SECOND EUROPEAN CONFERENCE ON EARTHQUAKE ENGINEERING
AND SEISMOLOGY, ISTANBUL AUG. 25-29, 2014

AN EXPERIMENTAL INVESTIGATION OF A MULTIPLE-TUNED-MASS-DAMPER-STRUCTURE SYSTEM WITH SOIL-STRUCTURE INTERACTION

Raz JABARY[1] and Gopal MADABHUSHI[2]

ABSTRACT

A tuned mass damper (TMD) is a popular vibration control device commonly used in buildings to resist wind and seismic loading. This is achieved through tuning the natural frequency of the damper to that of the building in the case of fixed-base conditions, or to that of a soil-structure system when soil-structure interaction (SSI) plays a significant role. The conventional design of a TMD is based on the control and reduction of the largest modal structural response which may cause the most damage to the structure in which it is to be installed (Ghosh & Basu, 2004). A single-tuned-mass-damper (STMD) can be used to reduce the peak accelerations and displacements of a multiple-degrees-of-freedom (MDOF) structure. However, with the aim of reducing the likelihood of de-tuning from occurring through distributing the natural frequencies of several TMDs around the pre-dominant modal frequency of the structure, the use of multiple-tuned-mass-dampers (MTMDs) is recommended. Through the conduct of small-scale shaking table tests on a MDOF sway frame structure overlying a level sand bed of high relative density, this study aims to investigate the performance of a range of MTMD configurations in the context of a MDOF soil-structure-MTMD system with higher modes. This paper demonstrates the potential effectiveness of tuning MTMDs to higher mode frequencies and shows that when tuning MTMDs to the frequency of the dominant mode of excitation and placing these on consecutive storeys: (i) they can be more effective in attenuating a soil-structure system's response than when using a greater number of MTMDs, and (ii) they result in significantly more damping than when placing the same number of MTMDs on non-adjacent storeys.

KEYWORDS: tuned mass damper, multiple-tuned-mass-damper, sway frame structure, seismic loading, dynamic soil-structure interaction

INTRODUCTION

A popular method of mitigating risks from earthquakes to structures and limiting inconvenience to its occupants is the use of vibration resisting devices. One such device which has been widely installed in many structures around the world to reduce their response to seismic loading is the passive tuned mass damper (TMD), which comes in many shapes and sizes and reacts solely in response to the motion of the storey to which it is installed (i.e. it is not externally driven). It consists of three components which are a spring, a damper and a mass directly attached to the spring and damper. TMDs operate through dissipation of the vibrational energy induced to a structure in which they are fitted. This is accomplished through the combined action of inertial dissipation and damping (Liu et al., 2008). A

---

[1] Ph.D. Student, University of Cambridge, Cambridge, United Kingdom, rnj22@cam.ac.uk
[2] Professor of Civil Engineering, University of Cambridge, Cambridge, United Kingdom, mspg1@cam.ac.uk

TMD operates at its best when it is 'tuned'. This is when its natural frequency is set to be equal to the most pre-dominant modal frequency of the structure (in the event of fixed-base conditions) or to that of the soil-structure system (in the event of underlying soil), given that in the latter case a modification in response takes place as a result of soil-structure interaction (Jabary & Madabhushi, 2014).

Though identical to a TMD, throughout this study the term multiple-tuned-mass-damper (MTMD) is used to emphasise that a tuned mass damper is installed along with one or more tuned mass dampers within the same structure. Fig. 1 below shows a schematic illustration of the MDOF MTMD-structure system that was tested for this study. The installation of MTMDs can be used to address one of the following two purposes: (i) the control of multiple structural modes through tuning different TMDs to different frequencies, and (ii) distributing the natural frequencies of the MTMDs around the pre-dominant modal frequency of the system to alleviate potential system de-tuning effects from occurring (Lee et al., 2005).

Zuo and Nayfeh (2005) conducted numerical analyses on a MDOF structure fitted with MTMDs and arrived at the conclusion that an increase in the number of TMDs used in a structure to control the dominant mode of excitation generally yields better robustness and is associated with less likelihood of de-tuning. However, Rana and Soong (1998) performed numerical studies on a 3-d.o.f. fixed-base model structure fitted with TMDs and found that the use of a MTMD system to control multiple modes of a structure does not lead to significant response reduction in addition to what can already be achieved with the use of a single-tuned-mass-damper (STMD) tuned to the frequency of the dominant mode. In line with the findings from Rana and Soong (1998) this study focuses on the use of a MTMD system to control one modal frequency at a given time.

Past studies into MTMDs overwhelmingly focused on the development of analytical expressions for the optimisation of the TMD parameters mass, stiffness and damping with the aim of reducing one or more structural response parameters, typically specified by means of a transfer function (Rana & Soong, 1998; Zuo & Nayfeh, 2005). Occasional parametric verifications of such analytical expressions have made reference to very specifically defined model structures with a limited amount of variables in structural and soil properties. However, often in such studies numerous unrealistic assumptions were made to arrive at simplified forms of analytical expressions. In order to overcome the limitations of such studies in understanding the performance of a wide range of MTMD configurations fitted into a soil-structure system, there is a need for experimental testing.

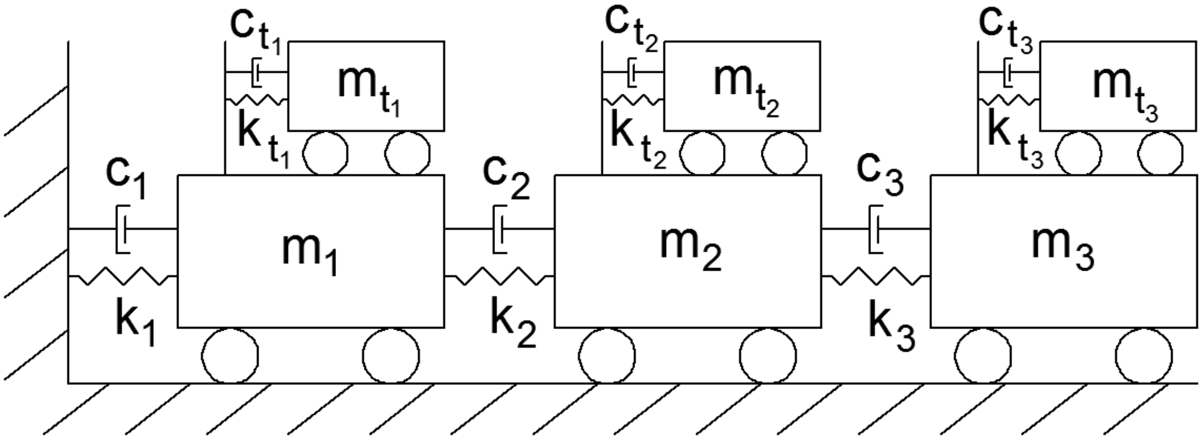


Figure 1. A schematic illustration of a MDOF structure fitted with MTMDs

In Fig. 1 above m_i, c_i and k_i are the mass, damping coefficient and stiffness of the i^{th} storey of the frame, and m_{t_i}, c_{t_i} and k_{t_i} are the mass, damping coefficient and stiffness of the TMD installed on the i^{th} storey of the frame. In the small-scale model used as part of this study and shown in Fig. 2-4 the values of m_1, m_2 and m_3 are 1.75 kg each; m_{t_1}, m_{t_2} and m_{t_3} are 0.15 kg each; k_1 , k_2 and k_3 are 707 N/m each, and k_{t_1}, k_{t_2} and k_{t_3} vary depending on the frequency configuration used and range from 40 N/m to 801 N/m as part of this study. $EI = 0.1677\ Nm^2$ for each storey of the structure and $EI = 0.0522\ Nm^2$ for each MTMD studding.

SHAKING TABLE TESTING

Experimental model tests of geotechnical structures can be divided into two categories, namely those performed under Earth’s gravity (shaking table tests) and those performed under higher gravitational environments (centrifuge tests). Each of the two methods has its own advantages that could be used to justify its application in research. Model testing under 1g conditions has become an integral part in geotechnical earthquake engineering research that helps understand the behaviour of geotechnical structures and their performance during earthquakes (Prasad et al., 2004). A shaking table (shown in Fig. 2) provides the opportunity to conduct experiments at 1g and take measurements at relative ease and little cost. The shaking table operates through the conversion of rotational motion generated by an electric motor into simple near-sinusoidal motion via a crank. The shaking table features used to control the input motion are easily adjustable and the model structure under testing is easily accessible for the purpose of instrumentation and modification of system properties. This was particularly of relevance to the study conducted as part of this research and presented within this paper, for which a wide range of MTMD configurations was installed in a sway frame structure and tested.

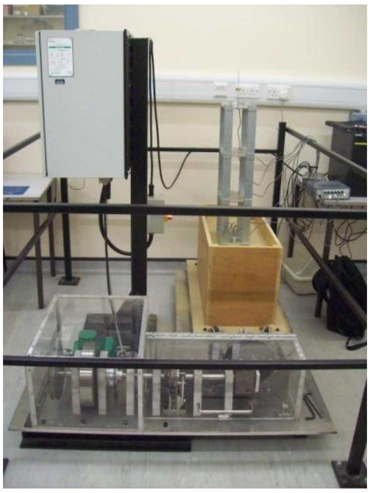


Figure 2. A shaking table

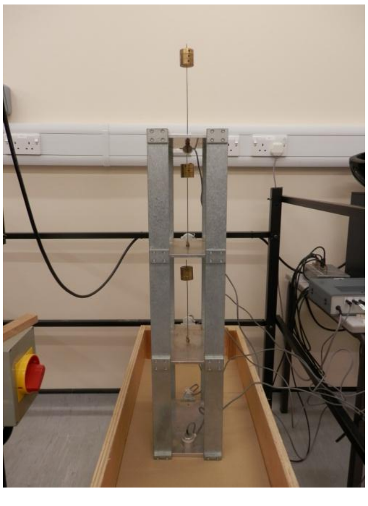


Figure 3. A soil-structure-MTMD system

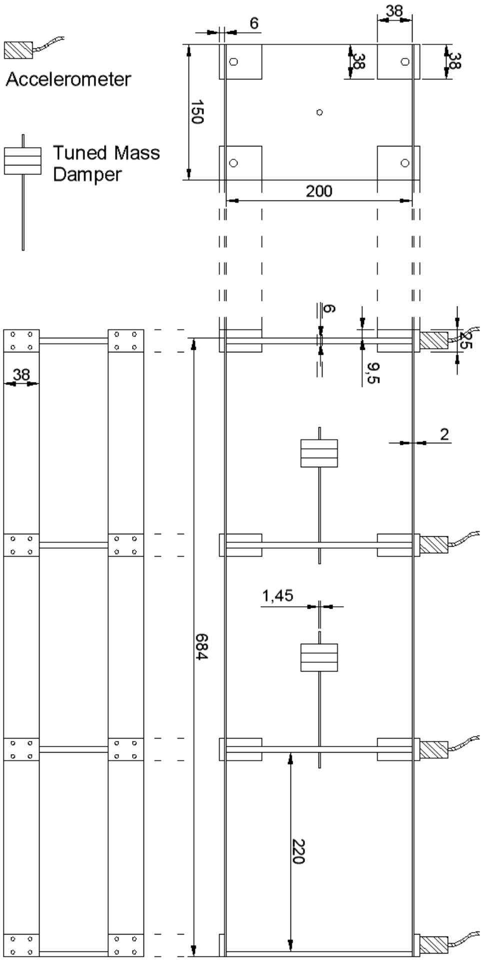


Figure 4. A schematic illustration of a 3-d.o.f. sway frame model structure (all dimensions in mm). The MTMD configuration shown is arbitrary.

As part of this study a 3-d.o.f. sway frame model structure exerting linear-elastic behaviour was rested on a flat uniform dry sand bed and fitted with a range of MTMD configurations. The sand used was silica sand fraction E with properties $G_s = 2.65$ and $D_{50} = 0.14\,mm$ (Tan, 1990), at $D_r = 0.9$ and with a total depth of 300 mm. The soil-structure system fitted with an arbitrary MTMD configuration is shown in Fig. 3. Every storey of the 3-d.o.f. structure with the inclusion of the base was fitted with an accelerometer that was aligned with the horizontal direction of movement by the shaking table. A full schematic illustration of the structural model showing its dimensions and its instrumentation is shown in Fig. 4. Full joint fixity at the connections between the storeys and the walls remained in place throughout testing so as to minimise structural damping.

Attaching two or more TMDs to the sway frame structure rested on a dry sand bed creates a soil-structure-MTMD system. Various MTMD configurations were installed and their effects on the system response was computed using a combination of Fast Fourier Transforms (FFTs), acceleration-time histories and amplification factor plots. FFTs transform data from the time domain into the frequency domain. The amplification factor is defined here as the ratio of Fourier Transform amplitude at the shaking frequency of a particular storey of the structure to the Fourier Transform amplitude at the shaking frequency of the base of the structure. Different MTMD configurations were achieved according to the following three alterations: (i) varying the number of TMDs used, (ii) setting the frequency to which the TMDs are tuned, and (iii) changing the positioning of the TMDs on the storeys. The latter of the three adjustments was only of relevance when two TMDs were considered.

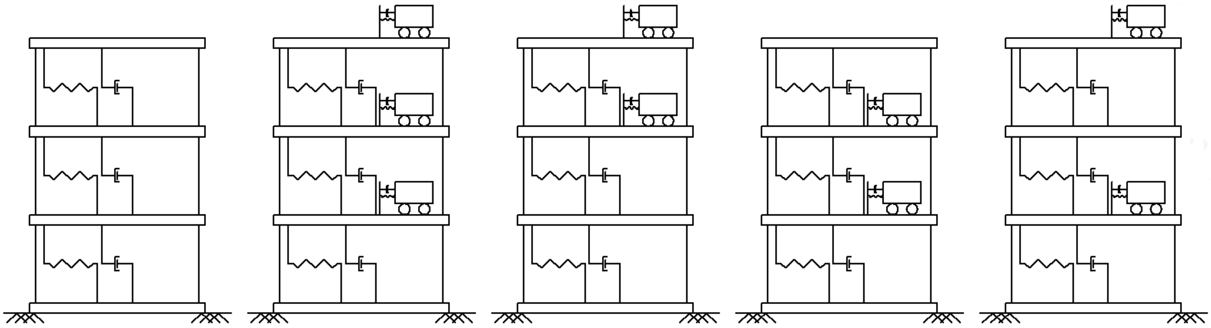
By means of amplification factor plots obtained through experimental investigation, the first, second and third-mode frequencies of the soil-structure system were determined to be 2.6 Hz, 8.0 Hz and 11.6 Hz respectively.

The 1g shaking table is limited in the sense that it can only carry out shakes in one lateral horizontal direction with an excitation frequency range of 0-5 Hz, meaning that a resonance condition between the input frequency and the soil-structure-system's natural frequency can only be replicated for the first-mode frequency of the system. However, given that this limited excitation frequency range of the shaking table resembles a practical earthquake excitation range, this ought not be much of a concern in the consideration for the use of the shaking table. Although the shaking table input amplitude can be varied, a small constant amplitude of 1mm was considered throughout all tests to prevent the narrow TMD studdings from deflecting. The use of the relatively large TMD mass and the positioning of that mass far up on the TMD studdings in order to achieve tuning conditions namely makes the TMD studdings very susceptible to plastic deformation if violent shaking were to be applied.

Shakes were consistently carried out for 14 s in duration and data was logged at a sampling frequency of $f_s = 1000\ Hz$.

RESULTS

Fig. 5(a)-(e) below show the range of MTMD-structure configurations that were tested using the 3-d.o.f. sway frame structure specified in Fig. 4. These are all the MTMD configurations that are possible (MTMD positioning-wise that is, given that there is an unlimited range of configurations possible in light of the natural frequencies of MTMDs that could be set).



(a) (b) (c) (d) (e)

Figure 5. Soil-structure-MTMD configurations: (a) the isolated soil-structure system with no external damping devices (b) MTMDs on all storeys (c) MTMDs on the upper two storeys (d) MTMDs on the bottom two storeys (e) MTMDs on the first and third storeys

Fig. 5(a) shows the soil-structure system which is isolated from any external damping devices (no MTMDs). This configuration shall be referred to throughout this paper as the isolated soil-structure system.

The performance of MTMD configurations is evaluated using amplification factors, defined here as the ratio of Fourier Transform amplitude (Fourier Component) at the shaking frequency of a particular storey of the structure to the Fourier Component at the shaking frequency of the base of the structure.

Fig. 6(a)-(c) below show the amplification factors associated with the range of MTMD configurations (consisting of either two or three TMDs) with a variation in their positioning on the different storeys of the sway frame structure. In Fig. 6(a)-(c) all the TMDs making up the MTMD configurations shown in Fig. 5(b)-(e) are tuned to the first-mode system frequency of 2.6 Hz.

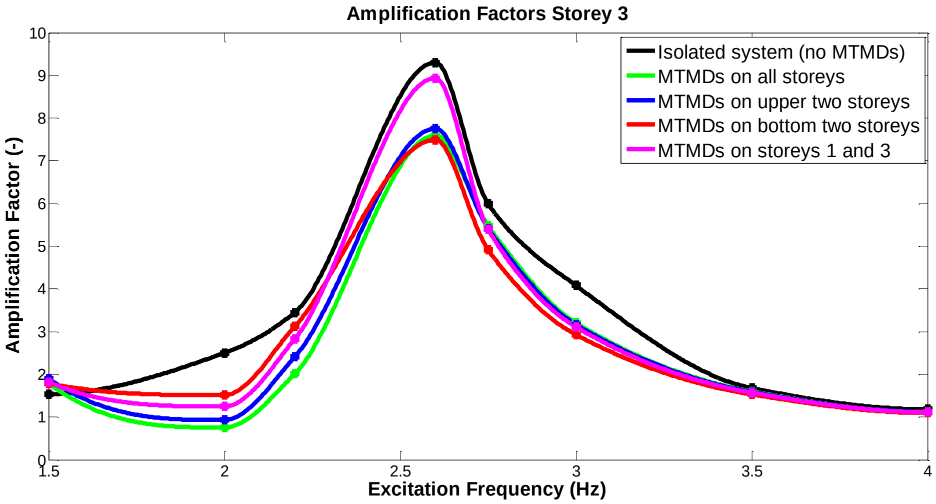


Figure 6(a). Amplification factors of MTMD configurations tuned to the first-mode system frequency of 2.6 Hz for the third storey

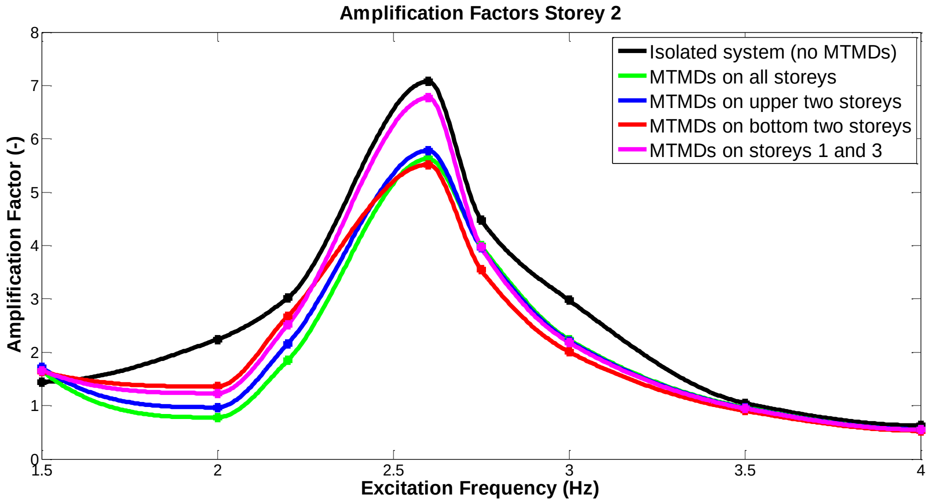


Figure 6(b). Amplification factors of MTMD configurations tuned to the first-mode system frequency of 2.6 Hz for the second storey

As expected, given that all MTMDs are tuned to the isolated soil-structure-system's first-mode frequency of 2.6 Hz, no modification of the system's natural frequency has taken place and the peak amplification factors shown in Fig. 6(a)-(c) correspond to a frequency of 2.6 Hz.

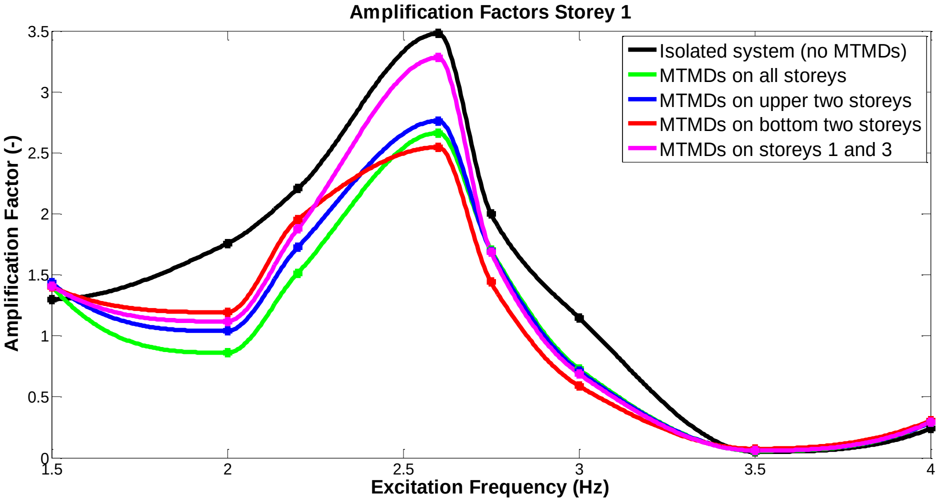


Figure 6(c). Amplification factors of MTMD configurations tuned to the first-mode system frequency of 2.6 Hz for the first storey

Analysis of the storeys' amplification factors results in the following findings, listed below in the order of significance:

1) For the aim of attenuating the isolated system response, it is more important to position multiple TMDs on consecutive storeys than using a greater number of TMDs. Regardless of which of the three storey responses is considered, the same trend of peak amplification factors holds. It is found that positioning MTMDs on consecutive storeys of the structure results in significantly more system damping than when placing MTMDs on the first and third storeys of the structure and leaving the middle storey isolated. On the other hand, there is not much variation in amplification factor peaks associated with the case for which all three storeys of the structure are fitted with MTMDs and the two cases for which two MTMDs are positioned on consecutive storeys.

2) In this particular case the use of MTMDs on the bottom two storeys of the structure is seen to result in the greatest attenuation of system response, even more so than placing an additional MTMD on the upper storey. Therefore, the addition of a third MTMD on the upper storey in this case results in a reduction of the overall MTMD damping effect. This result contradicts the findings of numerical analyses conducted by Zuo and Nayfeh (2005), who concluded that an increase in the number of TMDs in a structure to control the dominant mode of excitation yields better robustness. Nevertheless, the case considering three MTMDs still shows better performance than the configuration in which MTMDs are installed in the upper two storeys of the frame. In conclusion, placement of a MTMD on the first and on the second storey is of much greater significance than placement of a MTMD on the third storey of the frame.

3) All MTMD combinations in the soil-structure-MTMD system considered with each of the TMDs tuned to the fundamental soil-structure-system frequency of 2.6 Hz result in smaller amplification factors in comparison to the responses of the isolated soil-structure-system. This is true for all storeys.

4) Absolute differences between the amplification factor peaks of the different MTMD configurations increase as one moves up the storeys of the structure. However, the percentage differences between the peaks diminish as one moves up the structure. This holds true for any two comparable amplification factor peaks and can be explained by the fact that absolute differences in peaks between different storeys are significantly greater than those between configurations for the same storey.

In addition to the findings above which are clear from direct observation of Fig. 6(a)-(c), comparison of the amplification factor peaks with those obtained by Jabary and Madabhushi (2013) in which the structure shown in Fig. 4 was fixed to a base plate highlights the significant role played by the soil in damping the structural response. Despite the fact that the dry silica sand fraction E considered in this study is of very high relative density ($D_r = 0.9$), amplification factor peak magnitudes observed in

Fig. 6(a)-(c) are significantly lower than those found by Jabary and Madabhushi (2013) for a fixed-base case.

Following on from Fig. 6 from which it was evident that the configuration involving MTMDs on the two bottom storeys of the structure was the most effective in reducing peak amplification factors, the extent of damping is illustrated in the acceleration-time histories shown in Fig. 7 below.

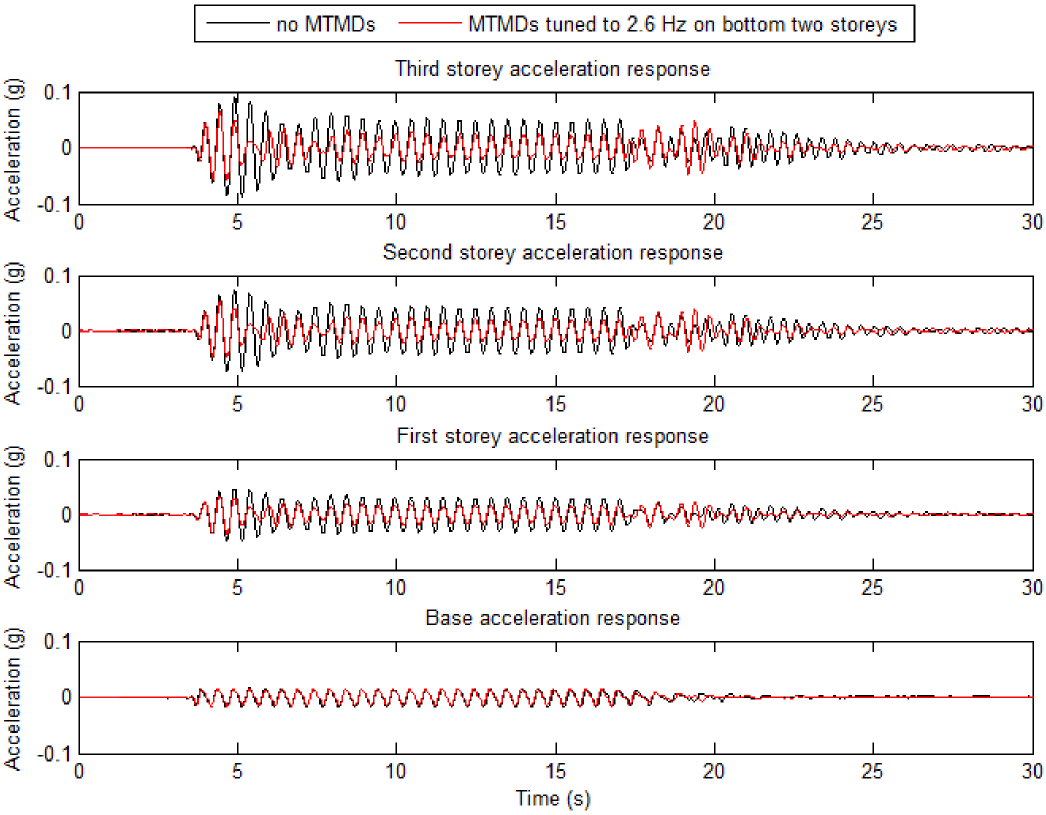


Figure 7. Acceleration-time histories for MTMD-damped and undamped cases

The acceleration-time histories in Fig. 7 were obtained for an excitation frequency of 2.0 Hz and a shake duration of 14 s (between 3.5 s and 17.5 s into the records shown above). In Fig. 7:

(i) The base acceleration response remains unaffected by the installation of MTMDs

(ii) Acceleration magnitudes are consistently damped throughout the duration of shaking. However, as soon as shaking stops at 17.5 s into the records shown, the MTMDs cause slight amplification to the isolated soil-structure system response. Nevertheless, the peak acceleration magnitudes which occur early on in the records are effectively damped.

Corresponding FFTs were obtained and are shown in Fig. 8. The sampling frequency at which data was logged was $f_s = 1000\ Hz$. As noted earlier and is evident from the peak Fourier Components, the first, second and third-mode frequencies of the soil-structure system were experimentally determined to be 2.6 Hz, 8.0 Hz and 11.6 Hz respectively. In addition, a significant peak Fourier Component is consistently shown at the applied excitation frequency of the shaking table, which in this case is 2.0 Hz. Two important observations can be made from Fig. 8:

(I) For the case of the isolated soil-structure system response (no MTMDs) the Fourier Component observed at the second-mode frequency of 8.0 Hz towers above those observed at the first- and third-mode system frequencies of 2.6 Hz and 11.6 Hz respectively. This is true for all storeys.

(II) The installation of MTMDs tuned to the first-mode frequency of the soil-structure system diminishes the effects of the second and third-mode responses.

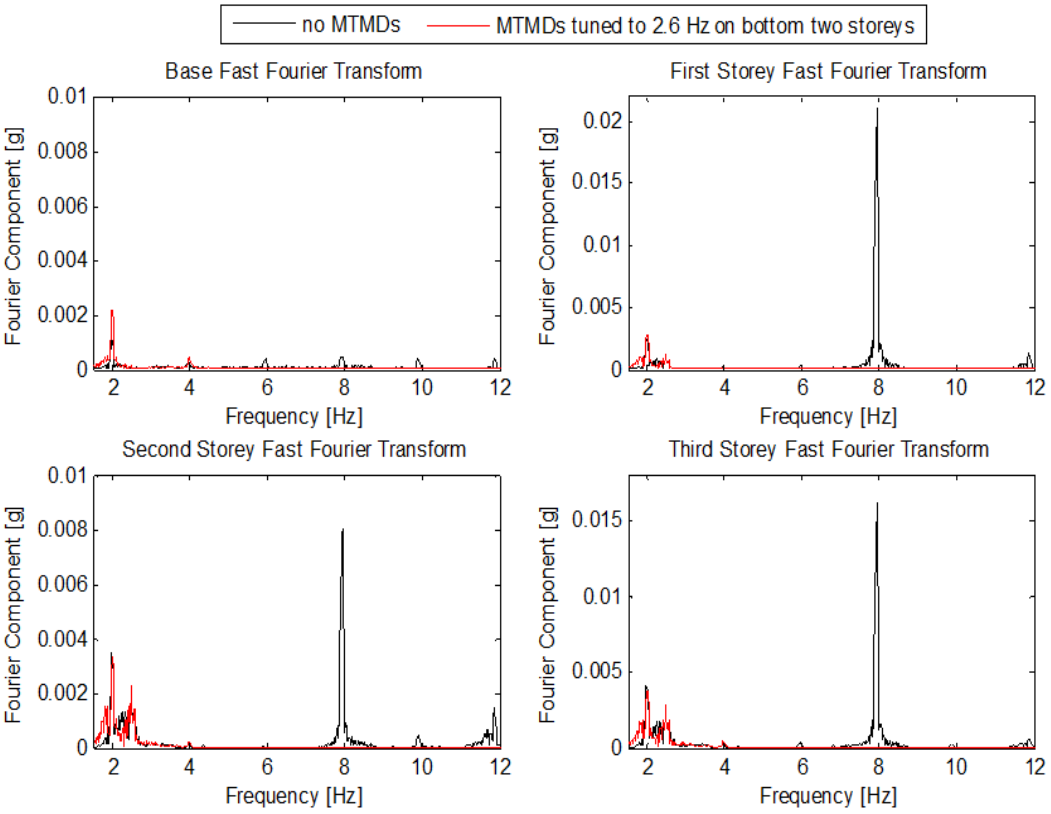


Figure 8. Fast Fourier Transforms for MTMD-damped and undamped cases

The FFTs in Fig. 8 illustrate the significance of the installation of a MTMD system tuned to the first-mode system frequency on higher mode effects. The finding described in (I) above prompts the further investigation of higher mode effects. Fig. 9(a)-(c) below show the amplification factors associated with the isolated system response and the range of MTMD configurations tuned to the second-mode frequency of 8.0 Hz.

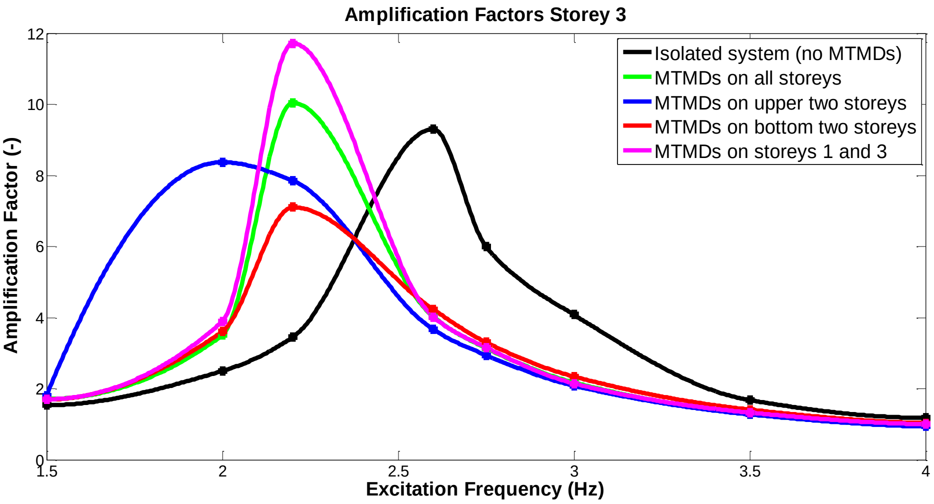


Figure 9(a). Amplification factors of MTMD configurations tuned to the second-mode system frequency of 8.0 Hz for the third storey

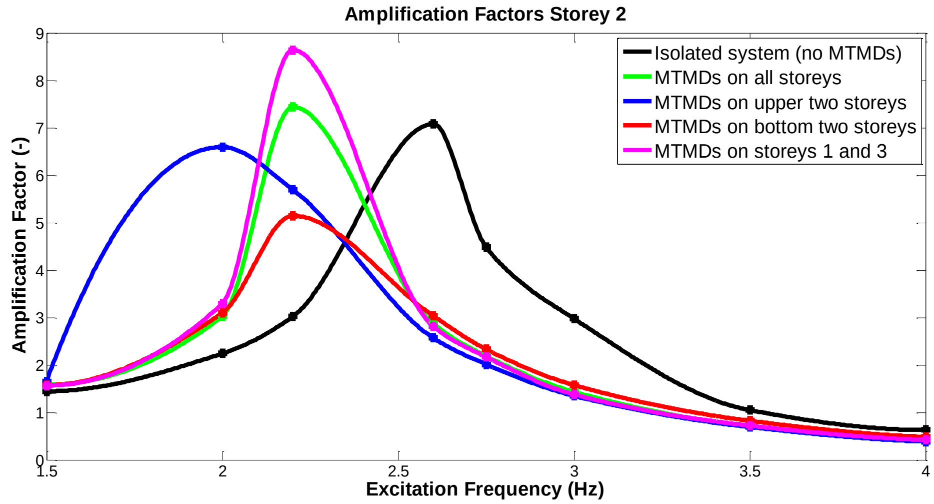


Figure 9(b). Amplification factors of MTMD configurations tuned to the second-mode system frequency of 8.0 Hz for the second storey

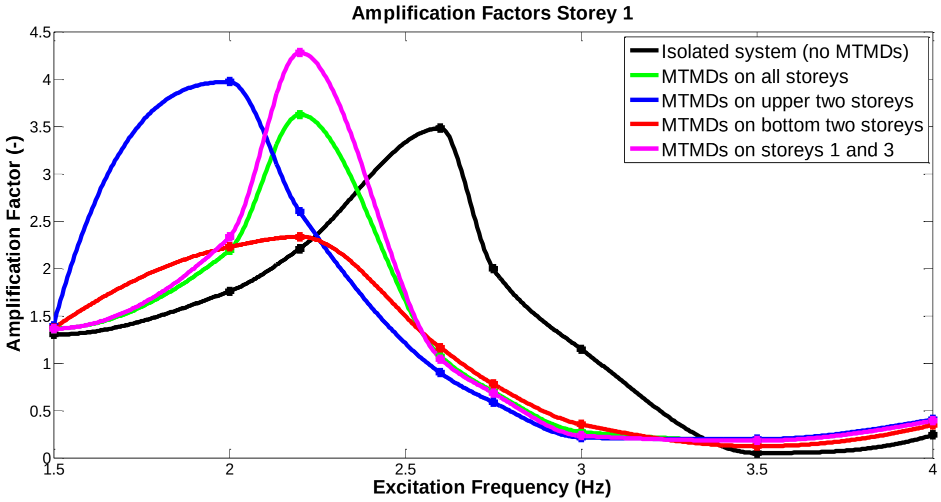


Figure 9(c). Amplification factors of MTMD configurations tuned to the second-mode system frequency of 8.0 Hz for the first storey

Based on Fig. 9(a)-(c), the following remarks can be made:

1) As opposed to when the MTMDs are tuned to the fundamental system frequency, tuning the MTMDs to the second-mode system frequency is seen to result in response amplification of the system corresponding to several of the MTMD configurations considered. The only configuration which is consistently observed to damp the system response of all storeys is whereby MTMDs are placed on the bottom two storeys of the frame.

2) Placement of MTMDs on the bottom two storeys while they are tuned to the second-mode system frequency of 8.0 Hz results in slightly more attenuation of the amplification factor magnitudes than when tuning the MTMDs to the first-mode system frequency of 2.6 Hz. This is in line with the findings in Fig. 8 in which the FFTs showed the significance of the second-mode system frequency.

3) In addition to the configuration for which MTMDs are placed on the two bottom storeys of the frame, the only other configuration which is observed to attenuate the isolated system response is for which MTMDs are placed on the upper two storeys of the frame. However, the extent of attenuation is less than for the case in which the MTMDs are tuned to the fundamental system frequency of 2.6 Hz. Furthermore, this configuration achieves system response attenuation for only two out of three storey responses of the frame and actually amplifies the first storey response relative to the isolated frame response.

4) Since the TMDs used in the MTMD configurations are de-tuned from the fundamental system frequency of 2.6 Hz, amplification factor peaks are not observed at 2.6 Hz anymore. Instead, a downward shift in the frequencies corresponding to the amplification factor peaks can be observed for all MTMD configurations considered.

5) The largest extent of response amplification of the isolated system is observed for the MTMD configuration in which TMDs are placed on storeys 1 and 3 of the frame. The response amplifications are 23%, 22% and 26% for storeys 1, 2 and 3 respectively. This indicates that tuning TMDs in this MTMD configuration to the second-mode rather than to the first-mode frequency could bring about potentially damaging effects onto a structure in a real case scenario.

Fig. 10(a)-(c) below show the amplification factors associated with the isolated system and the range of MTMD configurations tuned to the third-mode frequency of 11.6 Hz.

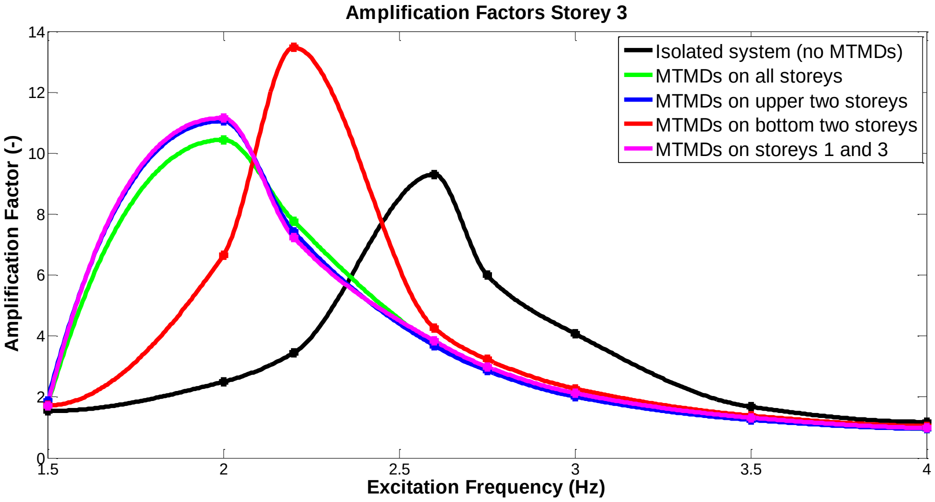


Figure 10(a). Amplification factors of MTMD configurations tuned to the third-mode system frequency of 11.6 Hz for the third storey

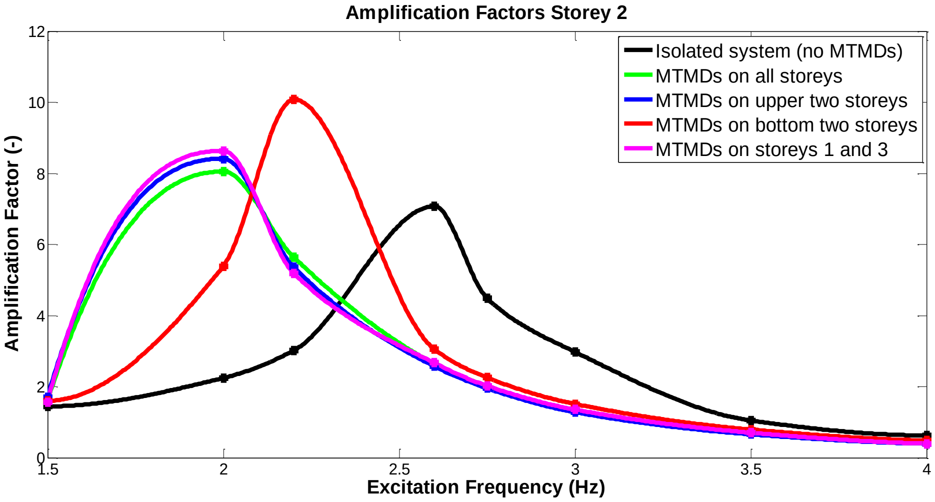


Figure 10(b). Amplification factors of MTMD configurations tuned to the third-mode system frequency of 11.6 Hz for the second story

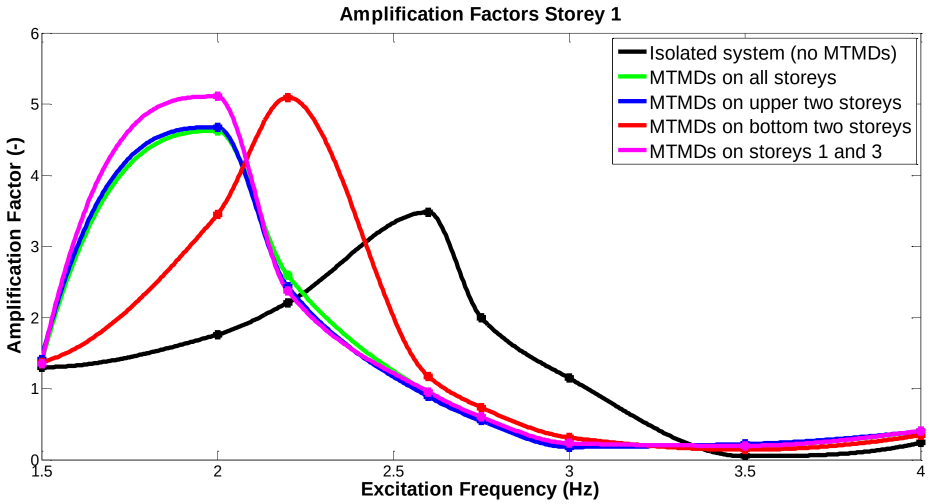


Figure 10(c). Amplification factors of MTMD configurations tuned to the third-mode system frequency of 11.6 Hz for the first storey

Based on Fig. 10(a)-(c), the following remarks can be made:

1) Tuning the MTMDs to the third-mode system frequency of 11.6 Hz causes amplification of the isolated system response for all MTMD configurations considered. This holds true for all storeys.

2) Contrary to the amplification factor plots corresponding to the cases in which MTMDs were tuned to the first- and second-mode frequencies for which the greatest damping of the system response was consistently observed for the case whereby only the bottom two storeys of the frame were fitted with MTMDs, the amplification factor plots shown in Fig. 10(a)-(c) indicate that the greatest system response amplification is caused by this exact same configuration, but with the MTMDs tuned to the third-mode frequency. This suggests that the relative performance of a particular MTMD configuration is not independent of the frequency to which the MTMDs are tuned. In fact, the extent of amplification of the configuration with MTMDs on the two bottom storeys of the frame (compared to the isolated system) and tuned to the third-mode system frequency is a significant 46%.

3) As was observed in Fig. 9(a)-(c), amplification factor peaks for all of the MTMD configurations shown in Fig. 10(a)-(c) are observed at frequencies lower than 2.6 Hz.

In the overwhelming majority of cases studied by Jabary and Madabhushi (2014), the isolated structural response was found to be amplified through de-tuned STMD configurations and a downward shift in the amplification factor peak frequency was observed similar to that seen in Fig. 9(a)-(c) and Fig. 10(a)-(c). Following the results of Fig. 9(a)-(c) and Fig. 10(a)-(c), it may be concluded that tuning MTMDs around a modal frequency which does not correspond to the dominant mode does in no way provide any better structural robustness than a STMD fitted into the same structure which is neither tuned to the dominant mode frequency nor the higher mode frequencies.

CONCLUSIONS

Based on the 1g shaking table tests conducted on a 3-d.o.f. sway frame model structure rested on a level uniform dry sand deposit at high relative density, the following main conclusions can be drawn:

- With the aim of attenuating structural response, it can be more effective to position MTMDs on consecutive storeys than using a greater number of MTMDs. This contradicts findings from a numerical study conducted by Zuo and Nayfeh (2005) on a MDOF-MTMD system in which it is claimed that an increase in the number of MTMDs in a structure to control the dominant mode of excitation yields better robustness.
- Installing MTMDs on consecutive storeys results in significantly more damping than when installing MTMDs on the first and third storeys and leaving the storey in between isolated.

• The relative performance of a particular MTMD configuration is not independent of the frequency to which the MTMDs are tuned. And, although a particular MTMD configuration may consistently perform well in attenuating structural response when tuned to the first- and second-mode system frequencies, it could lead to very significant structural response amplifications when tuned to the third-mode system frequency.
• Tuning MTMDs around a modal frequency which does not correspond to the dominant mode does in no way provide any better structural robustness than a STMD fitted into the same structure which is de-tuned from both the dominant mode frequency as well as the higher mode frequencies.
• A comparison of amplification factor peak magnitudes found in this study for a structure rested on a soil deposit and those found by Jabary and Madabhushi (2013) for a structure fixed to a base plate highlights the significant role played by the soil in damping structural response. Despite the very high relative density of the dry sand used in this study, amplification factor peak magnitudes are found to be significantly lower than those found for a fixed-base case.
• When tuning MTMDs to the second and third-mode system frequencies, a downward shift is observed in the frequencies to which the amplification factor peaks correspond.
• Placing MTMDs in a particular configuration and tuning these to a higher mode frequency of the system can be more effective than when tuning the MTMDs to the first-mode system frequency.

Results found as part of this study are associated with the specific structural specifications and model set-up described and shown in this paper. Experiments on other models may yield different results.

ACKNOWLEDGEMENTS

The first author would like to thank his supervisor Prof. S. P. G. Madabhushi and his advisor Dr. S. K. Haigh for the invaluable insight they have provided at all stages of his research.

Furthermore, the first author is greatly indebted to the Engineering and Physical Sciences Research Council (EPSRC) Doctoral Training Account (DTA) scholarship for funding his study at the University of Cambridge.

REFERENCES

Ghosh A and Basu B (2004) "Effect of soil interaction on the performance of tuned mass dampers for seismic applications," *Journal of Sound and Vibration*, 274(3-5):1079-1090

Jabary RN and Madabhushi SPG (2013) "An experimental investigation of tuned mass dampers on the seismic response of a multi-storey structure," In: Elia G., Rouainia M., Vemury C. and Wilkinson S. (eds.) *SECED 2013: Young Engineers Conference*, *3-4 July 2013*, *Newcastle upon Tyne*, *United Kingdom*. Newcastle upon Tyne, Newcastle University. pp. 76-81

Jabary RN and Madabhushi SPG (2014) "An experimental investigation of soil-structure interaction in a sway frame structure with tuned mass dampers," In: *EURODYN 2014: IX International Conference on Structural Dynamics*, *Young Engineers Conference*, *30 June-2 July 2014*, *Porto*, *Portugal*. Porto, University of Porto

Lee CL, Chen YT, Chung LL and Wang YP (2005) "Optimal design theories and applications of tuned mass dampers," *Engineering Structures*, 28(1):43-53

Liu MY, Chiang WL, Hwang JH and Chu CR (2008) "Wind-induced vibration of high-rise building with tuned mass damper including soil-structure interaction," *Journal of Wind Engineering and Industrial Aerodynamics*, 96(6-7):1092-1102

Prasad SK, Towhata I, Chandradhara GP and Nanjundaswamy P (2004) "Shaking table tests in earthquake geotechnical engineering," *Current Science*, 87(10):1398-1404

Rana R and Soong TT (1998) "Parametric study and simplified design of tuned mass dampers," *Engineering Structures*, 20(3):193-204

Tan FSC (1990) Centrifuge and theoretical modelling of conical footings on sand, Ph.D. Thesis, Cambridge University, United Kingdom

Zuo L and Nayfeh SA (2005) "Optimization of the Individual Stiffness and Damping Parameters in Multiple-Tuned-Mass-Damper Systems," *Journal of Vibration and Acoustics*, 127(1):77-83