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SIMPLE ANALYTICAL MODEL FOR RUBBER BEARINGS

Igor GJORGJIEV[1], Mihail GAREVSKI[2], Kemal EDIP[3], Julijana BOJADJIEVA[4]

ABSTRACT

Rubber bearings are among the most frequently applied devices for base isolation of structures. In conventional analysis of base isolated structures, the behavior of rubber bearings is interpreted through the bilinear constitutive law. The laboratory investigations performed over last 30 years, confirmed that the actual behavior of rubber bearings under large shear strains is highly nonlinear. Therefore, in this investigation, a simple polynomial model is used to take a bearing nonlinear behavior into account. The first part of the paper covers testing of small rubber bearings and results obtained. The last part deals with development of a simple nonlinear mathematical model of a rubber bearing involving a polynomial function and eight additional parameters. The analytical model can simulate the behavior of natural rubber bearings in case of small and large deformations. The model is capable of covering the strengthening of the rubber in conditions of large deformations, including the loading history effect. The results from performed comparison between the analytical and experimental ones, point out that the proposed model is capable enough to simulate the force-displacement relationship of rubber bearings.

INTRODUCTION

Seismic isolation is a widely applied technique for protection of structures against earthquakes and it is considered to be an effective strategy to reduce damage in structures during earthquakes. The wide application of seismic isolation by use of rubber bearings has become a reality during the last 3 decades with the development of the high damping rubber bearings (HDRB). Only a decade ago, this technique was the privilege of the economically developed countries, but today, it can be said that a number of smaller countries like Macedonia (Garevski & Kelly, 2008) and Armenia (Melkumyan, 2006) are successfully developing and applying it.

Isolation systems are generally effective due to their flexibility and energy dissipation characteristics. Flexibility in the horizontal direction will increase the fundamental period of the building after the range of periods which usually dominate in earthquake input. The response of base isolated structures mainly depends on lateral behaviour of isolation devices. The lateral behaviour of rubber bearings, which are mostly used, is quite complex because it is mainly dependent on rubber compound. The factors that have an influence upon the hysteretic behaviour of rubber bearings are

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[1] Ass. Prof. Dr., Institute for Earthquake Engineering and Engineering Seismology, University “Ss. Cyril and Methodius”, Skopje, igorg@pluto.iziis.ukim.edu.mk

[2] Prof. Dr, Institute for Earthquake Engineering and Engineering Seismology, University “Ss. Cyril and Methodius”, Skopje, garevski@pluto.iziis.ukim.edu.mk

[3] Ass. Dr., Institute for Earthquake Engineering and Engineering Seismology, University “Ss. Cyril and Methodius”, Skopje, kemal@pluto.iziis.ukim.edu.mk

[4] Ass. MSc, Institute for Earthquake Engineering and Engineering Seismology, University “Ss. Cyril and Methodius”, Skopje, jule@pluto.iziis.ukim.edu.mk

stiffness hardening at large strains, strain-rate dependence, the presence of the Mullins effect, the axial load, ambient temperature, ageing effects, etc. Because of such complex behaviour, the generally accepted modelling of bearings in horizontal direction by a bilinear model (Naeim & Kelly, 1999), (EC8, 2004) cannot completely cover the actual bearing behaviour. However, on the other hand, if all these factors are included in the analytical model of a rubber isolator, then this model will be ideal but also very complex. To enable a more realistic dynamic analysis of structures isolated with rubber bearings, simple polynomial analytical models have been proposed (Gjorgjiev, 2013).

The first phase of the investigations covered the development of small rubber bearings and their testing. A comprehensive study was performed to develop the technological process of production of rubber bearings. The quality of the end products was verified through a series of dynamic tests. In this part, the results from the vertical and horizontal tests carried out on a few characteristic bearings are presented through force-displacement diagrams.

The simple analytical model presented in the second part of this paper was developed based on a series of tests on rubber isolators. The model can simulate the behavior of natural rubber bearings in conditions of small and large deformations. The model is defined by a polynomial function and eight parameters and it is able to cover the strengthening of the rubber in conditions of large deformations. It includes the loading history effect and enables adaptation to different shapes of loading/unloading. A least-square regression was used to determine the “best” coefficients in order to minimize the sum of the squares in an n’th order polynomial model. This model can substitute the bilinear simplification of rubber bearings in the design procedure and was implemented in the finite element nonlinear program (Gjorgjiev 2013).

TESTING OF NATURAL RUBBER BEARINGS

One of the main goals of this investigation was development of a high damping rubber (HDR) compound, adoption of production of rubber bearings and their testing. The production of rubber bearings with different rubber compounds was carried out in a small local workshop in Macedonia (Figure 1). Eighteen natural rubber compounds were developed and used for production of more than 100 bearings. Two types of bearings were produced. The first was of a square shape, side length of 200mm and total height of 75mm. The second was of a circular shape, external diameter of 150mm and total height of 100mm.

Figure 1. Vulcanization process of isolator

After the production of the bearings, two types of tests were performed, namely vertical and biaxial tests. All the tests on rubber bearings were carried out in the Dynamic Testing Laboratory of the Institute of Earthquake Engineering and Engineering Seismology (IZIIS), Skopje. To perform the tests, the existing biaxial dynamic frame with a dynamic load capacity of F_{MAX}=100kN and stroke of Δ=±75mm, was used. The static load capacity of the testing frame is F_{MAX}=200kN and the stroke is Δ=±50mm. The set-up of the vertical and biaxial test is shown in Figure 2a & 2b.

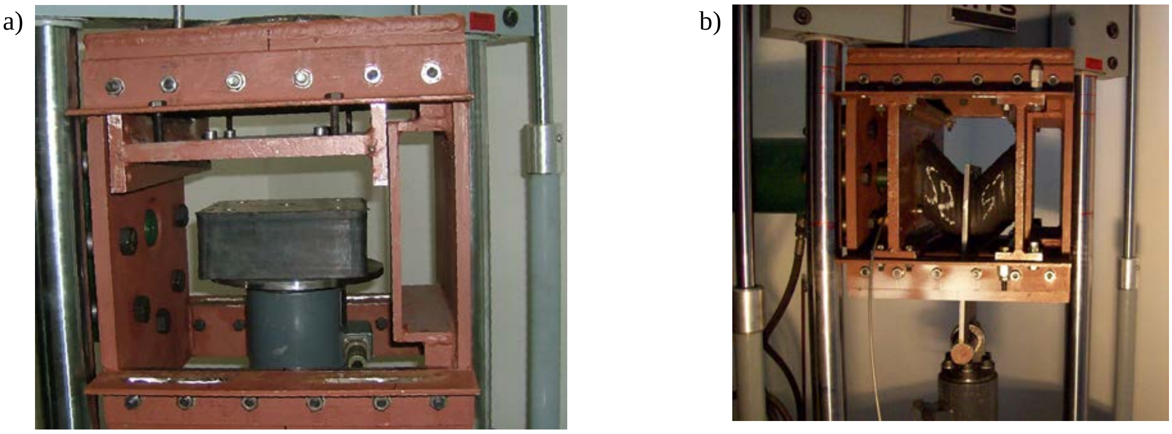


Figure 2. a) Set-up of axial test of squared bearing b) Set-up of biaxial test of two circular bearings

A detailed description of the selected tested specimens is given in Table 1.

Table 1. Specimen dimensions and rubber properties

No.	Label	shape	size b/d/h or D/h [mm]	No. Layers	G,50 [kPa]	βeff,50 [%]
1.	spec-1	squared	200/200/70	3	890.0	0.00
2.	spec-2	squared	200/200/70	4	1100.0	2.50
3.	spec-3	squared	200/200/70	0	1100.0	2.50
4.	spec-4	squared	200/200/70	3	1200.0	4.50
5.	spec-5	squared	200/200/70	3	370.0	4.50
6.	spec-6	squared	200/200/70	3	1500.0	7.20
7.	spec-7	squared	200/200/70	3	1000.0	7.80
8.	spec-8	squared	200/200/70	3	750.0	8.50
9.	spec-9	squared	200/200/70	3	700.0	8.60
10.	spec-10	squared	200/200/70	0	640.0	9.80
11.	spec-11	circular	200/100	5	465.0	8.70
12.	spec-12	circular	200/100	4	1350.0	12.50

Prior to each testing of the isolators, the materials were stabilized by keeping them under room temperature of (22±2°C) with a duration of 24 hours. First, the isolators were pre-loaded with three cycles at low frequency. Such a pre-loading is practiced in testing rubber elements for the purpose of eliminating the Mullins effect (Mullins, 1969). Pre-loading is carried out exclusively for elements not loaded in the course of the preceding 24 hours.

The procedure for the axial tests involved loading of the element with vertical compressive force at different axial stress levels. The specimens were monotonically loaded up to the achievement of the necessary stress level when harmonic excitation was applied. The purpose of these tests was to define the effect of the number of internal layers upon the vertical stiffness of the bearings at different load levels. Figure 3 shows the vertical force-displacement relationship for the rubber bearings produced by different number of internal steel plates. The presented graph provides a thorough insight into the behavior of the bearings with and without layers. As expected, the bearing with the greatest number of layers exhibited the highest vertical stiffness.

The biaxial tests consisted of application of horizontal load while the element was exposed to vertical load. The bearings were tested under two different harmonic loads: (1) a sinusoid function with constant amplitude defined by amplitude and frequency, (2) a sinusoid function with a linear increasing/decreasing amplitude. These tests provided accurate information about the horizontal stiffness and the damping of the bearing for each hysteretic cycle.

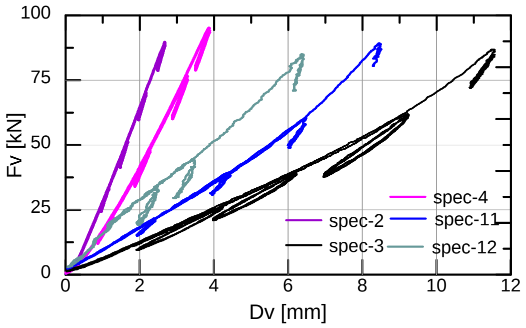


Figure 3. Comparison of axial behavior of rubber bearings

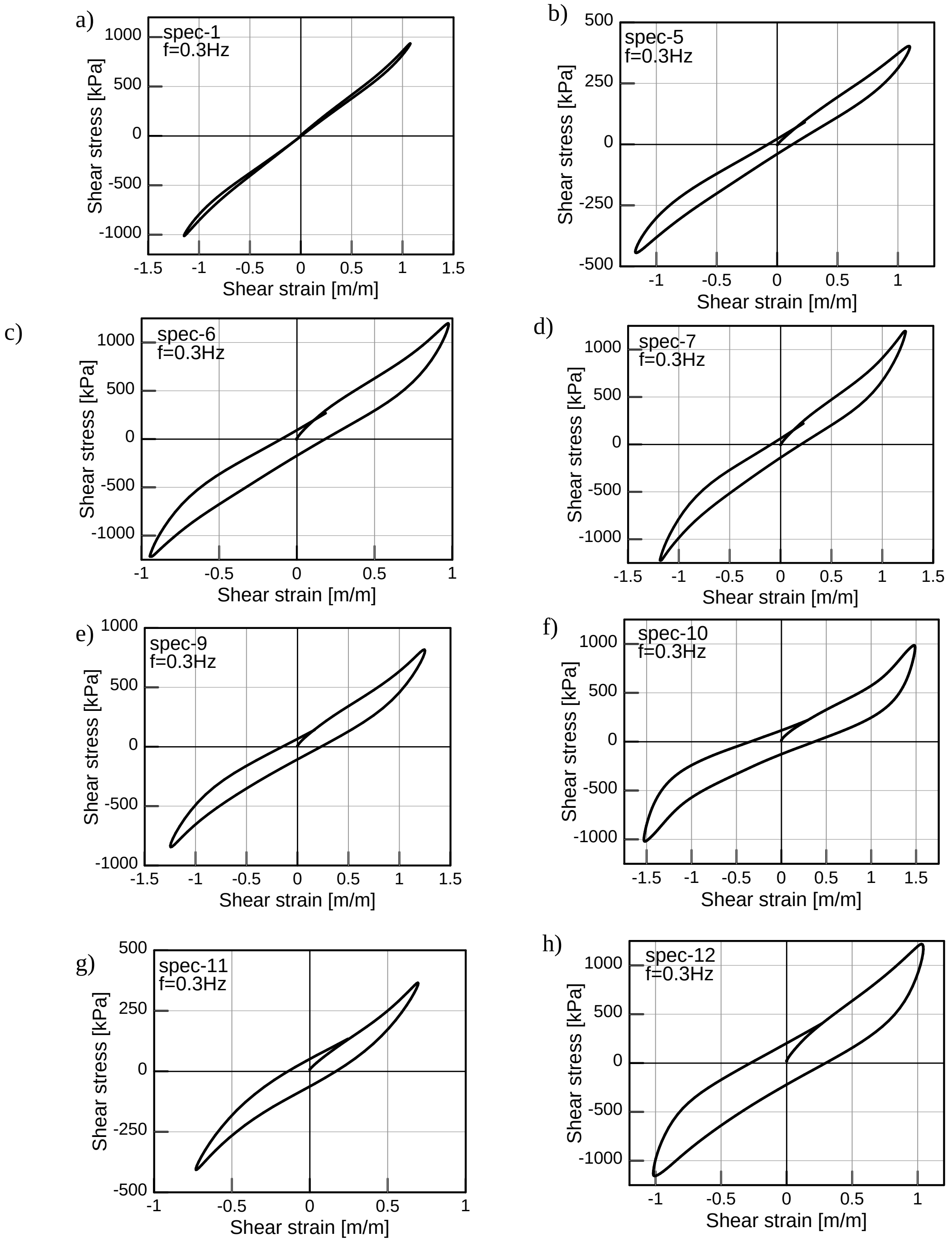


Figure 4. Comparison of horizontal behaviour of bearings made of eight different rubber compounds

Figure 4 shows comparison of the behavior of the rubber bearings made of eight different rubber compounds exposed to horizontal sinusoidal excitation with constant amplitudes. The graph provides a clear insight into the difference in the damping abilities and the difference in shear behavior. From the graphs presented in Figure 4, it can be concluded that the shear behavior mainly depends on the rubber compound. Analysing these types of behavior, it can be said that some bearings start to behave in the nonlinear range at lower strains, while others start to behave nonlinearly at higher strains. During unloading, almost all bearings exert high nonlinearity accompanied by different damping values.

MATHEMATICAL FORMULATION OF THE PROPOSED MODEL

Based on the performed tests on the bearings made of 18 different natural rubber compounds, an analytical model was proposed to mathematically describe the behavior of the natural rubber bearings. The model includes the (1) linear-elastic behavior, (2) post-elastic behavior at loading and (3) post-elastic behavior at unloading (Figure 5).

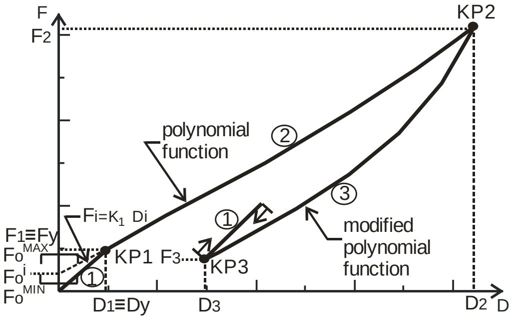


Figure 5. Polynomial analytical model of bearings made of rubber

The linear elastic state includes elastic behaviour of the bearing at low strains and it is presented by the following expression:

$$F = K_1 \cdot D \tag{1}$$

where F is the horizontal force, D is the horizontal deformation and K1 is the elastic (initial) stiffness of the bearing.

The post-elastic state at loading involves the behavior of the bearing after the yielding point. Once the yielding point is exceeded, the force-displacement relationship is defined by the polynomial function given in equation (2):

$$F_i = F_{om}^i + a_1 D_i + a_2 D_i^2 + a_3 D_i^3 + \dots \tag{1}$$

where Fi is the horizontal force, D_i – the horizontal deformation and F_{om}^i, a_1, a_2, a_3,... are the coefficients of the polynomial function which are calculated for best fitting the experimental curve. The polynomial coefficient F_{om}^i is defined based on the current and previously experienced horizontal deformation.

The mathematical force-displacement relationship for post-elastic behavior at unloading is formulated by the same polynomial function for post-elastic behavior at loading which is given in equation (2). To include different types of behavior of the bearings at unloading, modification is made in the computation of coefficient F_{om}^i given by equation (3).

$$F_{om}^i = F_o^i (2 \cdot kF_o^i - 1) \tag{3}$$

where kF_o^i is the decay coefficient and it depends on the current deformation of the bearing and the history of deformations.

VERIFICATION OF THE MODEL

The aim of the verification was to prove the stability of the model in different strain conditions. To validate the proposed model, the values of forces and displacements obtained from the polynomial model were compared to the experimental ones. The presented experimental results (Figure 6 & 7) were obtained from biaxial tests carried out on square and circular bearings, which were produced by use of different natural rubber compounds. The results from analytical solution were compared to results obtained from tests where the bearings were loaded with horizontal harmonic excitation of both constant and increasing amplitude.

The graph in Figure 6 shows the force-deformation relationship obtained from the rubber bearing test "spec-9" (loaded with horizontal harmonic excitation with increasing amplitude) and that obtained in the analytical solution. The dotted curve represents the data from the test, whereas the solid curves represent the analytical solution.

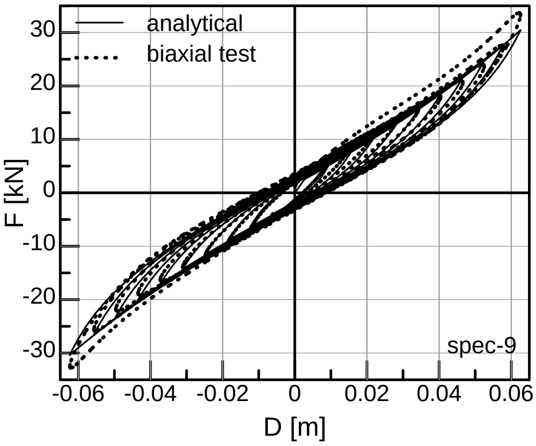
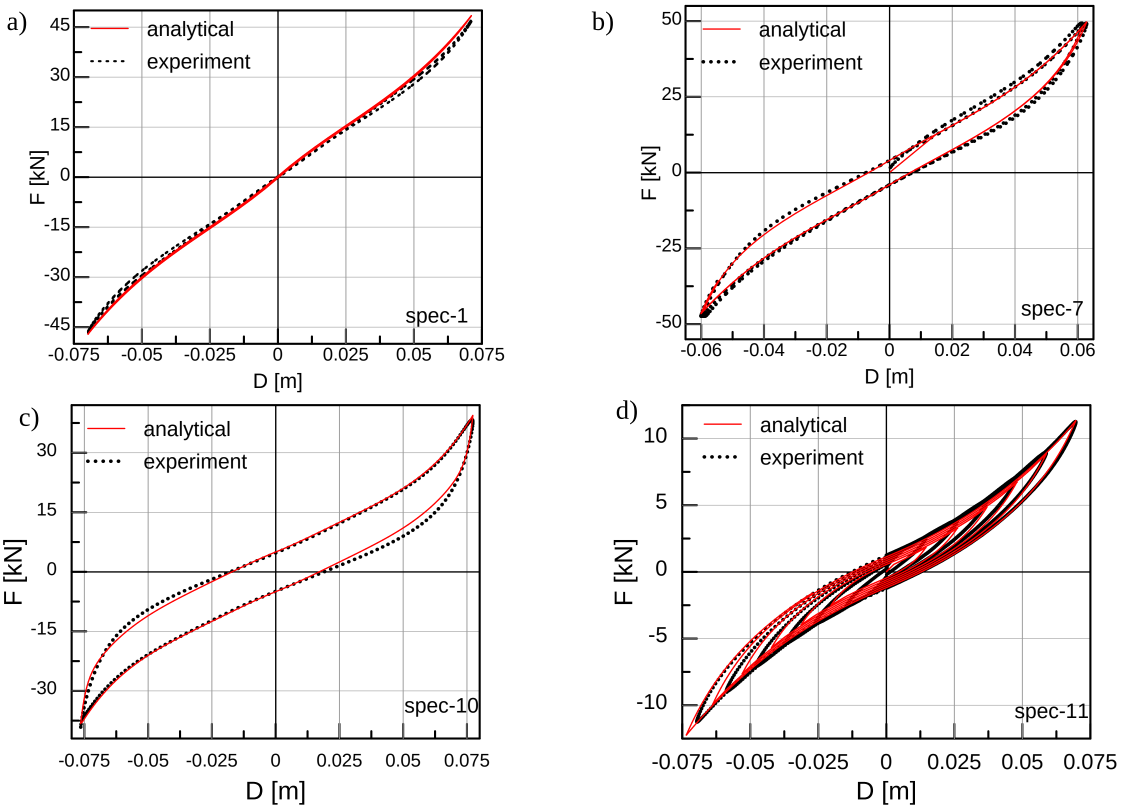


Figure 6. Force-deformation relationship obtained from test and the analytical solution



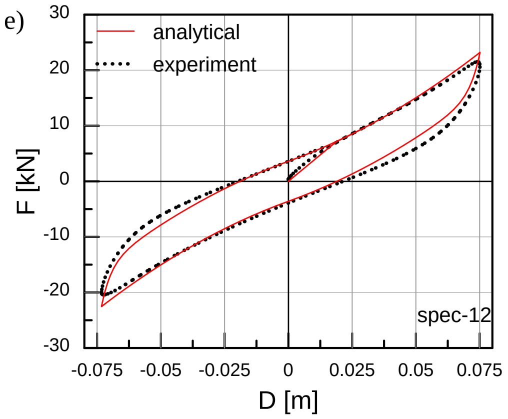


Figure 7. Verification of the analytical model with the most characteristic force-deformation relationship obtained by experimental testing

The applicability of the proposed analytical model for the bearings produced of different rubber compounds is presented through comparison of five force-displacement relationships (Figure 7a through Figure 7e). Figure 7a shows the nonlinear behaviour of the rubber without damping, while Figure 7b through Figure 7e show different forms of behavior in the case when the compound possesses a certain internal damping.

In order to investigate the mathematical stability of the polynomial model at low and large strains, the artificial harmonic excitations were generated (Figure 8). The results obtained from analytical solution are presented in graphs from Figure 8a through Figure 8c.

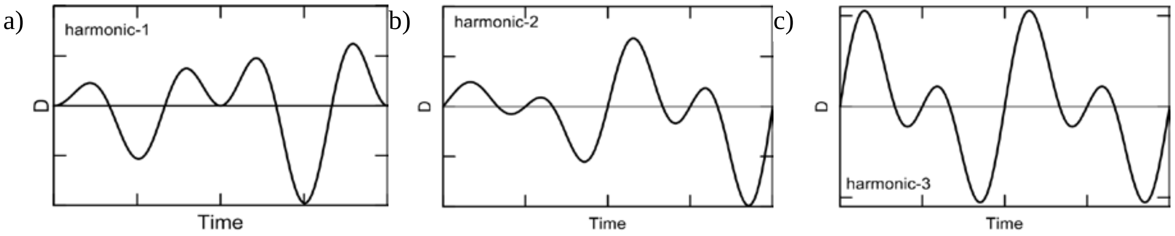


Figure 8. Input excitations

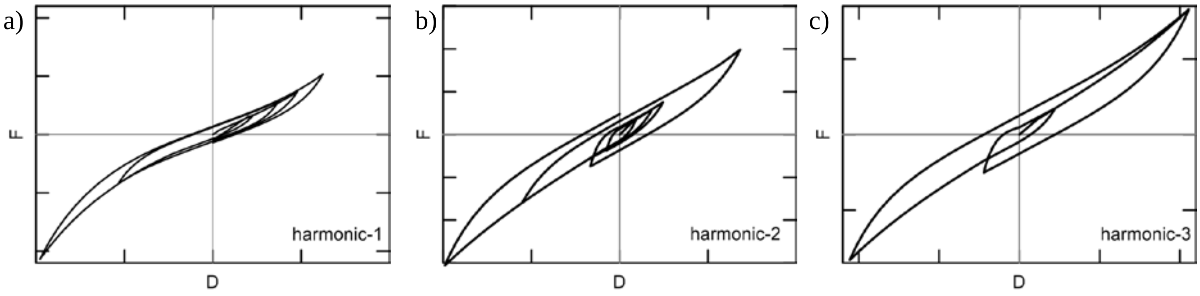


Figure 9. Force-deformation relationship obtained from analytical solution

From the presented graphs, it can be concluded, that the polynomial model is sufficiently flexible to be applied for modeling of bearings made of different rubber compounds. The model shows satisfactory accuracy not only in the case of high damping rubber bearings but also in the case of low damping rubber bearings.

The stability and accuracy of the polynomial model was also verified by nonlinear dynamic analysis of a seismically isolated steel reservoir (Gjorgjiev, Igor; Garevski, Mihail, 2013). The time histories of acceleration analytically obtained from several earthquake excitations (Gjorgjiev, Igor; Garevski, Mihail, 2013) were compared with the experimental ones, whereat the analytical model proved to be sufficiently precise.

CONCLUSIONS

This paper deals with production of rubber bearings, their testing and development of an analytical model of a rubber bearing. The process of production involved development of eighteen rubber compounds and establishment of a stable bond between the rubber and the reinforcement. Vertical and biaxial tests were carried out for square and circular bearings. Based on the performed tests, it was concluded that the behaviour of the bearings was bilinear until moderate shear strain, while at higher strains, the rubber bearings behaved in the nonlinear range. It was also concluded that the horizontal stiffness at zero horizontal deformation of the bearing depended on the loading history. In the case of change of horizontal deformation at the beginning of the unloading, the force at zero deformation changes, as well.

The simple analytical model of a rubber bearing is based on a polynomial function. The nonlinear shear behavior of the rubber bearing is presented through eight parameters plus the polynomial coefficients. The analytical model and the methodology presented in this paper are based on establishment of empirical model parameters by matching the actual bearing test results. This analytical model was developed on the basis of the results of experimental tests on bearings made of different rubber compounds. The model included large strain behavior and loading history effect. The Mullins effect, the strain rate and the dependence on the axial load were not taken into account in these investigations.

It can be concluded, that the polynomial model is sufficiently flexible to be applied for modeling of bearings made of different rubber compounds. The model shows satisfactory accuracy not only in the case of high damping rubber bearings but also in the case of low damping rubber bearings.

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