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A MULTIMODE PUSHOVER PROCEDURE FOR ASYMMETRIC BUILDINGS UNDER BIDIRECTIONAL GROUND MOTION

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ABSTRACT

In this paper a recently developed multimode pushover procedure for the approximate estimation of structural performance of asymmetric in plan buildings under biaxial seismic excitation is presented and evaluated. Its main idea is that the seismic response of an asymmetric multi-degree-of-freedom system under biaxial excitation can be related to the responses of 'modal' equivalent single-degree-of-freedom (E-SDOF) systems under uniaxial excitation. The steps of the proposed methodology are quite similar to those of the well-known Modal Pushover Analysis. However, the establishment of the equivalent single-degree-of-freedom systems is based on a new concept, in order to take into account multidirectional seismic effects. The proposed methodology does not require independent analysis in the two orthogonal directions and therefore the application of simplified superposition rules for the combination of seismic component effects is avoided. After a brief outline of the theoretical background, a series of applications to single-storey buildings is presented, which shows that, in general, the proposed methodology provides a reasonable estimation for the calculated response parameters.

INTRODUCTION

Static Pushover Analysis (SPA) is a widely accepted procedure for the approximate estimation of the inelastic performance of buildings under seismic excitations. Initially, SPA has been developed in some more or less similar variants called 'conventional' procedures. All of these variants are based on the assumption that the inelastic response of a structure can be related to the response of an equivalent single degree of freedom (E-SDOF) system. SPA was shortly adopted by several seismic codes and prestandards (ASCE/SEI 41-06, ATC-40, Eurocode 8, etc.) under the name 'Nonlinear Static Procedure' (NSP) and became a very popular and useful tool for the earthquake resistant design of new, as well as the seismic rehabilitation of existing buildings.

However, as it has already been stressed by many researchers, e.g. Krawinkler and Seneviratna (1998) and Goel and Chopra (2004), this procedure involves many shortcomings and can provide reasonable results only for low- and medium-rise planar systems. This is mainly due to the fact that the determination of the structural response is based on the assumption that its dynamic behaviour depends only on a single elastic vibration mode. In addition, this elastic mode is supposed to remain constant despite the successive formation of plastic hinges during the seismic excitation. Also, the choice of the roof displacement as the target displacement instead of any other displacement is arbitrary and it is doubtful whether the capacity curve is the most meaningful index of the nonlinear response of a structure, especially for irregular and spatial systems. Therefore, various 'advanced' pushover procedures have been proposed to overcome some of these shortcomings, e.g., Modal

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Pushover Analysis (MPA) (Chopra and Goel 2001), Energy-based Pushover Analysis (Hernadez-Montes et al. 2004), etc.

Nevertheless, the aforementioned ‘advanced’ pushover procedures - in their initial version - can be rigorously applied only to very simple structures which can be modeled by planar models, since they do not take into account multidirectional seismic effects. It is well known that the very common in current practice plan-asymmetric buildings have to be designed or assessed for concurrent action at least of the two horizontal components of the seismic excitation. In literature only few investigations concerning this issue can be found, e.g. (Fujii 2007), (Lin and Tsai 2008), (Fajfar et al. 2005), (Reyes and Chopra 2011a), (Reyes and Chopra 2011b). For example, on the basis of several assumptions, Fujii (2007) determines two orthogonal principal directions of an equivalent single-storey model of the multi-storey building under consideration and applies proper lateral loads simultaneously along them. The inelastic behaviour of the building is correlated to the response of the equivalent single-storey model. Lin and Tsai (2008) use pushover analysis to establish three-degree-of-freedom modal sticks, each one corresponding to a vibration mode of a multi-storey asymmetric building under biaxial excitation. The response of the building is then determined by modal superposition of modal sticks’ responses, calculated by means of uncoupled modal response history analysis (Chopra and Goel 2001). On the other hand, some researchers, e.g. Fajfar et al. (2005), Reyes and Chopra (2011a), Reyes and Chopra (2011b) apply pushover analyses independently in two horizontal directions and use one of the widely used directional combination rules (e.g., ASCE 41-06, Section 3.2.7 / EC-8, Section 4.3.3.5.1(6)) to take into account the multidirectional seismic effects. However, these rules are based on the superposition principle, while it is well known that this approach lacks a theoretical basis in the domain of inelastic response.

Recently, a new multimode pushover procedure for the approximate estimation of the seismic response of asymmetric in plan buildings under biaxial seismic excitation has been developed (Manoukas et al. 2012). Its main idea is that the seismic response of an asymmetric multi-degree-of-freedom (MDOF) system with N degrees of freedom under biaxial excitation can be related to the responses of N ‘modal’ equivalent single-degree-of-freedom (E-SDOF) systems under uniaxial excitation. The whole procedure is quite similar to the well-known MPA (Chopra and Goel 2001) as extended for asymmetric buildings (Reyes and Chopra 2011a), (Reyes and Chopra 2011b), (Chopra and Goel 2004). However, the establishment of the E-SDOF systems is based on an essentially different concept. In particular the properties of the E-SDOF systems are determined by proper equations which take into account bidirectional seismic effects. The proposed methodology does not require independent analysis in each direction of excitation, hence directional combination is avoided.

The preliminary evaluation of the proposed procedure, comprising applications to single-storey buildings consisting of quite simple structural system, indicated that, in general, provides conservative results and relatively small mean errors with regard to the NDA (Manoukas et al. 2012). The objective of this paper is the further evaluation of the procedure for single-storey asymmetric in plan buildings with realistic structural system, in order to check its accuracy and to identify possible limitations or/and shortcomings.

Firstly, the theoretical background and the assumptions of the proposed methodology are briefly outlined. Secondly, the sequence of steps to be followed for its implementation is systematically presented. The accuracy of the proposed methodology is evaluated by a parametric study, which comprises implementation of the procedure to five single-storey asymmetric in plan buildings with varying values of normalized eccentricity. The whole investigation shows that, in general, the proposed methodology provides a reasonable estimation of the response parameters calculated. Finally, the paper closes with comments on results and conclusions.

THEORETICAL BACKGROUND

Concerning the linear range of behaviour, it has been demonstrated that the proposed methodology can accurately determine the modal response of MDOF systems under two proportional horizontal seismic components (Manoukas et al. 2012). However, in the nonlinear range some fundamental assumptions have to be made:

- The seismic response of a MDOF system is expressed as superposition of the responses of appropriate SDOF systems just like in the linear range.
- Each SDOF system corresponds to a vibration 'mode' i with 'modal' vector $\boldsymbol{\varphi}_i$ (the quotation marks indicate that the application of the superposition principle is not strictly valid).
- The displacements $\mathbf{u}_i$ and the inelastic resisting forces $\mathbf{F}_{si}$ are supposed to be proportional to $\boldsymbol{\varphi}_i$ and $\mathbf{M}\boldsymbol{\varphi}_i$, respectively (where $\mathbf{M}$ is the mass matrix).
- The 'modal' vectors $\boldsymbol{\varphi}_i$ are supposed to be constant, despite the successive development of plastic hinges.
- It is supposed that Rayleigh damping is present.

Of course, such assumptions violate the very logic of nonlinearity, as the superposition principle does not hold for nonlinear systems. However, keeping always in mind that our main intention is the development of an approximate simplified procedure, the recourse to these assumptions is inevitable. They must be thought as a fundamental postulate, which constitutes the basis on which many simplified pushover procedures are built (Manoukas et al. 2011).

The only additional assumption introduced is that the two horizontal seismic components $\ddot{u}_g(t)_X$ and $\ddot{u}_g(t)_Y$ are proportional to each other, i.e.:

$$\ddot{u}_g(t)_Y = \kappa \ddot{u}_g(t)_X = \kappa \ddot{u}_g(t) \tag{1}$$

where κ is a constant factor. Of course, this is not true for recorded earthquake ground motions. However, this approximation is in accordance with the very common assumption adopted by seismic codes which specify that - within the framework of NSP as well as the linear analysis methods - the two horizontal seismic components are represented by the same design spectrum, while directional combination may be conducted using the percentage combination rule (e.g., ASCE 41-06, Section 3.2.7.1) which implies a constant factor (0.3) similar to κ. Obviously, the evaluation of this assumption, as well as the definition of specific values of κ is beyond the objective of the present study.

Given the aforementioned assumptions, the nonlinear response of an L-story MDOF system with N degrees of freedom (in the usual case of rigid diaphragms N = 3L) to a biaxial earthquake ground motion ($\ddot{u}_g(t)_X$ and $\ddot{u}_g(t)_Y = \kappa\ddot{u}_g(t)_X = \kappa\ddot{u}_g(t)$ along X and Y axes, respectively) is described by the following equation (for the sake of simplicity (t) is left out in all following expressions) (Manoukas et al. 2012):

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{F_s} = -\boldsymbol{M}(\boldsymbol{\delta}_{,X} + \kappa\boldsymbol{\delta}_{,Y})\,\ddot{u}_g \Rightarrow \boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{F_s} = -\boldsymbol{M}\boldsymbol{\delta}_{,XY}\ddot{u}_g \tag{2}$$

where $\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}$ are the displacement, velocity and acceleration vectors of order N, $\mathbf{M}$ is the $N \times N$ diagonal mass matrix, $\mathbf{C}$ is the $N \times N$ symmetric damping matrix, $\mathbf{F_s}$ the resisting forces vector and $\boldsymbol{\delta}_{,X}$, $\boldsymbol{\delta}_{,Y}$ are the influence vectors that describe the influence of support displacements on the structural displacements for independent uniaxial horizontal seismic excitations along X and Y axes, respectively. Vector $\mathbf{u}$ is written as follows:

$$\boldsymbol{u} = [\boldsymbol{u}_X, \boldsymbol{u}_Y, \boldsymbol{\theta}_z]^T \tag{3}$$

where $\mathbf{u}_X$, $\mathbf{u}_Y$, $\boldsymbol{\theta}_z$ are the vectors of order L of displacements along X axis, along Y axis and rotations around Z (vertical) axis, respectively. The influence vectors $\boldsymbol{\delta}_{,X}$ and $\boldsymbol{\delta}_{,Y}$ are:

$$\boldsymbol{\delta}_{,X} = [\boldsymbol{I}, \boldsymbol{0}, \boldsymbol{0}]^T \tag{4}$$

$$\boldsymbol{\delta}_{,Y} = [\boldsymbol{0}, \boldsymbol{I}, \boldsymbol{0}]^T \tag{5}$$

where $\mathbf{I}$, $\mathbf{0}$ are vectors of order L with each element equal to unity and zero, respectively. Due to the aforementioned assumptions, vectors $\mathbf{u}$ and $\mathbf{F_s}$ can be expressed as the sum of the 'modal' contributions (Anastassiadis 2004), (Chopra 2007):

$$\boldsymbol{u} = \sum_{i=1}^{N} \boldsymbol{u}_i = \sum_{i=1}^{N} \boldsymbol{\varphi}_i q_i \tag{6}$$

$$\boldsymbol{F_s} = \sum_{i=1}^{N} \boldsymbol{F}_{si} = \sum_{i=1}^{N} \alpha_i \boldsymbol{M\varphi}_i \tag{7}$$

where α_i is a hysteretic function that depends on the 'modal' co-ordinate q_i and the history of excitation (Anastassiadis 2004). By substituting Eqs. 6 and 7 into Eq. 2 and applying well-known principles of structural dynamics, N uncoupled equations can be derived, each one corresponding to an E-SDOF system (Manoukas et al. 2012):

$$M^*_{XYi}\ \ddot{D}_i + 2M^*_{XYi}\,\omega_i\zeta_i\,\dot{D}_i + V_{XYi} = -M^*_{XYi}\ \ddot{u}_g \tag{8}$$

where $D_i = q_i / v_{XYi}$, $\dot{D}_i$, $\ddot{D}_i$ the displacement, velocity and acceleration of the i[th] (i = 1…N) E-SDOF system, ω_i and ζ_i are the natural frequency and damping ratio of the elastic vibration mode i and:

$$V_{XYi} = V_{Xi} + \kappa V_{Yi} \tag{9}$$

$$M^*_{XYi} = M^*_{Xi} + \kappa(v_{Xi}\, L_{Yi} + v_{Yi}\, L_{Xi}) + \kappa^2 M^*_{Yi} \tag{10}$$

$$v_{XYi} = v_{Xi} + \kappa v_{Yi} \tag{11}$$

where V_{Xi}, V_{Yi} are the 'modal' base shears parallel to X and Y axes respectively, M^*_{Xi}, M^*_{Yi} and v_{Xi}, v_{Yi} are the effective modal masses and the modal participation factors of the elastic vibration mode i due to independent uniaxial excitations along X and Y axes respectively, while $L_{Xi}=\boldsymbol{\delta}_{,X}^T\boldsymbol{M\varphi}_i$ and $L_{Yi}=\boldsymbol{\delta}_{,Y}^T\boldsymbol{M\varphi}_i$.

Eq. 8 shows that, due to the aforementioned assumptions, the nonlinear response of a MDOF system with N degrees of freedom subjected to a biaxial seismic excitation $\ddot{u}_{gX}$ and $\ddot{u}_{gY} = \kappa\ddot{u}_{gX} = \kappa\ddot{u}_g$ along X and Y axes, respectively, can be expressed as the sum of the responses of N SDOF systems under uniaxial excitation $\ddot{u}_g$, each one corresponding to a vibration 'mode' having mass equal to M^*_{XYi}, displacement equal to D_i and inelastic resisting force equal to V_{XYi}, i.e. the sum of 'modal' base shear parallel to X axis plus 'modal' base shear parallel to Y axis multiplied by κ (see Eq. 9) (Manoukas et al. 2012).

THE PROPOSED METHODOLOGY

The application process of the proposed methodology resembles the one of MPA. However, the definition of the E-SDOF systems is essentially different, in order to take into account multidirectional seismic effects. In Table 1 the properties of the i[th] 'modal' E-SDOF system are tabulated, along with the properties that it would have in case of uniaxial excitation (parallel to X axis).

The proposed methodology should be implemented for all possible combinations of the seismic components. In particular, the following four combinations should be examined:

$$\ddot{u}_{gX} + \kappa\ddot{u}_{gY} \tag{12}$$

$$\ddot{u}_{gX} - \kappa\ddot{u}_{gY} \tag{13}$$

$$\ddot{u}_{gY} + \kappa\ddot{u}_{gX} \tag{14}$$

$$\ddot{u}_{gY} - \kappa\ddot{u}_{gX} \tag{15}$$

The equations derived by the process presented in the previous paragraph have to be modified proportionately for each combination. It can be easily proved - by simple implementation of the process - that the consideration of the four combinations with opposite sign (e.g., $-\ddot{u}_{gX} - \kappa\ddot{u}_{gY}$ instead of $\ddot{u}_{gX} + \kappa\ddot{u}_{gY}$) leads to identical properties for the E-SDOF systems, so they can be skipped.

Table 1. Properties of the i[th] E-SDOF system

Property	Uniaxial excitation $\ddot{u}_{gX}$	Biaxial excitation $\ddot{u}_{gX} + \kappa\ddot{u}_{gY}$
Mass	M^*_{Xi}	$M^*_{XYi} = M^*_{Xi} + \kappa(\nu_{Xi} L_{Yi} + \nu_{Yi} L_{Xi}) + \kappa^2 M^*_{Yi}$
Resisting force	V_{Xi}	$V_{XYi} = V_{Xi} + \kappa V_{Yi}$
Displacement	$D_i = u_{Ni} / \nu_{Xi} \varphi_{Ni}$ (roof displacement u_{Ni})	$D_i = u_{Ni} / \nu_{XYi} \varphi_{Ni} = u_{Ni} / (\nu_{Xi} + \kappa\nu_{Yi}) \varphi_{Ni}$ (roof displacement u_{Ni})
Damping factor	$2 M^*_{Xi} \omega_i \zeta_i$	$2 M^*_{XYi} \omega_i \zeta_i$

The steps needed for the implementation of the proposed methodology are as follows (Manoukas et al. 2012):

Step 1: Create the structural model.

Step 2: Calculate ν_{XY1} (Eq. 11) and M^*_{XY1} (Eq. 10) of the fundamental elastic vibration mode 1 for the first combination of seismic components ($\ddot{u}_{gX} + \kappa\ddot{u}_{gY}$).

Step 3: Apply to the structural model a set of lateral incremental forces (and moments) proportional to the vector $\mathbf{M}\boldsymbol{\varphi}_1$ of the fundamental elastic vibration mode 1 and determine the (resisting force)-(displacement) curve V_{XY1}-u_{N1} of the MDOF system. u_{N1} can be chosen to correspond to any degree of freedom, but usually the roof displacement parallel to X or Y axis is used.

Step 4: Divide the abscissas of the V_{XY1}-u_{N1} diagram by the quantity $\nu_{XY1}\varphi_{N1} = u_{N1}/D_1$ and determine the (resisting force)-(displacement) curve V_{XY1}-D_1 of the E-SDOF system.

Step 5: Idealize V_{XY1}-D_1 to a bilinear curve using one of the well known graphic procedures (e.g., ASCE/SEI 41-06, Section 3.3.3.2.5) and calculate the period T_1 and the yield strength reduction factor R_1 of the E-SDOF system corresponding to mode 1, from the following Eq. 16:

$$T_1 = 2\pi \sqrt{\frac{m_1 D_{y1}}{V_{y1}}} \rightarrow S_a\,(T_1) \rightarrow R_1 = \frac{m_1 S_a(T_1)}{V_{y1}} \tag{16}$$

where $m_1 = M^*_{XY1}$, D_{y1}, V_{y1} are the mass, the yield displacement and the yield strength of the system, respectively, and $S_a\,(T_1)$ is the spectral acceleration.

Step 6: Calculate the target displacement of mode 1 using one of the well known procedures of displacement modification (e.g., ASCE/SEI 41-06, Section 3.3.3.3.2 / FEMA 440, Section 10.4). If the procedure is applied for research purposes using recorded earthquake ground motions, it is recommended to estimate the inelastic displacement of the E-SDOF system by means of nonlinear dynamic analysis, instead of using the relevant coefficients (e.g., C_1 in ASCE/SEI 41-06 and FEMA 440). This is due to the fact that the coefficient values given by codes are based on statistical processing of data with excessive deviation and, therefore, great inaccuracies may result (Manoukas et al. 2006).

Step 7: Calculate the 'modal' values of the other response quantities of interest (drifts, plastic rotations, etc.) of mode 1 by conducting pushover analysis up to the already calculated target displacement.

Step 8: Repeat steps 3 to 7 applying the incremental forces (and moments) in the opposite direction.

Step 9: Repeat steps 2 to 8 for an adequate number of modes.

Step 10: Calculate the extreme values of response parameters by utilizing one of the well established formulas of modal superposition (SRSS or CQC).

Step 11: Repeat steps 2 to 10 for all possible combinations of the two horizontal components of the seismic excitation (Eqs. 12-15).

EVALUATION STUDY

The implementation of the proposed methodology to single-storey buildings consisting of quite simple structural system produced satisfactory results (Manoukas et al. 2012). In the present study, the procedure is further evaluated for single-storey asymmetric in plan buildings with realistic structural systems, in order to check its accuracy and to identify possible limitations or/and shortcomings.

In particular, a parametric study is carried out comprising applications to five single-storey asymmetric in plan reinforced concrete buildings with different values of normalized structural eccentricity $e/r = e_X/r = e_Y/r$ (where e_X, e_Y are the distances between center of mass CM and center of rigidity CR, and *r* is the radius of gyration) ranging between 0.10 and 0.50. The predefined values of normalized eccentricities are achieved by proper selection of the CM position.

The plans of the analysed buildings are shown in Fig. 1. Their structural system consists of moment frames in normal grid with bay width 5m and storey height 3m. The concrete is of class C16/20 (f_{ck}=16 MPa) and the reinforcement steel bars B500C (f_{yk}=500 MPa) according to the Greek standards. The cross-sections' dimensions and the reinforcement are shown in Fig. 2. The floor mass is equal to 150t and the mass moment of inertia equal to 4062.5tm^2.

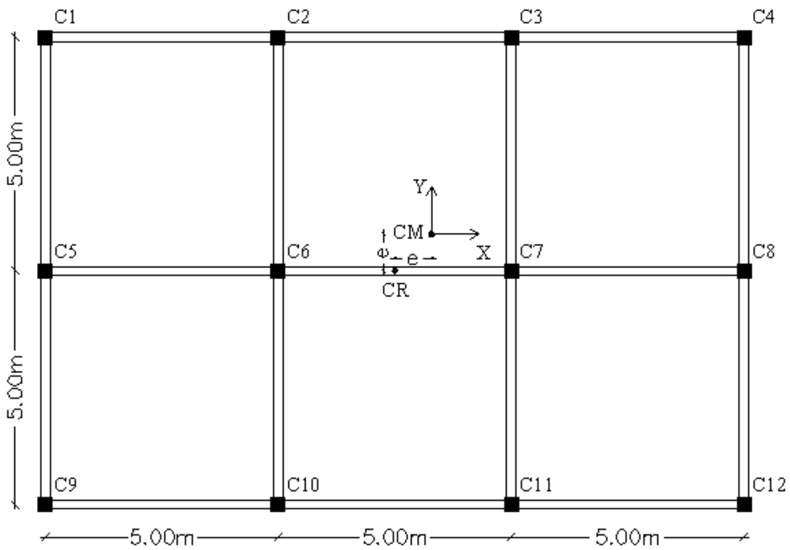

Figure 1. Floor plans of the analyzed buildings

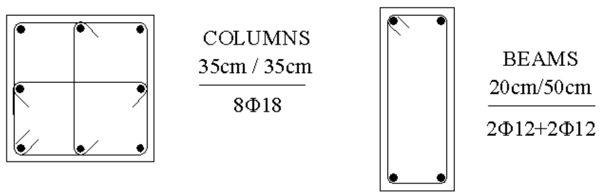

Figure 2. Cross-sections of columns and beams

All analyses are performed using the program SAP 2000 v10.0.7. The modeling of the inelastic behaviour is based on the following assumptions:

- Shear failure is precluded.
- The inelastic deformations are concentrated at the critical sections, i.e. at the ends of the frame elements (plastic hinges).
- Plastic hinges are modeled by bilinear elastic-perfectly plastic moments-rotations diagrams with practically unlimited available plastic rotations and yield moments calculated automatically by the program.
- The moment-axial force interaction is taken into account by appropriate interaction surface incorporated in SAP 2000.

The whole investigation conducted here comprises a number of 12 accelerograms, which is considered adequate to obtain preliminary conclusions for the accuracy of the proposed methodology.

These accelerograms correspond to strong earthquake motions recorded in Greece and are tabulated in Table 2. The excitations with relatively low ground accelerations (3, 4, 9 and 11) are scaled using an amplification factor equal to 1.5. Thus, the analyzed buildings sustain excessive nonlinear deformations for all excitations. It is considered that each ground motion acts simultaneously along the two horizontal axes of the buildings with the same intensity.

Table 2. List of seismic excitations

No	Excitation	Date	Magnitude (Ms)	Peak Ground Acceleration (m/sec^2)	Peak Spectral Acceleration (m/sec^2)
1	Aeghio (longitudinal)	06/15/1995	6.4	4.918	12.099
2	Aeghio (transverse)			5.326	14.157
3	Thessaloniki (longitudinal)	06/20/1978	6.5	1.389	4.477
4	Thessaloniki (transverse)			1.430	4.809
5	Alkyonides (longitudinal)	02/24/1981	6.7	2.336	6.023
6	Alkyonides (transverse)			2.989	8.155
7	Kalamata (longitudinal)	09/13/1986	6.0	2.170	6.648
8	Kalamata (transverse)			2.913	10.125
9	Patras (longitudinal)	07/14/1993	5.5	1.402	4.455
10	Patras (transverse)			3.936	12.151
11	Pirgos (longitudinal)	03/26/1993	5.5	1.466	5.887
12	Pirgos (transverse)			4.455	7.705

For each building two sets of pushover analyses are performed:

- One based on the proposed methodology (PM). Given that each ground motion acts simultaneously along the two horizontal axes with the same intensity, i.e. $\kappa=1$ and $\ddot{u}_{gX} = \ddot{u}_{gY}$, the possible combinations of the seismic components are only two: $\ddot{u}_{gX} + \ddot{u}_{gY}$ (PM+) and $\ddot{u}_{gX} - \ddot{u}_{gY}$ (PM-).
- A second similar to MPA (Reyes and Chopra 2011a; Reyes and Chopra 2011b; Chopra and Goel 2004) (conventional procedure - CP), which comprises pushover analyses of the examined buildings for independent uniaxial excitations along X and Y axes and directional combination of the response quantities using the percentage combination rule. The assumptions and steps of the second procedure are nearly identical to those of the proposed method, except that step 11 is obviously skipped and in steps 2 to 4 ν_{Xi}, M^*_{Xi}, V_{Xi} or ν_{Yi}, M^*_{Yi}, V_{Yi} are used in place of ν_{XYi}, M^*_{XYi}, V_{XYi}.

In both sets of pushover analyses the first two (translational) vibration modes are taken into account. 1st mode dominates the response for excitation along Y axis, while 2nd mode dominates the response for excitation along X axis. The ‘modal’ superposition is conducted by applying the CQC formula. In Table 3 the modal periods as well as the modal participating mass ratios of the modes taken into account are shown. The maximum ‘modal’ response of each E-SDOF system is calculated by means of nonlinear dynamic analysis (NDA) for each excitation. Then, the target roof displacement is estimated by multiplication of the resulting response by the quantity $\nu_{XYi}\varphi_{Ni}$ (PM) and $\nu_{Xi}\varphi_{Ni}$ or $\nu_{Yi}\varphi_{Ni}$ (CP). For each building, the displacements at the center of mass (CM), at the flexible side (C4) and at the stiff side (C9) of the plan are determined.

Table 3. Modal periods (T) and modal participating mass ratios (pmr_x, pmr_y)

e/r	Mode 1			Mode 2		
	T_1 (sec)	pmr_{x1} (%)	pmr_{y1} (%)	T_2 (sec)	pmr_{x2} (%)	pmr_{y2} (%)
0.10	0.211	10.6	87.2	0.207	87.9	11.5
0.20	0.217	31.5	60.8	0.207	64.3	35.4
0.30	0.226	36.9	50.9	0.207	56.6	43.3
0.40	0.237	38.5	46.8	0.207	53.7	46.2
0.50	0.250	39.1	44.8	0.207	52.4	47.6

The response values obtained by the two variants of pushover analysis are compared to the results of NDA, which is considered as the reference solution. For the latter analysis, each accelerogram is considered acting simultaneously along the two horizontal axes in all possible combinations ($\ddot{u}_{gX} + \ddot{u}_{gY}$, $\ddot{u}_{gX} - \ddot{u}_{gY}$, $-\ddot{u}_{gX} + \ddot{u}_{gY}$ and $-\ddot{u}_{gX} - \ddot{u}_{gY}$). For each response parameter $R_{j,s}$ estimated by the two applied variants of NSP for an excitation j, the error with regard to the NDA results E_j is determined by the following relation:

$$E_j\ (\%) = 100\frac{R_{j,s} - R_{j,d}}{R_{j,d}} \tag{17}$$

where $R_{j,d}$ is the value of the response parameter obtained by NDA. Furthermore, the mean error ME for the 12 excitations used in this study and the corresponding standard deviation SD are determined using Eqs. (18) and (19) respectively:

$$ME\ (\%) = \frac{1}{12}\sum_{j=1}^{12} E_j = 100\frac{1}{12}\sum_{j=1}^{12}\left(\frac{R_{j,s} - R_{j,d}}{R_{j,d}}\right) \tag{18}$$

$$SD\ (\%) = \sqrt{\frac{1}{11}\sum_{j=1}^{12}\left(E_j - ME\right)^2} \tag{19}$$

In Figs. 3 to 8 the mean errors for the 12 excitations (referring to the maximum values obtained by NDA) of floor displacements and storey drifts at the selected points (CM – center of mass, C4 – flexible side, C9 – stiff side) along X and Y axis are shown. Notice that the positive sign (+) means that response parameters obtained by PM or CP are greater than those obtained by NDA. Conversely, the negative sign (-) means that the response parameters are underestimated. It is apparent that the two combinations of PM (PM+ and PM-) provide an upper bound and a lower bound value for each response parameter. The exact value (NDA) in most cases (24 of 30) lies in this range. For the vast majority of the calculated response parameters (25 of 30) PM leads to conservative results. The mean errors of the more conservative combination (PM+ or PM-) range between -5% and 43%, while the mean errors of CP range between -2% and 79%. In comparison with CP, the absolute values of mean errors resulting from PM are smaller for 27 of 30 calculated displacements. Finally, concerning the standard deviation of the results, the values range between 2% and 50% for PM and 10% and 37% for CP.

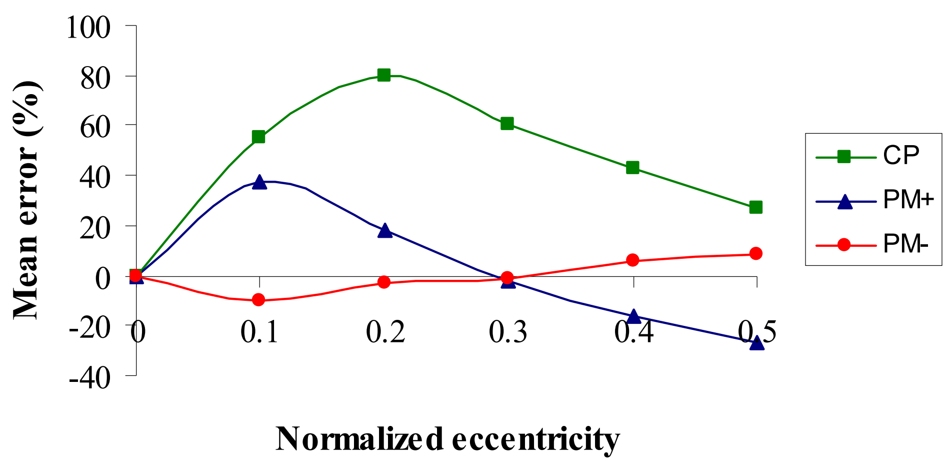

Figure 3. Mean errors (%) of displacements at the center of mass (CM) – X direction

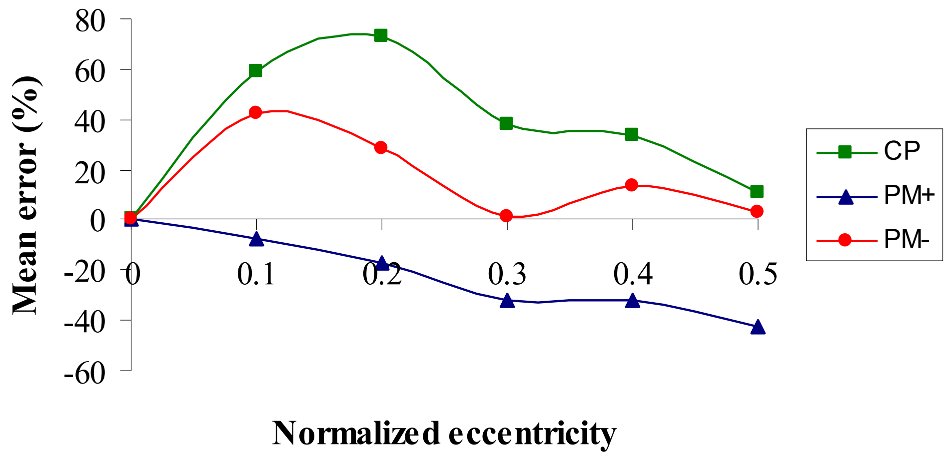

Figure 4. Mean errors (%) of displacements at the center of mass (CM) – Y direction

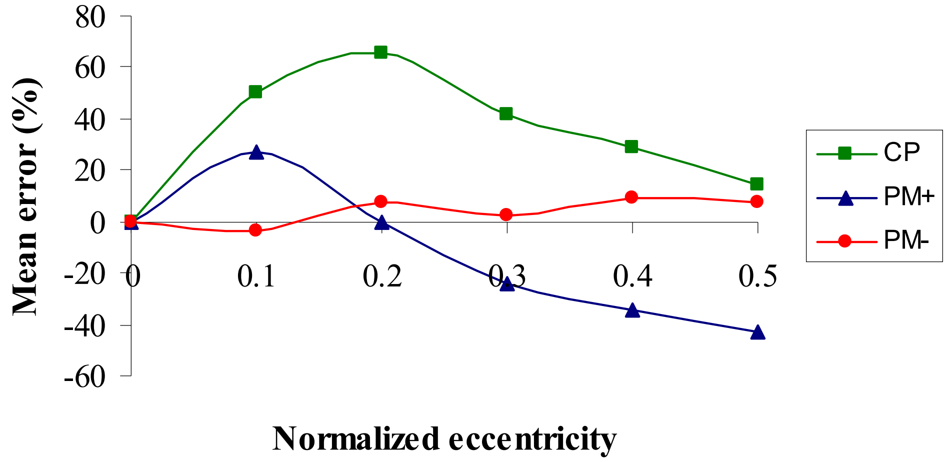

Figure 5. Mean errors (%) of displacements at the flexible side (C4) – X direction

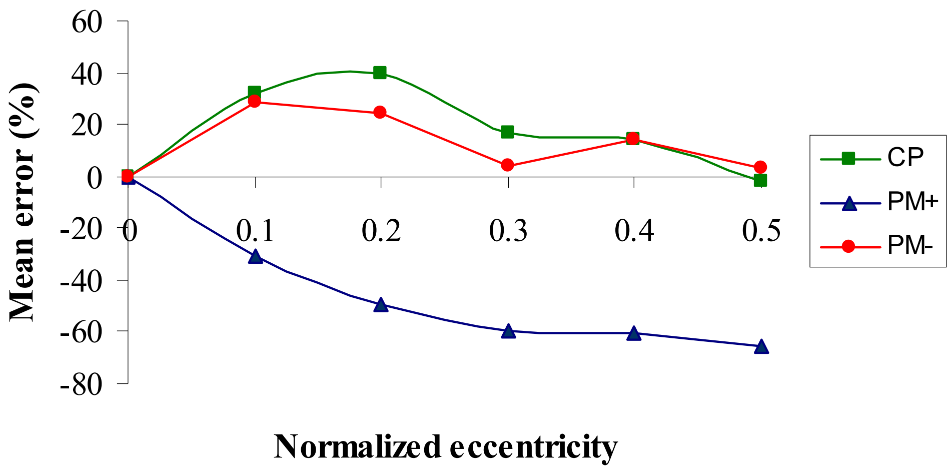

Figure 6. Mean errors (%) of displacements at the flexible side (C4) – Y direction

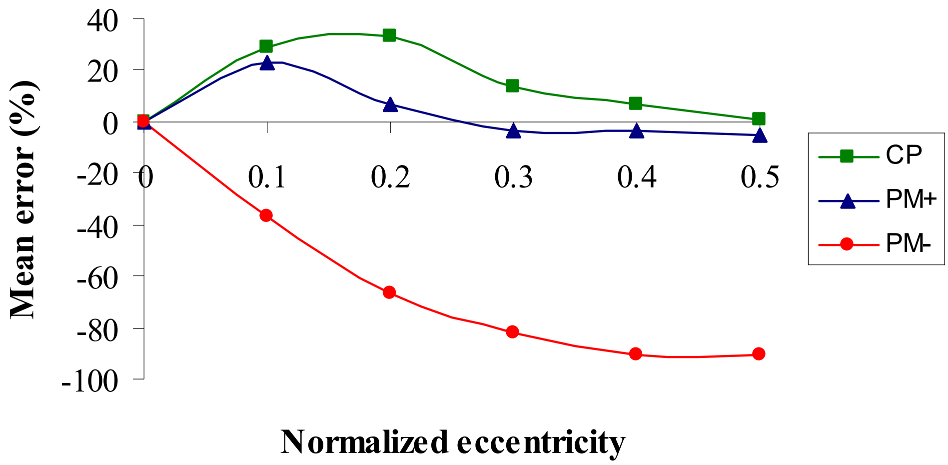

Figure 7. Mean errors (%) of displacements at the stiff side (C9) – X direction

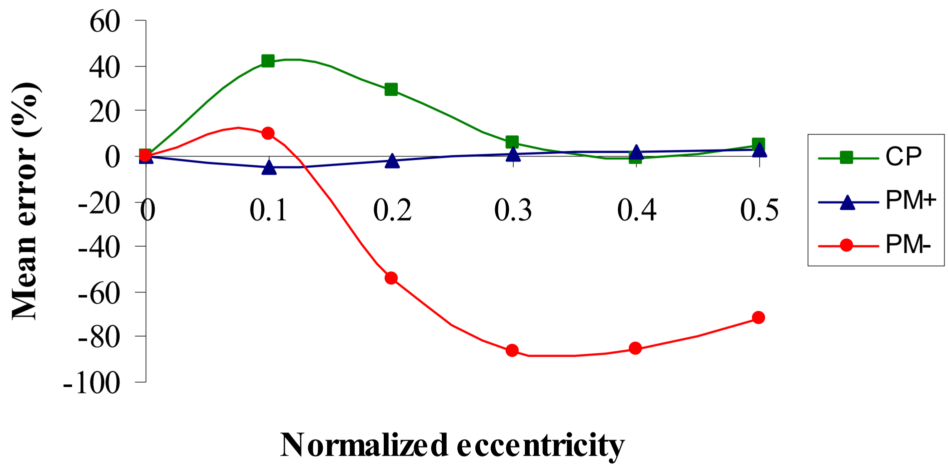

Figure 8. Mean errors (%) of displacements at the stiff side (C9) – Y direction

CONCLUSIONS

A recently developed multimode pushover procedure for the approximate estimation of the seismic response of asymmetric buildings under biaxial excitation is evaluated in this paper through a parametric study. The main idea of the procedure is that the seismic response of an asymmetric building under biaxial excitation can be related to the responses of a series of "modal" E-SDOF systems under uniaxial excitation. The whole procedure is quite similar to the well-known MPA. However, the establishment of the E-SDOF systems is based on an essentially different concept. From the presentation and the evaluation of the proposed method the following conclusions are derived:

- The proposed methodology does not require independent analysis in each direction of excitation, so application of simplified directional combination rules is avoided.
- The proposed methodology provides for each response parameter an upper limit and a lower limit which in the vast majority of cases envelope the corresponding value obtained by NDA.
- The mean errors with regard to the NDA results are significantly smaller than those resulting from a multimode pushover procedure comprising independent analysis along two horizontal axes and directional combination of the results (conventional procedure).

Finally, it is worth noticing that despite the fact that no restrictions are set to the development of the proposed methodology, generalization of the above conclusions for all types of asymmetric buildings requires further investigations, comprising application to a large variety of spatial structures and using an adequately high number of earthquake ground motions.

REFERENCES

American Society of Civil Engineers (2008) Seismic Rehabilitation of Existing Buildings, ASCE/SEI 41-06 Standard

Anastassiadis KK (2004) Approximate methods for the estimation of the inelastic response of buildings, Lecture notes (in Greek), Aristotle University, Thessaloniki

Applied Technology Council (1996) Seismic Evaluation and Retrofit of Concrete Buildings, Vol. 1, Report No. ATC-40, Redwood City

Chopra AK (2007) Dynamics of Structures – Theory and Applications to Earthquake Engineering, 3rd ed. Pearson Prentice Hall, New Jersey

Chopra AK and Goel RK (2004) "A modal pushover analysis procedure to estimate seismic demands for unsymmetric-plan buildings," *Earthquake Engineering and Structural Dynamics,* 33:903-927

Chopra AK and Goel RK (2001) A Modal Pushover Analysis Procedure to estimating seismic demands of buildings: theory and preliminary evaluation, PEER Report 2001/03, Pacific Earthquake Engineering Research Center, University of California, Berkeley

European Committee for Standardization (2004) Eurocode 8: Design of Structures for Earthquake Resistance, EUS: Brussels

Fajfar P, Marusic D and Perus I (2005) "The extension of the N2 method to asymmetric buildings," *Proceedings of the 4th European Workshop on the Seismic Behaviour of Irregular and Complex Structures*, 4EWICS, 26-27 August, Thessaloniki, Greece, Paper No. 41

Federal Emergency Management Agency - Applied Technology Council (ATC) (2004) Improvement of Nonlinear Static Seismic Analysis Procedures, Report No. ATC-55 (FEMA 440)

Fujii K (2007) "Prediction of seismic response of multi-story unsymmetric frame buildings," *Proceedings of the 8th Pacific Conference on Earthquake Engineering*, 5-7 December, Singapore, Paper No. 017.

Goel RK and Chopra AK (2004) "Evaluation of Modal and FEMA Pushover Analyses: SAC Buildings," *Earthquake Spectra* 20(6):225-254

Hernadez-Montes E, Kwon OS and Aschheim MA (2004) "An energy-based formulation for first- and multiple-mode nonlinear static (pushover) analysis," *Journal of Earthquake Engineering,* 8(1):69-88

Krawinkler H and Seneviratna GDPK (1998) "Prons and cons of a pushover analysis of seismic performance evaluation," *Engineering Structures*, 20:452–464

Lin JL and Tsai KC (2008) "Seismic analysis of two-way asymmetric building systems under bi-directional seismic ground motions," *Earthquake Engineering and Structural Dynamics*, 37:305-328

Manoukas GE, Athanatopoulou AM and Avramidis IE (2012) "Multimode Pushover Analysis for Asymmetric Buildings under Biaxial Seismic Excitation Based on a new Concept of the Equivalent Single Degree of Freedom System," *Soil Dynamics and Earthquake Engineering*, 38:88-96

Manoukas GE, Athanatopoulou AM and Avramidis IE (2011) "Static Pushover Analysis Based on an Energy-Equivalent SDOF System." *Earthquake Spectra*, 27(1):89-105

Manoukas GE, Athanatopoulou AM and Avramidis IE (2006) "Comparative evaluation of static pushover analysis' variations according to modern codes," (in Greek), *Proceedings of the 15th Hellenic Conference on R/C structures*, 25-27 October, Alexandroupoli, Greece, pp 410-421

Reyes JC and Chopra AK (2011a) "Three dimensional modal pushover analysis of buildings subjected to two components of ground motion, including its evaluation for tall buildings," *Earthquake Engineering and Structural Dynamics,* 40:789-806

Reyes JC and Chopra AK (2011b) "Evaluation of three-dimensional modal pushover analysis for unsymmetric-plan buildings subjected to two components of ground motion," *Earthquake Engineering and Structural Dynamics*, 40:1475-1494