SECOND EUROPEAN CONFERENCE ON EARTHQUAKE ENGINEERING AND SEISMOLOGY, ISTANBUL AUG. 25-29, 2014

Turkish Earthquake Foundation - Earthquake Engineering Committee
Prime Ministry, Disaster And Emergency Management Presidency

NEW COUPLED FINITE-INFINITE ELEMENT APROACH FOR WAVE PROPAGATION SIMULATION OF UNBOUNDED SOIL MEDIA

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ABSTRACT

In geotechnical earthquake engineering, wave propagation in semi infinite domains is of a great importance. In these kinds of problems, the elastic waves propagate from the interior to the exterior domain and require a special treatment at the computational boundaries in order not to reflect back to the interior domain. In this paper, a study on stable implementation of an absorbing layer in infinite elements is presented. The proposed algorithm is implemented in the ANSYS finite element software and tested on a two-dimensional wave propagation problem. The results obtained illustrate the usage and the effectiveness of the developed infinite elements in simulation of exterior domain.

INTRODUCTION

In geotechnical engineering, the earth under a foundation is a typical example of unbounded domain extending to infinity. From a numerical computational point, such domains extending to infinity can be divided into two parts; an interior and an exterior domain with the separating boundary named as computational boundary. The interior domain is the region of interest in which the field variables are looked for while the exterior is an unbounded domain whose effects are considered at the computational boundary. In dynamic analysis using the linear elastic material models, the calculations are usually performed in frequency domain. However, in more realistic cases, in order to account for nonlinearities in soil simulation, computations in time domain have to be considered. In the finite element model, unbounded soil media are presented by various procedures among which the infinite elements take an important role. In time domain simulations in most cases of finite element analysis boundaries, the infinity is simulated by artificial boundaries such as using of spring-dashpot elements [1, 2]. In the last decades the unbounded soil media are presented by the infinite elements which take an important role in boundary simulations. The efficiency of mapped infinite elements has been shown in the work of Zienkiewicz [3]. By providing the mapped infinite elements with absorbing properties,

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reasonable good dynamic characteristics of the boundaries are obtained as shown in the works of Haegblad [4] , Garevski [5], Edip [6] etc. This type of approximation is local in time and space and the infinite elements function as absorbers. In this work, the absorbing properties are added to all nodes of the infinite elements forming a new type of absorbing infinite elements.

FORMULATION OF ABSORBING INFINITE ELEMENTS

The formulation of mapped infinite elements is the same as for the finite elements in addition to the mapping of the domain. The main advantage of the mapped infinite elements is the usage of the conventional Gauss-Legendre abscissa and weights.
The main difference is in the increased number of nodes of the infinite element and the used mapping functions, whose verification has shown great accuracy of results. The infinite element has been obtained from an eight nodded finite element, as shown in Figure 1.

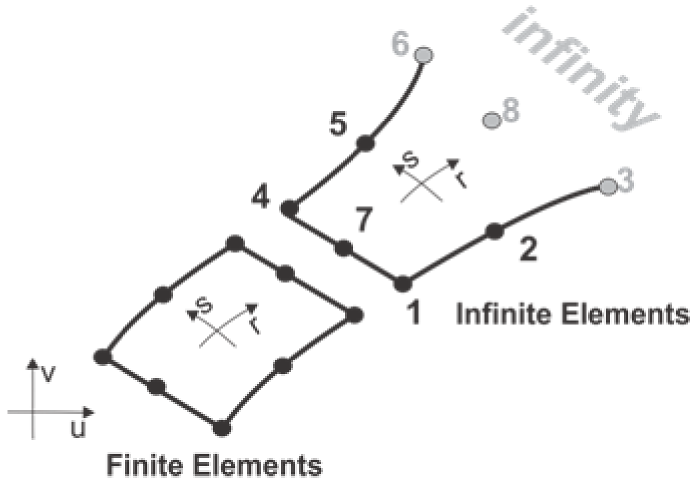

Figure 1. Coupling of finite and infinite elements

The element displacement in *u* and *v* direction is interpolated with the usual shape functions N^1, N^2, N^4, N^5 *and* N^7:

$$
\begin{aligned}
u &= [N^1 \quad N^2 \quad 0 \quad N^4 \quad N^5 \quad 0 \quad N^7 \quad 0]\mathbf{u} \\
v &= [N^1 \quad N^2 \quad 0 \quad N^4 \quad N^5 \quad 0 \quad N^7 \quad 0]\mathbf{v}
\end{aligned} \tag{1}
$$

In expression (1), **u** and **v** are vectors with nodal point displacements in global coordinates. The shape functions are given in expression (2) as:

$$
\begin{aligned}
N^1 &= -(r-1)(-1+s)(s+1+r)/4 \\
N^2 &= (r-1)(1+r)(-1+s)/2 \\
N^4 &= -(r-1)(1+s)(s-1-r)/4 \\
N^5 &= -(r-1)(1+r)(1+s)/2 \\
N^7 &= (-1+s)(1+s)(r-1)/2
\end{aligned} \tag{2}
$$

Based on the isoparametric concept, the infinite element in global coordinate is mapped onto an element in local coordinate system using the expression in (3). In the formulation of the infinite element, only the positive *r* direction extends to infinity.

$$
\begin{aligned}
r &= [M^1 \quad M^2 \quad 0 \quad M^4 \quad M^5 \quad 0 \quad M^7 \quad 0]\mathbf{r} \\
s &= [M^1 \quad M^2 \quad 0 \quad M^4 \quad M^5 \quad 0 \quad M^7 \quad 0]\mathbf{s}
\end{aligned} \tag{3}
$$

where

$$
\begin{aligned}
M^{1} &= -\frac{(1-s)rs}{1-r} \\
M^{2} &= -\frac{(1-s)(1+r)}{2(1-r)} \\
M^{4} &= -\frac{(1+s)rs}{1-r} \\
M^{5} &= -\frac{(1+s)(1+r)}{2(1-r)} \\
M^{7} &= -\frac{2r(1+s)(1-s)}{(1-r)}
\end{aligned} \tag{4}
$$

In expression (3), **r** and **s** are vectors of nodal point displacements in local coordinates where it is to be mentioned that, on the side of infinity (*r=1*), no mappings are assigned to the nodes as it is taken that no displacement is possible at infinity. The number and location of the nodes connecting finite and infinite elements must coincide to guarantee continuity condition between the elements. The main advantage of the proposed infinite elements is that the number of nodes on the infinite element allow coupling with finite elements with eight nodes which are used for displacement sensitive problems. Construction of element matrices is done by using the usual procedures as described in Bathe [7]. The new coordinate interpolation functions are taken into consideration in the Jacobian matrix as described in Bettess[8]. For the absorbing layer of the infinite element, the Lysmer-Kuhlmeyer [9] approach is used. In all cases, a plane strain two dimensional case is studied. For impact of plane waves on element sides, normal and tangential stresses are derived as follows:

$$
\begin{bmatrix} \sigma^{n} \\ \tau \end{bmatrix} = \begin{bmatrix} a\rho c^{p} & 0 \\ 0 & b\rho c^{s} \end{bmatrix} \begin{bmatrix} \dot{u}^{n} \\ \dot{u}^{t} \end{bmatrix} \tag{5}
$$

where c^P and c^S indicate compression and shear waves, ρ is the density of soil medium. In order to take into account the directions of the incident waves, coefficients a and b as suggested by Kouroussis [10] are used as multipliers for better wave absorption. Transformation from local to global coordinates is done automatically by the ANSYS software [11] such that there is no need of defining transformation matrices. By adding together the parts from each element, the governing incremental equations for equilibrium in dynamic analysis are obtained. The programming of the infinite element has been done using the Programmable Features of ANSYS[11]. For the sake of verification of the presented infinite elements, a two dimensional unbounded soil medium is taken into consideration.

WAVE PROPAGATION IN SOIL MEDIUM

In order to verify the absorbing properties of the newly developed infinite elements, a two dimensional wave propagation problem is simulated. Figure 2 shows the soil domain with a total width of 40m while the height is 20m.

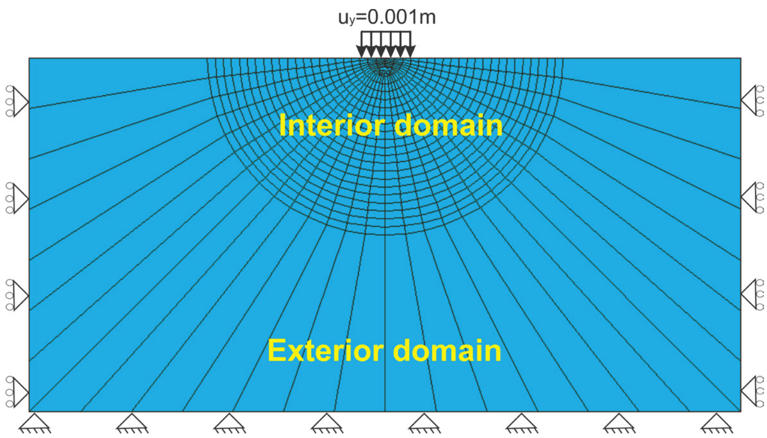

Figure 2. Soil domain

The soil medium is simulated in two alternative ways. First, spring-dashpot elements are considered as boundaries and then absorbing infinite elements are considered to be the exterior domain of the soil medium. The finite elements are used for discretization of the near field while infinite elements are used for discretization of the far field. The soil domain discretization is shown in Figure 3.

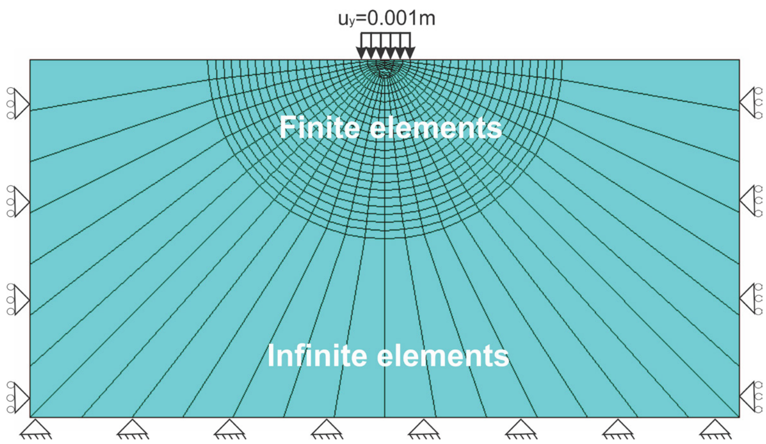

Figure 3. Soil domain of finite and infinite elements

Properties of the soil medium are given in Table 1. The soil medium is considered to be linear elastic due to its simplicity. The main focus is given to the boundary simulation. The nonlinear material models are can be included since the absorbing infinite elements are defined in time domain.

Table 1 Soil properties

Young's modulus	E	50000	kPa
Poisson's ratio	υ	0.35	-
Density	ρ	2.70	Ton/m^3

The applied displacement is considered to be of the impulse type. In this case, displacement of u_y=1mm is applied as an impulse function at the very beginning of the time. (t=0sec)

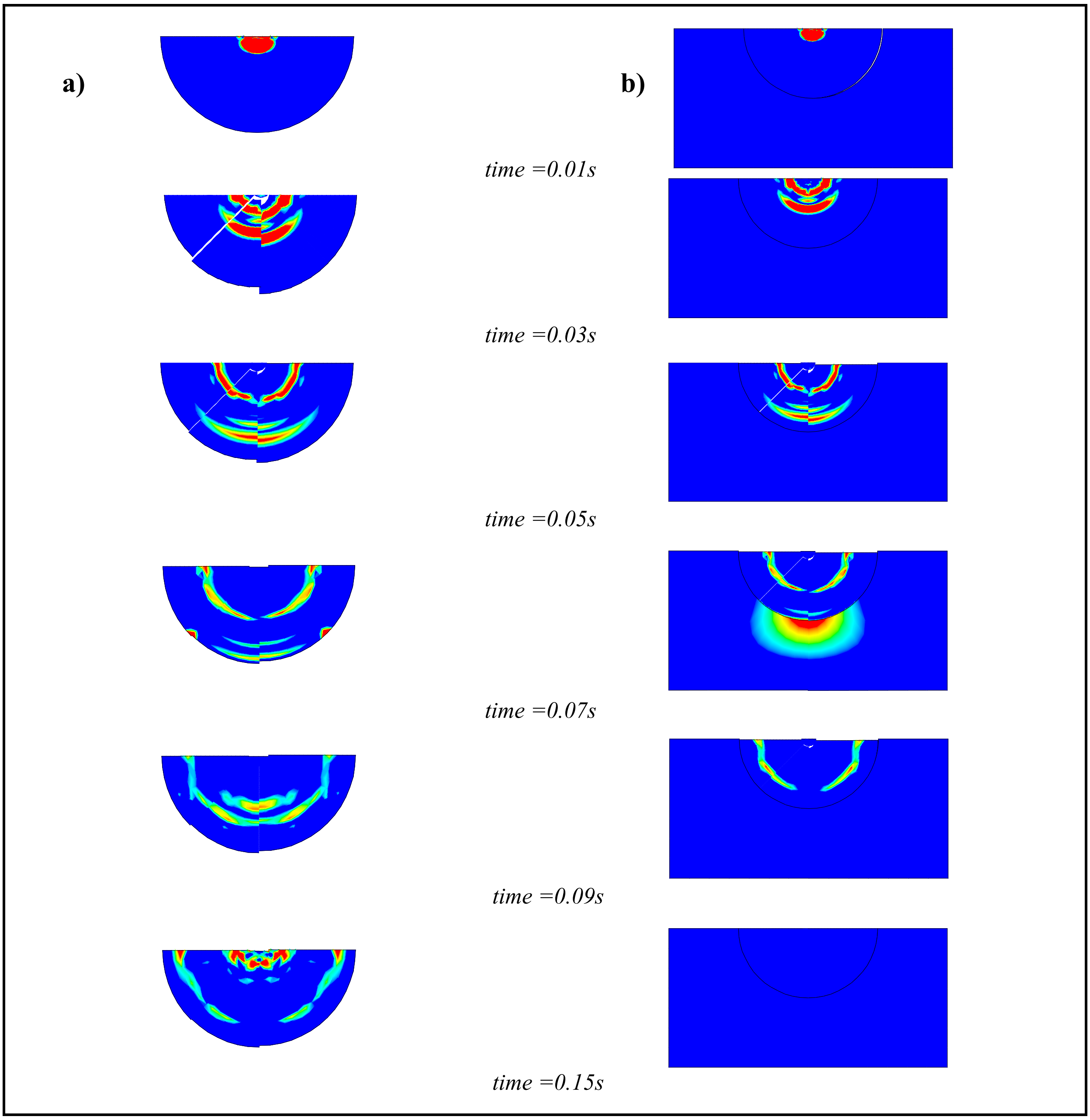

Figure 4. Wave propagation at different time spots when exterior domain is composed of a) springs and b) absorbing infinite elements

As can be seen from figure 4, there are two waves propagating in the domain. The first one is the compression p wave while the second one is the shear S wave. The used infinite elements are able to absorb both types of waves although their speeds of propagation are not the same. It is to be mentioned that, at time *t=0.07 seconds*, the P wave enters the infinite domain. In the case of absorbing infinite elements, the wave is almost completely absorbed while in the case of finite elements with spring-dashpot boundaries, the P wave reflects back to the finite element domain. At *time=0.09 seconds*, in the case of a domain surrounded by absorbing infinite elements, S-wave propagation at the finite element boundaries is observed. On the other hand, in the domain of finite elements with spring-dashpot boundaries, the outward propagating S-wave collides with the reflected P wave, as shown in the figure. At time *t=0.15seconds* in the domain composed of absorbing infinite elements, there is no sign of backward wave propagation since the waves have been absorbed in the infinite element region.

CONCLUSIONS

In this work, a coupled computational method of finite and infinite elements is presented and applied in the wave propagation problem considering the unbounded medium. The efficacy of the coupled finite and infinite elements is shown through this numerical example. This model uses mapping infinite elements to approximate the stiffness of the unbounded exterior domain. It can be said that by using the coupled finite-infinite element approach the accuracy of the results in the interior domain is considered to be reliable. Thus, the absorbing infinite elements which were implemented in this study provide a promising tool which is easy to implement in finite element analysis. The presented approach in this paper can be extended to three dimensional cases easily.

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