



EVALUATION OF A MULTIMODE PUSHOVER PROCEDURE FOR ELASTIC SYSTEMS

Grigorios MANOUKAS¹

ABSTRACT

In this paper a recently developed multimode pushover procedure for the approximate estimation of structural performance of asymmetric in plan buildings under biaxial seismic excitation is evaluated for linear elastic systems. Its main idea is that the seismic response of an asymmetric multi-degree-of-freedom system under biaxial excitation can be related to the responses of ‘modal’ equivalent single-degree-of-freedom systems under uniaxial excitation. The proposed methodology does not require independent analysis in the two orthogonal directions and therefore the application of simplified superposition rules for the combination of seismic component effects is avoided. After a brief outline of the theoretical background, a series of application to single- and three-storey buildings is presented, which shows that, in general, the proposed methodology provides a reasonable estimation for the calculated response parameters.

INTRODUCTION

In recent years a series of more or less similar variants of Static Pushover Analysis for the assessment and the design of structures under seismic excitations have been presented. All these variants are based on several assumptions concerning the inelastic range of behavior and aim at the approximate estimation of the inelastic performance of structures. However, a well-established pushover procedure should be ‘exact’ for linear elastic systems. For example, the well-known Modal Pushover Analysis (MPA) (Chopra and Goel 2001) in the linear range of behaviour is equivalent to the Response Spectrum Analysis.

Recently, a new multimode pushover procedure for the approximate estimation of the seismic response of asymmetric in plan buildings under biaxial seismic excitation has been developed (Manoukas et al. 2012). Its main idea is that the seismic response of an asymmetric multi-degree-of-freedom (MDOF) system with N degrees of freedom under biaxial excitation can be related to the responses of N ‘modal’ equivalent single-degree-of-freedom (E-SDOF) systems under uniaxial excitation. The whole procedure is quite similar to the well-known MPA (Chopra and Goel 2001) as extended for asymmetric buildings (Reyes and Chopra 2011a), (Reyes and Chopra 2011b), (Chopra and Goel 2004). However, the establishment of the E-SDOF systems is based on an essentially different concept. In particular the properties of the E-SDOF systems are determined by proper equations which take into account bidirectional seismic effects. The proposed methodology does not require independent analysis in each direction of excitation, hence directional combination is avoided.

The preliminary evaluation of the proposed procedure indicated that, in general, provides conservative results and relatively small mean errors with regard to the NDA (Manoukas et al. 2012). The objective of this paper is the further evaluation of the procedure for buildings which remain elastic under bidirectional ground motion.

¹ Dr, Aristotle University, Thessaloniki, Greece, grman7@otenet.gr

After a brief presentation of the theoretical background, an evaluation study is conducted. The whole investigation comprises applications to five single-storey and five three-storey reinforced concrete buildings with various structural systems and plan configurations (asymmetric in plan, symmetric in plan with one or two axis of symmetry, moment frames, dual frames, etc.). These buildings are considered to be representative of a large variety of common structures. The whole investigation shows that the proposed procedure provides a reasonable estimation of the seismic performance of elastic systems.

THEORETICAL BACKGROUND

Concerning the linear range of behaviour, the whole procedure is based on the fundamental assumption that the two horizontal seismic components $\ddot{u}_g(t)_X$ and $\ddot{u}_g(t)_Y$ are proportional to each other, i.e.:

$$\ddot{u}_g(t)_Y = \kappa \ddot{u}_g(t)_X = \kappa \ddot{u}_g(t) \quad (1)$$

where κ is a constant factor. Of course, this is not true for recorded earthquake ground motions. However, this approximation is in accordance with the very common assumption adopted by seismic codes which specify that - within the framework of the pushover procedure as well as the linear analysis methods - the two horizontal seismic components are represented by the same design spectrum, while directional combination may be conducted using the percentage combination rule (e.g., ASCE 41-06, Section 3.2.7.1) which implies a constant factor (0.3) similar to κ . Obviously, the evaluation of this assumption, as well as the definition of specific values of κ is beyond the objective of the present study.

Given the aforementioned assumption, the linear response of an L-story MDOF system with N degrees of freedom (in the usual case of rigid diaphragms $N = 3L$) to a biaxial earthquake ground motion ($\ddot{u}_g(t)_X$ and $\ddot{u}_g(t)_Y = \kappa \ddot{u}_g(t)_X = \kappa \ddot{u}_g(t)$ along X and Y axes, respectively) is described by the following equation (for the sake of simplicity (t) is left out in all following expressions) (Manoukas et al. 2012):

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}(\boldsymbol{\delta}_{,X} + \kappa\boldsymbol{\delta}_{,Y})\ddot{u}_g \Rightarrow \mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\boldsymbol{\delta}_{,XY}\ddot{u}_g \quad (2)$$

where $\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}$ are the displacement, velocity and acceleration vectors of order N, \mathbf{M} is the $N \times N$ diagonal mass matrix, \mathbf{C} and \mathbf{K} are the $N \times N$ symmetric damping and stiffness matrices respectively and $\boldsymbol{\delta}_{,X}, \boldsymbol{\delta}_{,Y}$ are the influence vectors that describe the influence of support displacements on the structural displacements for independent uniaxial horizontal seismic excitations along X and Y axes, respectively. By applying well-known principles of structural dynamics, N uncoupled equations can be derived, each one corresponding to an E-SDOF system (Manoukas et al. 2012):

$$M_{XYi}^* \ddot{D}_i + 2M_{XYi}^* \omega_i \zeta_i \dot{D}_i + V_{XYi} = -M_{XYi}^* \ddot{u}_g \quad (3)$$

where $D_i, \dot{D}_i, \ddot{D}_i$ the displacement, velocity and acceleration of the i^{th} ($i = 1 \dots N$) E-SDOF system, ω_i and ζ_i are the natural frequency and damping ratio of the elastic vibration mode i and:

$$V_{XYi} = V_{Xi} + \kappa V_{Yi} \quad (4)$$

$$M_{XYi}^* = M_{Xi}^* + \kappa(v_{Xi} L_{Yi} + v_{Yi} L_{Xi}) + \kappa^2 M_{Yi}^* \quad (5)$$

$$v_{XYi} = v_{Xi} + \kappa v_{Yi} \quad (6)$$

where V_{Xi}, V_{Yi} are the ‘modal’ base shears parallel to X and Y axes respectively, M_{Xi}^*, M_{Yi}^* and v_{Xi}, v_{Yi} are the effective modal masses and the modal participation factors of the elastic vibration mode i due

to independent uniaxial excitations along X and Y axes respectively, while $L_{Xi} = \delta_{,X}^T \mathbf{M} \boldsymbol{\phi}_i$ and $L_{Yi} = \delta_{,Y}^T \mathbf{M} \boldsymbol{\phi}_i$, where $\boldsymbol{\phi}_i$ is the modal vector of mode i .

Eq. 3 shows that the linear response of a MDOF system with N degrees of freedom subjected to a biaxial seismic excitation can be expressed as the sum of the responses of N SDOF systems under uniaxial excitation \ddot{u}_g , each one corresponding to a vibration ‘mode’ having mass equal to M_{XYi}^* , displacement equal to D_i and inelastic resisting force equal to V_{XYi} , i.e. the sum of ‘modal’ base shear parallel to X axis plus ‘modal’ base shear parallel to Y axis multiplied by κ (Manoukas et al. 2012). Based on this conclusion, the properties of the i^{th} ‘modal’ E-SDOF system are determined using the equations given in Table 1, where the properties that it would have in case of uniaxial excitation (parallel to X axis) are tabulated too.

Table 1. Properties of the i^{th} E-SDOF system

Property	Uniaxial excitation \ddot{u}_{gX}	Biaxial excitation $\ddot{u}_{gX} + \kappa \ddot{u}_{gY}$
Mass	M_{Xi}^*	$M_{XYi}^* = M_{Xi}^* + \kappa(v_{Xi} L_{Yi} + v_{Yi} L_{Xi}) + \kappa^2 M_{Yi}^*$
Resisting force	V_{Xi}	$V_{XYi} = V_{Xi} + \kappa V_{Yi}$
Displacement	$D_i = u_{Ni} / v_{Xi} \phi_{Ni}$ (roof displacement u_{Ni})	$D_i = u_{Ni} / v_{XYi} \phi_{Ni} = u_{Ni} / (v_{Xi} + \kappa v_{Yi}) \phi_{Ni}$ (roof displacement u_{Ni})
Damping factor	$2 M_{Xi}^* \omega_i \zeta_i$	$2 M_{XYi}^* \omega_i \zeta_i$

The proposed methodology should be implemented for all possible combinations of the seismic components. In particular, the following four combinations should be examined:

$$\ddot{u}_{gX} + \kappa \ddot{u}_{gY} \quad (7)$$

$$\ddot{u}_{gX} - \kappa \ddot{u}_{gY} \quad (8)$$

$$\ddot{u}_{gY} + \kappa \ddot{u}_{gX} \quad (9)$$

$$\ddot{u}_{gY} - \kappa \ddot{u}_{gX} \quad (10)$$

The equations derived by the process presented previously have to be modified proportionately for each combination. The sequence of steps needed for the implementation of the proposed methodology has already been presented in previous studies (Manoukas et al. 2012).

EVALUATION STUDY

In the present study, the proposed procedure is implemented to a series of reinforced concrete buildings with various structural systems and plan configurations, in order to check its soundness and its accuracy for elastic systems. In particular, an extensive study is carried out comprising applications to five single-storey and five three-storey buildings which are considered to be representative of a large variety of common structures. Each building is characterized by a string symbol comprising one or two letters and a number (1 or 3) indicating the number of its storeys. The meaning of the letters is as follows:

- S – Symmetric in plan buildings with moment frame structural system (Fig. 1).
- SX – Symmetric in plan buildings (X axis of symmetry) with dual structural system (Fig. 2).
- SD – Buildings with diagonal symmetry and dual structural system (Fig. 3).
- AC – Asymmetric buildings with dual structural system comprising a core wall (Fig. 4).
- AW – Asymmetric buildings with dual structural system comprising planar walls (Fig. 5).

For example, the symbol SD3 corresponds to a three-storey building with the plan configuration shown in Fig. 3. All storey heights are 3 m. The concrete is of class C20/25 ($f_{ck} = 20$ MPa) and the reinforcement steel bars B500C ($f_{yk} = 500$ MPa) according to the Greek standards. The slab thickness

is equal to 15 cm. All columns are square shaped with dimension of 35 cm. All beams have a height of 60 cm and a thickness of 25 cm. Finally, the length of the walls is equal to 2 m or 3 m and their thickness equal to 25 cm.

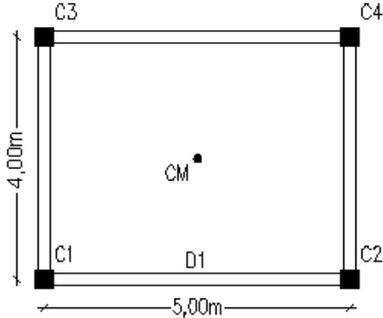


Figure 1. Floor plans of the symmetric buildings (S1 and S3)

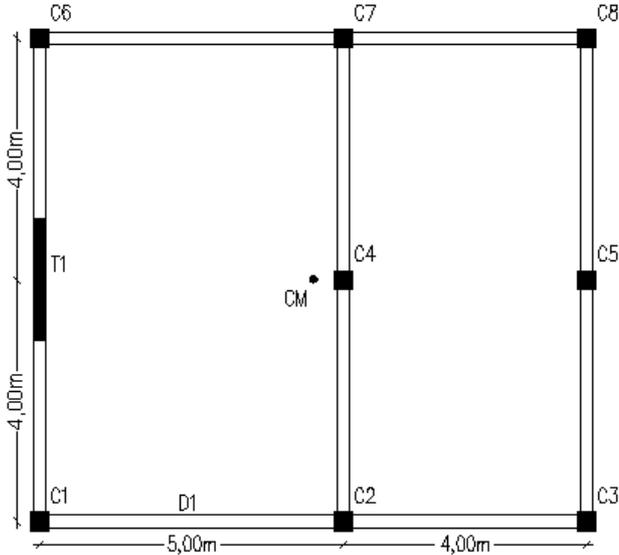


Figure 2. Floor plans of the one-way symmetric buildings (SX1 and SX3)

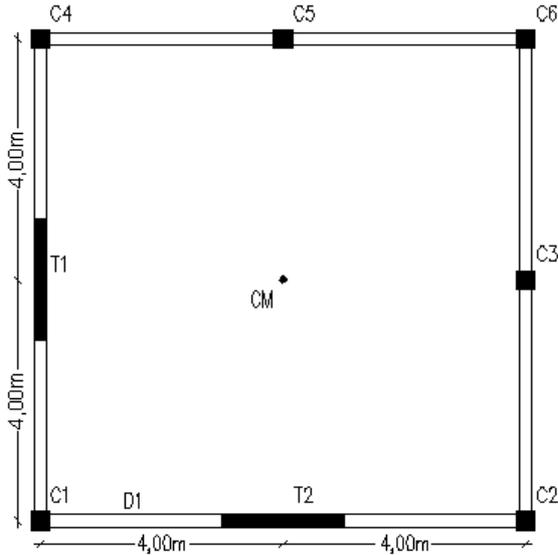


Figure 3. Floor plans of the buildings possessing diagonal symmetry (SD1 and SD3)

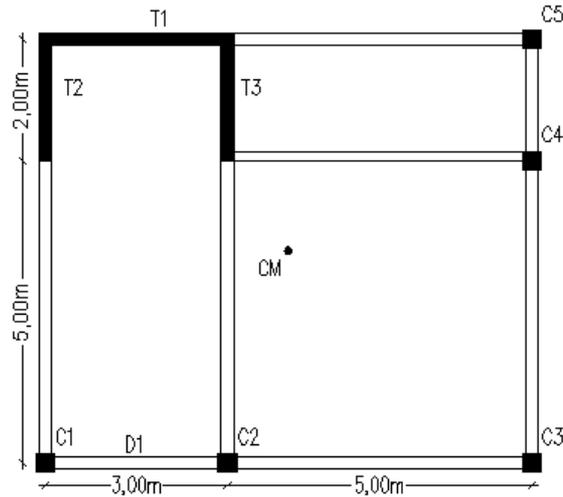


Figure 4. Floor plans of the asymmetric buildings with core wall (AC1 and AC3)

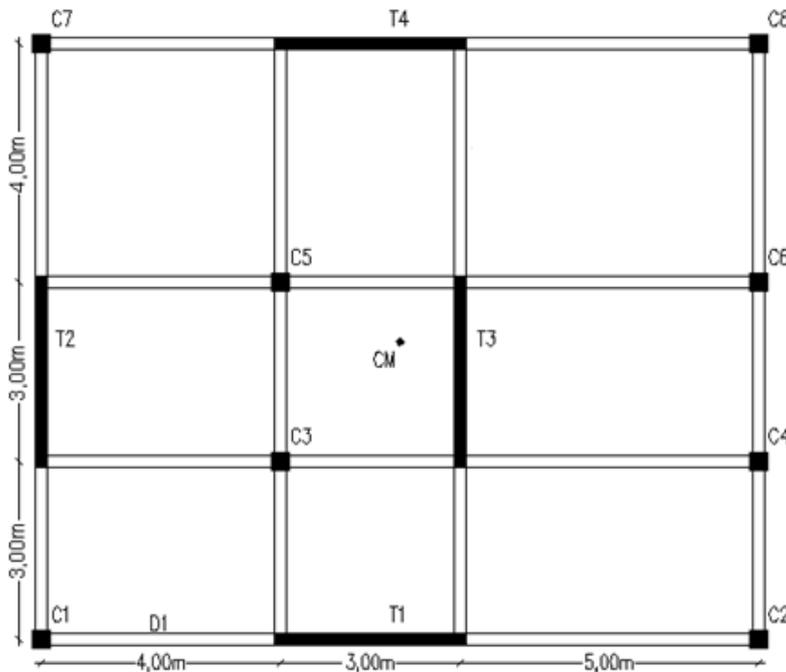


Figure 5. Floor plans of the asymmetric buildings with planar walls (AW1 and AW3)

The whole investigation conducted here comprises a number of 20 accelerograms, which is considered adequate to obtain concrete conclusions for the accuracy of the proposed method. These accelerograms are obtained from the PEER strong motion database (<http://peer.berkeley.edu/smcat/>) and tabulated in table C-3 of Appendix C of FEMA 440. It is considered that each ground motion acts simultaneously along the two horizontal axes of the buildings.

For each building three sets of analyses are performed using the program SAP2000 v10.0.7:

- Multimode pushover analysis according to the proposed methodology (PM). The value of κ is taken equal to 0.3 and as a consequence the possible combinations of the seismic components are the following four: $(\ddot{u}_{gX} + 0.3\ddot{u}_{gY})$, $(\ddot{u}_{gX} - 0.3\ddot{u}_{gY})$, $(\ddot{u}_{gY} + 0.3\ddot{u}_{gX})$ and $(\ddot{u}_{gY} - 0.3\ddot{u}_{gX})$.
- Response Spectrum Analysis (RSA). The directional combination of the response quantities is contacted using the percentage combination rule (ASCE 41-06, Section 3.2.7).
- Linear Dynamic Analysis (LDA). Each accelerogram is considered acting simultaneously along the two horizontal axes in the following combinations: $(\ddot{u}_{gX} + 0.3\ddot{u}_{gY})$, $(\ddot{u}_{gX} - 0.3\ddot{u}_{gY})$, $(-\ddot{u}_{gX} + 0.3\ddot{u}_{gY})$, $(-\ddot{u}_{gX} - 0.3\ddot{u}_{gY})$, $(\ddot{u}_{gY} + 0.3\ddot{u}_{gX})$, $(\ddot{u}_{gY} - 0.3\ddot{u}_{gX})$, $(-\ddot{u}_{gY} + 0.3\ddot{u}_{gX})$ and $(-\ddot{u}_{gY} - 0.3\ddot{u}_{gX})$.

In both PM and RSA all vibration modes are taken into account and the modal superposition is conducted by applying the CQC formula. In Table 2 the modal periods of the buildings are shown.

Table 2. Modal periods (sec) of the analyzed buildings

Building	T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇	T ₈	T ₉
S1	0.142	0.140	0.081	-	-	-	-	-	-
S3	0.379	0.367	0.210	0.122	0.119	0.074	0.073	0.069	0.042
SX1	0.198	0.148	0.055	-	-	-	-	-	-
SX3	0.529	0.381	0.183	0.167	0.125	0.095	0.078	0.049	0.025
SD1	0.161	0.084	0.044	-	-	-	-	-	-
SD3	0.421	0.278	0.149	0.133	0.082	0.074	0.039	0.037	0.019
AC1	0.123	0.049	0.026	-	-	-	-	-	-
AC3	0.344	0.179	0.106	0.078	0.060	0.044	0.024	0.023	0.013
AW1	0.060	0.055	0.037	-	-	-	-	-	-
AW3	0.210	0.203	0.139	0.054	0.050	0.033	0.028	0.025	0.017

For each building the displacements at the center of mass of the floors along X and Y axis, as well as the member forces of selected structural components are determined. In particular, the moment around the local axis 3 (horizontal) and the shear force along local axis 2 (vertical) of the selected beams are calculated. Concerning the columns and the walls, the moment around and the shear force along both local axes, as well as the axial force are calculated. The response values obtained by PM and RSA are compared to the results of LDA, which is considered as the reference solution. For each response parameter $R_{j,s}$ estimated by PM or RSA for an excitation j , the error with regard to the LDA results E_j is determined by the following relation:

$$E_j (\%) = 100 \frac{R_{j,s} - R_{j,d}}{R_{j,d}} \quad (11)$$

where $R_{j,d}$ is the value of the response parameter obtained by LDA. Furthermore, the mean error ME for the 20 excitations used in this study and the corresponding standard deviation SD are determined using Eqs. (12) and (13) respectively:

$$ME (\%) = \frac{1}{20} \sum_{j=1}^{20} E_j = 100 \frac{1}{20} \sum_{j=1}^{20} \left(\frac{R_{j,s} - R_{j,d}}{R_{j,d}} \right) \quad (12)$$

$$SD (\%) = \sqrt{\frac{1}{19} \sum_{j=1}^{20} (E_j - ME)^2} \quad (13)$$

In Tables 3 to 12 the mean errors for the 20 excitations (referring to the maximum values obtained by LDA) of the calculated response parameters are shown. For the calculation of these errors the maximum values for each applied procedure resulting from all possible combination of seismic components are considered. Notice that the positive sign (+) means that response parameters obtained by PM or RSA are greater than those obtained by LDA. Conversely, the negative sign (-) means that the response parameters are underestimated. It is apparent that the two compared procedures give similar error values ranging between -25% and 27.4% for PM and -13.9% and 27.9% for RSA.

As it has been demonstrated, PM as well as RSA are able to estimate accurately the seismic response of every single vibration mode separately. Thus, the estimations of response quantities that depend on only one mode (e.g. the displacements of CM of building S1, the axial force of the wall T1 of building SX1, etc.) are absolutely accurate (mean errors equal to 0.0).

On the other hand, for response quantities which depend on more than one modes, but on only one component of the seismic excitation (e.g. the displacements of CM of building S3, the axial force of the wall T1 of building SX3, etc.), PM and RSA lead to equal errors in almost all cases. Obviously, these errors result from the approximate nature of the modal superposition formulae. The slight differences between PM and RSA observed are due to:

- Numerical approximations

- The way applied the CQC rule for the modal superposition of the response quantities. In particular, when PM is applied the modal values of response quantities have no specified sign and the maximum extreme value results when all modal values have the same sign. On the contrary, when RSA is applied the sign of the modal values is specific. Thus, when there are at least two correlated modes with opposite signs, different extreme values occur. Obviously, when all modes are uncorrelated, i.e. the SRSS rule is applied, there is no difference between PM and RSA.

Concerning the response quantities which depend on more than one modes and on both components of the seismic excitation, PM and RSA lead to similar errors. The difference between PM and RSA is due to the different way of taking into account the multidirectional seismic effects.

Finally, concerning the standard deviation of the results, the values range between 0% and 24.2% for PM and 0% and 23.4% for RSA.

Table 3. Mean errors (%) of response quantities of building S1

Point or Component	Response Quantity		Mean Errors	
			PM	RSA
CM	Displacement	u_x	0.0	0.0
		u_y	0.0	0.0
D1	Moment	M_3	0.0	0.0
	Shear Force	V_2	0.0	0.0
C1	Moment	M_2	0.0	0.0
		M_3	0.0	0.0
	Shear Force	V_2	0.0	0.0
		V_3	0.0	0.0
	Axial Force	N	-0.3	0.1

Table 4. Mean errors (%) of response quantities of building S3

Floor	Point or Component	Response Quantity		Mean Errors	
				PM	RSA
1 st	CM	Displacement	u_x	-0.5	-0.4
			u_y	-0.6	-0.5
	D1	Moment	M_3	-0.2	-0.2
		Shear Force	V_2	-0.2	-0.2
	C1	Moment	M_2	-0.6	-0.6
			M_3	-0.6	-0.5
		Shear Force	V_2	-0.7	-0.6
			V_3	-0.7	-0.6
Axial Force	N	-0.3	1.1		
2 nd	CM	Displacement	u_x	-0.2	-0.2
			u_y	-0.2	-0.2
	D1	Moment	M_3	-0.1	-0.2
		Shear Force	V_2	-0.1	-0.2
	C1	Moment	M_2	0.5	0.4
			M_3	-0.1	-0.2
		Shear Force	V_2	0.0	0.0
Axial Force	N	0.3	1.7		
3 rd	CM	Displacement	u_x	0.0	0.0
			u_y	0.1	0.1
	D1	Moment	M_3	0.0	-0.1
		Shear Force	V_2	0.0	-0.1
	C1	Moment	M_2	2.3	2.1
			M_3	0.4	0.1
		Shear Force	V_2	0.2	0.0
			V_3	2.0	1.8
Axial Force	N	0.7	1.9		

Table 5. Mean errors (%) of response quantities of building SX1

Point or Component	Response Quantity		Mean Errors	
			PM	RSA
CM	Displacement	u_x	0.0	0.0
		u_y	-1.6	-1.6
D1	Moment	M_3	-1.8	4.8
	Shear Force	V_2	-1.8	4.8
C1	Moment	M_2	-14.4	-13.5
		M_3	-1.8	4.9
	Shear Force	V_2	-1.9	5.0
		V_3	-14.5	-13.3
	Axial Force	N	-5.0	7.1
T1	Moment	M_2	-14.3	-13.9
		M_3	0.0	0.0
	Shear Force	V_2	0.0	0.0
		V_3	-14.3	-13.9
	Axial Force	N	0.0	0.0

Table 6. Mean errors (%) of response quantities of building SX3

Floor	Point or Component	Response Quantity		Mean Errors	
				PM	RSA
1 st	CM	Displacement	u_x	6.0	6.1
			u_y	6.3	6.5
	D1	Moment	M_3	-0.9	8.7
		Shear Force	V_2	-0.9	8.8
	C1	Moment	M_2	-3.0	-1.5
			M_3	0.4	11.2
		Shear Force	V_2	0.4	11.5
			V_3	-3.2	-1.3
		Axial Force	N	-3.0	13.4
	T1	Moment	M_2	-3.1	-2.3
			M_3	3.0	3.2
		Shear Force	V_2	3.1	3.4
			V_3	-5.2	-4.4
Axial Force	N	-1.6	-1.7		
2 nd	CM	Displacement	u_x	3.2	3.2
			u_y	6.9	7.1
	D1	Moment	M_3	-4.0	3.6
		Shear Force	V_2	-4.1	3.6
	C1	Moment	M_2	-1.3	1.1
			M_3	-4.3	3.6
		Shear Force	V_2	-3.6	4.8
			V_3	-1.3	1.2
		Axial Force	N	-2.4	12.6
	T1	Moment	M_2	2.1	2.2
			M_3	-3.7	-2.6
		Shear Force	V_2	-6.9	-6.7
			V_3	-0.3	0.4
Axial Force	N	-1.8	-1.8		
3 rd	CM	Displacement	u_x	1.8	1.8
			u_y	7.5	7.6
	D1	Moment	M_3	-4.3	2.1
		Shear Force	V_2	-4.3	2.1
	C1	Moment	M_2	-0.3	3.2
			M_3	-3.3	2.7
		Shear Force	V_2	-3.9	2.4
			V_3	0.2	3.3
		Axial Force	N	-1.5	10.9
	T1	Moment	M_2	20.5	16.6
			M_3	3.5	3.4
		Shear Force	V_2	4.8	4.7
			V_3	5.6	4.5
Axial Force	N	-1.9	-2.0		

Table 7. Mean errors (%) of response quantities of building SD1

Point or Component	Response Quantity		Mean Errors	
			PM	RSA
CM	Displacement	u_x	-4.6	-2.3
		u_y	-4.6	-2.3
D1	Moment	M_3	-11.9	-3.9
	Shear Force	V_2	-11.7	-4.4
C1	Moment	M_2	-11.3	-5.1
		M_3	-11.3	-5.1
	Shear Force	V_2	-11.3	-5.1
		V_3	-11.3	-5.1
	Axial Force	N	0.0	0.0
T1	Moment	M_2	-11.3	-4.9
		M_3	-4.6	-2.3
	Shear Force	V_2	-4.7	-2.3
		V_3	-11.3	-4.9
	Axial Force	N	-8.7	-3.3

Table 8. Mean errors (%) of response quantities of building SD3

Floor	Point or Component	Response Quantity		Mean Errors	
				PM	RSA
1 st	CM	Displacement	u_x	-7.7	2.0
			u_y	-7.7	2.0
	D1	Moment	M_3	-6.2	-0.4
		Shear Force	V_2	-6.2	-0.3
	C1	Moment	M_2	-6.6	-1.7
			M_3	-6.6	-1.7
		Shear Force	V_2	-6.7	-1.7
			V_3	-6.7	-1.7
		Axial Force	N	0.3	0.3
	T1	Moment	M_2	-6.6	-2.0
			M_3	-5.9	1.7
		Shear Force	V_2	-2.6	1.2
			V_3	-7.8	-3.5
Axial Force	N	-8.7	6.3		
2 nd	CM	Displacement	u_x	-9.8	2.4
			u_y	-9.8	2.4
	D1	Moment	M_3	-5.0	1.5
		Shear Force	V_2	-5.0	1.5
	C1	Moment	M_2	-5.4	0.3
			M_3	-5.4	0.3
		Shear Force	V_2	-5.4	0.4
			V_3	-5.4	0.4
		Axial Force	N	0.8	0.8
	T1	Moment	M_2	-2.8	2.5
			M_3	10.3	5.0
		Shear Force	V_2	-2.9	10.2
			V_3	-4.6	0.5
Axial Force	N	-8.6	6.2		
3 rd	CM	Displacement	u_x	-10.7	3.4
			u_y	-10.7	3.4
	D1	Moment	M_3	-4.6	2.6
		Shear Force	V_2	-4.6	2.6
	C1	Moment	M_2	-4.4	2.4
			M_3	-4.4	2.4
		Shear Force	V_2	-4.4	2.4
			V_3	-4.4	2.4
		Axial Force	N	1.3	1.2
	T1	Moment	M_2	14.6	19.1
			M_3	0.4	-0.7
		Shear Force	V_2	4.8	0.4
			V_3	-0.7	4.3
Axial Force	N	-8.5	6.1		

Table 9. Mean errors (%) of response quantities of building AC1

Point or Component	Response Quantity		Mean Errors	
			PM	RSA
CM	Displacement	u_x	-1.7	-1.5
		u_y	2.7	3.9
D1	Moment	M_3	-0.4	-0.3
	Shear Force	V_2	-0.4	-0.3
C1	Moment	M_2	-0.8	5.1
		M_3	-0.4	-0.3
	Shear Force	V_2	-0.4	-0.3
		V_3	-1.0	4.7
	Axial Force	N	0.3	0.4
T1	Moment	M_2	-7.0	-1.3
		M_3	-13.7	-9.8
	Shear Force	V_2	-25.0	-13.0
		V_3	-7.1	-1.3
	Axial Force	N	-6.7	-1.5

Table 10. Mean errors (%) of response quantities of building AC3

Floor	Point or Component	Response Quantity		Mean Errors	
				PM	RSA
1 st	CM	Displacement	u_x	-1.4	-0.4
			u_y	-2.9	5.1
	D1	Moment	M_3	-0.2	0.5
		Shear Force	V_2	-0.2	0.5
	C1	Moment	M_2	-6.0	8.7
			M_3	-1.5	-1.0
		Shear Force	V_2	-1.7	-1.3
			V_3	-6.1	8.7
		Axial Force	N	0.1	1.3
	T1	Moment	M_2	-4.7	-3.1
			M_3	-7.4	3.0
		Shear Force	V_2	-15.8	-1.1
			V_3	-8.6	-6.5
Axial Force	N	-0.2	0.7		
2 nd	CM	Displacement	u_x	-0.2	1.2
			u_y	-3.6	6.6
	D1	Moment	M_3	1.9	3.4
		Shear Force	V_2	1.9	3.5
	C1	Moment	M_2	-0.5	10.0
			M_3	1.2	2.5
		Shear Force	V_2	1.0	2.1
			V_3	-1.0	10.1
		Axial Force	N	0.9	2.9
	T1	Moment	M_2	11.9	12.2
			M_3	-3.7	9.8
		Shear Force	V_2	-8.4	6.4
V_3			9.6	10.2	
Axial Force	N	4.8	5.3		
3 rd	CM	Displacement	u_x	0.6	2.5
			u_y	-4.3	8.1
	D1	Moment	M_3	2.7	4.9
		Shear Force	V_2	2.7	5.0
	C1	Moment	M_2	3.0	7.8
			M_3	2.8	5.5
		Shear Force	V_2	2.8	5.2
			V_3	2.9	7.9
		Axial Force	N	0.6	6.8
	T1	Moment	M_2	27.4	27.9
			M_3	8.9	19.2
		Shear Force	V_2	1.4	14.8
			V_3	21.4	21.7
Axial Force	N	12.6	12.7		

Table 11. Mean errors (%) of response quantities of building AW1

Point or Component	Response Quantity		Mean Errors	
			PM	RSA
CM	Displacement	u_x	2.0	1.4
		u_y	-2.0	-2.6
D1	Moment	M_3	0.7	4.7
	Shear Force	V_2	0.6	4.7
C1	Moment	M_2	-20.4	-19.8
		M_3	0.5	4.7
	Shear Force	V_2	0.6	4.7
		V_3	-20.5	-19.9
	Axial Force	N	-16.9	-10.0
T3	Moment	M_2	1.2	0.7
		M_3	0.3	-0.4
	Shear Force	V_2	0.9	0.1
		V_3	1.2	0.8
	Axial Force	N	-1.7	2.3

Table 12. Mean errors (%) of response quantities of building AW3

Floor	Point or Component	Response Quantity		Mean Errors	
				PM	RSA
1 st	CM	Displacement	u_x	-3.1	1.6
			u_y	2.6	1.1
	D1	Moment	M_3	-1.0	0.8
		Shear Force	V_2	-1.0	0.9
	C1	Moment	M_2	-8.8	-4.2
			M_3	-2.4	-0.9
		Shear Force	V_2	-2.5	-1.0
			V_3	-9.2	-4.4
		Axial Force	N	-0.2	-1.5
	T3	Moment	M_2	-3.5	1.3
			M_3	3.3	1.6
		Shear Force	V_2	1.0	-0.4
			V_3	-3.6	1.1
Axial Force	N	0.3	0.4		
2 nd	CM	Displacement	u_x	-1.8	2.9
			u_y	4.0	2.5
	D1	Moment	M_3	0.6	2.8
		Shear Force	V_2	0.6	2.8
	C1	Moment	M_2	-7.2	-2.1
			M_3	0.0	1.9
		Shear Force	V_2	-0.1	1.9
			V_3	-7.4	-2.2
		Axial Force	N	1.4	-0.9
	T3	Moment	M_2	-1.1	3.9
			M_3	8.9	7.2
		Shear Force	V_2	7.1	4.9
			V_3	-1.1	3.8
Axial Force	N	0.7	1.0		
3 rd	CM	Displacement	u_x	-1.0	3.6
			u_y	4.7	3.2
	D1	Moment	M_3	1.3	3.7
		Shear Force	V_2	1.2	2.8
	C1	Moment	M_2	-6.6	-1.3
			M_3	1.3	3.8
		Shear Force	V_2	1.3	3.7
			V_3	-6.6	-1.4
		Axial Force	N	2.9	-0.5
	T3	Moment	M_2	0.3	5.2
			M_3	19.9	24.6
		Shear Force	V_2	11.8	11.0
			V_3	0.2	5.2
Axial Force	N	1.0	1.4		

CONCLUSIONS

A recently developed multimode pushover procedure for the approximate estimation of the seismic response of asymmetric buildings under biaxial excitation is evaluated for linear elastic systems. The main idea of the procedure is that the seismic response of an asymmetric building under biaxial excitation can be related to the responses of a series of “modal” E-SDOF systems under uniaxial excitation. The whole procedure is quite similar to the well-known MPA. However, the establishment of the E-SDOF systems is based on an essentially different concept. Also, the proposed methodology does not require independent analysis in each direction of excitation, so application of simplified directional combination rules is avoided. From the presentation and the evaluation study conducted the following conclusions are derived:

- The proposed methodology is able to estimate accurately the linear seismic response of every single vibration mode separately.
 - For response quantities which depend on a single mode, PM, just like RSA, leads to accurate results.
 - For response quantities which depend on more than one modes, but on only one component of the seismic excitation, PM and RSA lead to equal errors (except the case described in the previous paragraph). These errors result from the approximate nature of the modal superposition formulae.
 - For response quantities which depend on more than one modes and on both components of the seismic excitation, PM and RSA lead to similar errors. The difference between PM and RSA is due to the different way of taking into account the multidirectional seismic effects.
- Conclusively, it is demonstrated that the proposed procedure provides a reasonable estimation of the seismic performance of elastic systems.

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