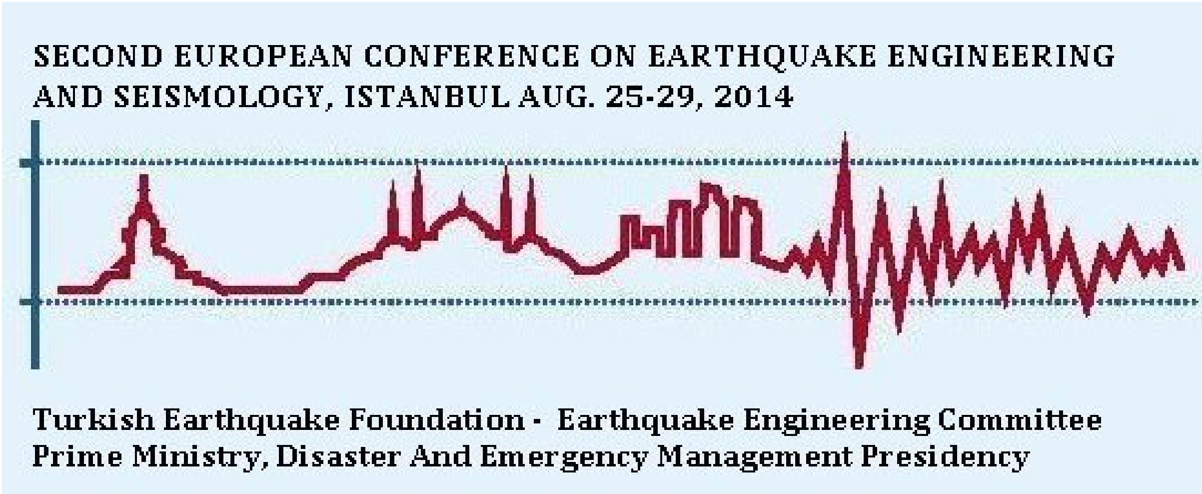

RAPID AND CONCURRENT EPI- & HYPOCENTRE LOCATION USING TABULAR DATA STRUCTURES

George Rowett DAGLISH[1]   Yurii Pavlovitch SIZOV[2]

ABSTRACT

Two kinds of interpolative tabular scans are described in this paper. The first provides a hypocentre location given the epicentral co-ordinates of the event. The second represents a simultaneous, or concurrent, location of an epicentre/hypocentral pair using P-wave first arrivals, and without prior knowledge of the epicentre.

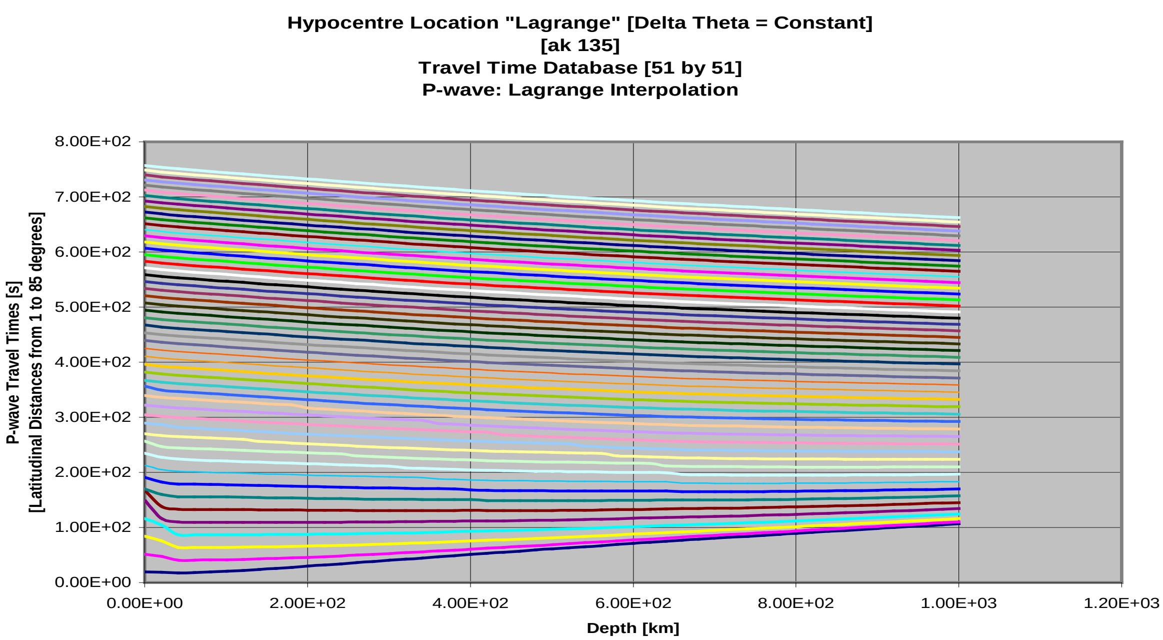

Figure 1 Travel Time (1st Arrivals) Database

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Each of these methods can, in fact, use P- or S-wave first arrivals and each method uses a set of first arrival tables constructed by point-to-point (P2P) ray tracers, following any of a given set of earth velocity models. Graphical representations of such tables appear as Figure 1 (above) and Figure 2.

The software which implements these scanning procedures represents an intermediate stage in the production of a prototype system model, (Daglish GR and Sizov YuP 2011), which, it is hoped, will help to suggest a software structure for the rapid location of earthquakes within an early warning context (EEW).

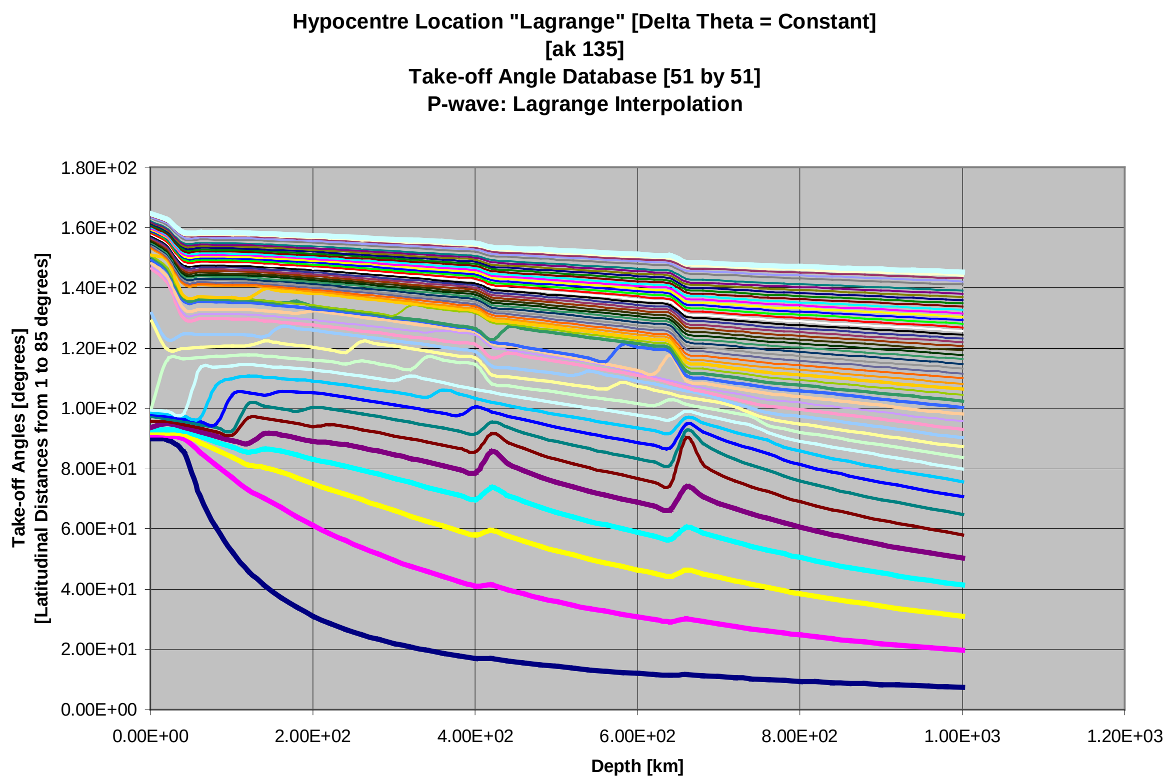

Figure 2 Take-off Angle Database

With regard to the speed of these two location types, the first type (**epicentre known**) can perform a single location using 15 to 20 stations, within 0.25s, scanning 2000 depth points from a prescribed depth. Normally four scans are performed in tandem, exercising combinations of linear, cubic, and Lagrangian interpolation methods. A track of such a scan is shown as Figure 3. The second type (**epicentre unknown**) will find both the epicentre and hypocenter of an event, using up to 10 - 15 stations, within 6 – 7 seconds. This scheme uses only linear interpolation and also produces clear minima as in Figure 3. However the reason for the relative slowness of the latter routines is the larger amount of time spent in evaluating a "cost function" ("error indicator") which monitors the fit (internal consistency) of an epicentre calculation, together with the match on a hypocentral depth. These timings are based on the throughput for a 3.2 Mega Hertz machine.

The paper finds that both methods can rapidly find results in close agreement with currently received values (Daglish GR and Sizov YuP, 2013) and owing to their speed might form strong assets as components in the context of an Earthquake Early Warning (EEW) scheme.

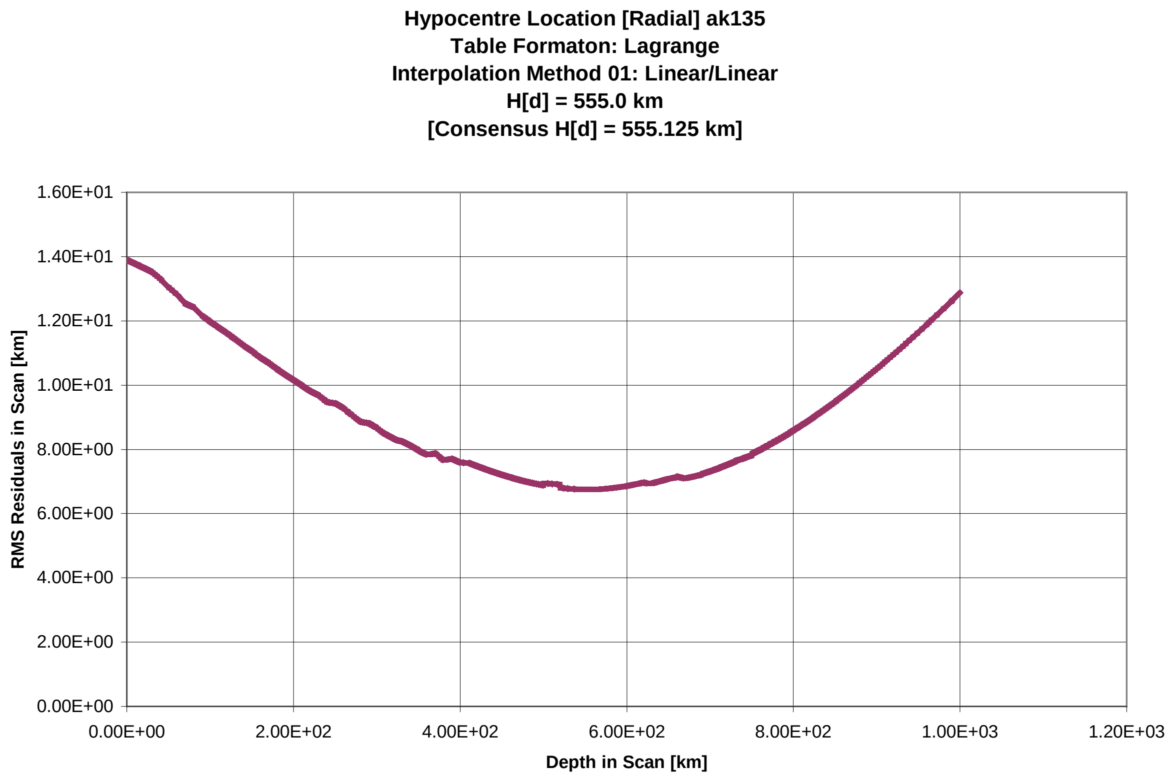

Figure 3. Track of a Tabular Scan (Epicentre Known at 49.78N, 145.13E)

Key Words ~ Seismology; Epicentre; Hypocentre; Location; P-wave

INTRODUCTION

The overall method presented here differs from the conventional since it does not employ Matrix Algebra such as that implicit in Geiger's Method (Lee WHK, Stewart SW 1981) and in Progressive Multiple Event Location (Pavlis GL, Booker JR 1981 and Pujol J 1988) and later methods (Richards PG et al. 2006 and Waldhauser F, Schaff DP 2008). Instead it relies entirely on interpolation.

The authors' motivation and work on Interpolative Tabular Scanning with respect to Earthquake Location problems have been mentioned in Daglish GR, Sizov YuP 2013, 2012 and 2011.

In this paper two types of interpolative scan will be described:

- The **first location procedure** which provides Hypocentre Depth, H_d, (or "H[d]") only, given Epicentre co-ordinates
- The **second location procedure** returning both Epicentre co-ordinates & Hypocentre Depth from P-wave (or S-wave) 1st arrivals.

Each type, by tabular extension, will provide the Take-off angles for each event identified.

TABULAR STRUCTURE

The table generation routines provide matrix tables of the order of 101×101 to 51×51 (Daglish GR, Sizov YuP 2012) by steering any one of the point-to-point (P2P) Ray Tracing algorithms of which there are several currently available (Thurber CH, Rabinowicz N 2000).The main output from this process consists of 5 objects:

1. P-wave, (S-wave), 1st arrival travel time matrix.
2. Corresponding Take-off angles matrix.
3. Calibration Data matrix.
4. Error Data matrix.
5. Error Log – a list.

Each of the matrices at 1 to 4, above, is dimensioned by Depth and Latitudinal Distance. The Latitudinal Distance, referred to as Co-latitude, starts from a conceptual "pole" at $Zero$ degrees and extends in the current set (for this instance) to 87^{O}. Earth velocity models "PREM", "iasp91" or "ak135" can be used to generate these tables (Kennett BLN et al. 1995).

The contents of the fourth element in this structure are directed at each point in matrices 1 to 3, above, and the fifth element is of the form of an error log, recording where the chosen P2P algorithm does not achieve the required level of accuracy.

HYPOCENTRE LOCATION

This is the **first location procedure** as mentioned above. We deal with the associated **cost function and scanning process** and the description of this procedure here follows that given in Daglish GR and Sizov YuP 2014 (Feb).

The Onset timings are made up as follows, from the set, T :

$$T \equiv \{t_j\}; \quad j = 0,(n-1) \tag{3.1}$$

Here, the t_j are elements of a universal time running on a continuous time base, and represent arrivals of whatever wave species at Stations $(j+1)$. Thus:

$$\bar{t} = \frac{1}{n}\sum t_j \tag{3.2}$$

We generate a spread of deviations about this mean as:

$$\tau_j = \{t_j - \bar{t}\}; \quad \text{j=0,}(n-1) \tag{3.3}$$

This forms the fixed "Time Template" for the scan. Similarly, using the matrices of travel times generated by the point-to-point Spherical Ray Tracing routines, from any position on the upward/downward scanning trajectory, we can find, by forward interpolation, using the active station Co-latitudes, a set of time deviations, indexed, at the given depth-point, as i. These are:

$\left\{\nu_j\right\}_i$; $j = 0,(n-1)$. In this, i indexes a depth-point in the scan and j the timings associated with the set of Co-latitudes for the active Seismic Stations.

For each depth-point in the scan, relative to the known Epicentral co-ordinates, we get:

$$RMS_i = \sqrt{\frac{1}{n}\sum_j \left(\tau_j - \left(\nu_j\right)_i\right)^2} \tag{3.4}$$

It is such a set of values that is graphed against depth in Figure 3, and the infimum is taken to correspond to a Hypocentral Depth, in this case 555.0 kilometers. Such a method forms an example of a "type 1" cost function. This type of cost function, or indicator, is characterized by making a comparison primarily of timings leading to an RMS score. Up to six variants of "type 1" cost function are used in this work.

Chauvenet's principle for the rejection of outliers (Neville AM, Kennedy JB 1964) is installed in the main scanning routine, which implements this **first location procedure**, as it is also in the routines that implement the **second location procedure**.

Two examples of the **first location procedure** locations are given here at Figures 4 and 5. Using WILBUR III (provided to the public by courtesy of the Incorporated Research Institutes for Seismology – "IRIS") some data was chosen corresponding to an M7.3 event in the Kuril Islands stated to be at: epicentre: 46.224N; 150.783E. $H_d = 112.20$ km on 19th April 2013. Some results previously quoted in Daglish GR, Sizov YuP 2013[2] follow:

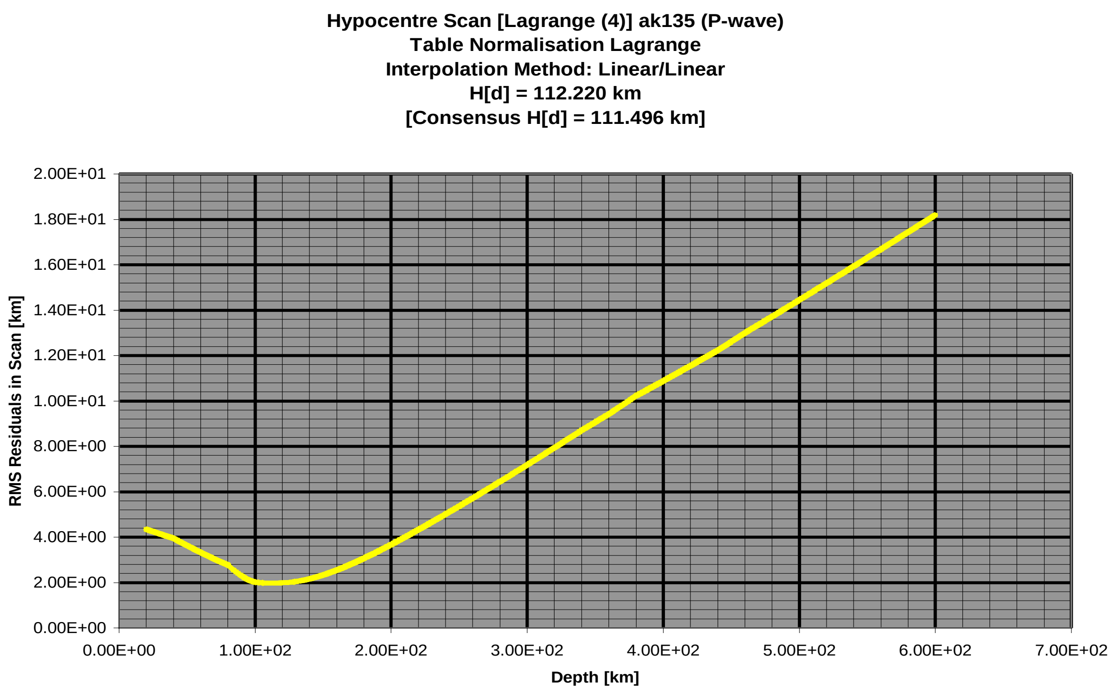

Figure 4. 1st Example of **first location procedure**.

[2] When quoted in this reference, the results in Fig 4 and Fig 5 included tables of Station co-ordinates and Take-off angles directly derived from the location process, as well as tables of the final sets of residuals for each of the 4 interpolation methods used. However space will not allow. See below for a brief explanation of these.

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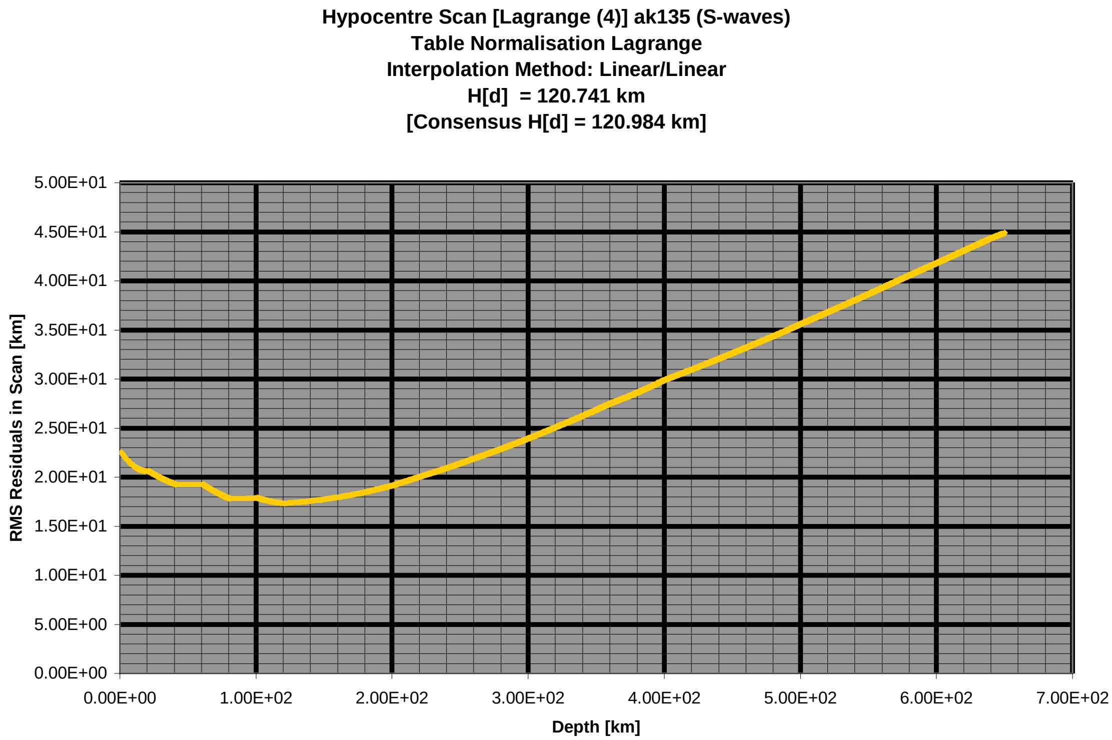

Figure 5. 2nd example of **first location procedure**.

These results comprise one by P-wave and one by S-wave. The programming allows four interpolative scanning methods, listed below. These methods are generally run in tandem and a consensus result is generated from their findings.

1. [Linear; Linear]
2. [Linear; Cubic]
3. [Cubic; Cubic]
4. [Lagrange; Lagrange]

The first method in each of the parentheses, above, interpolates with Depth and the second with Co-latitude.

CONCURRENT EPI- & HYPOCENTRE LOCATION

This section deals with the **second location procedure** as mentioned at the outset. We begin with the **Scanning Process.** Although there appear to be two possible types of scanning method within this context:

1. that which makes a prior estimate of the Latitude and Longitude of the Epicentre of the event,
2. that which makes no such prior estimation,

space dictates that only the second, and more important, type will be dealt with by this paper. This second method is considered more powerful and important since it performs the complete concurrent calculation from scratch without the need for the help of an initial approximation to the Epicentral position. Each of these two methods has its own “cost function” which are designated: “type 1” (as described above with the **first location procedure**) and “type 2” indicator systems.

We now discuss **method 2** and its corresponding “type 2” indicator system, which is in fact a measure of the internal consistency of a least squares location calculation.

The bulk of this explanation, up to equation (4, 6), is quoted from Daglish GR and Sizov YuP (Feb, 2014):

“The co-latitudes and longitudes of the set of active stations are formed into a set of Cartesian co-ordinates within the Earth space-frame. These are to be used later in the localisation calculation which is used to derive the Epicentral position.

The next step is the organisation of the set of P-wave first arrival times into a set of differences forming a fixed “Timing Template”. Having formed these two sets of information the scan commences by:

- Interpolating an entire co-latitudinal row of timings (P-wave 1st arrivals from the tabular Structure referred to above) for a given depth-point.
- Laterally scanning the fixed “Timing Template” along this interpolated row to generate co-latitudes corresponding to its elements by reverse interpolation.
- The above two processes are repeated for each of the set of depth points which form the scan and the smallest local minimum of the “type 2” indicator so found is taken to define the Epicentral co-ordinates, $\underline{\varepsilon}$, and the Hypocentre Depth, H_d.

To repeat this in plainer language: an actual fixed “Time Template” for the lateral scanning procedure consists of a set of differences:

$$\delta t_j = t_j - t_0; \quad j = 0, n-1 \tag{4.1}$$

The base in time for the lateral scan is defined as T_0, and the template is shifted across the depth-interpolated time row as:

$$\begin{aligned} \tau_i &= T_0 + i \cdot \Delta t; \quad i = 0, N-1 \\ t_j &= \tau_i + \delta t_j; \quad j = 0, n-1 \end{aligned} \tag{4.2}$$

Here N is the granularity of the scan and $\Delta t = \frac{T_{\max} - T_0}{N}$. T_0 and $T_{\max}$ are the limiting values of the depth-interpolated time row. These new t_j are used to reverse interpolate to a set of values for Co-latitude. At each point, τ_i, in the scan the Epi- & Hypocentre are localised directly forming a “type 2” indicator value in a manner to be described below.

The **second method**, then, with its “type 2” cost function, proceeds without an estimate of an Epicentral location. As explained above, it forms a “Timing template” from sorted P-wave first arrivals and uses this in a lateral scan (across the timings associated with the set of Co-latitudes) and an upward or downward scan (over the set of defined depth-points) to derive, by inverse interpolation, co-latitudes from the Tabular Structure, as stated above. This explanation and Figure is derived from an earlier exposition in Daglish GR, Sizov YuP 2014.

At each point in the scan, these Co-latitudes are used to form radii (as depicted in Figure.6, below). These radii are translated and are then subtended from the known station locations. A least-squares calculation for the Epicentral co-ordinates ensues. This is the "within scan Epicentre Calculation". Since the calculation is by least-squares and can be well over determined, then a "goodness-of-fit" or "self-consistency" assessment can be made for the result. This occurs for each positioning of the fixed "timing template" in the scan. That location calculation which provides the best "self-consistency" score is deemed to indicate the sought after Epicentre and Hypocentre pair. This is the "type 2" indicator.

This method can avail itself of Chauvenet's principle as a procedure for the rejection of outliers, as described in Neville AM, Kennedy JB 1964, in order to attempt to refine the results. This provides an outer controlling loop on both of the scan types. On detecting an outlier among the set of residuals associated with a given Epi & Hypocentral location, the input data is consolidated to exclude the information associated with the rejected station and the scan recycles. An upper limit is placed for the number of possible recycles and termination may occur, when no further outliers are found, prior to this limit being reached.

We now describe the within-scan Epicentre Calculation. This description refers to Figure. 6. This figure depicts the geometry associated with a single time point match out of a set which corresponds to a local or global minimum of a "type 2" cost function or indicator.

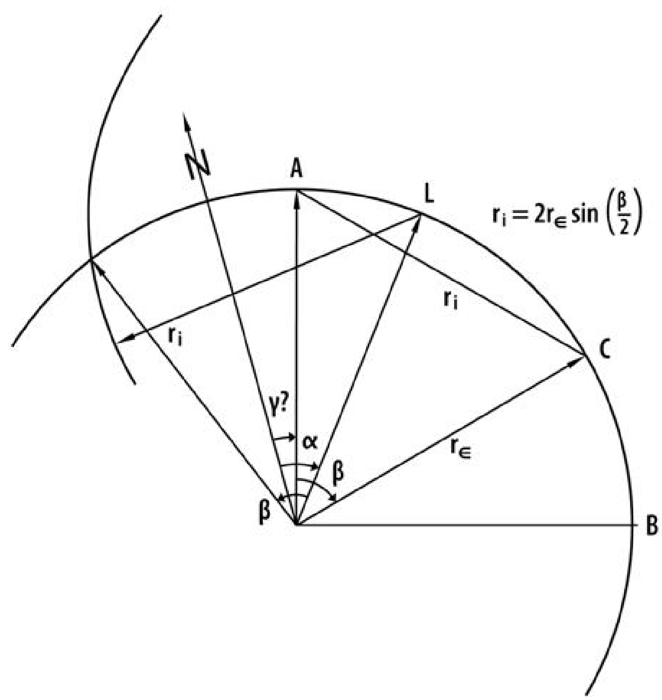

Figure 6. Geometrical basis for within-scan Epicentre location

The arc AB represents the latitudinal extent, starting at zero degrees at A, of the P-wave 1st arrival Tables. C represents a point of match for the object time position from the moving time-template, now associated with the minimum. This object time-position corresponds to a particular station, whose co-latitudinal position is at L. The angle "γ?" is the unknown angle giving the immaterial orientation of the frame of the Table to the Earth Space Frame. AC is a chord subtended from the point of match, C, to the "epicentre pole" of the table at A. This chord is then translated and placed with one end at L which now subtends it as the spherical radius, r_i, of the i^{th} sphere centred at the i^{th} station co-latitude, L.

$$r_i = 2 \cdot r_e \sin\left(\frac{\beta}{2}\right) \tag{4.3}$$

Once all these radii have been assembled from the matched positions of timings within the template, then the following system of equations is solved for $(x, y, z)_\varepsilon$, which will be a possible location for the epicentre in the Earth Space Frame:

$$a_i x_\varepsilon + b_i y_\varepsilon + c_i z_\varepsilon = -\frac{1}{2}\left(r_i^2 - r_e^2 - \left(a_i^2 + b_i^2 + c_i^2\right)\right); \quad i = Zero, (n-1). \tag{4.4}$$

In the above, the $(a, b, c)_i$ are the co-ordinates of the station positions within the Cartesian Earth Frame. r_e is a value for the Earth radius, while n, (≥ 3), is the number of stations with which the scan is undertaken. The $(x, y, z)_\varepsilon$ is converted to Latitude and Longitude.

The "type 2" residuals can now be found from this calculation. Each residual is calculated as:

$$q_i = \left(\sqrt{(x_\varepsilon - a_i)^2 + (y_\varepsilon - b_i)^2 + (z_\varepsilon - c_i)^2} - r_i\right) \tag{4.5}$$

which gives an rms score of:

$$\sqrt{\frac{\sum_{i=0}^{n-1} q_i^2}{n}}. \tag{4.6}$$

Some **examples of Earthquake localisations** performed by this method are now given. The data used for these trials is taken, using WILBER III (supported by Incorporated Research Institutes for Seismology - IRIS), from four distinct Earthquakes. Their parameters are tabulated at Table 1. The concept of "error", in this paper, refers to the deviation of a result from its received value, which is quoted at Table 1.

Table 1. Earthquake Parameters.

Number	Region	Magnitude	Lat/Long. (degrees)	H_d (km)
1	Fiji Islands region	4.5	17.8997S/ 178.5269W	558.0
2	Just East of Honshu	5.4	35.621N/ 140.6862E	36.0
3	South Mid Atlantic Trench	5.2	47.1861S/ 13.4304W	10.0
4	West of North American Coast	5.4	41.7136N/ 126.8446W	9.9

The following four tables give the results of locations of the four Earthquakes in Table 1 using two Earth Velocity models: the "iasp91" and the "ak135" model. Each initial set of localisations under any one of these Velocity models, is followed by the use of the Chauvenet Principle to attempt to refine the initial values so found. (The Latitude and Longitude in these tables refer to the located Epicentres).

Table 2 Localisations of the 4 Earthquakes using the "iasp91" Earth Velocity Model.

Earthquake Number	Latitude [degrees]	Longitude [degrees]	H[d] [km]	"Error" in H[d] [km]	"Error" in Epicentre Position [km]
1	17.8562S	178.4960W	521.9306	36.0694	9.5971
2	35.6757N	140.6234E	52.4873	16.4873	8.5219
3	47.1840S	13.4522W	26.9599	16.9599	2.4353
4	41.7189N	126.8445W	47.0790	37.0790	0.5894

Table 3 Chauvenet's Principle applied to the Localisations of the 4 Earthquakes using the "iasp91" Earth Velocity Model.

Earthquake Number	Latitude [degrees	Longitude [degrees]	H[d] [km]	"Error" in H[d] [km]	"Error" in Epicentre Position [km]
1	17.8221S	178.5400W	543.1310	14.869	8.5270
2	35.6286N	140.6990E	39.0747	3.0797	2.20
3	47.1463S	13.4800W	13.98	3.980	5.3870
4	41.734N	126.803W	45.132	35.132	3.8101

Table 4 Localisations of the 4 Earthquakes using the "ak135" Earth Velocity Model.

Earthquake Number	Latitude [degrees	Longitude [degrees]	H[d] [km]	"Error" in H[d] [km]	"Error" in Epicentre Position [km]
1	17.7715S	178.5250W	498.012	59.984	14.2791
2	35.734N	140.7050E	42.880	6.880	12.6683
3	47.1952S	13.4280W	32.4115	22.4115	1.0253
4	41.5231N	127.5400W	117.156	107.256	65.3806

Table 5 Chauvenet's Principle applied to the Localisations of the 4 Earthquakes using the "ak135" Earth Velocity Model.

Earthquake Number	Latitude [degrees	Longitude [degrees]	H[d] [km]	"Error" in H[d] [km]	"Error" in Epicentre Position [km]
1	17.8422	178.5430W	549.336	8.684	6.6336
2	35.734N	140.7050E	42.880	6.880	12.6683
3	47.1887S	13.4040W	36.898	22.4115	1.8927
4	41.533N	127.5400W	117.156	107.256	65.3806

The content of Tables 4 & 5 is derived from earlier work (Daglish GR, Sizov YuP 2014). It would appear that the use of "iasp91" makes the results move closer to those given by the IRIS data-base and tabulated at Table 1. In a large number of cases the effect of using the Chauvenet Principle proved to be beneficial.

One result is noteworthy for its intransigence. That is Earthquake 4, Table 1. This is the shallowest Earthquake (and also with the closest grouping of stations. See Daglish GR, Sizov YuP 2014). This perhaps is due to the local velocity structure not being adequately modelled by either the "iasp91" or the "ak135" systems in this case. It is possible that for shallow Earthquakes (<200 km) this may always be a problem. However in Table 3 the depth of the first 3 Earthquakes appears to be returned in a more satisfactory manner.

The location of the Epicentres by routines using the "iasp91" model appears, on this showing, to be superior to those using the "ak135".

It should be stated at this juncture that we have refrained in this paper, for reasons of lack of space and for simplicity, from using those routines which perform an estimate of the Epicentral co-ordinates before commencing the scan. A treatment of this aspect of the system is found in Daglish GR, Sizov YuP 2014.

CONCLUSIONS

The fact that the **second location procedure** uses P-wave 1st arrivals only, would appear to obviate the need for picking, if Earthquake Early Warning (EEW) is the concern, since detection of the initial P-wave onset is quite reliable and well-established (Thurber CH, Rabinowicz N 2000 and Lee WHK, Stewart SW 1981).Therefore in an EEW context such a method as this would appear have some merit, being able to return the Epicentre, Hypocentre and, by extension in the tabular structure, Take-off angles, within $\approx 4,5$ seconds of receiving notification of the latest P-wave 1st arrival in the sequence from which data is to be utilised. It would seem, from factors such as:

- The low self-noise levels recorded for the **first location procedure**
- The close agreement with IRIS quoted results
- Speed of execution

being demonstrated in some previous trials with the two methods, and also quoted here, that there is a strong possibility they would be "adequate for purpose" as algorithms usable within an Earthquake Early Warning (EEW) context.

Further trials of software structures are planned for the **second location procedure**, which will simulate real time conditions by allowing the location system to react to incremental input and to "evolve" solutions in an analogous manner to Progressive Multiple Event Location (Pavlis GL, Booker JR 1981).

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