



DUCTILITY SPECTRUM METHOD FOR DESIGN AND VERIFICATION OF STRUCTURES: SINGLE-DEGREE-OF-FREEDOM BILINEAR SYSTEMS

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ABSTRACT

In this study a Ductility Spectrum Method (DSM) applicable to the evaluation and design of structures is developed, illustrated, and compared with the Capacity Spectrum Method adopted by the Applied Technology Council (ATC-40). This method uses inelastic response spectra and gives peak responses consistent with those obtained when using the nonlinear time history analysis (NLTHA). The DSM can be used to estimate the seismic deformation of Single-Degree-of-Freedom (SDOF) bilinear systems based on inelastic response spectrum.

INTRODUCTION

Current seismic codes require the seismically designed structures to be capable to withstand inelastic deformations. Many studies dealt with the development of different inelastic spectra with the aim to simplify the evaluation of inelastic deformation and performance of structures. Recently, the concept of inelastic spectra has been adopted in the global scheme of the Performance-Based Seismic Design through the capacity spectrum based methods. The capacity-spectrum method CSM (ATC-40, 1996) gives an overall view of the inelastic behavior of structures subjected to ground motions. However, to be as more accurate as possible it requires a realistic capacity curve for the structure that is in accordance with its dynamic behavior when subjected to the design earthquake. Many refinements and corrections have been made to the pushover analysis to make it in accordance with the nonlinear time history analysis (NLTHA). For instance, the adaptive pushover analysis (Elnashai 2001, among others) actually represents the best choice. Based on the deficiencies in ATC-40 procedures (ATC-40, 1996), Chopra and Goel (1999) proposed an improved Capacity-Demand-Diagram method that uses the constant ductility design spectrum. Their method has been evaluated using SDOF elastoplastic systems, and to retain the attraction of the graphical implementation of the ATC-40 procedures (A and B) Chopra and Goel (1999) developed an alternative graphical implementation of their improved method. In this paper, we first present the development of the ductility demand response spectrum (DDRS). Based on the inaccuracy of ATC-40 (1996) procedures and the need to a graphical approach for the seismic design and verification of structures, an improved procedure were proposed and investigated in this paper.

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PUSHOVER CURVE

Using a Pushover analysis, a characteristic nonlinear force-displacement relationship of the SDOF bilinear system can be determined. The selection of an appropriate lateral load distribution is an important step within the Pushover analysis. Once the pushover analysis over, a pushover (capacity) curve is obtained and it is characterized by some parameters as in Fig. 1. These parameters will be used for the construction of the demand diagram.

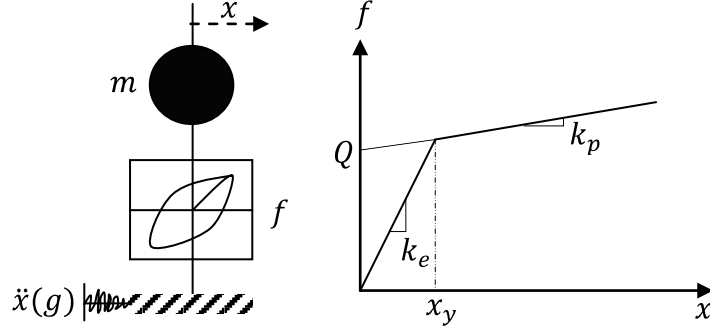


Figure 1. (a) SDOF System and (b) Capacity curve

SEISMIC DEMAND

The seismic demand in this paper is determined by using the Ductility Demand Response Spectrum (DDRS) developed in earlier works (Chikh et al., 2012). In this response spectrum we consider an inelastic SDOF system as shown in Fig.1, its dynamics when subjected to an earthquake ground motion is governed by the following equation:

$$m\ddot{x} + c\dot{x} + f(x, \dot{x}) = -m\ddot{x}_g(t) \quad (1)$$

Where m, c and f represent the mass, damping, and the resisting force of the system, respectively, $\ddot{x}_g(t)$ denotes the earthquake acceleration. The resisting force f is defined as the sum of a linear part and a hysteretic part:

$$f = k_p x + Qz \quad (2)$$

In the above, k_p is the postyielding stiffness, Q is the yielding strength, and z represents the dimensionless variable that characterizes the Bouc-Wen model of hysteresis (Wen, 1976), it is given by:

$$\dot{z} = \frac{\dot{x}}{x_y} [A - |z|^\lambda (B \text{sgn}(\dot{x}z) + \beta)] \quad (3)$$

In the above equation, x_y is the yield displacement and A, B, λ , and β are parameters that control the shape of the hysteresis loop. Substituting Eq. (2) into Eq. (1) and dividing by m yields:

$$\ddot{x} + 2\xi\omega\dot{x} + \alpha\omega^2 x + qgz = -\ddot{x}_g(t) \quad (4)$$

In which ξ, ω, α and q represent the damping ratio, circular frequency, post-to-preyielding stiffness ratio, and the yielding strength coefficient (defined as yielding strength divided by the system weight $w, w = mg, g$ stands for the gravity), respectively.

Eq. (4) is solved for a single ground acceleration (Fig. 2a) to obtain the deformation history $x(t)$ (Fig. 2b). Also shown is the yielding history through $z(t)$ (Fig. 2c) and the variation of the system force coefficient f/w with deformation (Fig. 2d).

Next, Eq. (4) is rewritten in terms of ductility factor, μ . Substituting: $x = x_y \mu$, $\dot{x} = x_y \dot{\mu}$, and $\ddot{x} = x_y \ddot{\mu}$ in Eq. (4) and dividing by x_y gives:

$$\ddot{\mu} + 2\xi\omega\dot{\mu} + \alpha\omega^2\mu + \omega^2(1-\alpha)z = -\frac{\omega^2(1-\alpha)}{qg}\ddot{x}_g(t) \quad (5)$$

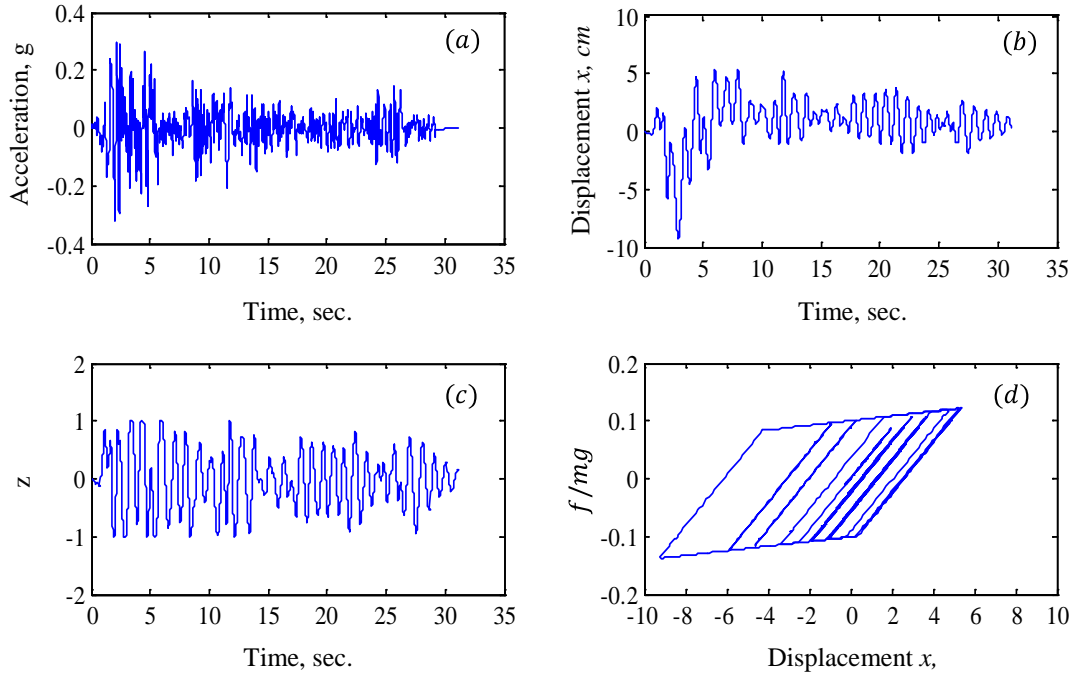


Figure 2. (a) Strong component of El Centro 1940 ground motion, (b) system deformation, (c) yield function $z(t)$, and (d) force-deformation relation. System parameters are $x_y = 2.5$ cm, $T = 1.0$ sec, $k_e = 17488.9$ N/cm and $\alpha = 10\%$

We observe from Eq. (5) that for a given ground acceleration, $\mu(t)$ depends on ξ , ω , α , and q . To obtain meaningful system response to an ensemble of ground motions, the system yield strength coefficient has to be defined relative to the intensity of individual ground motions. Using the parameter η introduced by Mahin and Lin (1983) as:

$$\eta = \frac{qg}{PGA} \quad (6)$$

Where, PGA stands for the Peak Ground Acceleration. Incorporating η into Eq. (5) results (Chikh et al., 2012):

$$\ddot{\mu} + 2\xi\omega\dot{\mu} + \alpha\omega^2\mu + \omega^2(1-\alpha)z = -\frac{\omega^2(1-\alpha)}{\eta}\overline{\ddot{x}}_g(t) \quad (7)$$

In which, $\overline{\ddot{x}}_g(t)$ represents the ground acceleration normalized with respect to the PGA .

The ground acceleration has been normalized such that its value varies from -1 to 1 (Fig.3a). Eq. (7) implies that for a given inelastic system, if α and η are fixed, the intensity of the ground motion has no effect on the peak normalized deformation, μ (Fig.3b). This permits the construction of the ductility response spectrum for an ensemble of ground motions with common frequency content but variable intensity.

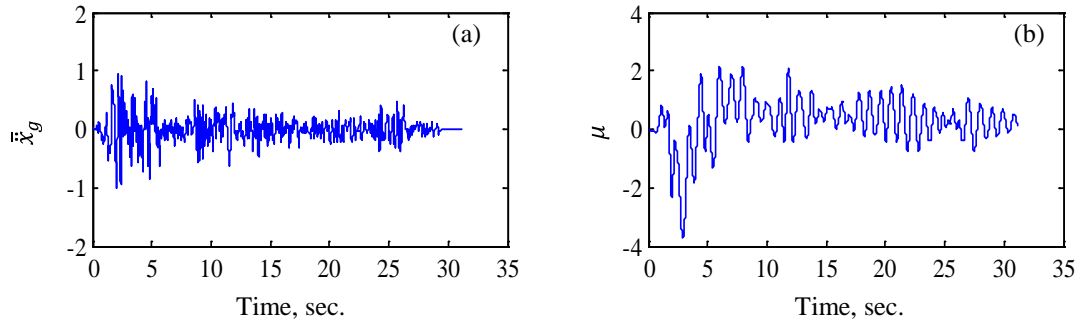


Figure 3. (a) Strong component of normalized ground acceleration of El Centro 1940 (N/S) and (b) ductility demand μ for a system with $x_y = 2.5$ cm, and $\eta = 0.25$

➤ Constant- η Ductility Demand Response Spectrum

The procedure to construct the ductility demand response spectrum for inelastic systems corresponding to specified levels of normalized yield strength η , is summarized in the following steps, (Chikh et al., 2012):

- 1- Define the ground motion $\ddot{x}_g(t)$;
- 2- Select and fix the damping ratio ξ and the post-to-preyield stiffness ratio α .
- 3- Specify a value for η ;
- 4- Select a value for elastic period T ;
- 5- Determine the ductility response $\mu(t)$ of the system with, T, ξ and α equal to the values selected by solving Eq. (7). From $\mu(t)$ determine the peak ductility factor μ ;
- 6- Repeat steps 4 and 5 for a range of T , resulting in the spectrum values for the η value specified in step 3;

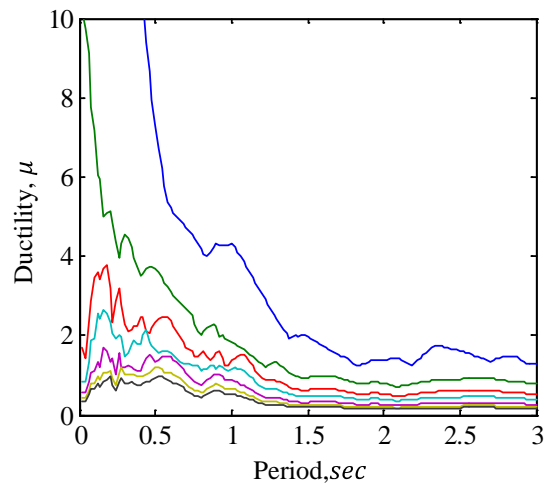


Figure 4. DDRS for inelastic system computed for El Centro 1940 (N/S) component ($\alpha = 10\%$, $\xi = 5\%$, $\eta = 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5$ from top line to bottom line).

The value of the ductility factor is read from the spectrum developed by the above procedure and multiplied by x_y to obtain the peak deformation, x_m . The DDRS is constructed for the El Centro 1940 ground motion (N/S) component ($PGA = 0.32g$, $PGV = 36.14$ cm/sec, and $PGD = 21.34$ cm) and is shown in Fig. 4.

The spectrum was divided logically into three period regions according to the last work described in (Chikh et al., 2012), where T_b marks the transition from the acceleration-sensitive region to the velocity-sensitive region which ends at $T \sim 3$ sec.

DUCTILITY SPECTRUM METHOD

A relatively simple method for determining the seismic demands of structures is presented. It combines the capacity curve represented as a bilinear model with a DDRS spectrum. The seismic demand can be obtained without iterations. The differences between the proposed method and the CSM method are discussed, and some examples are processed to address those differences.

The proposed procedure is a direct application of the principles described above and it consists of:

1. Developing the pushover curve of the considered structure, that is idealized later for simplification.

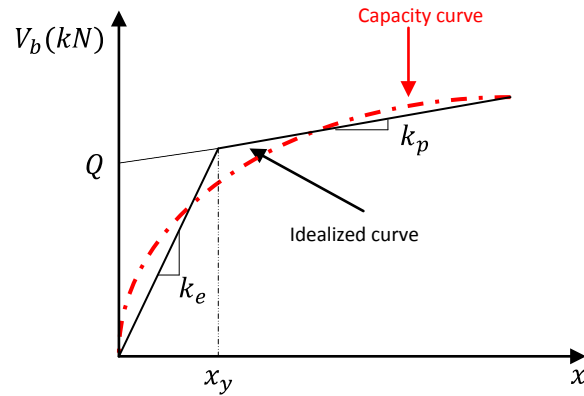


Figure 5. Capacity curve (Pushover curve)

2. The pushover curve undergoes the following transformations:

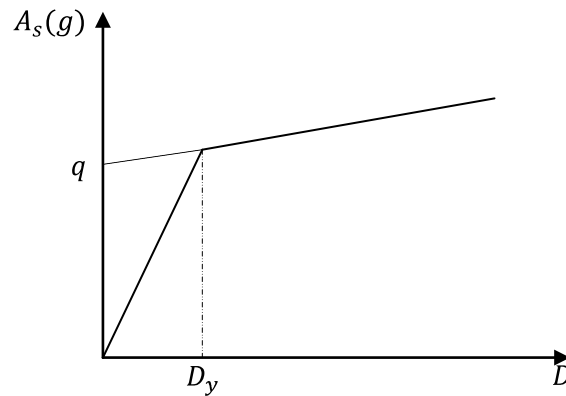


Figure 6. Capacity diagram

$$A_s(g) = \frac{V_b}{mg} \quad (8)$$

$$q = \frac{Q}{mg} \quad (9)$$

Where

A_s Spectral acceleration.

q Yield strength coefficient.

3. After the idealization, we determine the following parameters q , η , α and T .

4. Develop the inelastic spectrum DDRS.

5. The ductility demand is obtained directly using the inelastic spectrum and the fundamental period of the structure.

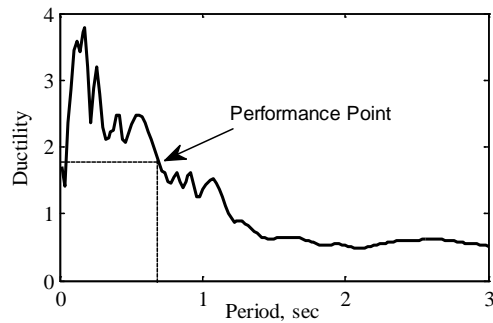


Figure 7. Performance point

CAPACITY SPECTRUM METHOD

ATC-40 (1996) specifies three different procedures of the CSM Method to estimate the earthquake-induced deformation and finding the performance point, all based on the same principles, but differing in implementation. Procedures A and B are analytical and amenable to computer implementation, whereas procedure C is graphical and most suited for hand analysis. Designed to be the most direct application of the methodology, Procedure A is suggested to be the best of the three procedures (ATC-40, 1996). The CSM Method compares the capacity of a structure to resist lateral forces to the demands of earthquake response spectra in a graphical presentation that allows a visual evaluation of how the structure will perform when subjected to earthquake ground motion. The method is easily understandable and is generally consistent with other methods that take into account the nonlinear behavior of structures subjected to strong motion earthquake ground movements (Freeman et al., 1975).

The following steps summarize the CSM procedure:

1. Capacity curve: Estimate or calculate the capacity curve in terms of roof displacement x , and base shear, V_b (i.e. total lateral force at base).
2. Dynamic characteristics: Estimate or calculate modal vibrational characteristics such as periods of vibration, mode shapes, modal participation factors, and effective modal mass ratios.
3. Capacity Spectrum: Convert the V_b vs. x capacity curve to a S_a vs. S_d capacity spectrum by the use of dynamic characteristics.
4. Response Spectra: Obtain or calculate response spectra for several levels of damping, including the 5% damped spectrum.
5. Graphical Solution: Plot capacity spectrum and family of damped response spectra on an ADRS format (i.e. S_a vs. S_d coordinates with period T lines radiating from origin). The intersection of the capacity spectrum with the appropriately damped response spectrum represents the estimated demands of the earthquake on the structure.

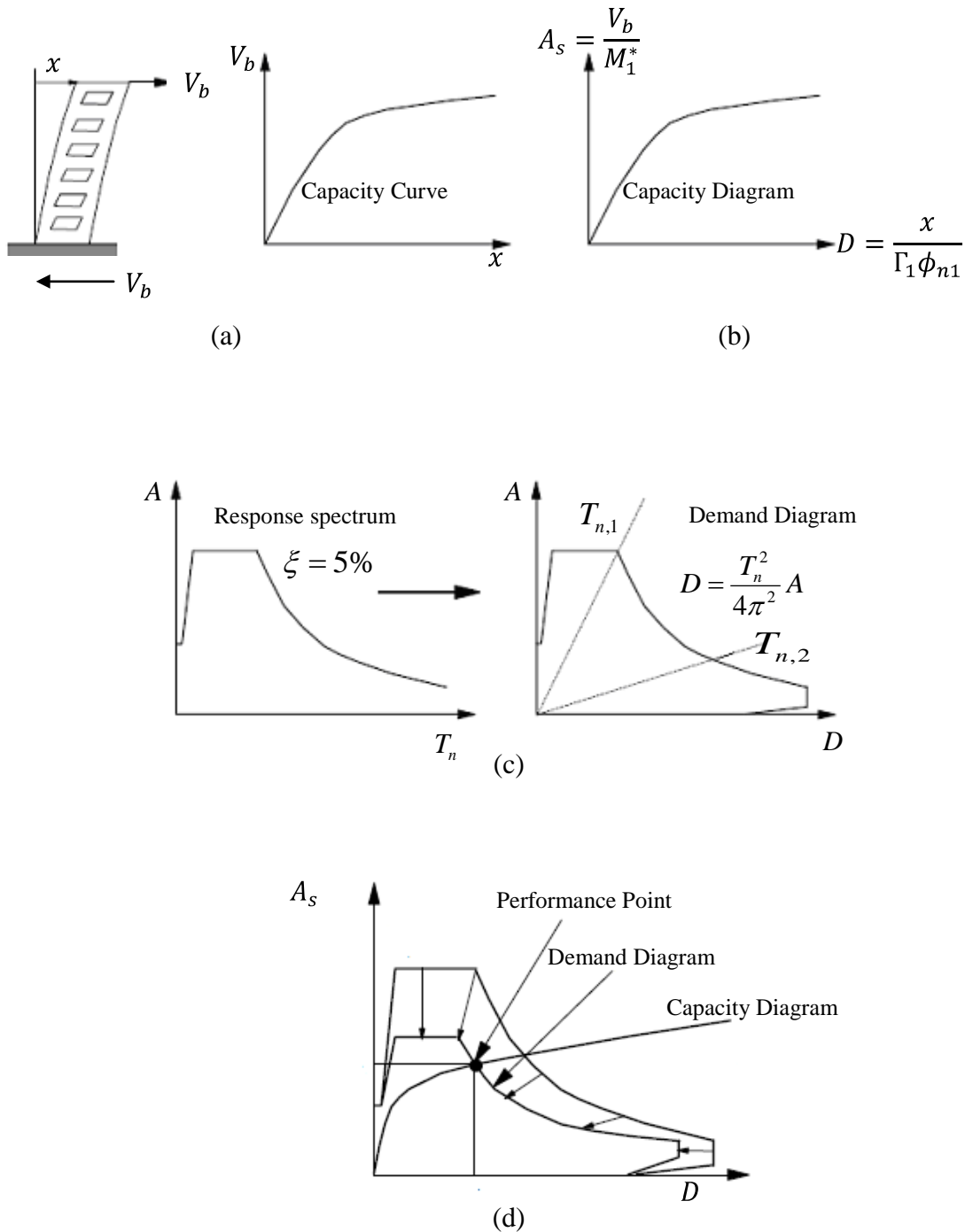


Figure 8. Capacity spectrum method: (a) development of pushover curve, (b) conversion of pushover curve to capacity diagram, (c) conversion of elastic response spectrum from standard format to A-D format, and (d) determination of displacement demand (Chopra *et al.*, 1999)

APPLICATIONS AND EVALUATION OF THE IMPROVED PROCEDURE

The procedure of ATC 40 (Procedure A) is used in this paper for multiple systems that account for a period interval of 0.1 sec to 0.9 sec; each system is characterized by its capacity curve.

Figures 9a and 9b compare the ductility and inelastic displacement demands determined by the Procedure A of ATC-40, DSM, and the NL-THA (for $\eta = 0.5, \alpha = 5\%$ and $\xi = 5\%$) associated with the 1940 El Centro earthquake. The percentage of errors in these approximations is shown in Fig.9c.

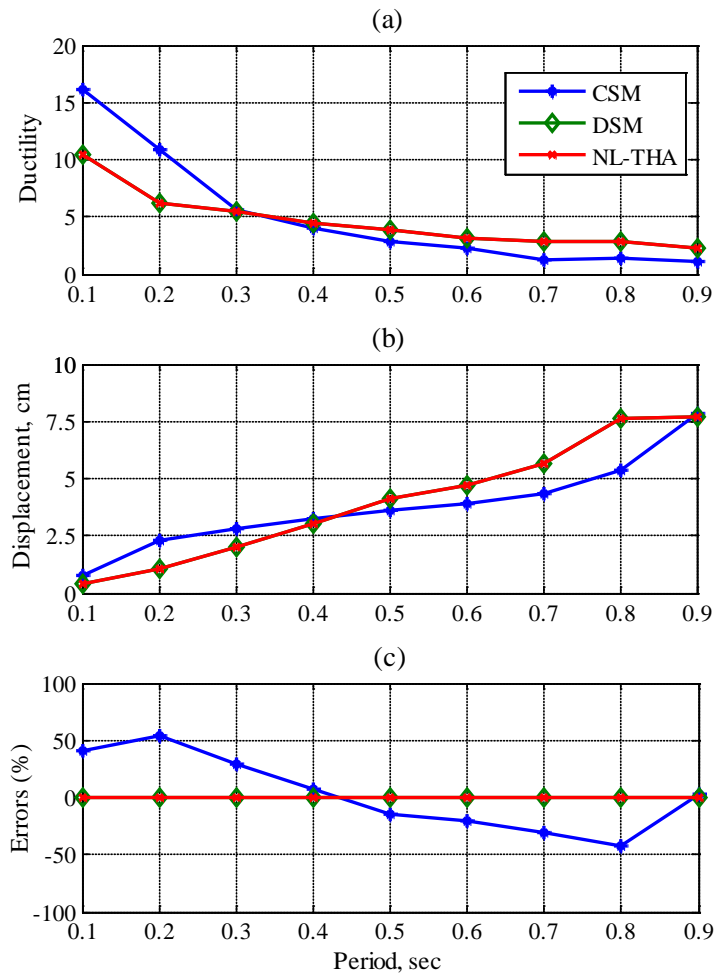


Figure 9. Comparison of different approaches: (a) Ductility; (b) Displacement; (c) Errors

We can observe that the procedure A (CSM) is not accurate; it overestimates the displacement for the systems with a period less than 0.42 sec with errors, in some cases exceeding 50%. However, for systems with a fundamental period greater than 0.42 sec, the procedure underestimates the displacement. Good results are obtained only for system with periods around 0.42 sec.

The use of elastic spectra (CSM) differs from the method based on the use of inelastic spectra (DSM) essentially through the calculation of the reduction factor of the demand spectrum that allow the evaluation of the corresponding reduced spectrum.

CONCLUSION

We developed an improved direct procedure for the seismic demands of SDOF bilinear system and its accuracy was verified by examples. The following conclusions are drawn:

- The efficiency of the DDRS is demonstrated; the designer needs only to have the DDRS for the design earthquake(s) to determine the peak response of any structure, namely, base displacement and base shear.
- The DDRS is applicable to a variety of uses such as a rapid evaluation technique for a large inventory of buildings, or as a design verification procedure for new construction, or as an evaluation procedure for an existing structure to identify damage states.
- Based on this study the ATC-40 Procedure A did not converge for some systems. It converged in other cases but not to the exact deformation determined by nonlinear response history

analysis of the inelastic system, nor to the value determined from the DDRS inelastic response spectrum.

- The ductility demand is given by the direct estimation where the ductility calculated from the DDRS diagram matches the value associated with the period of the system. This method gives the deformation value consistent with the selected DDRS inelastic response spectrum, while retaining the attraction of graphical implementation of the ATC-40 methods.

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