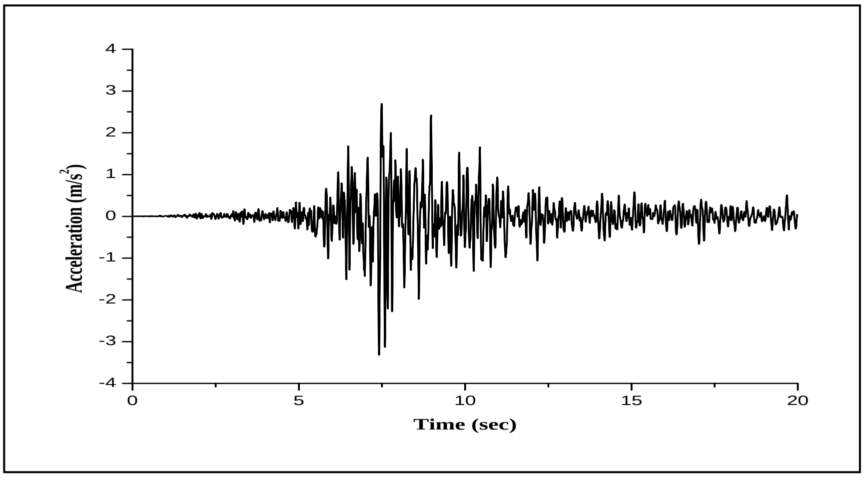


Figure 6. Acceleration records of 2003 boumerdes earthquake

Horizontal Displacements

The horizontal displacement of the crest of dam for both models without and with ISS is shown in Figures 7 and 8.

In linear analysis, time history of horizontal displacement at the crest of dam, is presented in Figure 7 for both case without ISS and case with ISS. The horizontal displacement of the crest increases from 1.96 cm for dam without ISS to 3.78 cm for dam with ISS. This indicates that there is about 93% rise in the magnitude of the crest displacement in case with ISS. Figures 9 and 10 show the contour diagram of the dam for the maximum horizontal displacement at crest for model without ISS and model with ISS.

According to nonlinear analysis (Figure 8), the horizontal displacement of the crest increases from 1.97 cm for dam without ISS to 3.61 cm for dam with ISS (% 83.25).

In the comparison of linear and nonlinear analyses, the horizontal displacement at crest decreases due to nonlinear response of the dam. The dam nonlinearity could decrease or increase the displacement at crest depending upon the ground motion characteristics and dam properties (Halabian and Naggar, 2002).

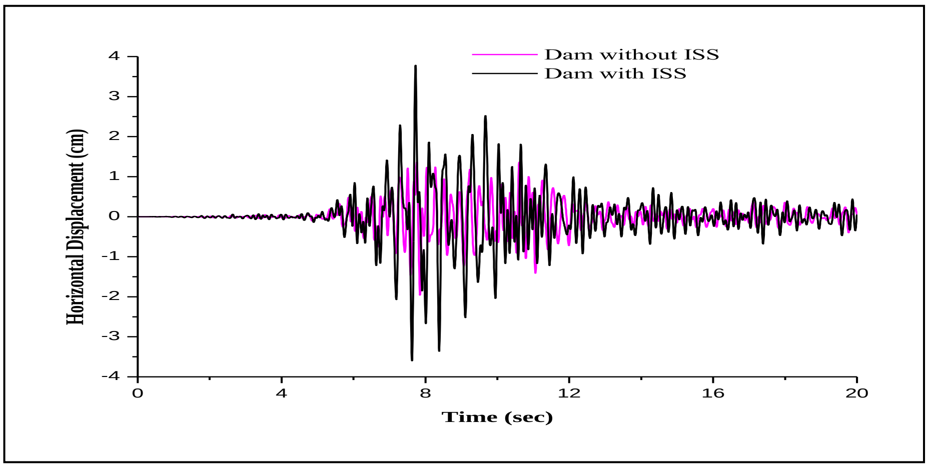


Figure 7. Time history of horizontal displacement at crest of dam in linear analysis

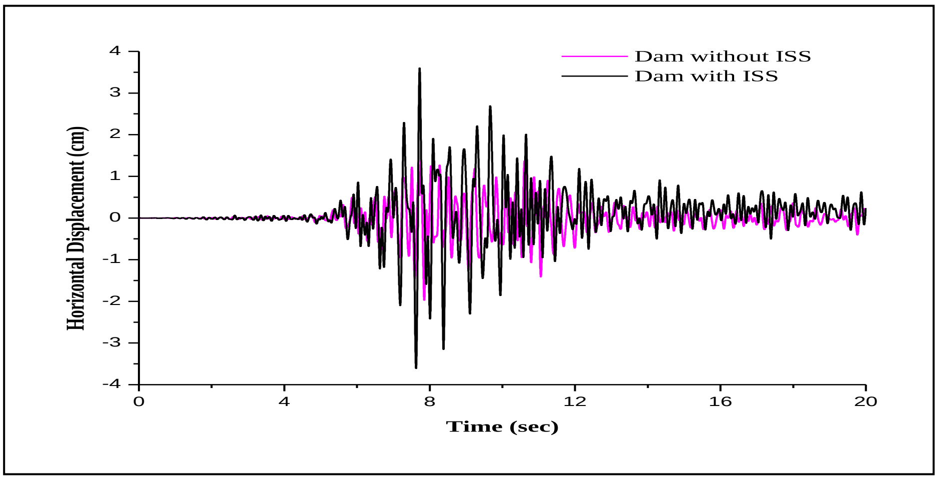


Figure 8. Time history of horizontal displacement at crest of dam in nonlinear analysis

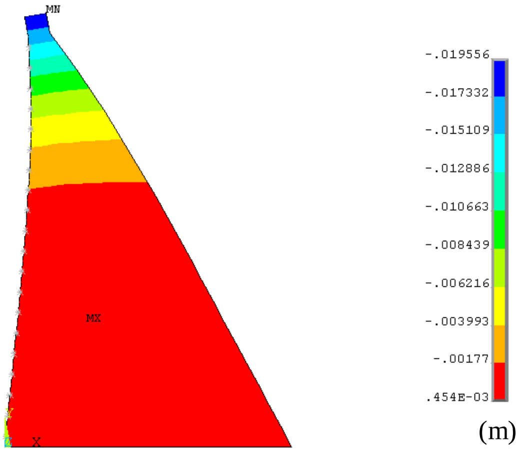


Figure 9**.** Horizontal displacement contour diagram of the dam for model without ISS

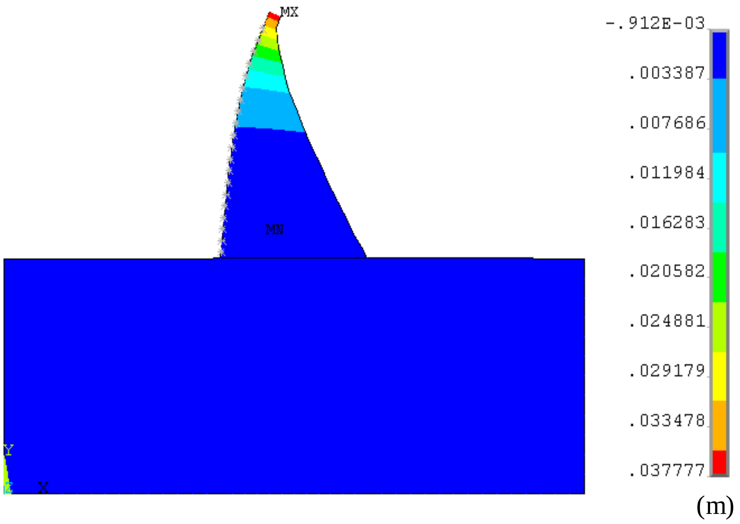


Figure 10. Horizontal displacement contour diagram of the dam for model with ISS

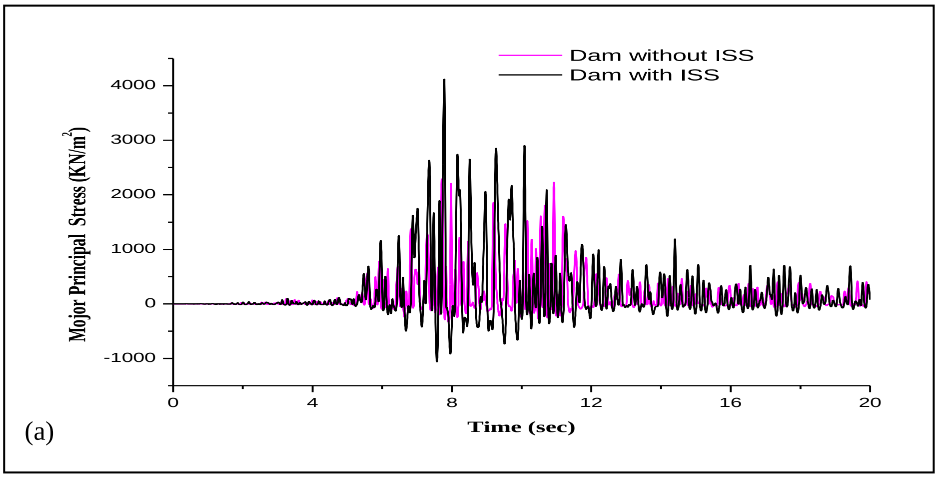
Stresses

Figures 11 and 12 show the time history of major and minor principal stress in the heel of the dam for both models without and with ISS.

In linear analysis shown in Figure 11, it has been observed that major and minor principal stresses at heel of the dam increase from 2290.94 KN/m^2 and -2693.20 KN/m^2 in case without ISS to from 4120.90 KN/m^2 and -4185.10 KN/m^2 in case with ISS (80 % and 50.4 %) respectively.

According to nonlinear analysis (Figure 12), the major and minor principal stresses at heel increase from 2267.1 KN/m^2 and - 2716.5 KN/m^2 in case without ISS to from 2993.2 KN/m^2 and -4189.6 KN/m^2 in case with ISS (32 % and 54.23 %) respectively.

It is observed that nonlinearity at the dam could decrease or decrease the overall stresses at heel compared to the case when dam material is considered to be linear. Therefore, dam nonlinearity could increase or decrease the stress values in the dam body depending upon the ground motion characteristics (Halabian and Naggar, 2002).



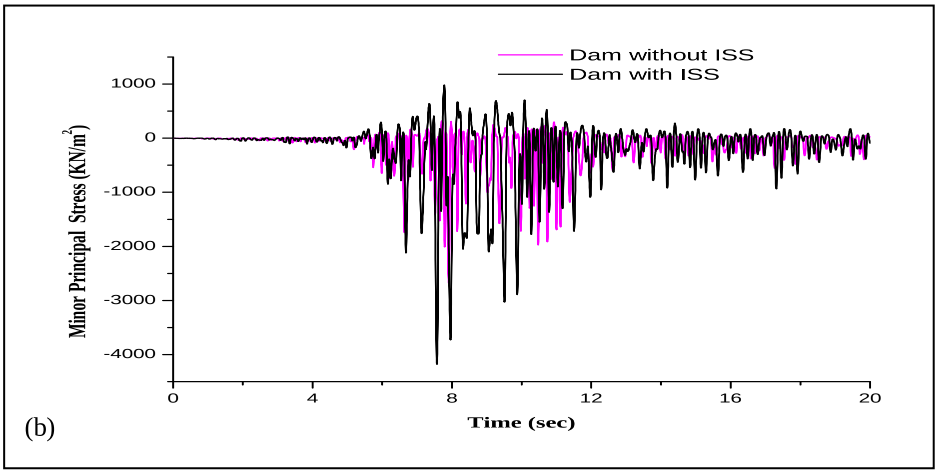


Figure 11. Time history of principal stress at heel of dam in linear analysis: (a) major principal stress; (b) minor principal stress

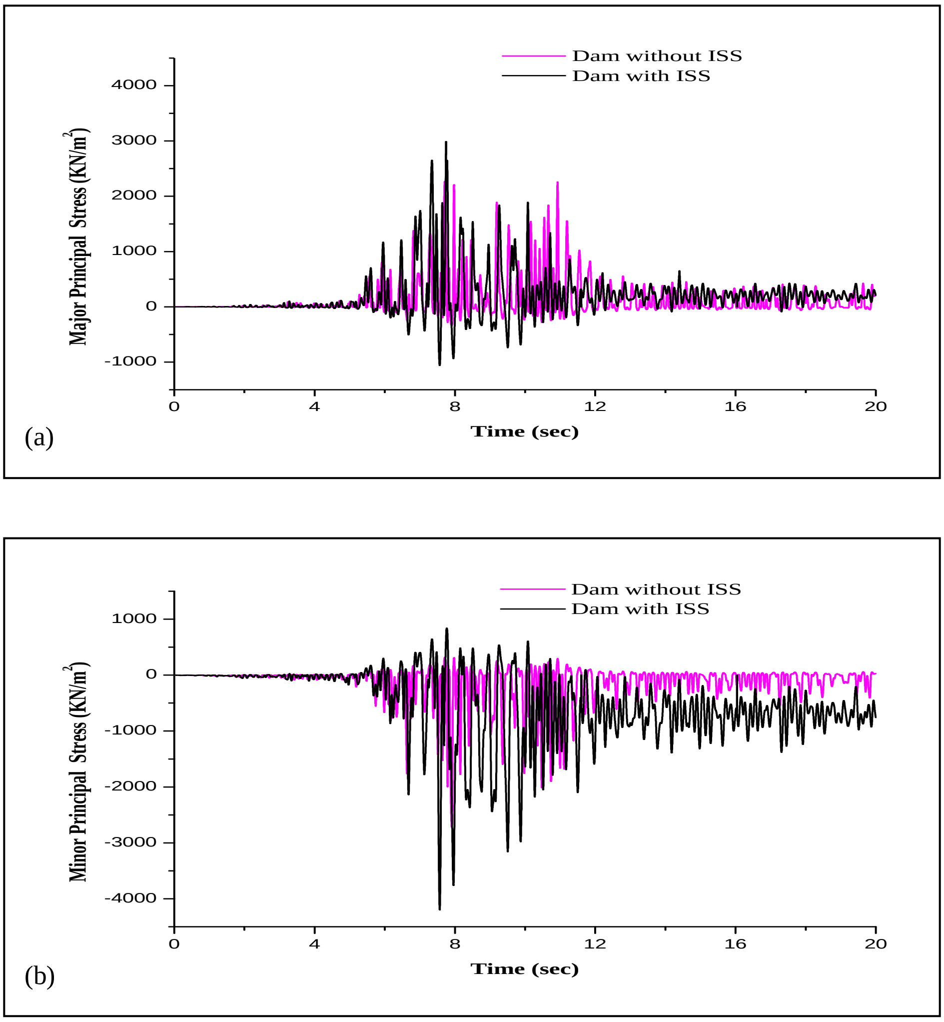


Figure 12. Time history of principal stress at heel of dam in nonlinear analysis: (a) major principal stress; (b) minor principal stress

CONCLUSIONS

The effect of soil-structure interaction on the seismic response of concrete gravity dams is investigated in this paper. For this purpose, two finite element models: dam alone without ISS and dam with ISS are conducted to simulate the behavior of concrete gravity dams. Linear and nonlinear analyses are performed. The Numerical results show that the displacements and stresses have increased for the model with ISS as compared to the model without ISS. Hence, it is advisable to carry out the soil-structure interaction analysis for massive structures. The dynamic soil-structure interaction phenomenon plays an important role in accurately estimating the concrete gravity dam response.

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