



## ON THE PROBABILISTIC SEISMIC ASSESSMENT OF STRUCTURES WITH ADDED VISCOUS DAMPERS

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### ABSTRACT

The purpose of this paper is to propose a simplified probabilistic procedure for the seismic assessment of nonlinear structures equipped with nonlinear fluid viscous dampers. The considered reference probabilistic approach is the 2000 SAC-FEMA method, which allows to obtain the probability of exceeding a specified performance level. Within this method, in general, the probabilistic seismic demand analysis is performed in terms of incremental dynamic analysis (IDA) by considering several earthquake records. This approach has been followed with reference to a set of case studies, characterized by RC frames with and without added dampers. The main characteristics of the proposed simplified approach are: the application of a direct procedure previously proposed by the authors for the response assessment of structures with dampers as an alternative to nonlinear incremental dynamic analyses; the study of the relation between the dispersion of seismic demand of structures with dampers with that of structures without dampers. The latter, in fact, can be estimated also on the basis of several available literature data. In this paper the direct procedure previously proposed by the authors has been applied to the case studies for increasing values of seismic intensity. The results of such applications have been then compared with those of the IDA analyses in terms of the relation between the roof displacement demand and the seismic intensity. The results of the IDA analyses have been also examined in order to determine the correlation between the dispersion of demand of structures with dampers with that of structures without dampers. Finally the values of probability obtained with the proposed simplified approach has been compared with those of the 2000 SAC-FEMA method based on IDA analyses.

### INTRODUCTION

A widespread method for the seismic risk evaluation of structures is the 2000 SAC-FEMA method (Cornell et al., 2002). This method gave the basis for FEMA350 (2000) guidelines for structural design of steel moment resisting frames under seismic action and allows to determine the probability of failure of a structure in a closed form. The application of the method has then been extended to RC structures (Lupoi et al., 2002). Within the framework of the SAC-FEMA method, a relationship between the seismic intensity measure and the considered engineering demand parameter has to be defined. The incremental dynamic analysis (IDA) (Vamvatsikos and Cornell, 2004) represents a powerful and realistic procedure to determine the mentioned relationship for multiple levels of seismic

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intensity. However, for the determination of an IDA curve, which represents the relation between a demand parameter and the seismic intensity, a large number of nonlinear time-history analyses have to be developed. This procedure requires several and detailed input data for a structural system. As a consequence, simplified tools to determine approximated IDA curves are required and can be very useful. In this paper a simplified probabilistic seismic assessment method of inelastic structures with nonlinear viscous dampers is proposed.

The nonlinear fluid viscous dampers are innovative seismic protection devices which can be applied to mitigate the effects of seismic actions on structures (Christopoulos and Filiatrault, 2006). The simplified criteria proposed in literature to quantify the effects of these devices are often based on the estimate of an effective damping (Ramirez et al., 2000), which can be evaluated by the sum of three terms: the inherent damping ratio, the supplemental damping ratio due to added dampers and the hysteretic damping ratio. By knowing these parameters it is possible to define the damping reduction factor to be applied to the elastic response spectrum. The supplemental damping ratio related to nonlinear fluid viscous dampers can be evaluated through the concept of equivalent linear viscous damping (Ramirez et al., 2000). In this way the parameter depends on the maximum displacement of the structure and iterative procedures have to be implemented to find it.

Several proposals have been presented in literature as alternative and simplified methods to determine the IDA curve, as for example the incremental N2 method (IN2) (Dolšek and e Fajfar, 2007). The IN2 method can be successfully applied to consider the nonlinear behaviour of nonlinear structures. If nonlinear fluid viscous devices are inserted in the structural system, the simplified methods commonly used have to be applied iteratively to account for the effects of the energy dissipation devices. However, by introducing a new dimensionless parameter defined as damper index (Diotallevi et al., 2012), and using the direct assessment method (DAM) for nonlinear structure equipped with nonlinear fluid viscous dampers proposed by Landi et al. (2014), it is possible to estimate directly the supplemental damping ratio and the structural response for a given value of seismic intensity. In this way the IDA curves can be directly determined by applying the DAM for different values of seismic intensity. The DAM method applied in this way is here called incremental direct assessment method (IDAM). In a probabilistic framework, the seismic response of a structure is characterized by dispersion due to the ground motion variability (randomness) and to the aleatory related to structural modelling and analysis (uncertainty). The first kind of dispersion can be determined by applying the IDA with several ground motions. Additional analyses have to be performed for the determination of the latter kind of dispersion. In order to simplify the probabilistic assessment procedure and to avoid the IDA analyses, several proposals have been presented in literature for the direct adoption of suggested dispersion values. However these suggested values have been proposed mainly for structures without dampers. Therefore, another objective of this study is the definition of dispersion values for nonlinear structures equipped with nonlinear viscous dampers.

The following paragraphs summarize the probabilistic basis for the 2000 SAC-FEMA method and illustrate the simplified procedure proposed by the authors to approximate the IDA curves. Then, the procedure is verified through several applications regarding a set of structures characterized by a variable number of storeys. The results of the procedure have been compared with those provided by several nonlinear dynamic analyses. Moreover, an extended study has been performed to define a relationship between the first mode spectral acceleration and the dispersion in the considered engineering demand parameter. Finally, a comparison in terms of probability of failure for the structures investigated, calculated by applying the basic approach for the probability assessment in closed form (Cornell et al., 2002), has been made between the procedure based on IDA curves and that based on IDAM curves.

## **PROBABILISTIC ASSESSMENT APPROACH**

Large aleatory and epistemic uncertainty characterize in general the seismic response of a structure, especially if a nonlinear behaviour characterizes the structural system. The 2000 SAC-FEMA method (Cornell et al., 2002) allows to perform the probabilistic seismic assessment of a structure in closed form. This method requires to determine a relation between the spectral acceleration at the fundamental period of vibration  $S_{a,1}$  and a seismic engineering demand parameter. This relationship

can be obtained through the incremental dynamic analysis (IDA) (Vamvatsikos and Cornell, 2004). However, this is a very expensive method because several nonlinear dynamic analysis (NLDA), with increasing values of peak ground acceleration (PGA), have to be performed for each considered accelerogram.

The 2000 SAC-FEMA method provides the annual probability of failure  $P_{F,LS}$  that characterizes a structure for a given limit state. This probability can be obtained as follows (Cornell et al., 2002; Benedetti et al., 2013; Bianchini et al., 2007):

$$P_{F,LS} = H(S_{a,1}^{LS}) \exp \left[ \frac{k^2}{2b^2} (\beta_D^2 (S_{a,1}^{LS}) + \beta_C^2) \right] \quad (1)$$

where  $S_{a,1}^{LS}$  is the spectral acceleration associated to the achievement of the considered limit state and  $H(S_{a,1}^{LS})$  represents the mean estimate of the hazard curve (Cornell et al., 2002). It should be remembered that the hazard function gives the annual probability that a specific random intensity  $S_a$  at the site will equal or exceed a specific level of spectral acceleration. Furthermore, the parameters  $\beta_D(S_{a,1}^{LS})$  and  $\beta_C$ , in Eq. (1), are respectively the dispersion in the seismic demand and capacity. The dispersion of the seismic demand can be directly derived through the results of the IDA. Otherwise the dispersion of the seismic capacity depends on the uncertainty that characterizes the materials, the geometric and the physic modelling of the actual structure. Moreover, the coefficients  $k$  and  $b$  are constant values which depend on the approximation of the mean hazard function and on the approximation of the IDA curve. The first one can be approximated as follows:

$$H(S_{a,1}^{LS}) = k_0 (S_{a,1}^{LS})^{-k} \quad (2)$$

where the parameters  $k$  and  $k_0$  are constant values that depend on the approximation of the hazard function. It should be noted that the parameter  $k$  is present also in Eq. (1). In a similar way, the IDA curve can be approximated through a power function. As engineering demand parameter it can be selected, for example, the displacement of a control point located at the top of a building ( $D_{roof}$ ). Then, the IDA curve, which represents the relationship between the spectral acceleration at the first mode of vibration and the roof displacement of the structure, can be approximated as follows:

$$D_{roof} = a S_{a,1}^b \quad (3)$$

where  $b$  is the exponent of the power function that approximates the IDA curves and is present in Eq. (1), while  $a$  is another constant value that depends on the approximation of the IDA curve.

Eq. (1) represents a primary result reached by Cornell et al. (2002). It should be noted that if the IDA curve is known, the relationship between the spectral acceleration at the first mode of vibration and the engineering parameter can be approximated by applying Eq. (3). Moreover, by developing the IDA, also a relationship between the spectral acceleration at the first mode of vibration and the dispersion in the engineering demand parameter can be determined. Then, if a prefixed limit state is defined (for example, in terms of roof displacement), the corresponding spectral acceleration at the fundamental mode of vibration can be calculated by means of the hazard function approximated by Eq. (2). By knowing the spectral acceleration at the fundamental mode of vibration for the fixed limit state, it is possible to determine a value of the parameter  $\beta_D(S_{a,1}^{LS})$ . With regard to  $\beta_C$ , values usually suggested in literature can be adopted. At this point, all the parameters needed in Eq. (1) are known and the probability of failure of the structure investigated can be determined.

As explained in the introduction, the determination of the IDA curve requires in general a great burden. As a consequence, simplified methods have been proposed to define the approximated IDA curve, as for example the IN2 method (Dolšek and e Fajfar, 2007). However, if nonlinear viscous dampers are inserted in the structures, the usual simplified methods have to be repeated iteratively. In this case the DAM method (Landi et al., 2014) previously proposed by the authors can be useful. This method allows to directly determine the seismic response of structures with added dampers and can be applied for increasing values of PGA (IDAM), thus obtaining an approximation of the IDA curve.

## INCREMENTAL DIRECT ASSESSMENT METHOD (IDAM)

The DAM procedure can be applied to obtain directly the seismic response of a given nonlinear structure equipped with nonlinear viscous dampers. It is composed by two steps that are briefly presented below.

The first step consists in the linearization of the nonlinear viscous dampers. It is aimed at the direct estimate of the supplemental damping provided by nonlinear viscous dampers applied to a linear elastic structure. To this purpose a dimensionless parameter called damper index (Diotallevi et al., 2012) is introduced and presented both for SDOF and MDOF systems. The damper index for a SDOF system can be determined as follows:

$$\varepsilon = \frac{\lambda c_{NL}}{\pi 2m} \left(\frac{T_e}{2\pi}\right)^\alpha (\ddot{u}_{g0})^{\alpha-1} \quad (4)$$

where  $c_{NL}$  is the damping coefficient,  $T_e$  is the elastic period of the system with mass  $m$ ,  $\ddot{u}_{g0}$  is the PGA of the seismic action and  $\lambda$  is a constant which depends on the exponent of the velocity  $\alpha$ . The damper index does not depend on the structural response and is related to the supplemental damping under elastic condition  $\xi_{ve}$  as follows:

$$\xi_{ve} = \varepsilon (R_d)^{\alpha-1} = \varepsilon \left(\frac{u_0 \omega^2}{\ddot{u}_{g0}}\right)^{\alpha-1} \quad (5)$$

where  $R_d$  is the deformation response factor, which is proportional to the product of the square of the natural frequency  $\omega$  of the SDOF system with the ratio between the maximum displacement  $u_0$  and the peak ground acceleration  $\ddot{u}_{g0}$ . By introducing the previous relations in the equation of motion of a SDOF system equipped with nonlinear viscous damper it is possible to obtain an expression which is independent from the structural response.

If a MDOF system is considered, the damper index for the first mode can be defined as:

$$\varepsilon_1 = \frac{T_{e1}^\alpha \lambda \sum_{j=1}^{N_D} c_{NL} f_j^{1+\alpha} \phi_{rj1}^{1+\alpha}}{(2\pi)^{1+\alpha} (\Gamma_1 \ddot{u}_{g0})^{1-\alpha} \sum_{i=1}^N m_i \phi_{i1}^2} \quad (6)$$

where  $N_D$  and  $N$  are respectively the number of devices and of the degrees of freedom (DOF),  $f_j$  is an amplification factor related to the geometrical arrangement of the damper,  $T_{e1}$  is the elastic period of the first mode of vibration,  $\phi_{rj1}$  is the difference between the first modal ordinates associated with the DOF  $j$  and  $j-1$ ,  $\Gamma_1$  is the first modal participation factor and  $m_i$  is the mass of the  $i$ -th DOF. The damper index and the supplemental damping under elastic condition are related through Eq. (5), but the deformation response factor depends on the amplitude of the roof displacement  $D_{roof}$  and on the natural frequency  $\omega_1$ :

$$R_d = \frac{D_{roof} \omega_1^2}{\Gamma_1 \ddot{u}_{g0}} \quad (7)$$

Once the damper index is known, it is possible to calculate the supplemental damping  $\xi_{ve}$  using particular spectra as a function of the elastic period (Fig.1). These spectra are derived for given values of the damper index and of the exponent of velocity (Diotallevi et al., 2012). They can be obtained for a single ground motion or on the basis of the code response spectra; the damper index, in fact, can be expressed as a function of the pseudo-acceleration which can be derived from the design response spectra provided by codes:

$$\varepsilon = \xi_{ve} [\bar{S}_a(T_e, \xi_{ve})]^{1-\alpha} \quad (8)$$

where  $\bar{S}_a$  is the acceleration normalized to the PGA. The spectra shown in Fig.1 derive from the response spectrum in terms of acceleration provided by the Italian Building Code (Min.LL.PP., 2008).

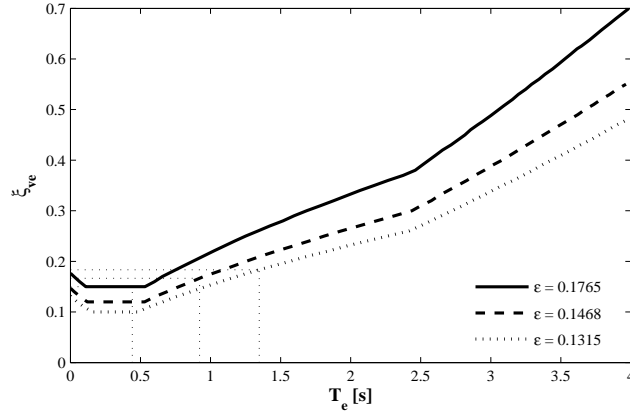


Figure 1. Spectra of the supplemental damping  $\xi_{ve}$  for given values of  $\varepsilon$  ( $\alpha = 0.5$ )

The second step of the DAM extends the procedure to consider the nonlinear behaviour of a structure, which can be studied through pushover analysis. In the acceleration-displacement response spectrum format (ADRS) two functions can be determined to represent a relationship between the spectral acceleration  $S_a$  and the spectral displacement  $S_d$  (Fig.2): the spectral capacity curve and the spectral demand curve. If the two functions intersect, the structure is able to resist the seismic effects, on the contrary a retrofit of the system is necessary. An effective period can be associated to the SDOF system equivalent to the actual structure. For an elastic-perfectly plastic system, the elastic and effective periods ( $T_e$  and  $T_{eff}$ ) are related through the ductility demand  $\mu_D$  (Ramirez et al., 2000) as follows:

$$T_{eff} = T_e \sqrt{\mu_D} \quad (9)$$

Let us consider Fig.2. In order to obtain an intersection between the capacity and demand spectrum it is necessary to increase the damping ratio of the structure. The damping reduction factor  $B$  of the spectral ordinates can be determined with the following equation:

$$B = \frac{S_{a,el}(T_{eff})}{S_{ay}} = B(\xi_{eff}) \quad (10)$$

where  $S_{a,el}$  is the elastic demand in terms of acceleration and  $S_{ay}$  is the maximum level of acceleration that the structure can bear. Eq. (10) also shows that the damping reduction factor is a function of the effective damping  $\xi_{eff}$ , which is the total damping of the equivalent SDOF system. By inverting Eq. (10), and considering Eq. (9), it is possible to obtain the constant capacity damping curves that correlate the effective damping with the ductility demand (unknowns of the problem) for a given value of the fundamental elastic period (Fig.3):

$$\xi_{eff} = B^{-1} \left( \frac{S_{a,el}(T_{eff})}{S_{ay}} \right) = B^{-1} \left( \frac{S_{a,el}(\mu_D)}{S_{ay}} \right) \quad (11)$$

Another relation between the effective damping and the ductility demand can be derived by examining the damping properties of the structural system with the damping devices, for a given value of the supplemental damping  $\xi_{ve}$ , which is the output of the first step of the DAM:

$$\xi_{eff} = \xi_i + \frac{C}{\pi} \left( 1 - \frac{1}{\mu_D} \right) + \xi_{ve} (\mu_D)^{1-\frac{\alpha}{2}} \quad (12)$$

where the first term is the inherent damping  $\xi_i$ , the second one represents the hysteretic damping in case of elastic-plastic nonlinear structural behaviour  $\xi_h$ , and the last one is the supplemental damping due to the nonlinear viscous dampers in presence of a nonlinear structural behaviour  $\xi_v$ . The term  $C$  depends on the type of hysteretic response of the structure. In general the hysteretic damping is proportional to the energy dissipated by the considered hysteretic model. By assuming an elastic-

perfectly plastic behaviour of the structure and by calculating the corresponding hysteretic damping with the area-based criterion, a coefficient  $C$  equal to 2.00 can be determined. Eq. (12) represents the constant supplemental damping ratio curve, which in Fig.3 is superimposed to the constant capacity damping curves.

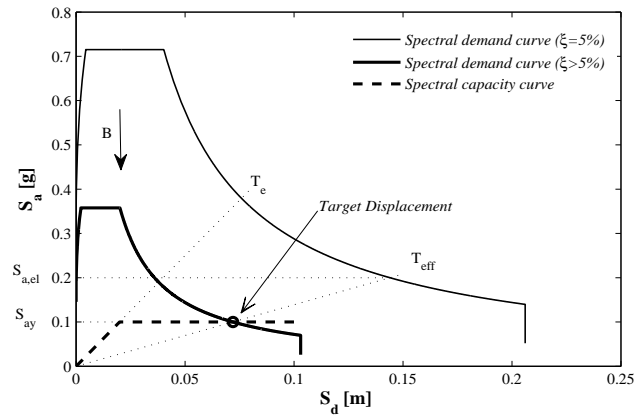


Figure 2. ADRS representation: spectral capacity and spectral demand curves

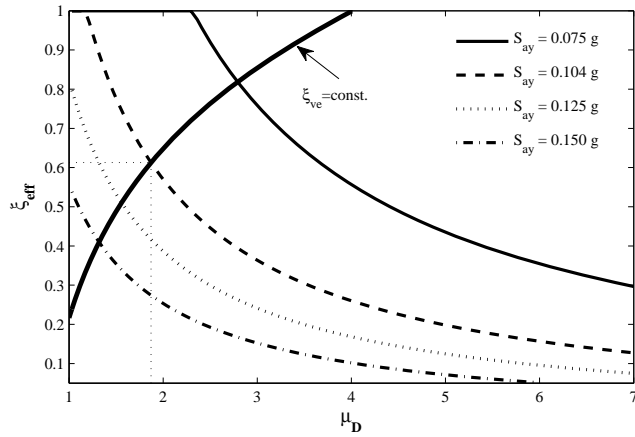


Figure 3. Constant capacity damping curves (for a given  $T_{e1} = 0.92$  s) and constant supplemental damping curve in terms of ductility demand.

If the supplemental damping  $\zeta_{ve}$  and the yield acceleration  $S_{ay}$  are known, it is possible to individuate these two curves and to directly assess the ductility demand and the effective damping as the intersection point. Even a direct analytical solution can be found by solving the equation that can be derived by combining Eq. (11) and Eq. (12) (Landi et al., 2014).

As explained in the introduction, the DAM illustrated above can be used to determine approximated IDA curves in place of nonlinear time-history analyses. The method should be applied through the following phases:

- assume a value of PGA and calculate, for a given structure and passive energy dissipation system, the damper index;
- apply the first step of the DAM and obtain the supplemental damping ratio under elastic condition using specific spectra as those reported in Fig.1;
- apply the second step of the DAM to determine the ductility demand due to the seismic action; consequently, calculate the displacement demand;
- repeat the procedure from the first phase by imposing a different value of PGA.

The above analyses should be repeated for increasing values of PGA until the achievement of the collapse condition. In this way it is possible to obtain an approximated IDA curve.

## CASES OF STUDY

The probabilistic seismic assessment method has been applied to three different RC frames. As shown in Fig.4, the structures considered are characterized by 3 bays and 1 floor (3B1F), 3 bays and 3 floors (3B3F) and 3 bays and 6 floors (3B6F). The characteristic material strengths are equal to 25 MPa for the concrete and 450 MPa for the steel. A seismic weight of 945 kN is considered for the frame 3B1F, while 510 kN and 825 kN are respectively the seismic weights for the roof and for the other storeys of both frames 3B3F and 3B6F (without the self weight of the columns). The structures are supposed to be designed to resist only gravity loads. Pushover analyses have been developed to investigate the nonlinear behaviour of the structures by applying modal distributed seismic action to each floor. The material nonlinearities have been represented through plastic hinges located at the ends of the structural members and defined by using the empirical expressions provided by the Italian code (Min.LL.PP., 2008). The resulting pushover curve has been approximated by matching the subtended area with that defined by an equivalent elastic – perfectly plastic capacity curve. Table.1 shows the parameters of the equivalent SDOF systems associated to the actual structures, where  $S_{dy}$  is the spectral displacement related to the yielding point,  $S_{du}$  is the spectral displacement related to the collapse condition and  $D_{roof,u}$  is the amplitude of the roof displacement corresponding to the collapse limit state. Nonlinear viscous dampers (with an exponent of velocity equal to 0.5) have been inserted in the central bay of each storey of the structures. The damping coefficient has been assumed equal to 236, 331 and 556 kNs<sup>0.5</sup>/m<sup>0.5</sup> respectively for the frames 3B1F, 3B3F and 3B6F.

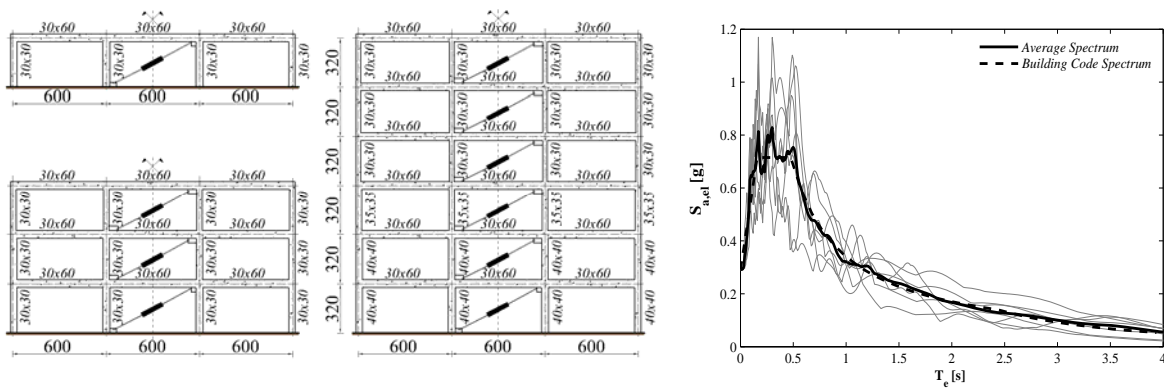


Figure 4. Geometrical characteristics of the three RC frames under study (dimensions in cm) and average spectrum in terms of acceleration of the set of selected ground motions.

Table 1. Characteristics of the SDOF systems equivalent to the actual structures.

Parameter	3B1F	3B3F	3B6F
$T_{el}$ [s]	0.44	0.92	1.35
$S_{av}$ [g]	0.16	0.10	0.09
$S_{dy}$ [cm]	0.80	2.21	3.58
$S_{du}$ [cm]	8.93	11.46	9.87
$D_{roof,u}$ [cm]	8.93	14.48	13.31

The proposed method has been validated through comparisons with nonlinear dynamic analyses (NLDA) of the structures under study. A set of seven real ground motions has been identified through the software REXEL (Iervolino et al., 2010) in order to obtain an average spectrum consistent with the one provided by the Italian code (Min.LL.PP., 2008) for soil type C and PGA equal to 0.291 g (Fig. 4).

It can be useful and interesting to illustrate the application of the DAM for the case studies and to compare the method with the results obtained with NLDA, for a single value of PGA.

With regard to the determination of the hysteretic damping, it is known that in general the area-based criterion tends to provide an overestimation of the actual dissipative capacity, and that such overestimation is significant for an elastic-plastic hysteretic response (Dwairi et al., 2007). In the study developed by Landi et al. (2014) specific values of  $C$  for the elastic-plastic behaviour have been determined through NLDA with reference to the set of selected ground motions. The mean of the

values obtained for structures with different periods has been called  $C_a$ . The value of  $C_a$  obtained considering 3 levels of PGA, equal to 1.256, has been used in the application of Eq. (12) and of the direct assessment method here. With regard to the damping reduction factor, the relation provided in the EC8 (CEN, 2003) and in the Italian code (Min.LL.PP., 2008) has been used in Eq. (10) and (11):

$$B = \sqrt{\frac{5+\xi_{eff}}{10}} \quad (13)$$

If a structure is not equipped with nonlinear fluid viscous dampers the first step of the DAM can be avoided by imposing the term  $\zeta_{ve} = 0$  in Eq. (12). However, if dampers are inserted in the structure, the supplemental damping  $\zeta_{ve}$  has to be considered. By applying the first step of the DAM, the following values of damper index can be obtained for the frames 3B1F, 3B3F and 3B6F: 17.65%, 14.68% and 13.15. By using the design spectra (Fig.1) and Eq. (13), the following values of the supplemental damping  $\zeta_{ve}$  have been determined for the frames 3B1F, 3B3F and 3B6F: 15.00%, 16.65% and 18.33%. Subsequently, in the second step, the values of  $\zeta_{ve}$  determined in the first step have been employed in Eq. (12). As an example, Fig.3 shows the estimate of the ductility demand and of the effective damping for the frame 3B3F.

The results of the procedure for the structures without and with dampers are summarized in Table.2. It should be noted that the results obtained with the DAM are all conservative, in particular for the structures equipped with nonlinear fluid viscous dampers. It is known, in fact, that Eq. (13) tends to underestimate the damping reduction factor for increasing values of effective damping ratio. As explained in Landi et al. (2014), if a calibrated relationship between the damping reduction factor and the effective damping ratio is adopted, it is possible to reduce the difference between the results of the DAM and those of NLDA to few percentage point.

Table 2. Comparison between the results obtained with the DAM and the NLDA

Analysis	Structures without dampers			Structures with dampers		
	3B1F [cm]	3B3F [cm]	3B6F [cm]	3B1F [cm]	3B3F [cm]	3B6F [cm]
NLDA <sub>MEAN</sub>	3.63	7.31	11.61	1.68	4.60	7.00
NLDA <sub>ST.DEV</sub>	1.38	1.75	2.22	0.44	0.90	1.19
DAM <sub>EC8</sub> ( $C_a$ )	4.13 (+13.7%)	9.18 (+25.5%)	13.25 (+14.2%)	2.44 (+45.1%)	5.82 (+26.4%)	8.63 (+23.3%)

## PROBABILISTIC ASSESSMENT RESULTS

As explained in the first paragraphs, the 2000 SAC-FEMA method has been applied by calculating the annual probability of failure for a given structure by means Eq. (1). Fig.5 illustrates the seismic hazard function, which has been calculated, for the reference site, on the basis of the Italian code criteria (Min.LL.PP., 2008). The Italian code provides for each site different spectra according to the return period of the seismic event. From these data it has been possible to derive the hazard curve for the considered site. This curve has then been approximated through Eq. (2), which tends to be a straight curve in the logarithmic plane.

Nonlinear incremental dynamic analyses have been performed for the structures illustrated in Fig.4 with the set of selected ground motions. As previously mentioned, the engineering demand parameter considered here is the roof displacement, while the intensity measure assumed is the spectral acceleration relative to the fundamental period. Therefore the output of the IDA is represented in terms of roof displacement versus spectral acceleration. The log-normal distributed IDA data have been interpolated, as shown in Fig.6-8, with Eq.(3). In these figures, the IDA curves and the curve based on Eq.(3) are compared with the results obtained by applying the proposed IDAM. It should be noted that the IDAM curves are conservative, as expected due to the conservative nature of Eq. (13) used in the application of the assessment method. However the IDAM curves are consistent with those obtained with nonlinear incremental dynamic analyses.



The probability to exceed the near collapse limit state (NC) has been then investigated. By knowing the collapse displacement for the three structures (reported in Table 1), the corresponding value of spectral accelerations  $S_{a,1}^{NC}$  can be obtained from the IDA curves. From these values of spectral acceleration, the associated values of the annual probability of failure can be obtained considering the hazard function illustrated in Fig.5.

Moreover, using the results of the IDA it is possible to determine the dispersion  $\beta_D$  in the seismic demand both for the structure without and with dampers as a function of the spectral acceleration (Fig.9a-9c).

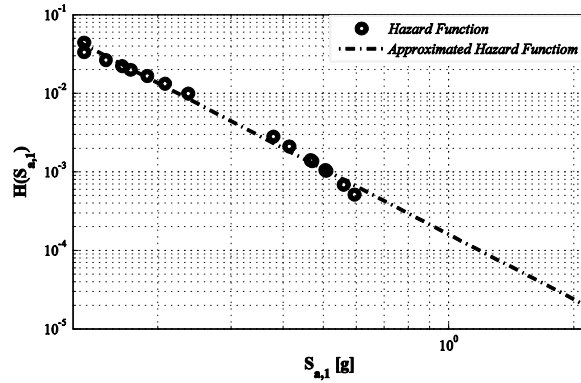


Figure 5. Hazard function determined according to the Italian building code criteria.

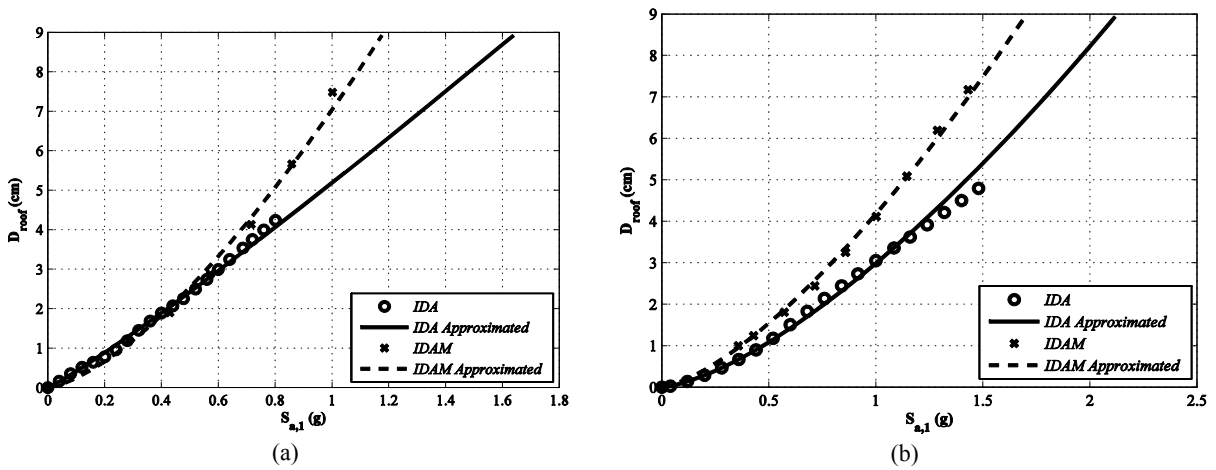


Figure 6. Comparison between IDA and IDAM curves: 3B1F structures without (a) and with (b) dampers.

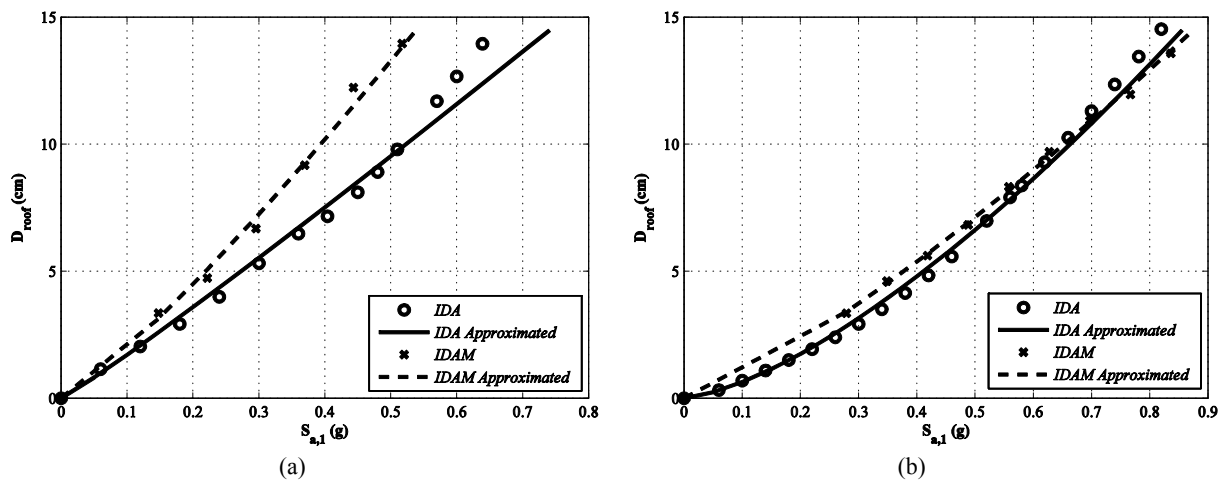


Figure 7. Comparison between IDA and IDAM curves: 3B3F structures without (a) and with (b) dampers.

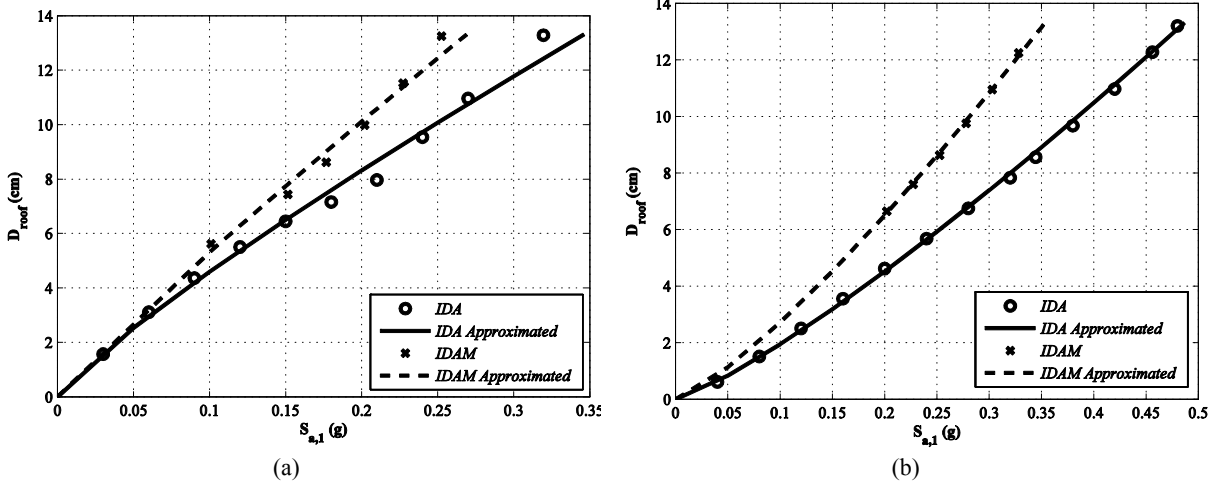


Figure 8. Comparison between IDA and IDAM curves: 3B6F structures without (a) and with (b) dampers.

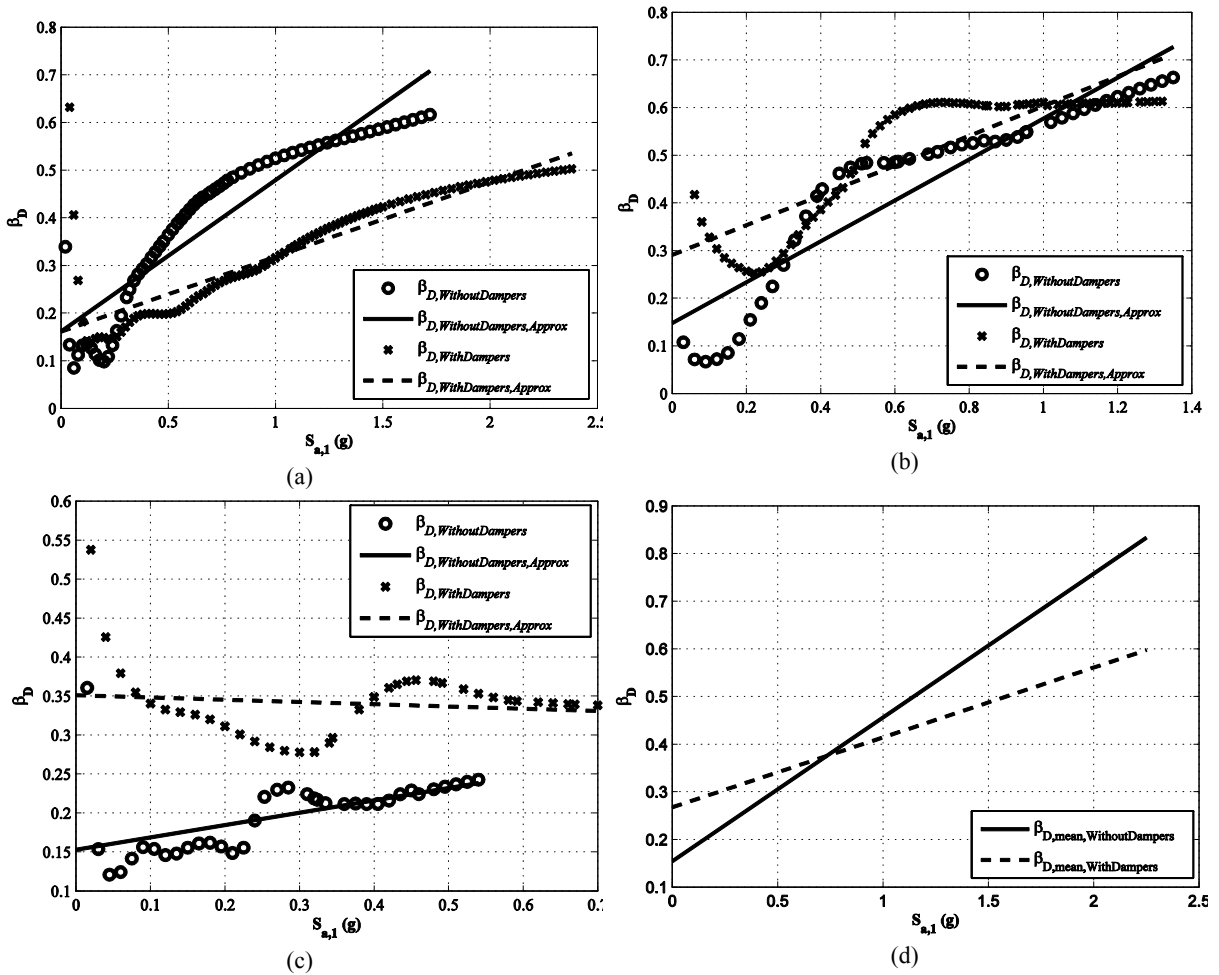


Figure 9. Dispersion in the seismic demand: (a) 3B1F structure with and without dampers; (b) 3B3F structure with and without dampers; (c) 3B6F structure with and without dampers; (d) mean dispersion in the seismic demand for the three structures investigated.

It should be noted that the dispersion in the seismic demand increases with the spectral acceleration, more in structures without dampers than in structures with dissipative system. The values of dispersion in the demand parameter can then be approximated with a linear equation, as reported in Fig. 9. In this way, mean lines can be determined to uniquely represent the dispersion for structures without and with dampers (Fig.9d). The tangent of the angle between the average line and the

horizontal axis is equal to 0.3 in structures without dampers and to 0.15 in structures with dampers. The intersection of the average line with the ordinate axis is equal to 0.15 and to 0.26 for the structures without and with dampers, respectively. By knowing these values it is possible to correlate the dispersion for structures with dampers with that for structures without dampers. In Fig. 9, for large values of intensity, the mean dispersion with dampers tend to be lower than without dampers (on average, for values of  $S_{a,1}$  larger than about 0.7 g).

As can be found in literature, the capacity dispersion  $\beta_C$  can be set equal to 0.275 (Bianchini et al., 2007). In this way all of the parameters necessary to determine the seismic risk of the structures have been determined and Eq. (1) has been applied. The probabilistic seismic assessment of the structures with and without dampers is summarized in Table.3-5, by considering both the IDA and the IDAM results. As expected, the probability of failure of the structures decreases with dampers than without dampers. This reduction is mainly related to the increase of the collapse acceleration and to the consequent reduction of the hazard function.

Table 3. Annual probability of failure for the 3B1F systems (collapse displacement equal to 8.93 cm).

Parameters	Without dampers		With dampers	
	IDA	IDAM	IDA	IDAM
$S_{a,1}^{NC}$ [g]	1.648	1.182	2.121	1.7
$H(S_{a,1}^{NC})$	4.09E-05	1.02E-04	2.05E-05	3.76E-05
$k_0$	1.62E-04	1.62E-04	1.62E-04	1.62E-04
$k$	2.748	2.748	2.748	2.748
$a$	5.156	6.981	2.973	4.163
$b$	1.100	1.470	1.463	1.439
$\beta_D(S_{a,1}^{NC})$	0.651	0.511	0.579	0.517
$\beta_C$	0.275	0.275	0.275	0.275
$P_{F,NC}$	1.95E-04	1.84E-04	4.22E-05	7.02E-05

Table 4. Annual probability of failure for the 3B3F systems (collapse displacement equal to 14.48 cm).

Parameters	Without dampers		With dampers	
	IDA	IDAM	IDA	IDAM
$S_{a,1}^{NC}$ [g]	0.739	0.538	0.854	0.876
$H(S_{a,1}^{NC})$	3.71E-04	8.87E-04	2.49E-04	2.32E-04
$k_0$	1.62E-04	1.62E-04	1.62E-04	1.62E-04
$k$	2.748	2.748	2.748	2.748
$a$	19.980	30.190	18.220	17.120
$b$	1.067	1.186	1.456	1.266
$\beta_D(S_{a,1}^{NC})$	0.377	0.316	0.393	0.396
$\beta_C$	0.275	0.275	0.275	0.275
$P_{F,NC}$	7.63E-04	1.42E-03	3.75E-04	4.02E-04

Table 5. Annual probability of failure for the 3B6F systems (collapse displacement equal to 13.31 cm).

Parameters	Without dampers		With dampers	
	IDA	IDAM	IDA	IDAM
$S_{a,1}^{NC}$ [g]	0.346	0.269	0.487	0.353
$H(S_{a,1}^{NC})$	2.98E-03	5.957E-03	1.17E-03	2.823E-03
$k_0$	1.62E-04	1.62E-04	1.62E-04	1.62E-04
$k$	2.748	2.748	2.748	2.748
$a$	33.09	45.17	31.91	49.3
$b$	0.858	0.930	1.215	1.259
$\beta_D(S_{a,1}^{NC})$	0.258	0.235	0.339	0.31944
$\beta_C$	0.275	0.275	0.275	0.275
$P_{F,NC}$	6.19E-03	1.05E-02	1.90E-03	4.31E-03

## CONCLUSIONS

A new simplified method for the direct probabilistic assessment of the seismic risk of nonlinear structures equipped with nonlinear fluid viscous dampers has been proposed. This method is based on a simplified direct assessment method developed by Landi et al. (2014). The proposed procedure has then been applied to evaluate its effectiveness in comparison with NLDA and IDA. These applications have shown the validity of the proposed procedure.

The 2000 SAC-FEMA method has then been applied in order to perform probabilistic evaluations for the structures without and with added dampers. As expected, by using the dampers, a reduction of the probability of failure can be observed. The procedure proposed has then been implemented in this framework in order to avoid IDA in the estimate of the relation between the median demand and the intensity measure. Moreover, also a correlation between the dispersion for structures with dampers and for structures without dampers has been found. In this way a simplified and direct application of the 2000 SAC-FEMA method has been derived for structures with dampers.

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