



COUPLED AND DECOUPLED APPROACHES FOR SEISMIC STABILITY ASSESSMENT OF REINFORCED EARTH STRUCTURES

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ABSTRACT

The current study aims at examining the dynamic response and stability issues of reinforced soil structures and the potentially beneficial impact of geosynthetics for the prevention of seismic induced instabilities. For this purpose, efficient lumped mass models have been developed based on the well-known sliding block approach. Moreover, dynamic coupled and decoupled analyses were performed in order to investigate the impact of the most significant parameters involved, such as the flexibility of the sliding system, the mechanical properties of the soil and the geosynthetics material, the period of the harmonic excitation and the interface shear strength.

1. INTRODUCTION

Geosynthetics have been widely used for various applications in engineering practice for reinforcement, drainage, filtration, containment and separation. Reinforcement in slopes and embankments is used to enable the construction of steep slopes without endangering their stability. Over the last decades increased research effort has been placed on the dynamic response and stability issues of geosynthetically reinforced geostructures. Various methods have been used to investigate these phenomena analytically, numerically and experimentally (Seed and Whitman 1970; Nova-Roessig and Sitar 1999; Paulsen 2002; Abramson et al. 2002, Tzavara et al. 2009; Trandafir et al. 2009; among others).

Seismic stability of slopes is commonly assessed in terms of permanent deformation. Newmark (1965) firstly introduced the concept of the sliding block model for the calculation of permanent deformation of an earth dam caused by a seismic event. It is a relatively simple analytical model, in which the displacement of a soil mass above a slip surface behaves as a rigid block sliding on a flat surface. The basic concept of the method is that permanent displacements accumulate along a well-defined plane, when the inertia forces of the block resting on this plane exceed the shear resistance of the interface. The computation of the permanent displacement is achieved by double integration of the relative acceleration, i.e., the difference between the applied and the critical acceleration. Up to now Newmark's approach has been extensively applied for the seismic stability assessment of earth structures, even though the accuracy of the method is limited by the following assumptions: (a) the

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soil behaves in a rigid, perfectly plastic manner, (b) displacements occur along a single, well-defined slip surface, (c) the stress-strain behaviour of interface's shear strength is rigid-plastic, (d) the uphill resistance is infinitely large, (e) the input motion is horizontal, and (f) the sliding surface is a plane.

Nevertheless, soil slopes are flexible systems characterized by relatively large fundamental period. Therefore, design procedures which are used to evaluate earthquake-induced sliding displacements typically refer to three different approaches: (a) simplified dynamic analysis by means of the conventional Newmark rigid block model, (b) dynamic analysis accounting for the flexibility of the oscillating mass, where the dynamic response and the sliding block displacements are computed separately, referred as decoupled approach (Makdisi and Seed 1978), and (c) dynamic analysis where the dynamic response and slip displacement accumulation are considered simultaneously, commonly referred to as coupled approach (Rathje and Bray, 1999; Ausilio et al. 2009).

The application of coupled methods in the stability analysis of earth structures was based on research studies on base isolation systems in the early 80's. Initially, the dynamic behavior of a single-degree-of-freedom (SDOF) system was examined (Westermo and Udawadia 1983; Mostaghel et al. 1983). In particular, Westermo and Udawadia (1983) after deriving the mathematical expressions of the sliding motion, considered a periodic slip-slip response to evaluate the dynamic behaviour of the examined system. Mostaghel et al. (1983) concluded that sliding displacement increases for lower friction coefficients and higher mass ratios, while the level of response is decreasing for the same conditions. In addition, Iura et al. (1992) presented the occurrence conditions of sliding, which consists of three different modes: stick-stick, slip-slip and stick-slip of the aforementioned system, in conjunction with the impact of the main parameters that affect the response of the system.

In the sequence, the comparison of decoupled versus coupled analysis for geotechnical structures has gained attention. In the majority of the related studies, the main aim was to investigate of the conservativeness of the decoupled methods. Chopra and Zhang (1991) used a SDOF model, with mass and elasticity distributed along the height in order to examine the dynamic response of a dam. It was also found that the displacements predicted by the decoupled analysis were more conservative compared to the corresponding ones of the coupled procedure. Moreover, Lin and Whitman (1983) analyzed three lumped mass-systems corresponding to sliding masses of different depth and the dynamic response and the corresponding slip displacements were calculated. It was concluded that the decoupling assumption is related to over-conservative estimates of the sliding displacements, especially for resonance conditions.

Several related studies concluded that the difference between coupled and decoupled approaches lies on the following two parameters: (a) the ratio of critical to maximum acceleration, and (b) the tuning ratio (denoted as β), which represents the ratio of the eigenperiod of the structure to the period of excitation (Kramer and Smith 1997; Matasovic et al., 1997; Bray and Rathje 1998, Rathje and Bray 2000, Wartman et al. 2003; among others). The results of these studies indicate that compared to coupled analysis the decoupled analysis is significantly more conservative for tuning ratios lower than unity.

Under this perspective, the current study examines the dynamic response and stability issues of reinforced earth structures and the beneficial role of geosynthetics for the prevention of seismically induced instabilities. For this purpose, efficient models have been developed based on Newmark's sliding block approach. In addition, the present work aims to demonstrate the most important issues related to the simple models that have been proposed, in order to highlight the importance of the seismic stability of deep-seated reinforced and unreinforced soil slopes, taking into account the most significant aspects of the problem such as the dominant parameters, the mechanical properties of the soil and the geosynthetics, the period of the harmonic excitation and the interface shear strength.

These factors control the flexibility, yield acceleration, dynamic loading, reinforcement and the overall response of the problem at hand. Parametric analyses were performed and the permanent slip displacements of the aforementioned models were computed. The scope of this study is to compare permanent slips displacements computed using the decoupling assumption with those calculated by coupled analysis and to assist developing the basis for an improved procedure for seismic design of reinforced slopes. Moreover, the normalization of the accumulated seismic displacements with respect to the most important parameters is examined.

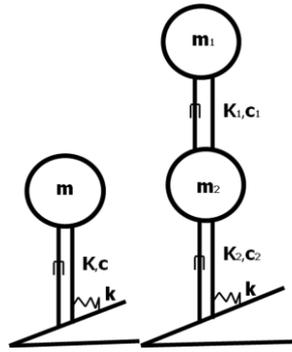


Figure 1. Sketches of the one and two lumped mass reinforced soil models.

2. DESCRIPTION OF THE MODELS

The prototype model that has been used as reference for this work was initially proposed by Westermo and Udawadia (1983), who examined the occurrence conditions of sliding for a single-degree-of-freedom (SDOF) system. In the current study the two beam models shown in Fig. 1 have been developed, namely a lumped mass SDOF (denoted for simplicity as 1M in the plots) and a multiple-degree-of-freedom (MDOF) model consisting of two lumped masses (denoted in the plots as 2M). For the SDOF system, the stiffness (K) was simulated by beam elements, viscous damping (c) was modeled via a dashpot, the reinforcement by a linear elastic spring (k) and the frictional interface at the base of the model was represented by a contact element with pre-determined shear strength defined via Coulomb friction ($\tan\phi$) conditions.

Several values of model stiffness, K , were selected in order to analyse models with different eigenperiods and resulting tuning ratio (denoted as β), which represents the ratio of the eigenperiod of the structure to the period of excitation. Damping was assumed to be constant, with damping ratio (ξ) equal to 5% or 10%. The properties of the soil and the reinforcement were regarded constant, defined by the density, ρ , the shear modulus, G , Young modulus, E , and Poisson's ratio, ν . The model which represents the unreinforced soil structure comprises of all the abovementioned components apart from the reinforcement spring. Moreover, in order to capture higher modes of vibration of the reinforced slope and assess their impact on the stability, a two-lumped mass model was also analyzed and compared to the corresponding one mass (Fig. 1). Hence, the two-lumped mass model consisted of: a) two concentrated masses (m_1 , m_2), b) two dashpots (c_1 , c_2), and c) two beam elements (K_1 , K_2). The mechanical behavior of the interface is characterized as absolutely rigid in the vertical direction and perfectly plastic (i.e., zero strains) in the horizontal direction, with shear strength characterized by Coulomb friction, identical to the one mass model. In order to minimize any inconsistency among the two models, the first modal response properties of the two models were matched, thus, the effective modal mass (m_{eff}) and the eigenfrequency (ω) of the first mode are identical for both models.

The dynamic analyses of the abovementioned models were performed utilizing the finite element software ABAQUS (2010). Four-cycle sinusoidal pulses were used in the dynamic time domain analyses characterized by period equal to 0.288sec or 0.576sec, and scaled to peak acceleration equal to 0.4g and 0.8g. The sinusoidal acceleration time history provides a displacement time history of non-zero final value, but on the other hand, every cycle is characterized by identical velocity variation assists in obtaining a fully symmetrical loading. Nevertheless, the use of real earthquake records would also be fruitful, however, the explanation and justification of the much more complicated results wouldn't be straightforward. For this reason, analytical procedures have been proposed to generate equivalent sinusoidal pulses for a large set of earthquake records and to use them in dynamic sliding analysis (Jafarian-Manzouni and Baziar 2005).

The numerical configuration of this study replicated, as closely as possible, the physical model developed in the geotechnical centrifuge by Nova-Roessig and Sitar (1999), providing, thus, a realistic basis for the parameters of the developed models. Detailed parametric analyses were performed taking into account the factors that influence the flexibility of the system. As it is well-known, the rigidity of

the sliding mass is the most important of all the assumptions that are used in the sliding-block model. Under this perspective, the permanent seismic displacement of the unreinforced model has been shown to be efficiently normalized with respect to: (a) the ratio of critical to maximum acceleration ($\tan\phi \cdot g/a_{\max}$), and (b) the tuning ratio (denoted as $\beta=T_{\text{str}}/T$), the latter representing the ratio of the eigenperiod (T_{str}) of the structure to the period (T) of the excitation (Zania et al. 2010).

This study illustrates the significance of the aforementioned ratios in the dynamic response of the examined geostructures, which is in accordance with results of previous investigations (Kramer and Smith 1997; Bray and Rathje 1998; Rathje and Bray 2000). For this purpose, the dynamic response of the examined models has been assessed for several models of different flexibility, by calculating different values for the tuning ratio (β) equal to 0.6, 0.8, 1.0, 1.5 and 2.0, while for the ratio of critical to maximum acceleration ($\tan\phi \cdot g/a_{\max}$) the values of 1.0 and 0.5 were considered.

In the sequence, the seismic stability of reinforced and unreinforced soil structures is assessed examining the impact of the most significant parameters involved, such as the flexibility of the sliding mass, the mechanical properties of the reinforcement, the period of the sinusoidal pulse excitation and the interface shear strength. The permanent displacements of the coupled analysis are compared with the corresponding ones of the decoupled analysis in terms of slip displacements and displacement ratio ($d_{\text{decoupled}} / d_{\text{coupled}}$) for different values of the ratio of the yield acceleration to the maximum applied acceleration, as well as for various characteristic values of the so-called tuning ratio, which as previously mentioned represents the ratio of the eigenperiod of the structure to the period of the excitation (T_{str}/T).

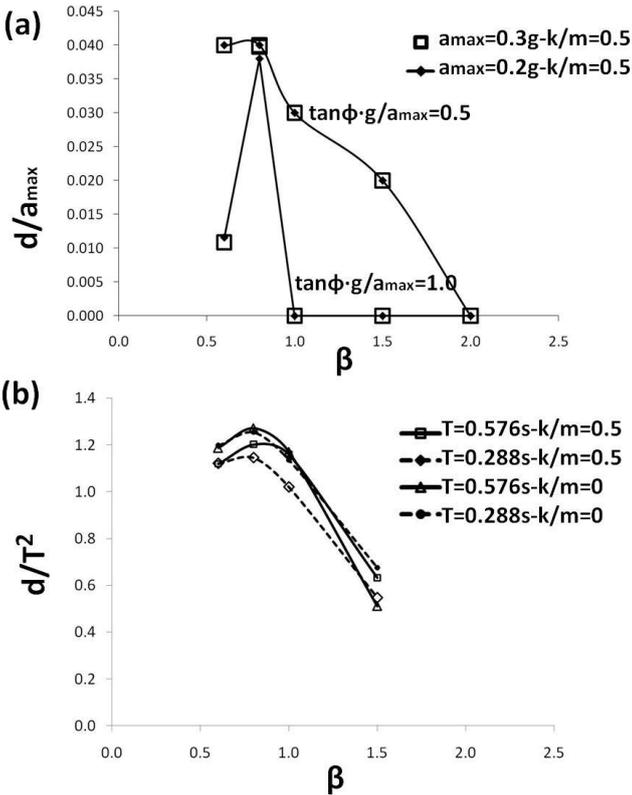


Figure 2. Variation of normalized displacement with respect to: (a) maximum applied acceleration, and (b) the square of the period of the excitation. Results correspond to the mass that slides on an inclined plane.

In addition, the normalization of the accumulated seismic displacements is examined with respect to the most important parameters of the reinforced slope models. More specifically, the normalization was attempted with respect to: a) the maximum applied acceleration, and b) the square of the period of the excitation. Hereafter, the validity of this normalization is examined both for a reinforced and an unreinforced SDOF model. For this purpose, the SDOF model was analyzed considering two values of the ratio $\tan\phi \cdot g/a_{\max}$, namely 0.5 and 1.0, and several values of tuning

ratios. Firstly, the normalization to the maximum applied acceleration was verified for the reinforced SDOF model and the results are illustrated in Fig. 2a. By observing Fig. 2a, it is evident that when referring to the same $\tan\phi^*g/a_{\max}$ ratio and tuning ratio values, the normalized displacements to maximum acceleration corresponding to two values of a_{\max} (equal to 0.3g and 0.2g) are identical. The normalization of the results was also made with respect to the square of the period of the excitation. Two different values of sinusoidal pulse period, equal to 0.29sec and 0.576sec, were selected. The presented results refer to the same value of ratio $\tan\phi^*g/a_{\max}$, which is equal to 0.5 in this case.

In Figure 2b it is shown that for the unreinforced lumped mass model ($k/m=0$), the calculated normalized displacements coincide both at a 0.29sec and a 0.576sec period. On the other hand, the normalization is less successful, but within an acceptable range for reinforced SDOF models ($k/m=0.5$). Therefore, it is reasonable to assess that: a) for reinforced and unreinforced models, at least for sinusoidal pulses, permanent displacements can be normalized to the maximum applied acceleration, and b) only for the unreinforced model permanent displacements can be normalized to the square of the period of the excitation referring to same values of the ratio $\tan\phi^*g/a_{\max}$ and tuning ratio.

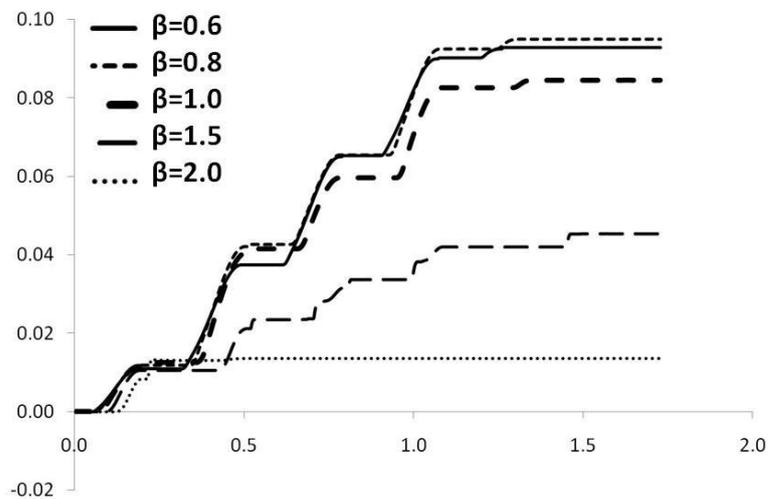


Figure 3. Accumulation of slip displacements for tuning ratio (β) equal to 0.6, 0.8, 1.0, 1.5 and 2.0, while $\tan\phi^*g/a_{\max}$ is equal to 0.5 and reinforcement stiffness (k) is equal to 8.3kN/m.

3. PARAMETRIC ANALYSES

In an extensive parametric analysis the two lumped mass models (SDOF & MDOF shown in Fig. 1) with and without geosynthetics have been examined and the consistency of the decoupled and coupled approaches has been investigated. The sliding plane of the deformable mass was assumed to be inclined to capture more realistically potential slope instability, while the spring that represents the reinforcement is considered to behave rigid-plastic and to deform only in extension. As aforementioned the lumped mass models shown in Fig. 1, were considered to be subjected to sinusoidal pulses and the impact of the dominant parameters was investigated. The positive impact of reinforcement illustrates the beneficial role of the geosynthetics to prevent slope instability.

More specifically, the results of the coupled analysis were compared with the results of the decoupled analysis in terms of slip displacement and displacement ratio ($d_{\text{decoupled}}/d_{\text{coupled}}$) for different values of the ratio of the yield acceleration to the maximum applied acceleration, as well as for various values of the tuning ratio (β) equal to 0.6, 0.8, 1.0, 1.5 and 2.0, while the ratio of critical to maximum acceleration ($\tan\phi^*g/a_{\max}$) and the stiffness of the reinforcement (k) for the results presented in the subsequent plots were equal to 0.5 and 8.3kN/m, respectively.

Fig. 3 depicts the time-histories of slip displacement accumulation of the SDOF (1M) lumped mass reinforced model for tuning ratio (β) equal to 0.6, 0.8, 1.0, 1.5 and 2.0. The sliding displacement (denoted as d in the vertical axis of the subsequent plots), is defined as the difference between the displacement at the base of the deformable sliding slope mass and the ground displacement. It is

obvious that for small tuning ratio values ($\beta < 1.0$), the increase of ratio β (i.e., the increase of model flexibility) leads to higher slip displacements, while the opposite trend occurs for higher ratios ($\beta > 1.0$). Moreover, the increased slip displacements has also led to the development of much higher permanent displacements.

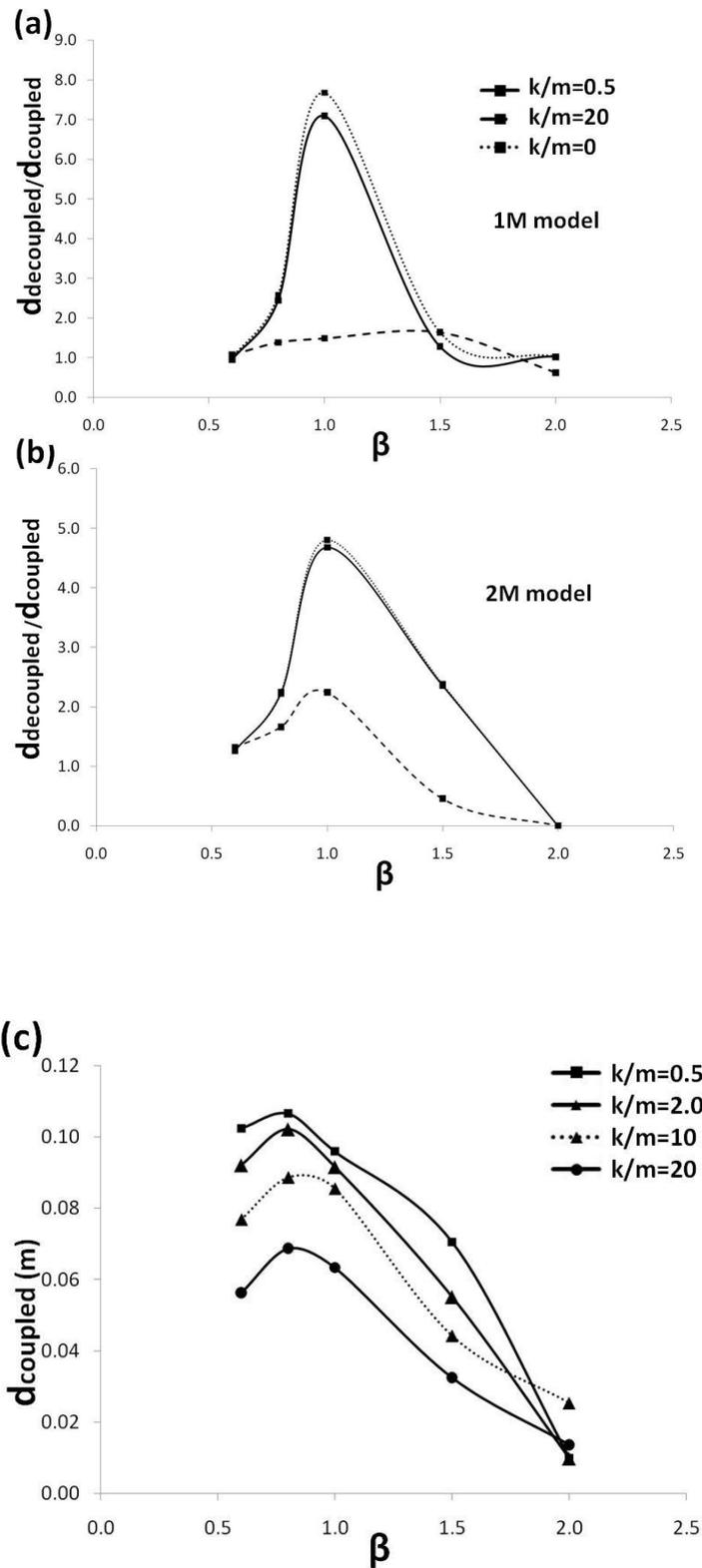


Figure 4. Displacement ratio of decoupled displacements to coupled displacements of a sliding reinforced: a) one lumped mass; b) two-mass lumped system and damping ratio (ξ) is equal to 5% and yield acceleration ratio ($\tan\varphi \cdot g/a_{max}$) is equal to 1.0; c) coupled displacement of a sliding reinforced one lumped mass system for various

values of the tuning ratio β (T_{str}/T) and damping ratio (ξ) is equal to 5% and yield acceleration ratio ($\tan\phi^*g/a_{max}$) is equal to 0.5.

The effect of reinforcement in geotechnical soil structures is significant and this is evident in Fig. 4, which illustrates the positive role of reinforcement on slip displacement reduction. The value of ratio k/m is equal to 0.5 and 20, which correspond to a value of reinforcement stiffness 150 kN/m and 5652 kN/m, respectively. Note that when the value of ratio k/m is set equal to zero it represents the unreinforced model. It is obvious, that for ratio k/m equal to zero the value of ratio of decoupled to coupled displacements of 1M models is closer to unity. Moreover, only in the case of unreinforced models the decoupled approach becomes unconservative for tuning ratios larger than 1.5, and this is attributed to the much smaller permanent displacement (see also Fig. 3).

In general, the increase of ratio k/m leads to the decrease of slip displacements. Hence, reinforcement constitutes an important factor for ensuring seismic slope stability of soil structures. By further inspecting Figs. 4a and 4b it can be easily noticed that for low values of β (<1.0), increasing the flexibility of the system leads to higher slip displacements. In contrast, for higher flexibility ($\beta > 1.0$) the increase of the tuning ratio β reduces the slip displacements. On the other hand, comparing Figs. 4a and 4b one can assess the impact of the higher modes in the decoupling approximation. The ratio of the displacements of the two approaches decreases as the dynamic degrees of freedom increase. Hence, the decoupling approximation is less influential for the two-mass model (2M).

The impact of ratio k/m for the one and two lumped mass models was investigated and results are depicted in Fig. 4c, where k is the stiffness of the reinforcement and m the mass of the model which are equal to 149.50kN/m ($k/m=0.5$), 565.20kN/m ($k/m=2.0$), 2486.88kN/m ($k/m=8.8$), 2826kN/m ($k/m=10$) and 5652kN/m- ($k/m=20$), while $m=282.60$ tn. The ratio k/m was studied for several values which were equal to: a) zero representing the unreinforced model and b) 0.5, 2.0, 10 and 20. It can be noticed that as the ratio k/m increases, the value of slip displacements decreases. The increase of ratio k/m results to an increased reinforcement capacity, which in turn leads to reduced permanent slip displacements.

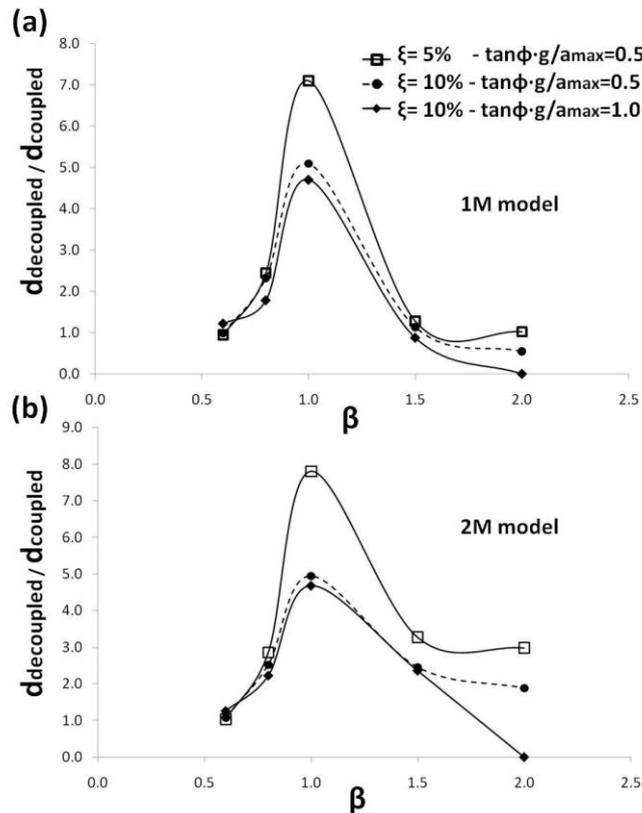


Figure 5. Displacement ratio of decoupled displacements to coupled displacements of reinforced models consisting of one lumped mass (a) and two lumped mass (b), for yield acceleration ratio ($\tan\phi^*g/a_{max}$) equal to 0.5 and 1.0 and damping ratio (ξ) equal to 5% and 10%.

Fig. 5 illustrates the variation of ratio of decoupled displacements to coupled displacements of a sliding reinforced system, having either one or two lumped masses, with respect to the tuning ratio β , for two different values of: a) damping (ξ) equal to 5% and 10%, and b) yield acceleration ratio ($\tan\phi \cdot g/a_{max}$) equal to 0.5 and 1.0. The ratio of decoupled to coupled displacements as well as slip and permanent displacements decrease as the damping increases. It is obvious that a reduction in yield acceleration ratio ($\tan\phi \cdot g/a_{max}$) increases the predicted slip and permanent displacements. Hence, the ratio of the permanent displacements as obtained from the decoupled and the coupled methods is also decreased. However, the decoupling approximation still overestimates the permanent displacements. Similar trends are observed for the two mass model, while the main influence of the higher modes is apparent only for tuning ratios higher than unity, where the ratio of the displacements is higher (for certain β values) than for the single mass model.

Finally, Fig. 6 depicts the decoupled and coupled displacements of a sliding reinforced models consisting of one and two lumped mass (1M and 2M, respectively) for yield acceleration ratio ($\tan\phi \cdot g/a_{max}$) equal to 0.5 and damping ratio (ξ) equal to 5% and 10%. It is evident that for both approaches (as expected) the values of slip displacements decrease as the damping ratio (ξ) increases, while the increase of damping also reduces the difference of the results of the two approaches. In addition, the increase of the number of degrees of freedom does not affect considerably the obtained displacements.

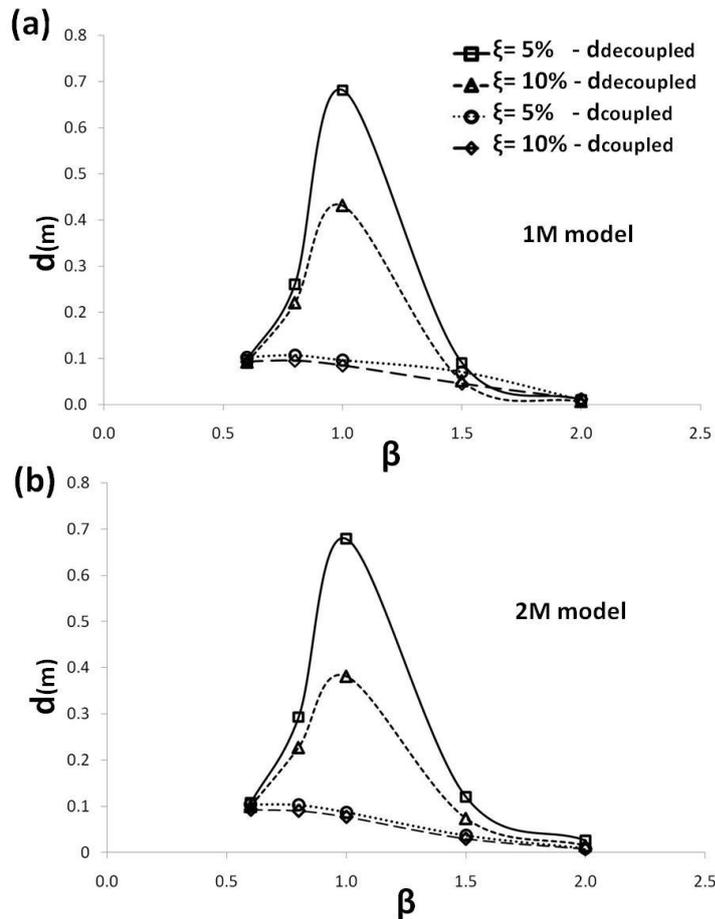


Figure 6. Decoupled and coupled displacements of a sliding reinforced a) one and b) two lumped mass model for yield acceleration ratio ($\tan\phi \cdot g/a_{max}$) equal to 0.5 and damping ratio (ξ) equal to 5% and 10%.

4. CONCLUSIONS

In the current study the seismic response of soil slopes with and without geosynthetics was investigated using simple, yet efficient, analytical approaches. For this purpose, a SDOF and a MDOF model were formulated and were subsequently used to calculate the magnitude of slip and permanent displacements using a coupled and a decoupled approach. These models were analyzed numerically utilizing the finite element software ABAQUS (2010). Additionally, it was proven that the permanent displacements can be efficiently normalized using the maximum applied acceleration (for soil slope models with and without reinforcement) and the square of the period of the excitation (only for unreinforced models), referring to identical values of $\tan\phi^*g/a_{\max}$ ratio and tuning ratio.

It was also demonstrated that slip displacements depend on the ratio of critical to maximum acceleration ($\tan\phi^*g/a_{\max}$) and the so-called tuning ratio (β) of the eigenperiod of the structure to the period of the excitation (T_{str}/T). By performing a detailed parametric analysis it was found that the impact of the flexibility of sliding mass, the yield acceleration ratio ($\tan\phi^*g/a_{\max}$), the degrees of freedom of the model, the reinforcement stiffness and the damping ratio (ζ) are very important factors that influence the results. In general, the increase of flexibility of the sliding mass leads to higher displacements for small tuning ratio values ($\beta < 1.0$), while the opposite trend occurs for higher tuning ratio values ($\beta > 1.0$). The increase of reinforcement stiffness and damping ratio resulted to a decrease of permanent displacements, while the number of model masses does not alter much the results. It was verified that the resulted displacements of the decoupled approach are more conservative compared to those of the coupled methodology.

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