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STRONG MOTION ATTENUATION RELATIONSHIPS FOR COLOMBIA

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ABSTRACT

A source spectrum model is used to derive spectral strong motion attenuation relationships for Colombia. We use a modified version of the formulation proposed by Singh et al (1989) to compute SH waves radiated spectra for different seismic moments and hypocentral distances, combining point and finite source models. Expected strong motion intensity (for example PGA or PSA) is computed from the theoretical radiated spectra using random vibration theory. 206 strong motion recordings form the national accelerograph network of Colombia (RNAC) were used to fit the source spectrum parameters, using a genetic algorithm (GA), to develop physically-based attenuation relationships for shallow crustal and subduction earthquakes separately. The obtained attenuation functions were compared with those used in the national seismic hazard assessment study of Colombia (AIS 2009). The comparison shows that the obtained attenuation functions give the lower residual bias, indicating reasonable agreement between them and the Colombian strong motion recordings.

INTRODUCTION

In the framework of probabilistic seismic hazard assessments (PSHA), ground motion prediction equations (GMPEs) provide the probability moments of a lognormal distributed strong motion random variable, for earthquake magnitudes and distances within their range of validity. They must adequately represent the seismological parameters associated to the rupture, the transformation processes suffered by seismic waves travelling through the earth's cortex and also be computed in terms of physical variables with engineering significance, such as peak ground acceleration (PGA) or spectral acceleration (SA).

Source, path and site parameters relevant in strong motion attenuation can be modeled using source spectrum models, which combine source scaling, geometrical spreading, transfer medium attenuation and site attenuation terms to provide a theoretical solution of spectral amplitudes of strong motion. Source spectrum formulations have been extensively used worldwide to develop GMPEs over the past decades (Ordaz and Singh, 1992; Atkinson 1995; Tavaloki and Pezeshk, 2005; Zafarani and Soghrat, 2012).

In the source spectrum formulation, sources are considered as discontinuities in a homogeneous and isotropic medium and earthquakes are assumed to be pure shear dislocations. Using these basic assumptions, the shape of the Fourier Amplitude Spectrum (FAS) at bedrock can be computed for any location. Once the FAS is computed, the mathematical expectation of peak strong motion parameters,

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in the time domain, is computed by applying random vibration theory.

The abovementioned approach allows calculating the mathematical expectancy of strong motion parameters, such as PGA, peak ground velocity (PGV) and/or peak ground displacement (PGD) at any location, given a seismic moment and hypocentral distance. When computing for several earthquake magnitudes and hypocentral distances, GMPEs can be obtained.

The source spectrum model depends on several seismological variables that define the rupture process and the quality of the medium, therefore, its use to compute GMPEs is not straightforward. Given the high non-linearity of the solution of strong motion expectancy, a numerical calibration process is proposed in this paper, in order to determine the set of seismological variables that best fits the observed strong motion intensities. In this study the objective calibration variable is set to PGA. Spectral acceleration is not used for calibration given that it is not a measured quantity. PGA on the other hand is measured and may be directly related to all the seismological variables in the source spectrum model.

The proposed procedure is applied to obtain attenuation relationships for Colombia. Gallego and Ordaz (1999) proposed attenuation relationships for shallow-crustal and subduction Colombian earthquakes, using an approach similar to the one followed in this work. Those attenuation functions were used in the national seismic hazard assessment study of Colombia, developed by the Colombian Association of Earthquake Engineering (AIS) in the framework of the update of the latest version of the national earthquake resistant building code (AIS 2010; Salgado et al. 2010; Salgado 2010).

MATHEMATICAL EXPECTANCY OF PGA

For any given magnitude and hypocentral distance, the radiated SH-waves acceleration FAS is calculated and the mathematical expectancy of PGA ($E\{PGA\}$) is computed by the application of random vibrations theory. We use as basis the source spectrum model proposed by Singh et al. (1989).

Fourier Amplitude Spectrum of PGA

Singh et al. (1989) SH-waves radiated spectrum model takes into consideration every relevant attenuation process. It is based on Brune's formulation of the source spectrum (Brune 1970), while including additional terms that account for anelasticity, site attenuation and wave-type predominance in geometrical spreading. Far-field and near-field spectra are defined by point and finite source models, respectively, and merged together using simple criteria. Singh et al. (1989) formulation is presented here with some notation differences for the purpose of this study.

Point source model

The theoretical acceleration Fourier amplitude spectrum, $A(f,R,M_0)$, assuming ω^{-2} source model, can be written as,

$$A(f,R,M_0) = C \cdot S(f,M_0) \cdot G(R) \cdot F_Q(f,R) \cdot F_\kappa(f,R) \tag{1}$$

where f is the frequency, R is the hypocentral distance, M_0 is the seismic moment, C is the constant term that reflects wave travelling medium characteristics, $S(f,M_0)$ is the source spectrum term, $G(R)$ is the geometrical spreading term, $F_Q(f,R)$ is the anelasticity filter, and $F_\kappa(f,R)$ is the site attenuation filter or kappa filter. The constant term is,

$$C = \frac{R_{\theta\phi}(2\pi)^2 F \cdot P \cdot A_{up}}{4\pi\rho\beta^3} \tag{2}$$

where $R_{\theta\phi}$ is the radiation pattern, F is a factor to account for free surface acceleration, P is a factor included for energy partition into two orthogonal components, ρ is the medium density and β the shear wave velocity. A_{up} is a factor that corrects for the amplification of S waves propagating upwards through materials of progressively lower shear wave velocity (Boore, 1986). Despite of being frequency dependent, it is approximately 2 for f >1Hz. A A_{up} = 2 is assumed for every frequency (same as Singh et al. 1989). The source spectrum term, for a ω^{-2} source model, is given by (Brune 1970),

$$S(f, M_0) = \frac{M_0 \cdot f^2}{1 + \left(\frac{f}{f_c}\right)^2} \tag{3}$$

$$f_c = 4.9 \cdot 10^6 \beta \cdot \sqrt[3]{\frac{\Delta\sigma}{M_0}} \tag{4}$$

where $\Delta\sigma$ is the stress drop in millibars. The geometrical spreading term may be written as

$$G(R) = \begin{cases} 1/R & for\ R \le R_x \\ 1/\sqrt{R \cdot R_x} & for\ R > R_x \end{cases} \tag{5}$$

This term accounts for body wave predominance when $R \le R_x$ and surface wave predominance when $R > R_x$. Therefore, R_x is the distance where wave-type predominance is expected to change. The anelasticity filter can be written as

$$F_Q(f, R) = \exp\left(\frac{-\pi f R}{\beta Q(f)}\right) \tag{6}$$

where $Q(f)$ is the frequency dependent Q. In this paper, we assume a dependency of Q to frequency of the form $Q = Q_0 \cdot f^{\varepsilon}$, which is a generalized version of the function assumed in previous works (Castro et al. 1990; Ordaz and Singh 1992; Atkinson 1995; Tavaloki and Pezeshk 2005; Singh et al. 2007; Hassani et al. 2011). It is important to note that in the case of $\varepsilon = 1$, Q is proportional to frequency and the anelasticity filter F_Q becomes constant for a given hypocentral distance. For values of $\varepsilon < 1$, the anelasticity filter works as a low-pass filter, which is its intention. Values of $\varepsilon > 1$ result in high-pass anelasticity filter. Such cases are considered impossible since they would result in a non ω^{-2} radiated spectrum. The site attenuation filter or kappa filter may be written as

$$F_\kappa = \exp(-\pi f \kappa) \tag{7}$$

The site attenuation filter refers to the fact that high frequency amplitudes are attenuated much rapidly than predicted by anelasticity, depending primarily of near-surface conditions that may vary from site to site. It accounts for the diminishing in spectral amplitude for $f > f_{max}$, being f_{max} the cut-off frequency introduced by Hanks (1982). Applying the above-mentioned definitions to Eq. 1 gives, for $R <= R_x$ (body waves predominance):

$$A_{point}(f, R, M_0) = C \cdot \frac{M_0 \cdot f^2}{1 + \left(\frac{f}{f_c}\right)^2} \cdot \frac{1}{R} \cdot \exp\left(\frac{-\pi f R}{\beta Q_0 f^{\varepsilon}}\right) \cdot \exp(-\pi f \kappa) \tag{8}$$

and for $R > R_x$ (surface waves predominance)

$$A_{point}(f,R,M_0)=C\cdot\frac{M_0\cdot f^2}{1+\left(\frac{f}{f_c}\right)^2}\cdot\frac{1}{\sqrt{R\cdot R_x}}\cdot\exp\left(\frac{-\pi f R}{\beta Q_0 f^{\varepsilon}}\right)\cdot\exp(-\pi f\kappa) \tag{9}$$

Finite source model

Singh et al. 1989 proposed a finite source model in which the rupture area is considered as circular and formed by an infinite set of area differentials, each of which can be modelled as a point source. Within that area, differentials rupture is assumed as uniform and randomly generated. The resulting near-field source spectrum, computed at an observation point located in the axis of the circle at a distance R above the hypocentre, is

$$A_{finite}(f,R,M_0)^2=2\cdot C^2\left(M_0 f_c{}^2\right)^2\frac{\exp(-2\pi f\kappa)}{r_0{}^2}\left[E1(\alpha R)-E1\left(\alpha\sqrt{r_0{}^2+R^2}\right)\right] \tag{10}$$

where $\alpha=2\pi/\beta Q_0$, r_0 is the circular rupture area radius and $E1$ is the exponential integral (Abramowitz and Stegun, 1964). The finite source model is only valid for $f>=f_c$. At lower frequencies the model predicts nearly constant spectral amplitude. This characteristic makes it impossible to apply the finite source model alone. The low frequency band of the radiated spectrum ($f<f_c$) will always be computed using the finite source model. On the other hand, point source model predicts very large amplitudes for the central frequency band ($f_c<f<f_{max}$) of the spectrum for high seismic moments. The finite source model tends to saturate the spectral amplitude given the finite nature of the rupture. In this model, all area differentials that compose the rupture area contribute to the final radiated spectrum at the observation point, but those differentials located near the perimeter of the rupture area will contribute less than those near the center. Based on all the previous arguments we define the radiated acceleration spectrum at a specific location as the minimum envelope of both finite and point source models. This is,

$$A(f,R,M_0)=Min\left\{A_{point}(f,R,M_0),A_{finite}(f,R,M_0)\right\} \tag{11}$$

Expected value of PGA

Given the random nature of strong motion records, random vibration theory can be used to determine $E\{PGA\}$ (i.e. the expected value of PGA) as a function of the acceleration FAS. From Cartwright and Longuett-Higgins (1956) and Davenport (1964) it can be established that,

$$E\{PGA\}=a_{rms}\cdot FP \tag{12}$$

where a_{rms} is the root mean square acceleration and FP is the expected value of the maximum ratio PGA/a_{rms}. a_{rms} and FP are defined respectively as

$$a_{rms}=\sqrt{\frac{m_0}{T_d}} \tag{13}$$

$$FP=\sqrt{2\ln\left(\frac{T_d}{\pi}\cdot\sqrt{\frac{m_2}{m_0}}\right)}+\frac{\gamma}{\sqrt{2\ln\left(\frac{T_d}{\pi}\cdot\sqrt{\frac{m_2}{m_0}}\right)}} \tag{14}$$

where T_d is the duration of the intense phase, γ is Euler's constant ($\gamma=0.577...$) and m_n are the n-

order moments of the FAS,

$$m_n = \frac{2^{n+1}\pi^n}{T_d}\int_{-\infty}^{\infty} f^n A(f,R,M_0)^2\, df \tag{15}$$

According to Herman (1985), the duration of the intense phase can be obtained as,

$$T_d = \frac{1}{f_c} + 0.05R \tag{16}$$

CALIBRATION PROCEDURE

The methodology presented allows the calculation of the mathematical expectancy of PGA as a function of the radiated FAS. Therefore, $E\{PGA\}$ can be expressed as a function of all parameters involved in its formulation.

$$E\{PGA\} = f\left(M_0, R, R_x, \Delta\sigma, Q_0, \varepsilon, \kappa, \rho, \beta, R_{\theta\phi}\right) \tag{17}$$

In this study, the parameters of the function in Eq. 17 are the calibration variables. However, not every parameter in Eq. 17 is susceptible of calibration. Since distance and magnitude are known quantities, M_0 and R are removed from the group of parameters. Particularly, M_0 is a function of magnitude (M_W) as given by Hanks and Kanamori, (1979). For this study is assumed that ρ = 2.5 Ton/m^3 and β = 3.5 Km/s. The final set of seismological parameters to be calibrated are: $\Delta\sigma$, ε, Q_0, k, R_x, and $R_{\theta\phi}$. The quantity $E\{PGA\}$ is defined by a highly non-linear function in a 7-dimension space, so the application of classical statistical fitting methods is extremely difficult and impractical. Therefore, a Genetic Algorithm (GA) was implemented in order to seek for the set of seismological parameters that best fits the observed PGA of the acceleration records.

In the formulation of the GA implemented in this study, individuals are defined as different strong motion attenuation models. The genotype of the individuals is the set of seismological parameters selected for calibration. The evolution starts from a population of randomly generated individuals, and follows an iterative process, in which the individuals are crossed-over and mutated (i.e. their genotype is modified) to give birth to the population of the next generation. In each generation, the fitness of every individual in the population (i.e. its bias to the PGA records) is evaluated. Moving away from the traditional approach in evolutionary programming, in this study we implemented what we call a Forced Evolution (FE) approach, in which the individual with the best fit in the population (which we call the "*champion*") is crossed-over with all the other individuals to give birth to the next generation. This approach guarantees that in every generation the obtained fit is, at least, as good as in the previous one. Faster convergence using the FE approach than with the traditional approach was observed. The new generation of individuals is used in the next iteration of the algorithm. The algorithm terminates when either the maximum number of generations is reached, or a satisfactory fitness level (low bias) is achieved.

Algorithm initialization

A set of several individuals (hundreds or thousands) are created and defined as the initial population, all generated by randomly selecting the values of the seismological parameters from within predefined ranges. These ranges allow constraining the genotype of the individuals to only physically logical values.

Selection

During each successive generation, the fitness of the individuals is determined by their capacity to lower the bias to the recorded PGA values. For each individual, $E\{PGA\}$ is computed and residuals are calculated for each record in the database. Residuals (Re) are computed as follows:

$$\mathrm{Re} = \ln\left(\frac{a_{rec}}{E\{PGA\}}\right) \tag{18}$$

where a_{rec} is the nominal recorded intensity of each accelerogram record,

$$a_{rec} = \sqrt{\frac{{a_x}^2 + {a_y}^2}{2}} \tag{19}$$

where a_x is the x orthogonal component (typically the E-W component) of strong motion (PGA) and a_y is the y orthogonal component (typically the N-S component). Vertical component is discarded for this procedure. Then, for each individual, the first central moment of residuals is computed (i.e. the bias).

$$bias = \frac{\sum_{i=1}^{N} \mathrm{Re}_i}{N} \tag{20}$$

The individual with the lower bias will be declared champion and will be crossed-over with all the other individuals of its generation.

Crossover and mutation

Crossover and mutation operations are performed after the selection of the champion individual. These operations allow breeding new individuals for the next generation. Crossover is performed following a random mix of the seismological parameters of the “parent” individuals (of which one of them is always the champion), so the resulting “child” individual has a genotype built with information from both parents. The population size remains constant.

Given the high non-linearity of the problem, it is not possible to ensure that the champion individual corresponds to the one with a global minimum bias. Therefore, individuals are randomly mutated during the evolution in order to avoid stagnation in a local minimum bias.

Termination

A minimum acceptable bias and a maximum number of generations must be defined in order to limit the total number of iterations. When either of them is achieved, the algorithm terminates. The champion individual of the last generation contains the optimum combination for the seismological parameters that define the source spectrum model, and will provide the lower bias.

APPLICATION OF THE PROPOSED METHODOLOGY TO COLOMBIA

The Colombian territory is a broad region of tectonic interaction, in which the Caribbean, South American and Nazca plates interact. The whole territory is located on the South American plate. To the west, the Nazca plate subduces the South American plate along the Colombian Pacific Coast, having associated considerable earthquakes of high magnitudes. The deformations induced in the cortex by the interaction between the Nazca and South American plates, are absorbed by a system of shallow-crustal faults from which highlights the Romeral fault system (strike slip), and the Frontal system of the Eastern Cordillera (reverse). To the north, the main seismotectonic feature is the compression zone between the South American and Caribbean plates, which induces deformations that are absorbed by the Oca system (strike slip) in Colombia and the San Sebastián (strike slip) system in Venezuela.

Strong motion database

Strong motion recordings were obtained from the National Accelerograph Network of Colombia (RNAC) operated by the National Geological Survey of Colombia (SGN). The basic database contains a total of 284 uncorrected accelerograms recorded at bedrock level over a 14 years period from 1994 to 2008 for earthquakes of *Mw* magnitude over 4.4. Latterly registered earthquakes were not included in this study but not significant events have occurred in the analysis area since then. This corresponds to an updated version of the database used by Gallego and Ordaz (1999).

The 284 strong motion recordings followed a correction process which included instrument and baseline correction (phase 1 processing) and bandpass filtering and decimation to a uniform time interval (phase 2 processing). After processing, the database was debugged in order to discard wrong data or records with low PGA values, deriving on a final database with 206 entries. Finally, the database was divided into two sets: subduction and shallow crustal earthquakes; 87 records associated to 22 shallow crustal earthquakes and 119 records associated to 15 subduction earthquakes (Table 1).

Table 1. Database of Strong-Motion Recordings

Event name	Type	Date	Mw	Hypocentral distance [Km]	Depth [Km]	Number of recordings
Páez	Shallow crustal	06/06/1994	6.1	297	1	1
Santa Isabel	Shallow crustal	9/19/1994	4.6	163	1	2
Tauramena	Shallow crustal	1/19/1995	6.2	95	25	5
Calima	Subduction	02/08/1995	6.4	191	102	7
Risaralda	Subduction	8/19/1995	6.4	112	110	4
Zaragoza	Shallow crustal	06/11/1996	5.1	204	5	4
Sipi	Subduction	09/11/1996	4.6	185	100	10
Juradó	Shallow crustal	11/04/1996	6.1	308	5	4
El Dovio	Subduction	11/17/1996	4.5	149	110	11
Bucaramanga Nest	Subduction	01/01/1997	5.8	337	160	5
Sipi	Subduction	2/19/1997	5.6	186	120	6
Bucaramanga Nest	Subduction	06/11/1997	5.8	347	170	3
Cubarral	Shallow crustal	7/17/1997	4.8	93	5	24
Génova	Subduction	09/02/1997	6.8	298	230	12
Argelia	Subduction	12/10/1997	4.4	165	100	1
Génova	Subduction	12/11/1997	6.4	279	220	1
Chameza	Shallow crustal	02/10/1998	4.5	138	5	6
Landazuri	Shallow crustal	03/06/1998	4.8	123	5	2
Cimitarra	Shallow crustal	03/08/1998	4.9	130	5	25
Armenia	Shallow crustal	1/25/1999	5.8	141	5	1
Guayabetal	Shallow crustal	06/01/1999	4.6	126	5	2
Sativasur	Shallow crustal	7/17/1999	5	114	5	7
Trujillo	Subduction	7/19/1999	4.9	137	130	1
Betulia	Subduction	11/08/1999	6.2	339	160	8
Fortul	Shallow crustal	1/17/2000	5.3	126	5	2
El Bagre	Shallow crustal	3/28/2000	4.7	179	5	5
Tuluá	Subduction	9/22/2001	5.6	195	180	1
La Uribe	Shallow crustal	11/23/2002	5	107	5	9
Colombia	Shallow crustal	1/22/2003	4.6	164	5	1
Sotará	Shallow crustal	8/18/2004	4.6	31	5	1
Pacific Ocean	Subduction	11/15/2004	6.6	374	24	2
Toro	Shallow crustal	03/08/2005	4.6	82	5	1
Tadó	Subduction	4/21/2005	4.7	110	90	4
Puracé	Shallow crustal	03/06/2007	4.4	88	5	1
Andes	Shallow crustal	4/24/2007	4.7	53	15	14
Cubará	Shallow crustal	6/20/2007	5.1	94	5	11
Quetame	Shallow crustal	5/28/2008	5.1	80	1	2

Earthquake magnitude was uniformly expressed in Mw scale, and focal distance was used as source-to-site distance. Magnitude ranges from 4.4 to 6.8 Mw and hypocentral distance ranges from 88 to 619 km. Figure 1 shows the number of recordings for magnitude ranges, and Figure 2 shows the number of recordings for distance ranges. Histograms in Figure 1 and 2 show that the number of recordings can be assumed to be evenly distributed over the magnitude range, but not over the distance range. The central distance interval (from 100 to 300 km) has more than 65% of all the recordings. For distances greater than 400 km, there are only 13 recordings available. Strong motion prediction for distances greater than 400 km is of the highest interest in Colombia given the possibility of occurrence of a high magnitude earthquake in the pacific coast (subduction zone), that affects the city of Bogotá due to its deep soft soil deposits. On the other hand, the highest magnitude available on the database is 6.8 Mw, which is low compared to the maximum expected magnitude of most Colombian seismic sources (AIS 2010; Salgado et al. 2010; Salgado 2010). These data voids limit the applicability of regression technics for deriving GMPEs in Colombia. The proposed procedure seems to provide a practical solution in this case.

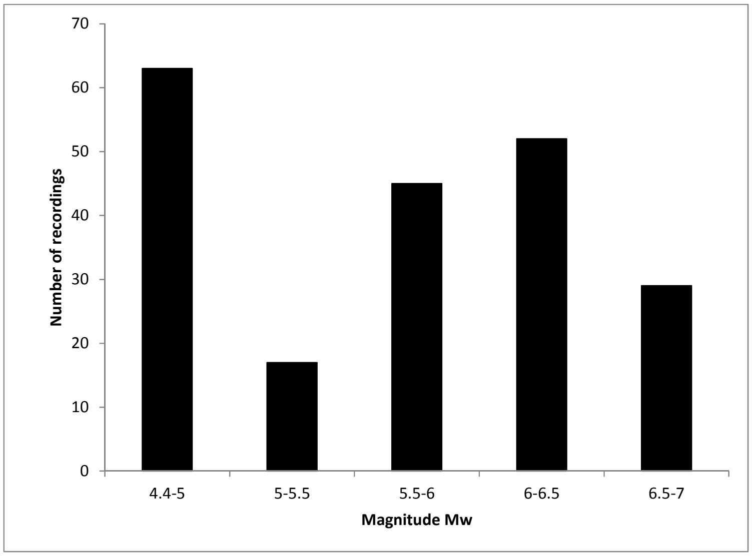

Figure 1. Number of recordings per magnitude range.

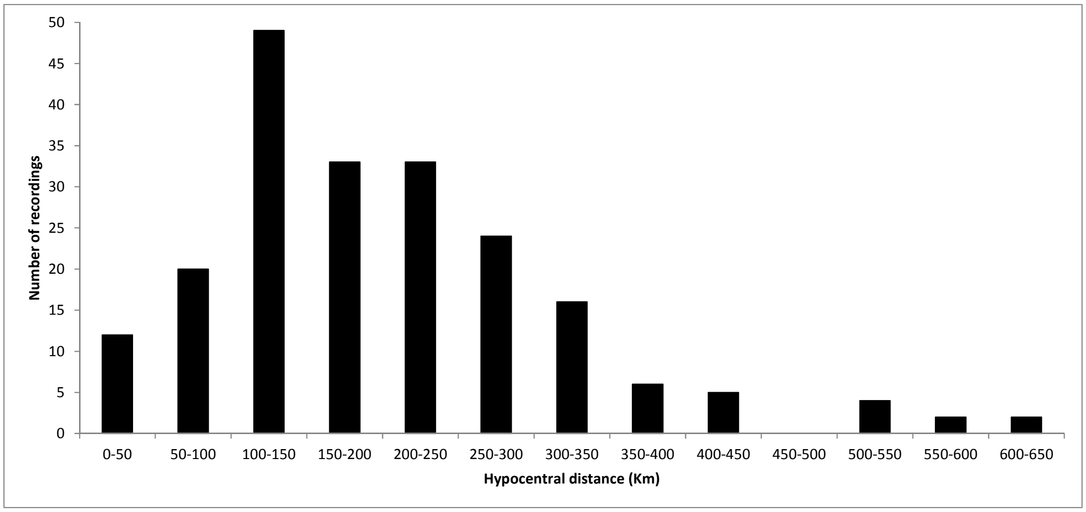

Figure 2. Number of recordings per distance range (b).

Ground motion prediction equations for Colombia

The calibration procedure in section 3 was applied to obtain the set of seismological parameters that define a source spectrum model that best fits the recorded PGA values for the Colombian earthquakes presented in this section. GMPEs were calibrated for shallow-crustal and subduction earthquakes separately. Table 2 shows the selected ranges for the seismological parameters selected for calibration. A fixed value of R_x=100 Km was used, according to Gallego and Ordaz (1999).

Table 2. Ranges for seismological parameters

Parameter	Units	Minimum	Maximum
$\Delta\sigma$	Bar	50	250
ε		0.8	1
Q_o	-	50	800
k_1	-	0.005	0.04
$R_{\theta\phi}$	-	0.55	0.65

The resulting seismological parameters are shown in Table 3 and those are physically feasible and meaningful. The obtained biases are very close to zero, meaning in practical terms that the estimation of PGA using the proposed attenuation procedure can be considered as unbiased. This is a very desirable characteristic in attenuation models.

Table 3. Resulting seismological parameters for shallow-crustal and subduction Colombian earthquakes

Focal mechanism	Bias	Standard deviation	$\Delta\sigma$ [bar]	Q_o	ε	k	$R_{\theta\phi}$
shallow-crustal	0.0009	0.63	235.9	723.1	0.9	0.0333	0.642
Subduction	0.0008	0.72	210.3	477.9	0.91	0.0346	0.623

Figures 3 and 4 present the obtained GMPEs in terms of PGA, for shallow-crustal and subduction earthquakes respectively, defined for hypocentral distances between 5 and 500 Km and moment magnitudes between 4 and 8, calculated with the parameters in Table 3.

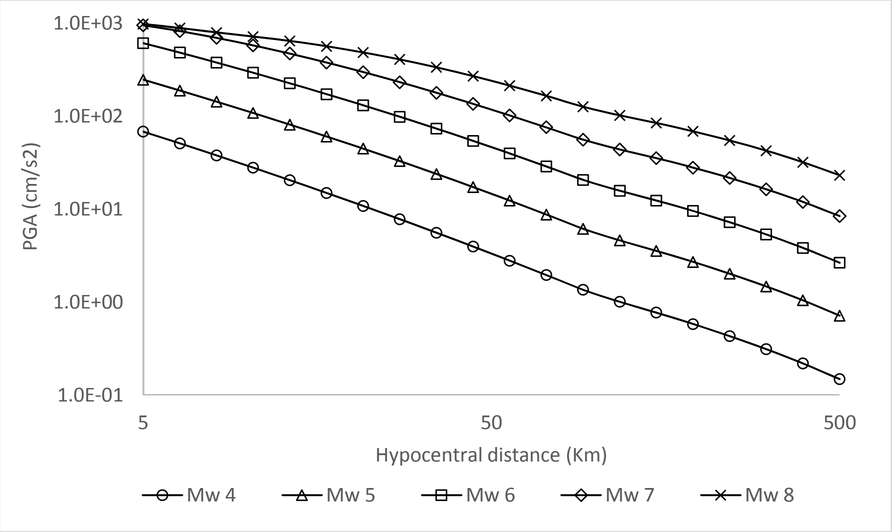

Figure 3. GMPEs obtained for Colombia, for shallow-crustal earthquakes

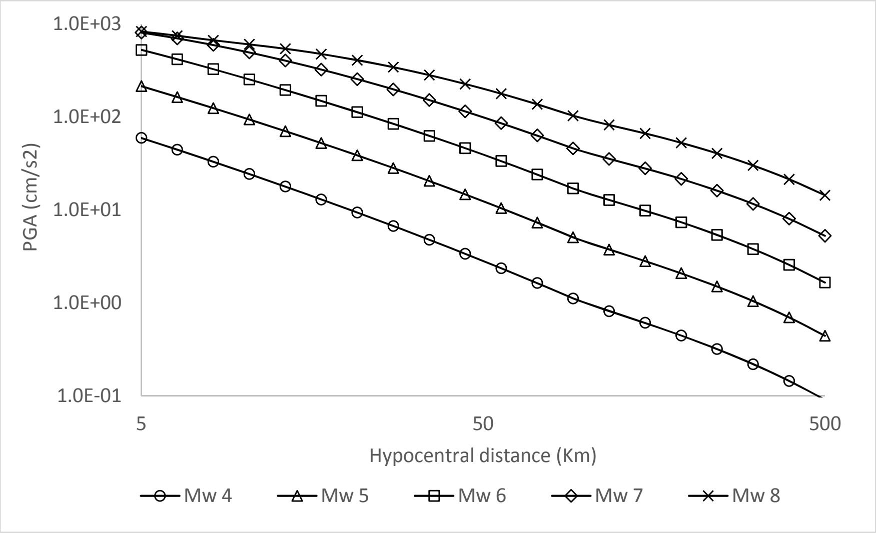

Figure 4. GMPEs obtained for Colombia, for subduction earthquakes

Comparison with GMPEs previously used in Colombia

The obtained attenuation relationships are compared to those used in the National Seismic Hazard Assessment of Colombia (AIS 2010; Salgado et al. 2010; Salgado 2010). Those GMPEs are: Campbell Strike (Campbell, 1997) and Gallego and Ordaz (1999) for shallow-crustal earthquakes, and Campbell Reverse (Campbell, 1997) and Gallego and Ordaz (1999) for subduction earthquakes; all of them at bedrock level. Residuals were calculated for those GMPEs using the strong motion recordings selected for calibration. The resulting biases and standard deviations are presented in Table 4, compared to those of the GMPEs obtained in this study.

Table 4. Biases and standard deviation for previously used GMPEs in Colombia compared to the ones obtained in this study.

Shallow-crustal			Subduction		
GMPE	Bias	Standard deviation	GMPE	Bias	Standard deviation
Campbell strike	0.018	0.79	Campbell reverse	0.02	1.56
Gallego and Ordaz	0.155	0.83	Gallego and Ordaz	1.248	1.66
This study	0.0009	0.63	This study	0.0008	0.72

Expansion to spectral ordinates

The procedure can be expanded to account for the attenuation of spectral acceleration (SA) by multiplying the FAS by the transfer function of a single-degree-of-freedom (SDOF) oscillator. Acceleration response spectra are of great importance for engineering proposes, given that they provide the expected maximum elastic acceleration induced over a SDOF oscillator during an earthquake. Buildings and structures in general are often designed by means of an elastic acceleration design spectrum which is used to apply seismic forces to the structure’s main vibration modes, each characterized by its vibration period. The transfer function of a SDOF oscillator of vibration period T and damping ratio ζ is given by,

$$H(f,T,\zeta)=\frac{1}{1-(f\cdot T)^2+2\zeta(f\cdot T)} \tag{21}$$

The oscillator acceleration FAS may be computed by multiplying the radiated acceleration spectra at a specific location with the transfer function of a single degree of freedom oscillator

$$A_{osc}(f,R,M_0,T,\zeta) = A(f,R,M_0)\cdot H(f,T,\zeta) \tag{22}$$

The expected value of SA is then calculated using random vibrations theory in the same way that for PGA, with a modification to account for the duration of the intense phase of the SDOF oscillator response. T_d is replaced in Eq. 15 by the duration of the SDOF oscillator response (T_r). T_r may be computed as defined by Joyner and Boore (1983).

$$T_r = T_d + \frac{\left(\frac{T_d}{T}\right)^3}{\frac{2\zeta\pi}{T}\left(\left(\frac{T_d}{T}\right)^3 + \frac{1}{3}\right)} \tag{23}$$

Figure 5 presents two examples of response spectra for ζ=5%, for different magnitudes and hypocentral distances, for both shallow-crustal and subduction environments. It can be seen that the shallow-crustal solution predicts higher accelerations in both cases than the subduction solution.

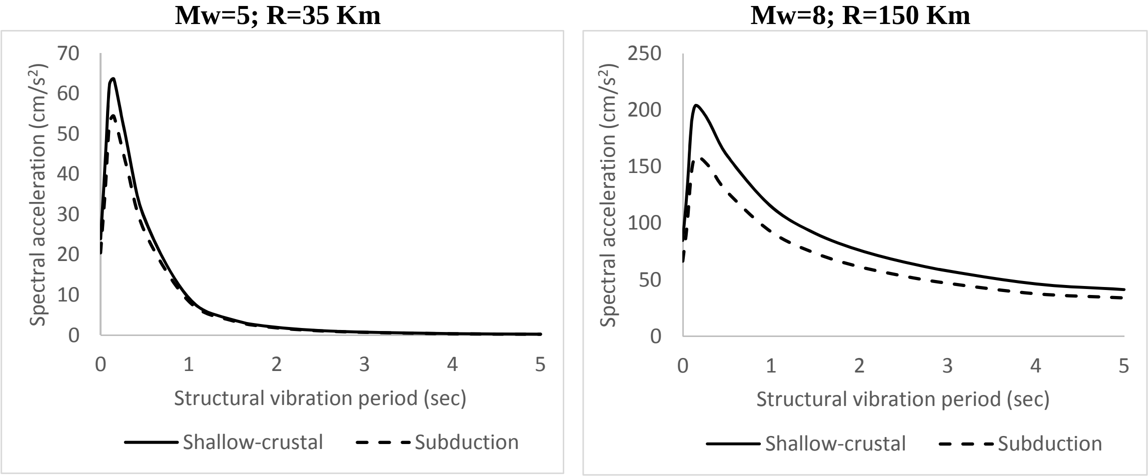

Figure 5. Response spectra for ζ=5% computed from calibrated attenuation relationships for Colombia

CONCLUSIONS

The presented source spectrum model is useful for representing the FAS of strong motion given an earthquake magnitude and hypocentral distance. Two kinds of source spectra are used: far-field and near-field. Then, by using random vibration theory, the mathematical expectancy of PGA can be computed from its FAS. The method can be expanded to account for the attenuation of SA. When computing for several earthquake magnitudes and hypocentral distances, GMPEs can be constructed.

Source spectrum parameters can be calibrated using the proposed methodology, which aims to minimize the bias between observations (acceleration records database) and predictions (source spectrum model). Source spectrum parameters were calibrated for Colombia, deriving in a new GMPE for the country, for shallow-crustal and subduction earthquakes. This GMPE has a lower bias compared to the ones previously used for hazard assessment in the country.

The proposed procedure can be used to derive GMPEs for any region of the world that has an operating accelerograph network and sufficient accelerogram records. The available records need to be debugged and processed before applying the methodology to avoid wrong bias estimation. The presented application in Colombia is the first study case in the use of this procedure, so further research in its application to other regions is required. The calibration GA was implemented in the software Strong Motion Analyst (Bernal 2013), which is free academic software that can be used to extend the application of this methodology.

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