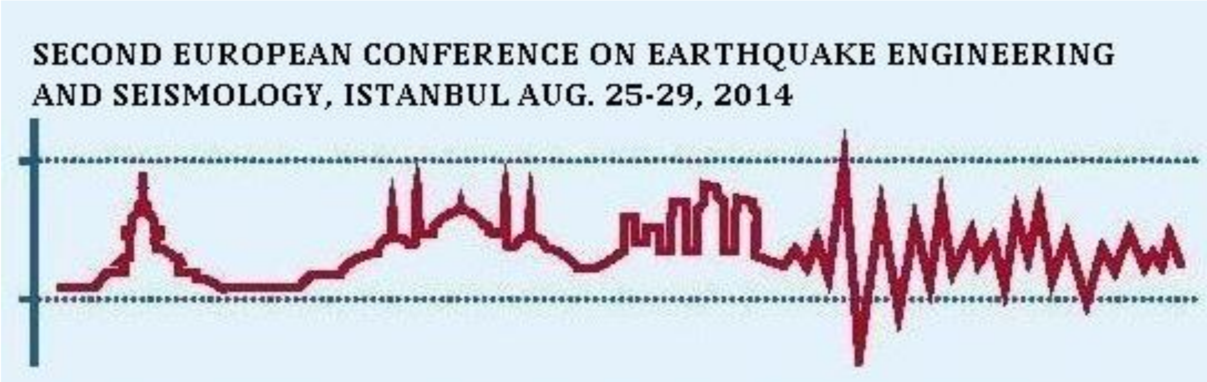

RESPONSE OF CIRCULAR FOUNDATION ON RANDOM SOIL PROFILE

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ABSTRACT

In this paper, the effect of spatial variability of soil properties on the dynamic response of rigid circular foundations resting on viscoelastic medium is studied. The foundation response is computed using the dynamic stiffness matrix formulated with conjunction of the discrete Green's functions. Soil properties of interest are: shear modulus, density, fraction of critical damping and Poisson's ratio, considered as spatially random fields. The presented numerical results indicate that the compliance components exhibit an increase in the amplitude when the degree of the non-homogeneity increases and that the spatial variability of the soil properties causes a filtering effect that shifts the fundamental frequency of the system soil-foundation to lower frequencies means that the heterogeneity makes the soil profile more flexible. Generally, obtained results indicate that shear modulus and Poisson's ratio are of prime importance. So, density and fraction of damping variability can be neglected in a dynamic analysis of foundation response.

INTRODUCTION

The study of the dynamic response of foundations is of significant importance in the design of foundations supporting vibrating machine as well as in designing structures taking into account the soil-foundation interaction. The damage caused by earthquakes indicate that the local soil properties such as shear modulus, damping, the ratio of impedance, the geometry of the foundation, soil layering, etc. play an important role in the dynamic behavior of the system soil-foundation. A crucial step in implementing a ground response analysis is the selection of the soil parameters values used to simulate the dynamic behavior of the soil profile. However, the soil parameters deal with many kinds of uncertainties which must be interpreted from limited observations and data availability. In recent publications (Fenton, 1990; Fenton and Vanmarcke, 1998; Rahman and Yeh, 1999; Nour et al., 2003), show that the randomness of the soil on earthquake ground response analysis has an important consideration and cannot be neglected for the development of design ground motions and response spectra, studies of soil liquefaction potential and dynamic soil-structure interaction. The authors have reported the successful implementation of a finite element method combined via Monte Carlo simulations for the solution of the probabilistic seismic response of heterogeneous elastic soil profile.

In this paper, using the successful technique developed in (Nour, 2004) for generating the random fields of bounded and unbounded values, the compliance function of a rigid circular foundation resting on homogeneous viscoelastic strata is analyzed. Soil properties of interest are shear modulus, density, fraction of critical damping and Poisson's ratio modeled herein as spatially random fields by considering the spatial Gaussian correlation. The shear modulus is modeled using the

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lognormal distribution and the bounded soil properties (density, fraction of critical damping and Poisson's ratio) are modeled using the Beta distribution. For calculation of the foundation response one used a formulation of stiffness matrix with conjunction of the discrete Green's functions (Kausel, 1982; Seale and Kausel, 1989).

DYNAMIC RESPONSE OF 3-D FOUNDATIONS

Considering a shallow foundation resting on multi-layered homogenous soil over rigid base, the dynamic response can be calculated rigorously based on the three-dimensional elastodynamics using the formulation of stiffness matrix with conjunction of the discrete Green's functions. These later are computed through the Thin Layers Method (TLM). Indeed, soil layers have subdivided into N sublayers in which the nodal displacement vector may be linearized (Kausel, 1981; Kausel, 2006). The principal advantage of this technique is the substitution of algebraic expressions in place of more involved transcendental functions. This approximation thus introduced is that no longer satisfying the wave equations due to residual stresses at each nod (interface). The equilibrium is preserved at nodes; therefore, is requiring that the variational equation of motion in an infinitesimal volume must vanish for any virtual displacement modal by applying the virtual work principle; the dynamic stiffness matrix of sub-layer (m) may be written as follow:

$$K_m = A_m\kappa^2 + B_m\kappa + G_m - \omega^2 M_m \qquad (2.1)$$

in which κ = wave number, ω = circular frequency, and A_m , B_m ,, G_m , M_m , are sublayer matrices that only depend on the soil properties of the sublayers (Kausel, 1981; Kausel, 2006). The global stiffness matrix is constructed by overlapping the contribution of layer matrices at each nod of system. In the case of bedrock, only part of the last sub-stiffness matrix N contributes to the formation of the global matrix is considered and this is due to zero displacement at the soil-bedrock interface. Hence, the eigenvalue problems for the natural modes of wave propagation are algebraic, and may be solved by standard techniques.

Propagation modes, Green's functions

The natural modes of wave propagation are obtained from the quadratic eigenvalue problem that shows from setting the load vector equal to zero. The Green's functions are obtained by inverting the global stiffness matrix K through a spectral decomposition in terms of the eigenvalues κ (Kausel, 1982), and after performing Fourier transform inversion, one gets displacement discrete Green's functions for uniform horizontal and vertical distributed loads in spatial domain as developed in (Pais, 1988; Hadid and Berrah, 2002; Berrah and Hadid, 2003).

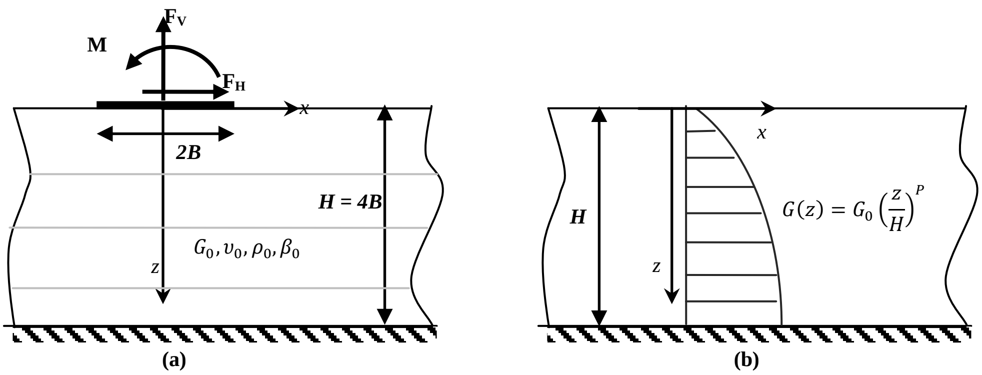

Figure 1. (a) Surface foundation on homogeneous strata over rigid base, (b) Power exponent variation of shear modulus

Dynamic compliance of foundations

Dynamic soil-structure interaction problems have a considerable importance in earthquake resistant design of structures. Therefore, the computation of the dynamic compliance of the foundations, i.e. the translatory or rotatory motion produced by unit dynamic forces or moment acting on a massless rigid foundation embedded in layered strata or in layered half space is of great interest. One can write

$$\begin{Bmatrix} U_H^x \\ \theta_R^y \\ U_H^y \\ \theta_R^x \\ U_V^z \\ \theta_T^z \end{Bmatrix} = \begin{bmatrix} F_X & F_{XR} & & & & \\ F_{RX} & F_R & & 0 & & 0 \\ & & F_Y & F_{YR} & & \\ & 0 & F_{RY} & F_R & & 0 \\ & & & & F_Z & \\ & 0 & & 0 & & F_T \end{bmatrix} \begin{Bmatrix} F_H^x \\ M_R^{xy} \\ F_H^y \\ M_R^{yx} \\ F_V^z \\ M_T^z \end{Bmatrix} \tag{2.2}$$

As mentioned, the terms of stiffness matrix (sometimes called impedance functions) are complex and depend to dimensionless frequency, a_0 which includes frequency excitation ω, foundation dimensions B and shear wave velocity, C_s, of site thereon. In addition, for the case of shallow foundations, the values of coupling terms $F_{RX,Y}$ are often small (especially for the typical values of Poisson's ratio, values between 0.3 and 0.5), which is usually ignored, unlike for the case of embedded foundations. The determination of the impedance matrix passes through the following steps:

- Mesh in finite element (disk) of the contact soil-foundation interface,
- Determination of elementary flexibility matrix between each two finite elements,
- Assembling the global flexibility matrix of foundation block,
- Reverse the flexibility matrix to get stiffness matrix.

RANDOM FIELD MODEL FOR SOIL PROPERTIES

Generally, the dispersion observed in soil data comes from both the spatial variability which greatly influences the behavior of large structures and from errors in testing. Thus, it is hardly surprising that mechanical properties of soils vary from place to place within resulting deposits. In principle, spatial variation of soil properties can be characterized in detail, but only if a large number of samples is available. In reality, the number of tests required far exceeds that which would be practical. Thus, for engineering purposes, one assumes that spatial variability of soil properties is decomposed into a deterministic trend, and a random component describing the variability about that trend (Fenton, 1990). In order to investigate the heterogeneous character of soil, the shear modulus, density, fraction of critical damping and Poisson's ratio are modeled herein as spatially random fields. For the random simulation of the medium, the spatial variation is considered only in the vertical direction and the chosen random variables are defined by their moments of order 1 and 2, which are respectively the mean, and the variance supposed estimated from in situ samples. Indeed, the mean shear modulus increases with some power exponent of depth (Hadid and Afra, 2000). Let the variable property $f_p(z)$ assumed to be a one-Dimensional homogeneous stochastic process defined as a function of the deterministic function $f_{0p}(z)$ describing the trend in space, taken in practice as the mean of measured values, and an added fluctuation random function $\Delta f_p(z)$ with zero mean and variance equal to unity. One can write (Fenton, 1990)

$$f_p(z) = \Re\left[f_{0p}(z) + \sigma_p\, \Delta f_p(z)\right] \tag{3.1}$$

$\Re$ is a transformation taking the Gaussian process $\Delta f_p(z)$, into the distribution appropriate for $f_p(z)$ and σ_p is the standard deviation. Considering the fluctuation component $\Delta f_p(z)$ of a soil property exhibiting spatial variability over the profile that is assumed to have zero mean and unite variance

$$\begin{cases} E\left[\Delta f_p(z)\right] = 0 \\ \left(\sigma_{\Delta f_p(z)}\right)^2 = 1 \end{cases} \tag{3.2}$$

and auto-correlation function

$$R(\xi) = \sigma^2 \exp\left(-\left(\frac{\xi}{a}\right)^2\right) \tag{3.3}$$

the corresponding spectral density function $S(\kappa)$ is

$$S(\kappa) = \frac{a\sigma^2}{2\sqrt{\pi}} exp\left(-\left(\frac{\kappa a}{2}\right)^2\right) \tag{3.4}$$

a is correlation distance. Using the spectral density function and Fast Fourier Transform (FFT) (Yamazaki and Shinozuka, 1988; Hadid and Berrah, 1995), the fluctuation function $\Delta f_p(z)$ can be written as follows (Shinozuka, 1987; Yamazaki and Shinozuka, 1988; Amin and Ang, 1968):

$$\Delta f_p(z) = \sqrt{2}\, Real\left[\exp\left(i\frac{\Delta\kappa z}{2}\right) \sum_{j=0}^{M} \sqrt{2S(\kappa_j)\Delta\kappa}\, \exp(i\,\Omega_j)\; \exp(i\Delta\kappa jz)\right] \tag{3.5}$$

Where

$$\begin{aligned} &\kappa_j = j\Delta\kappa, \;\; j = 1,..N \\ &z = r.\Delta\xi, \;\; \Delta\xi = \frac{2\pi}{\Delta\kappa\, M} = \frac{2\pi N}{\kappa_u\, M} \;; \quad M \geq 2N \quad \& \quad r = 1,2\ldots,(M-1) \end{aligned} \tag{3.6}$$

The upper bound wave number is $\kappa_u = N\Delta\kappa$, which represents an upper cut-off frequency beyond which $S(\kappa)$ may be assumed to be zero, for either mathematical or physical reasons (Rahman and Yeh, 1999; Nour, 2004). Ω_j is a set of phase angles distributed uniformly over the range $[0,2\pi]$. According to the central limit theorem, the simulated process is asymptotically Gaussian as N becomes large. The FFT technique is applied to speed-up the process (Shinozuka,1987) and produces simulations that are ergodic in the mean without any restriction assumption at the origin of $S(\kappa)$. In result, the application of the FFT in Eqn. (3.5) provides values for the simulation over half of the wavelength $L/2 = 2\pi/\Delta\kappa$, (Shinuzuka *et al.*, 1987) hence, the simulation should be extended for the entire wavelengths, and can be also easily extended to distances longer than the wavelength L.

Spatial variability of shear modulus

The shear modulus is assumed to be lognormally distributed, this choice is motivated by the fact that this soil property is positive parameter, and lognormal distribution enables analyzing its large variability. The shear modulus expression is given by

$$G(z) = \exp\left(\mu_{\ln G}(z) + \sigma_{\ln G}\, \Delta f_G(z)\right) \tag{3.7}$$

With

$$\begin{cases} \sigma_{\ln G}^2 = \ln\left(1 + \frac{\sigma_G^2}{\mu_G^2}\right) \\ \mu_{\ln G} = \ln(\mu_G) - \frac{1}{2}\sigma_{\ln G}^2 \end{cases} \tag{3.8}$$

μ_G, σ_G^2 respectively stand for shear modulus mean and variance. As proposed, the shear modulus mean, $\mu_G(z)$, is considered variable power exponent of depth whose average value $G_0 = 40$ MPa, at the interface layer of soil-bedrock(value of $\mu_G(z = H)$), Fig. 1.b.

$$\mu_G(z) = G_0 \left(\frac{z}{H}\right)^P \quad ; \quad 0 \le p \le 1 \tag{3.9}$$

The dimensionless parameter (p) represents the soil heterogeneity degree, varying between 0 and 1. Considering a viscoelastic characteristics, Lame constants will have complex values obtainable by introducing hysterical damping factor, β, as follow

$$G_0^* = G_0(1 + 2i\beta) \qquad , \; i = \sqrt{-1} \tag{3.10}$$

Spatial variability of bounded soil properties

It is well known that density ρ, fraction of critical damping β and Poisson's ratio υ for soil media are bounded in practice between two extreme values, so, the probability distribution appropriate for a random variable whose values are bounded is the Beta distribution. In this paper, one is using the technique well described by (Nour *et al.*, 2002), for which the expressions of variation of bounded soil properties are given respectively by

$$\begin{cases} \upsilon(z) = \upsilon_{min} + (\upsilon_{max} - \upsilon_{min})\, V_\upsilon(z) \\ \rho(z) = \rho_{min} + (\rho_{max} - \rho_{min})\, V_\rho(z) \\ \beta(z) = \beta_{min} + (\beta_{max} - \beta_{min})\, V_\beta(z) \end{cases} \tag{3.11}$$

The result, $V_i|_{i=\upsilon,\rho,\beta} \in [0\ 1]$, is the solution of the integral equation of the Beta cumulative distribution function (CDF) with parameters q and r where it is supplied the desired probability function $P_{\bar{V}_i}$ (Nour *et al.*, 2002; Nour *et al.*, 2002), e.g. :

$$B(V_i|q,r) = \frac{B_{V_i}(q,r)}{B(q,r)} = P_{\bar{V}_i} \tag{3.12}$$
$$q, r > 0\,; \quad i = \upsilon, \rho, \beta$$

In this study, because density, soil Poisson's ratio and fraction of critical damping are positive parameters, one prefers to map the Beta CDF with the lognormal CDF one, and $\bar{V}_i$ is one to one mapping of $\bar{V}_i \in [0\ \ +\infty]$ into $P_{\bar{V}_i} \in [0\ 1]$. $\bar{V}_i$ expressions are given by

$$\bar{V}_i(z) = \exp\left(\mu_{\ln i} + \sigma_{\ln i}\, \Delta f_i(z)\right) \tag{3.13}$$

With

$$\begin{cases} \sigma_{ln\,i}^2 = ln\left(1 + \frac{\sigma_i^2}{\mu_i^2}\right) \\ \mu_{ln\,i} = ln(\mu_i) - \frac{1}{2}\sigma_{ln\,i}^2 \end{cases} \quad , \quad i = \upsilon, \rho, \beta \tag{3.14}$$

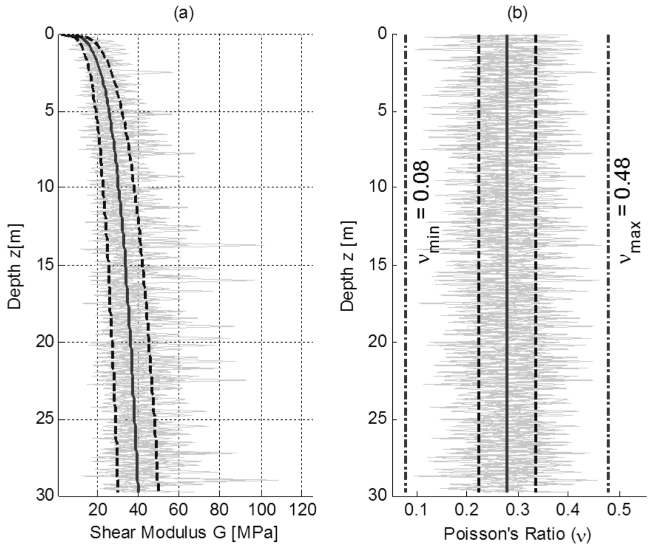

Figure 1. Spatial variability of shear modulus (a) and Poisson's ratio (b)

NUMERICAL RESULTS AND DISCUSSIONS

Preliminaries

In this section we use the above procedures for investigating the effect of spatial variability of the soil properties on the response of 3-D foundations. Therefore, the foundation is a circular form and considered as massless getting rigid body motion. The vertical, horizontal, rocking and torsion compliances functions are analyzed. The soil-foundation interface is discretized by ten (10) equal elements as constant (disc) leading the Green's functions applied. The stratum depth, $H = 4B$, is divided into 08 sublayers. The soil is modeled in first time as viscoelastic homogeneous medium with the following properties:

The mean of shear modulus is $G_0 = 40\ MPa$, Mean density $\rho_0 = 1800\ kg/m^3$ as limited by two values $\rho_{min} = 1400\ kg/m^3$ and $\rho_{max} = 2200\ kg/m^3$, the Poisson's ratio mean is to $\upsilon_0 = 0.28$, bounded by 0.08 and 0.48, and the critical damping ratio of mean equal to $\beta_0 = 6\%$, with $\beta_{min} = 2\%$ and $\beta_{max} = 10\%$. The coefficients of variation which is the standard deviation over the mean, of the four soil properties are given in table 4.1.

In second case, soil heterogeneity effects are investigated taking account the three values of power p (p = 0.25, 0.50 and 1), and with respect to the homogeneous case (p = 0).

Table 1. Soil Properties Values

Location	Mean	Min Values	Max Values	Coefficients of variation
Shear modulus (MPa)	40	-	-	0-0.25-0.50-1.00
Soil density (kg/m^3)	1800	1400	2200	0-0.10-0.15-0.20
Poisson's ratio	0.28	0.08	0.48	0-0.10-0.30-0.50
Critical damping ratio (%)	6	2	10	0-0.10-0.30-0.50

Soil properties variation effects

It is investigated the influence of variabilities in shear modulus, density, fraction of critical damping and Poisson's ratio on foundations response statistics of a rigid circular foundation resting on homogeneous viscoelastic strata.

- Fig. 3 shows the influence of shear modulus variability on the mean compliance functions. One observes that as CV_G increases as the fundamental frequency of the system soil-foundation is shifted to left. This result indicates that as heterogeneity of the medium increases, the frequency content is dominated by the lower frequencies and the simulated soil becomes softer. Moreover, one observes that increase in CV_G also induces an increase in the mean foundation response and this is most noticeable for large values of CV_G.
- Fig. 4 presents the influence of the variability in the Poisson's ratio CV_υ on the mean compliance functions. The obtained results indicate that variability in the Poisson's ratio has a more pronounced effect on the vertical and rocking displacement than on the horizontal displacement. Also one can observe that when CV_υ increase, vertical and rocking displacement decrease, implying that the soil became more rigid.
- Fig. 5 shows the influence of the variability in the density CV_ρ on the mean compliance functions. The variability in the density CV_ρ exhibits a slight influence on the mean foundation response, practically these influence can be neglected. Also, one can observe that when CV_ρ increases displacements decrease particularly in resonance frequencies
- The curves of Fig. 6 are plotted for different values of CV_β. The results indicate that the variability in fraction of critical damping does not exhibit any significant influence on mean foundation response.

Soil heterogeneity effects

The variation of shear modulus in depth defined by a dimensionless parameter 'p' affects the frequency content and amplitude of foundation response (fig. 7). Indeed, the flexibility functions exhibit increasing in amplitudes when the degree of inhomogeneity increases. Moreover, a reduction in displacement amplitudes is found to be the result of non-homogeneity. In other words, the displacement is greater for loose soils ('p' large) for coherent and soft soils ('p' small).

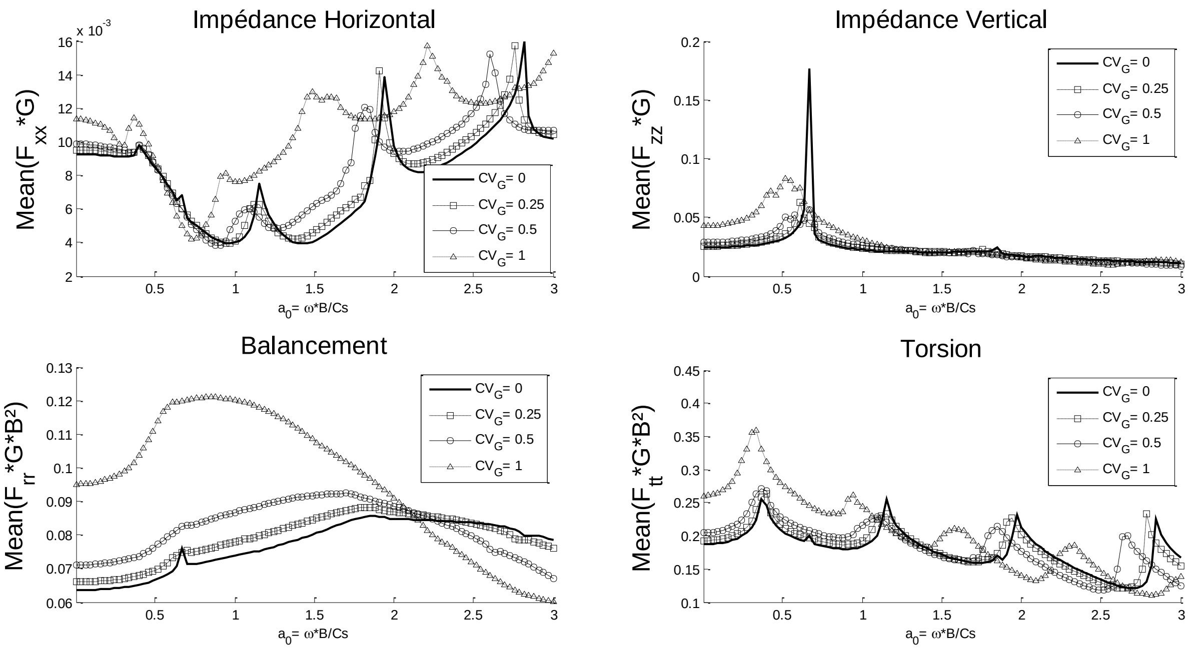

Figure 3. Compliance functions of rigid circular foundation for a homogeneous viscoelastic layer for different values of CV_G

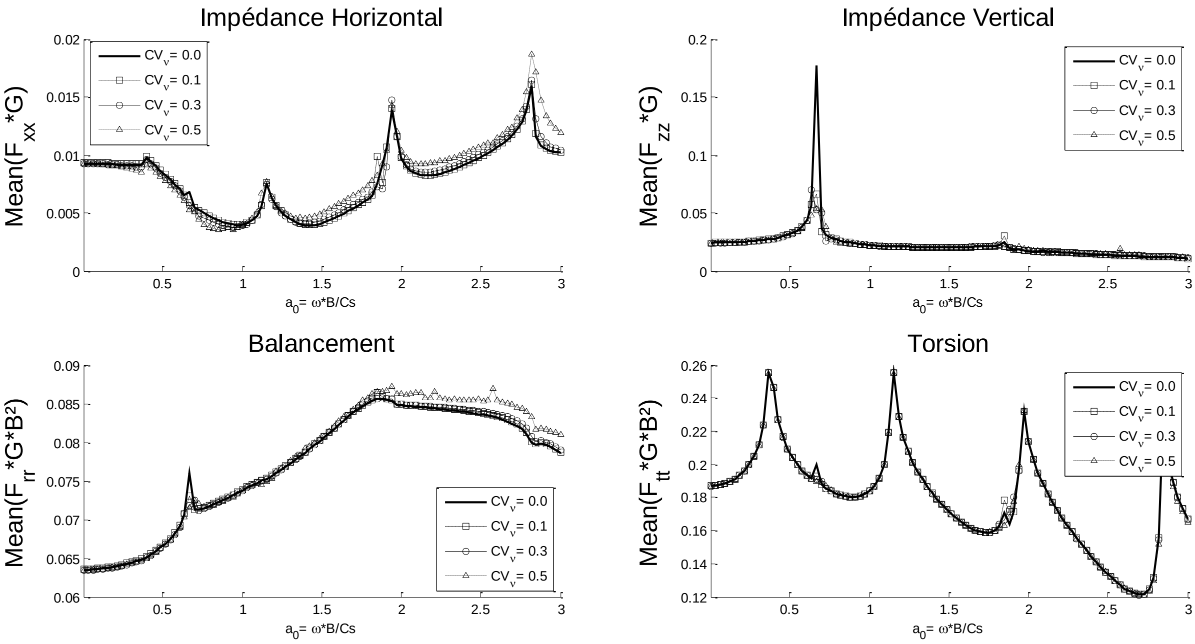

Figure 4. Compliance functions of rigid circular foundation for a homogeneous viscoelastic layer for different values of CV_υ

CONCLUSION

In this paper, the response of a rigid circular foundation resting on homogeneous viscoelastic strata is analyzed. Soil properties of interest are shear modulus, density, fraction of critical damping and Poisson's ratio modeled as spatially random fields by considering the spatial Gaussian correlation. The Shear modulus is modeled using the lognormal distribution, so that the density, fraction of critical damping and soil Poisson's ratio by Beta distribution.

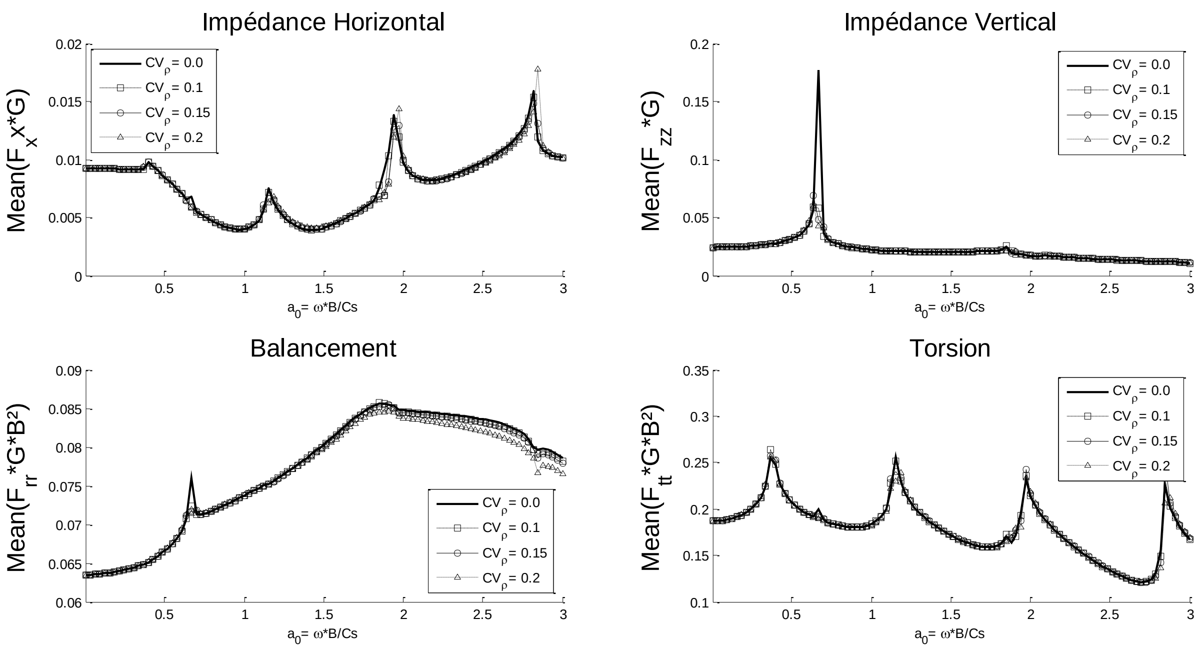

Figure 5. Compliance functions of rigid circular foundation for a homogeneous viscoelastic layer for different values of CV_ρ

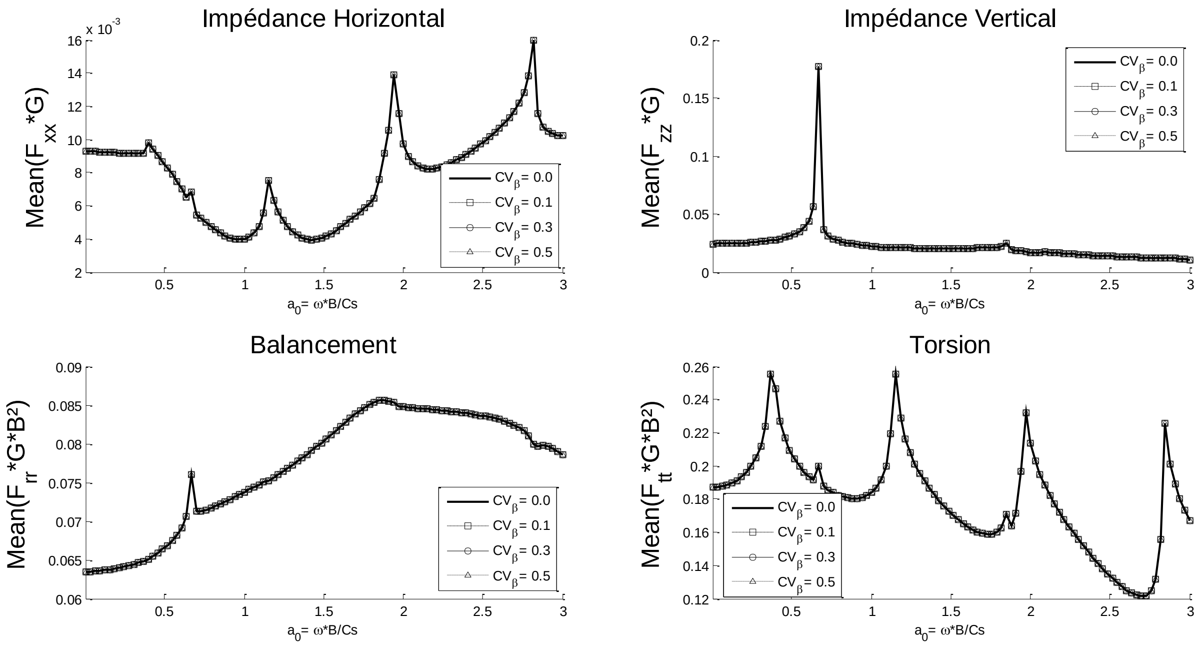

Figure 6. Compliance functions of rigid circular foundation for a homogeneous viscoelastic layer for different values of CV_β

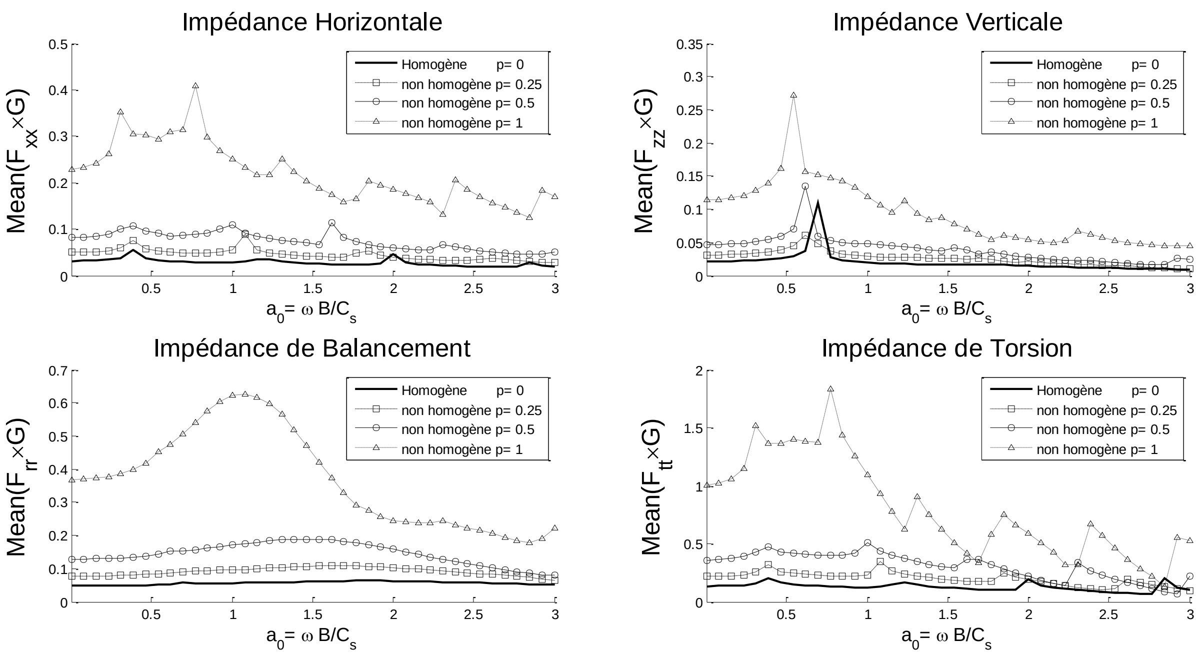

Figure 7. Soil heterogeneity effects on compliance functions of rigid circular foundation

CONCLUSION

In this paper, the response of a rigid circular foundation resting on homogeneous viscoelastic strata is analyzed. Soil properties of interest are shear modulus, density, fraction of critical damping and Poisson's ratio modeled as spatially random fields by considering the spatial Gaussian correlation. The Shear modulus is modeled using the lognormal distribution, so that the density, fraction of critical damping and soil Poisson's ratio by Beta distribution.

Obtained results indicate that increase in coefficient of variation of shear modulus CV_G shifts the fundamental frequency of the system soil-foundation to lower frequencies and increases the foundation response amplitude that explains that the heterogeneity makes the soil profile more flexible. The variability in the density CV_ρ has a slight effect on the mean foundation response, practically these influence can be neglected. The displacement decreases when the variation of CV_ρ increases particularly at resonance frequencies. The variability in fraction of critical damping has no influence on the mean foundation response. The results show that when the variability of Poisson's ratio CV_υ increases, vertical and rocking displacement decrease, implying that the soil becomes more rigid. Practically the influence of Poisson's ratio on the horizontal displacement can be neglected.

To compare the effect of soil heterogeneity, the flexibility functions exhibit increasing in amplitudes when the degree of inhomogeneity increases. Therefore, the displacement is greater for loose soils for soft soils.

The analysis carried out in this paper indicates that shear modulus and Poisson's ratio are of prime importance. So, density and fraction of critical damping variability can be neglected in a dynamic analysis of a foundation response.

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