SECOND EUROPEAN CONFERENCE ON EARTHQUAKE ENGINEERING
AND SEISMOLOGY, ISTANBUL AUG. 25-29, 2014

PROPOSAL OF HYSTERESIS MODEL OF STEEL COMPRESSION MEMBERS CONSIDERING DETERIORATION OF STRENGTH AND RIGIDITY SUBJECTED TO CYCLIC LOADINGS

Shoma SUGIYAMA[1], Teruhumi INOUE[2] and Takumi ITO[3]

ABSTRACT

For steel framed structures, high rigidity and strength can be obtained by use of steel braces. However, steel braces subjected to inelastic cyclic loadings show the complicated behavior and deterioration of strength and rigidity, because of lateral and local buckling occurrence. In order to simulate accurately the behavior in its ultimate states, it is necessary to consider the difference of sectional type, slenderness ratio and width-thickness ratio of steel compression members. In this paper, the effects of these parameters to restoring force characteristics are investigated with enormous past test data. And also, the effects of plastic amplitude history are investigated too. Furthermore, the global hysteresis model is formulized by regression analysis.

INTRODUCTION

The braces in a steel framed structure are one of main component of seismic resistant members, and it affects rigidity and strength of whole frame. However, it is recognized that the braces during its ultimate state show unstable behavior because buckling and crack are occurred. During the design process, a skeleton curve of perfect elastic-plastic and the hysteresis rule of progressive slip are widely adopted for the restoring force model of brace. It is often considered that its model underestimates the maximum strength and response deformation comparing with actual behavior with strength deterioration after buckling and fracture occurred. However, it is suggested that the deterioration behaviors will be affect the dynamic response and it could lead the local story collapse mode and concentrated plastic energy input.

In Japan, it is widely recognized that Wakabayashi model can approximately purchase the inelastic behavior of steel compression member considering with deterioration of strength and rigidity during cyclic loadings, and also it is expressed with simple formulation. However, this model is restricted with the applicable range of slenderness ratio. In particularly, hysteresis curve simulated by this model is overestimated in case of steel brace with small slenderness ratio. Herein, the reason is investigated by reference of enormous past test data of database. Furthermore, it is necessary that the hysteresis rule is modified by considering with deterioration of strength and rigidity by inelastic cyclic loadings.

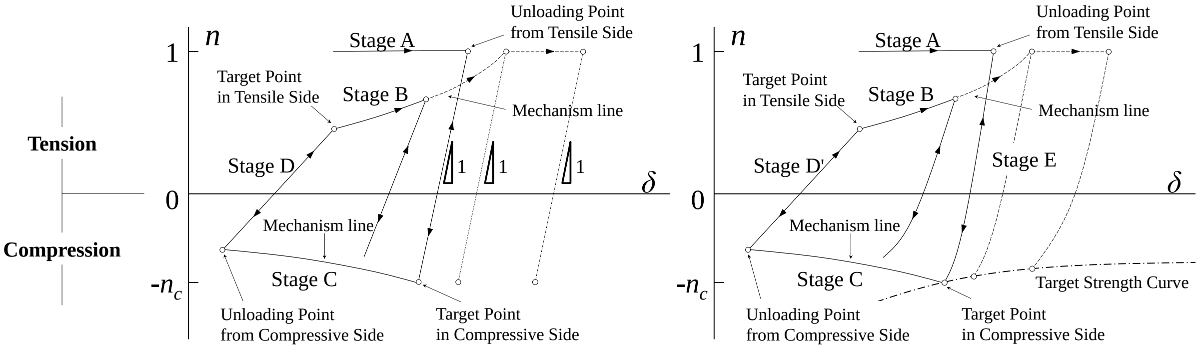
---

[1] Tokyo Univ. of Sci., 6-3-1, Niijuku, Katsushika-ku, Tokyo, Japan, shoma.0120@gmail.com
[2] Tokyo Univ. of Sci., 6-3-1, Niijuku, Katsushika-ku, Tokyo, Japan, teru.since1990g1@gmail.com
[3] Tokyo Univ. of Sci., 6-3-1, Niijuku, Katsushika-ku, Tokyo, Japan, t-ito@rs.kagu.tus.ac.jp

INELASTIC CHARACTERISTICS AND PAST PROPOSED HYSTERESIS MODEL

Restoring force characteristics of steel compression members

According to the reviews with the past hysteresis model and references, the hysteresis rule and inelastic restoring force characteristics of steel compression members are categorized into some stages, considering with each deformation and plasticity as shown in Fig.1 (a) and Table 1 (Wakabayashi et al., 1982). Where the vertical axis n is the axial load normalized with the yield strength, the horizontal axis δ is the axial deformation normalized with the yield deformation. And also, it is presented that the hysteresis behavior is expressed with 1) mechanism line and 2) its translation rule, 3) hysteresis behavior during unloading. In Japan, it is widely recognized that the Wakabayashi model can approximately purchase the inelastic behavior of steel compression member, and also it is expressed with simple formulation whose variable is only slenderness ratio. However, local buckling of steel compression member occurs during the ultimate states. It means that the ultimate behavior and restoring force characteristics are determined with not only slenderness ratio but width-thickness ratio. And also this model cannot express the deterioration of the limit compressive strength $-n_c$ and stiffness when the member is unloaded from full plastic state in tensile side. So, in this research, in order to distinguish the different deterioration behavior between the compressive side and tensile side, elastic unloading states are categorized into Stage D' and Stage E. This paper verify accuracy of Wakabayashi model and propose the new hysteresis model. Then, the hysteresis rules and the restoring force characteristics are arranged based on an above mentioned element deformation states.



(a) Past hysteresis model (Wakabayashi model)

(b) Proposal of this paper

Figure 1. Restoring force characteristics of steel compression members subjected to inelastic cyclic loadings

Table 1. Categorized stages of steel braces considering its state of deformation and damage

Stage A :	Full plastic state in tension
Stage B :	The section near the center of the member is in the yield state by a tensile force and a moment. The member is a stage where it has been extended from the state where it was crooked.
Stage C :	The section near the center of the member is in the yield state by a compressive force and a moment. The member is a stage where it is crooked rapidly.
Stage D	Elastic unloading state
Stage D' :	Elastic unloading state from the compressive side.
Stage E :	Elastic unloading state from the tensil side.

Outline of Wakabayashi model

This model focuses on braces having a rectangular or H section designed to buckle about its weak axis and a slenderness ratio λ of between 30 and 150. The idealized loop is composed of one mechanism line in tensile and compressive ranges respectively. Deterioration of strength and stiffness under load reversal is evaluated by translation rules of the mechanism lines and hysteresis behavior during unloading. The outline of the Wakabayashi model is described as shown in Fig.2.

Mechanism line

Where the mechanism line in tensile and compressive side is given as;

$$n = \frac{1}{(a \cdot \delta + b)^r} \tag{1}$$

a and b is function of nondimensional slenderness ratio λ^*, r is the coefficient showing the ratio of roundness.

Translation rule of mechanism line

Deterioration of strength and stiffness under load reversal is evaluated by translation rules of the mechanism lines. When one mechanism line in tensile and compressive ranges are translated, the translation rules of the target point of tensile side (point A in Fig.2 (a)) and compressive side (point B in Fig.2 (a)) are formulated statistically. The expression is a function of the ductility factor. Where the expression of translation rule for mechanism line in tensile side is given as;

$$x = (q_1 \cdot \bar{\delta}_a + 1) - q_2 \cdot s \tag{2}$$

Where q_1, q_2 are the function of nondimensional slenderness ratio, $\bar{\delta}_a$ is the ductility factor of compressive side, s is a variable of displacement from the unloading point in tension to the point which reaches yield strength.

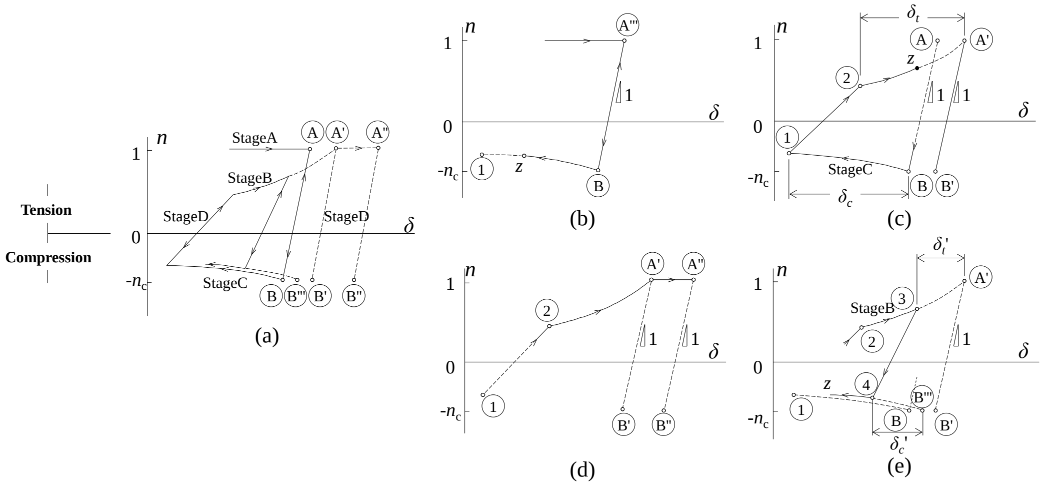
On the other hand, translation rule for mechanism line in compressive side changes with states of the member which receive tensile force. First, the case of unloading from Stage A, the mechanism line on the compressive side is translated as it pass along the intersection of limit compressive strength $-n_c$ and the elastic unloading line which is drawn from point A. Next, the case of unloading from Stage B, in order to maintain the consistency of hysteresis rules, a target point in compressive side must be located in the halfway point between point B' and point B like Fig.2 (e) shows. This movement is expressed the following equation as the movement y is proportional to δ_b.

$$y / y_0 = \delta_b / \delta_{b0} \tag{3}$$

Hysteresis behavior during unloading

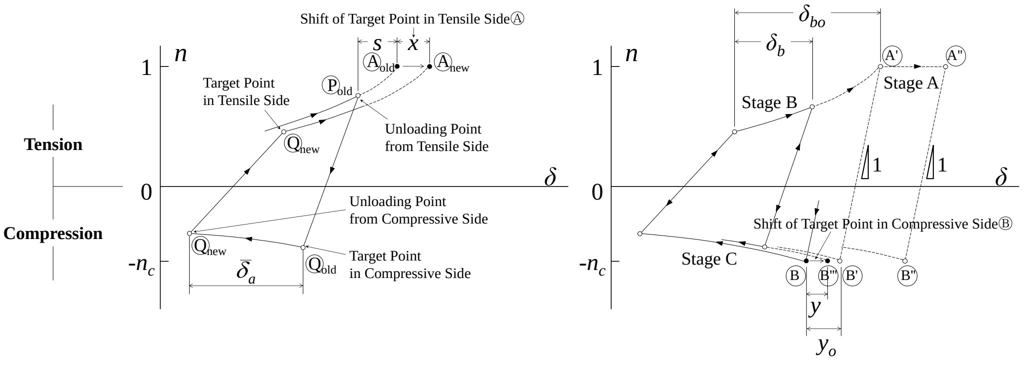
Hysteresis behavior of Wakabayashi model changes with the stage of deformation and plasticity. First, the case of unloading from Stage A, hysteresis loop reach the limit compressive strength $-n_c$ using initial elastic stiffness. On the other hand, the case of unloading form Stage B or C, Wakabayashi and others observed, from past experiments, that the relation of the ratio between the tensile side plastic deformation δ_t (δ'_t) with the compressive side plastic deformation δ_c (δ'_c) was not based on the value of amplitude but rather fixed. Where the ratio of the deformation q_3 ($=\delta_t/\delta_c=\delta'_t/\delta'_c$) is given as;

$$q_3 = 0.3\sqrt{n_E} + 0.24 \tag{4}$$



(a) (b) (c) (d) (e)

Figure 2. Hysteresis rule of Wakabayashi model



(a) Tensile side (b) Compressive side

Figure 3. Translation rule for mechanism line

ACCURACY VERIFICATION OF WAKABAYASHI MODEL

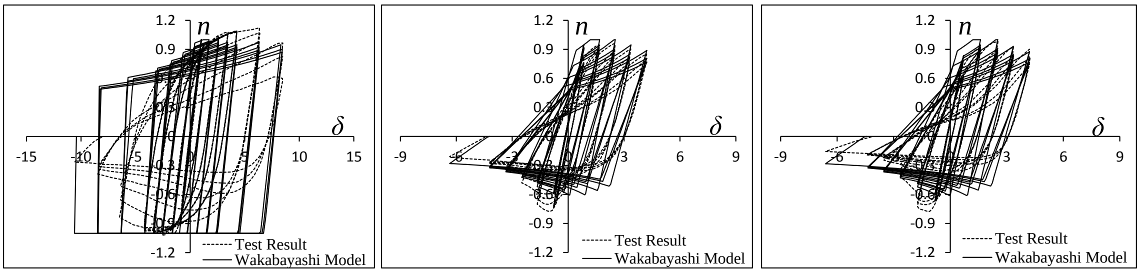
Data of past experimental results

In this chapter, by using reference of past experimental results, the accuracy of Wakabayashi model is verified, and the influence of each variable which give to the factor of errors and inelastic behavior is analyzed. Then the author has structured a database by reference of past reference published in Japan from 1973 to 1999. Its database stores 99 test results (18: rectangular sections, 33: H sections, 28: circular sections, 20: box sections), and these experimental conditions as length, size of section, boundary condition, yield strength and Young's modulus etc. are stored also. And also stored nondimensional slenderness ratio λ^*, and nondimensional width-thickness ratio β^*. Where λ^* and β^* is given as following. In addition, since local buckling occurred in the flange part first in all the experimental results stored in the database about H section, the width-thickness ratio of the flange part β_f^* is used.

λ^*	All	$\lambda^* = (l_b/i)\cdot\left(\sqrt{\varepsilon_y}/\pi\right)$	l_b : effective buckling length, i : radius of gyration, ε_y : yield strain
β^*	H	$\beta_f^* = b/t_f \cdot \sqrt{\varepsilon_{yf}}$	b : half of flange width, t_f : flange thickness, ε_{yf} : yield strain of flange
	Circular	$\beta^* = D/t \cdot \varepsilon_y$	D : diameter, t : thickness, ε_y : yield strain
	Box	$\beta^* = B/t \cdot \sqrt{\varepsilon_y}$	B : width, t : thickness, ε_y : yield strain

Comparison of hysteresis loop of Experimental result vs. Wakabayashi model

Here, examples of the experimental results of axial compression loading examinations about H sections are extracted from a database, and they are compared with the analytical results using the Wakabayashi model as shown in Fig.4. Although the Wakabayashi model is provided with the range of application, the case besides the range is also shown here for comparison and examination.



(a) λ^*=0.31 (λ=20), β_f^*=0.31 (b) λ^*=0.62 (λ=40), β_f^*=0.31 (c) λ^*=0.62 (λ=40), β_f^*=0.51

Figure 4. Comparison of hysteresis curve of past proposed model vs. past test result of H section

Mechanism line

For the analytical result by Wakabayashi model, it is observed that strength is large in the case of λ^* is small (Fig.4 (a)), and overestimated the mechanism line.

In the case of β_f^* is large (Fig.4 (c)), it is observed that strength is small than the case of β_f^* is small (Fig.4 (b)) in large deformation. And it is observed that the Wakabayashi model overestimates strength in the case of β_f^* is large. The model cannot consider the difference of width-thickness ratio. Therefore, it is required that hysteresis rules are considered of width-thickness ratio, since each experimental result shown in Fig.4 has produced local buckling in the flange part after lateral buckling. However, mechanism lines of the model are formulated only λ^*, so strength is overestimated.

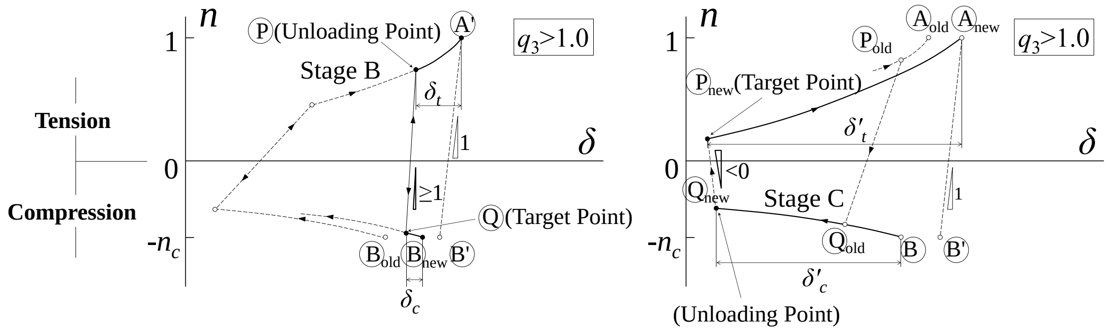
Translation rule of mechanism line

From the comparison of Fig.4, strength of unloading point in tensile side in each cycle is smaller than the test result. In brief, it was underestimated.

Hysteresis behavior during unloading

From Fig.4 (a), it is confirmed that Wakabayashi model overestimates strength and stiffness during unloading from compression to tension. If the Eq. (4) becomes large in case of small slenderness ratio, it gives the larger move for the target point on the mechanism line in tensile side (Fig.5(b), P_{new}). When Eq. (4) exceeds 1.0 as shown in Fig.5, the stiffness during unloading decided from a target point exceeds elastic stiffness. From these reasons, the stiffness during unloading of the Wakabayashi model is overestimated.

And also, as shown in Fig.6, if the unloading strength of tensile side does not exceed yield strength, the analytical result with Wakabayashi model can evaluate the deterioration of strength and rigidity on the test result accurately. But on the other hand, if the unloading strength of tensile side exceeds yield strength, it is observed that the deterioration of rigidity cannot be evaluated accurately and overestimate test result. The target point in the compressive side serves as the limit compressive strength ($-n_c$) which is not concerned plastic amplitude history, and overestimates strength in large deformation domain.



(a) Unloading from Stage B (b) Unloading from Stage C

Figure 5. Hysteresis behavior during unloading of Wakabayashi model

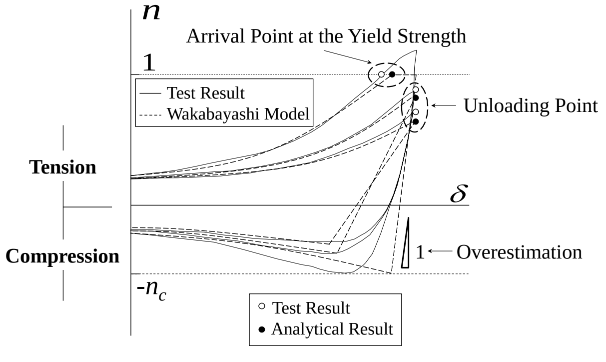


Figure 6. Hysteresis behavior during unloading on tensile side

PROPOSAL OF HYSTERESIS MODEL

Formulization of hysteresis model

In this paper, the hysteresis rule and inelastic characteristics of steel compression members are expressed with 1) mechanism line and 2) its translation rules, 3) hysteresis behavior during unloading based on each deformation and plasticity as shown in Fig.1 (b). By the examination of previous chapter, the Wakabayashi model may be able to pursue experimental results accurately, but error may be produced in unloading strength, stiffness and the hysteresis behavior.

Then this chapter, mechanism line and its translation rule and hysteresis behavior during unloading are analyzed about the influence of slenderness ratio, width-thickness ratio, and also plastic amplitude history based on consideration on the previous chapter and analysis of the database. Furthermore formulization of the hysteresis rule and inelastic characteristics is attempted using those variables. In this paper, those formulations are structured by regression analysis of test results of database based on each deformation and plasticity as shown in Fig.7.

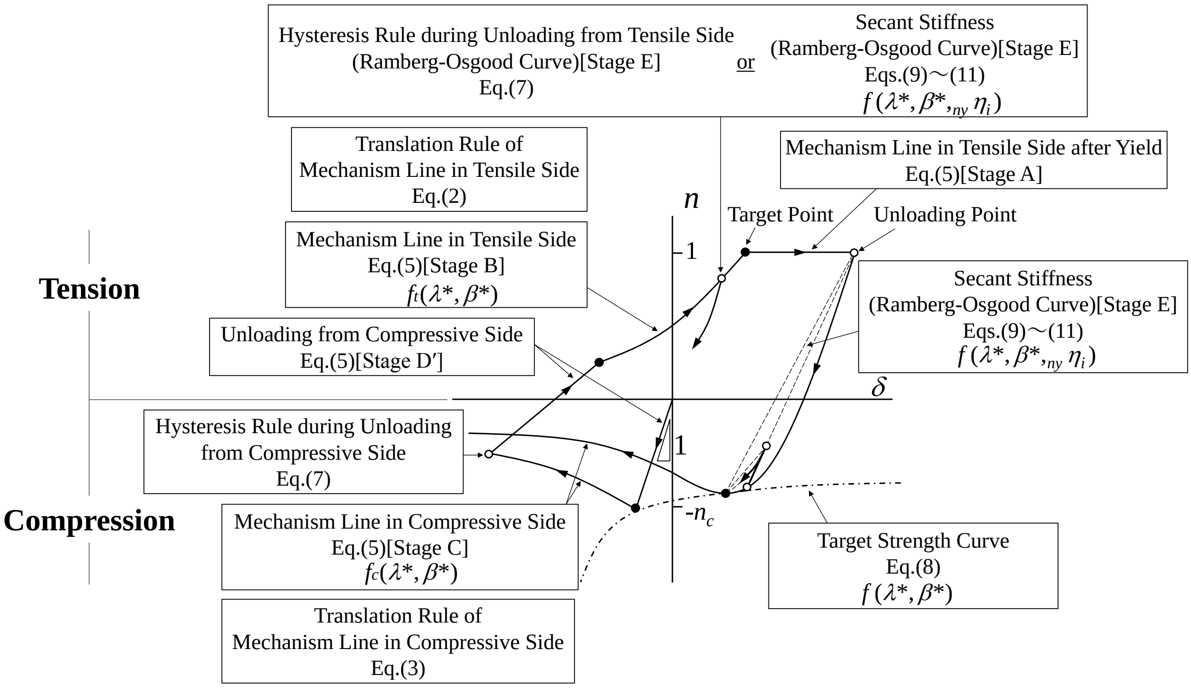


Figure 7. Proposal of hysteresis model considering deterioration

Mechanism line

Mechanism lines are expressed following equations which categorized into some stages considering with each deformation and plasticity as shown in Fig.1 (b).

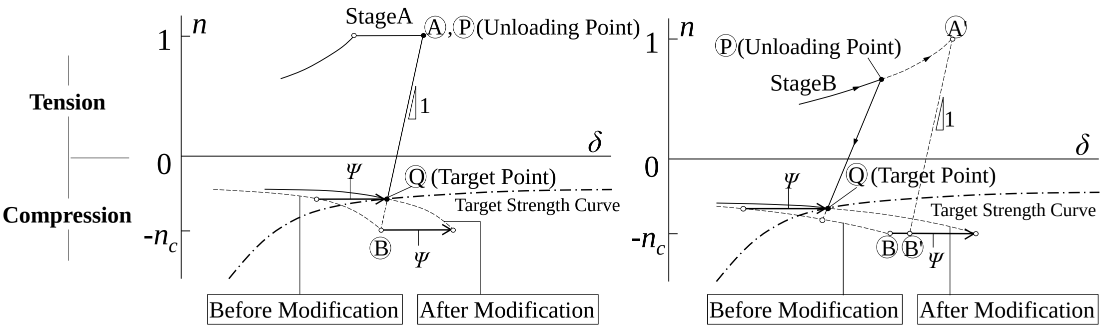
$$n = \begin{cases} 1 & \text{[Stage A]} \\ f_t\left(\delta^A - \delta\right) & \text{[Stage B]} \\ -f_c\left(\delta^B + \Psi + n_c - \delta\right) & \text{[Stage C]} \\ n^p + \left(\delta - \delta^P\right)\left(n^P - n^Q\right)/\left(\delta^P - \delta^Q\right) & \text{[Stage D']} \end{cases} \tag{5}$$

Where stage E is expressed with Ramberg-Osgood equation. In order to simplify explanation in a figure, Stage E is expressed with linear which connected an unloading point and a target point. δ^A, δ^B, δ^P and δ^Q is displacement of the target point or base point as shown in Fig.8. Ψ is a shift coefficient for moving target point in order to maintain the consistency of hysteresis rule as shown in Fig.8. And then, $f_c()$ and $f_t()$ in Eq. (5) is given as;

$$\begin{aligned} f_c(\mathrm{X}) &= \left(a_c\ \mathrm{X} + b\right)^{-r_c} \\ f_t(\mathrm{X}) &= \left(a_t\ \mathrm{X} + 1\right)^{-r_t} \end{aligned} \tag{6}$$

In this section, the relations between a_c, b, a_t of Eq. (6) and slenderness ratio, width-thickness ratio are analyzed for every experimental result of a database, and showed for every sectional type in Fig.9. γ_c and γ_t are fixed by sectional type not to relate with slenderness ratio or width-thickness ratio. And also they are calculated as n-δ relation of skeleton curve for experimental results shows linear relation.

From Fig.9, the correlations between a_c, b, a_t and slenderness ratio, width-thickness ratio are observed. Then regression analysis is applied these parameters, and the result is shown as Table 2.



(a) Unloading from Stage A (b) Unloading from Stage B

Figure 8. Hysteresis rule considering deterioration

Translation rule of mechanism line

From the examination of previous chapter, it is not necessary to correct about a translation rule. So then, the translation rule of Wakabayashi model is adopted.

Hysteresis behavior during unloading

From the examination of previous chapter, in the case of a specimen with small slenderness ratio, the stiffness during unloading sometimes exceeds the elastic stiffness. So the limitation is imposed on its model as presented in Eq. (7). This limitation makes that stiffness during unloading does not exceed the elastic stiffness. As a result, inelastic behavior during unloading is expressed as shown in Fig. 10.

$$q_3 = 0.3\sqrt{n_E} + 0.24 \leq 1.0 \tag{7}$$

Furthermore, it is considered that the hysteresis behavior during unloading from tensile side is difference between unloading from Stage A and Stage B. From the previous examination, hysteresis behavior during unloading from Stage B can evaluate accurately. But if cumulative deformation becomes large, error will produce between Wakabayashi model and experimental result about the behavior during unloading from Stage A. Because, if the member unloads from Stage A, unloading stiffness is recovered to elastic stiffness and also the target point on the compressive side is recovered to the limit compressive strength (Fig.6 ($-n_c$)). But actually, since the hysteresis curve during unloading softens and its upper limit in compressive side deteriorates along target strength curve as shown in Fig.11, the error has produced between analytical result and experimental result. Then in this paper, about during unloading from Stage A, deterioration of the unloading stiffness and target strength in compressive side affected with plastic amplitude history is formulized by analyzing the experimental results of a database.

First, it is explained about the deterioration of target strength in compressive side. It is formulized based on experimental results. The limitation named target strength curve shown in Fig.11 is assumed. And also this curve is defined following equation.

$$n = -c_1 - \frac{c_3}{(\delta + c_2)} \tag{8}$$

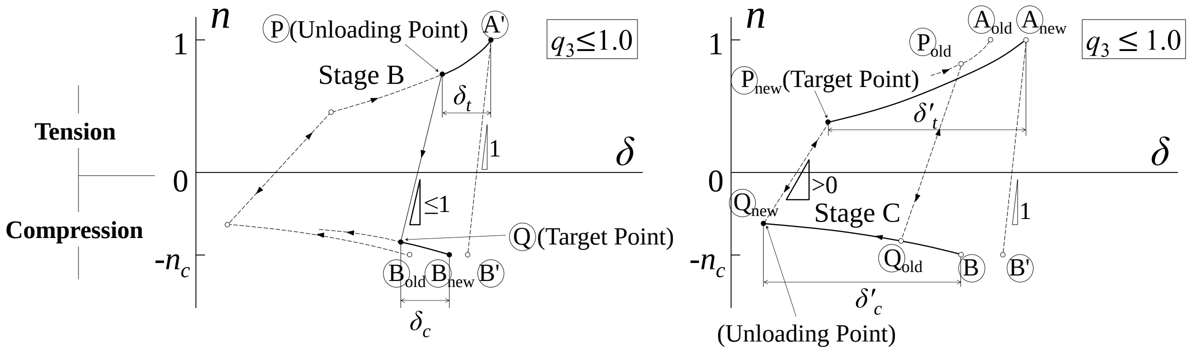
In this section, the relations between c_1, c_2, c_3 of Eq. (8) and slenderness ratio, width-thickness ratio are analyzed for every experimental result of a database, and showed for every sectional type in Fig.13. From the figure, the correlations between c_1, c_2, c_3 and slenderness ratio, width-thickness ratio are observed. Then regression analysis is applied these parameters, and the result is shown as Table 3.

Second, it is explained about the deterioration of stiffness during unloading from Stage A. In order to examine the hysteresis behavior and hysteresis rule during unloading from tensile side, as shown in Fig.12, unloading point in tensile side and the maximum strength point on the compressive side are connected with a straight line, and the slope of the line is defined secant stiffness k_a, it is calculated from hysteresis loop of experimental results. As shown in the figure, by paying attention to the only loop which reached yield strength in tensile side, the relations between cumulative tensile plasticity rate $_{ny}\eta$ and secant stiffness k_a are shown in Fig.14 by the use of the different symbol depending on the value of nondimensional slenderness ratio and nondimensional width-thickness ratio for every sectional type. About H sections, it is observed that the tendency which secant stiffness k_a decreases as cumulative tensile plasticity rate $_{ny}\eta$ increases from the result of the Fig.14. On the other hand, it is observed that box sections and circular sections are not depended on the value of cumulative tensile plasticity rate, but generally shown fixed value. And also, it is observed that the tendency which secant stiffness k_a increases as the value of nondimensional slenderness ratio or nondimensional width-thickness ratio becomes small about all sectional type. Based on the above analytical results, the relation between secant stiffness during unloading beyond yield strength and cumulative tensile plasticity rate is assumed by the following equations for every sectional type. And also, the result of regression analysis is shown in Fig.15 and Table 4.

(H sections) $$k_a = (1 + c_5 / c_4) + c_5 / ({}_{ny}\eta_i - c_4) \tag{9}$$

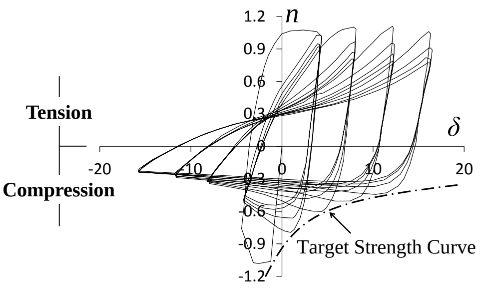
(Box sections) $$k_a = c_4 \tag{10}$$

(Circular sections) $$k_a = c_4 \tag{11}$$



(a) Unloading from Stage A (b) Unloading from Stage B

Figure 10. Hysteresis behavior during unloading of proposed model



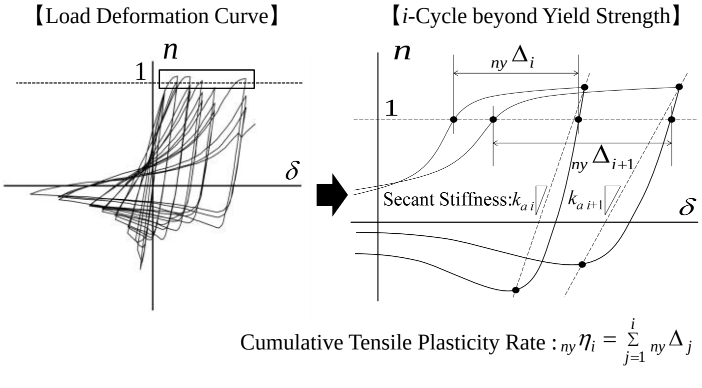
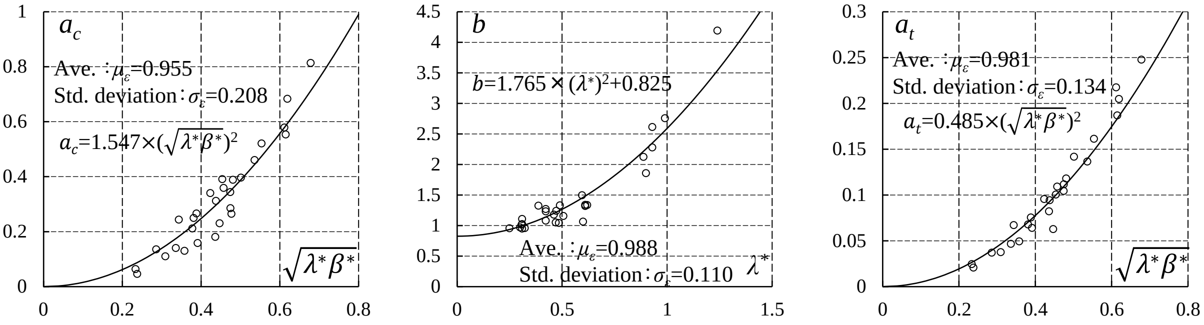
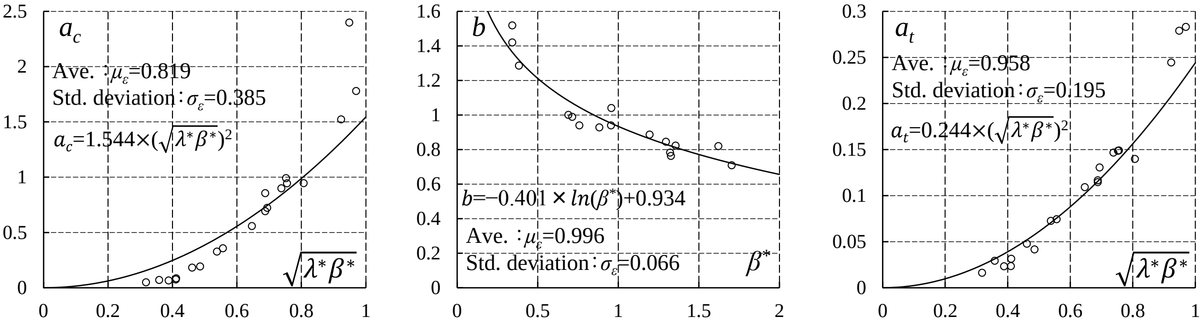


Figure 11. Target strength curve of compressive side

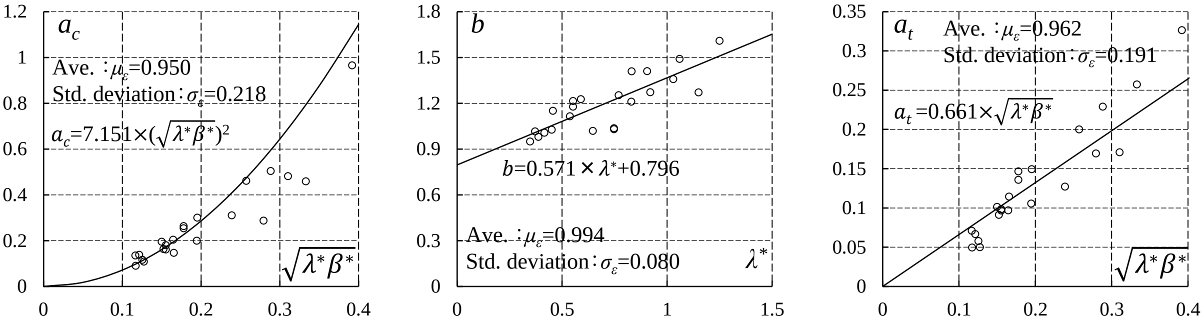
Figure 12. Hysteresis behavior during unloading and secant stiffness of tensile side



(a) H sections



(b) Box sections

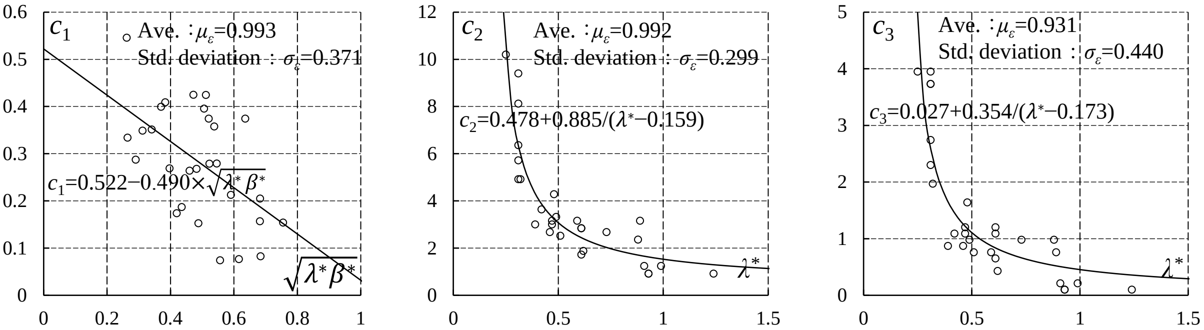


(c) Circular sections

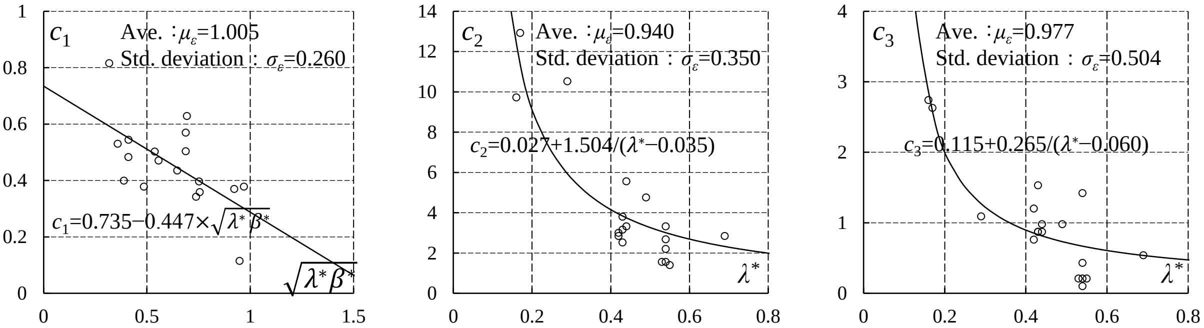
Figure 9. Relation of parameters of mechanism line vs. slenderness ratio, width-thickness ratio

Table 2. Regression equation of parameters of mechanism line

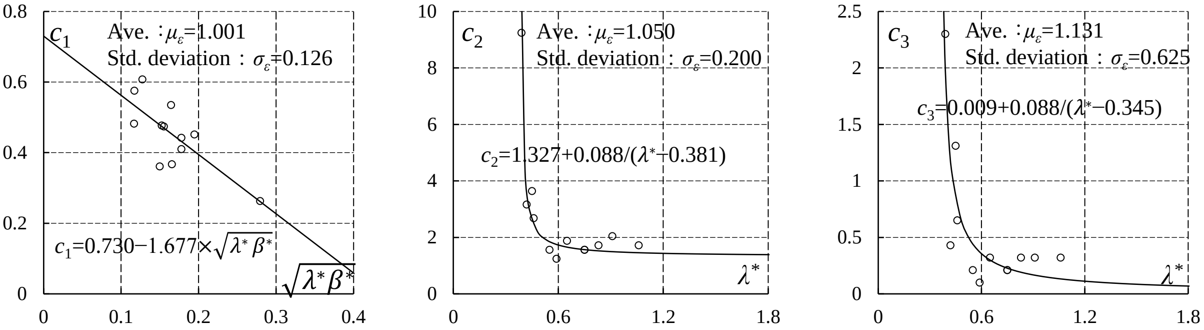
	a_c	a_t	b	r_c	r_t
H	$1.547\times\left(\sqrt{\lambda^*\beta^*}\right)^2$	$1.765\times\left(\lambda^*\right)^2+0.825$	$0.485\times\left(\sqrt{\lambda^*\beta^*}\right)^2$	1.0	3.0
Box	$1.544\times\left(\sqrt{\lambda^*\beta^*}\right)^2$	$-0.401\times\log\left(\beta^*\right)+0.934$	$0.244\times\left(\sqrt{\lambda^*\beta^*}\right)^2$	1.0	3.0
Circular	$7.151\times\left(\sqrt{\lambda^*\beta^*}\right)^2$	$0.571\times\lambda^*+0.796$	$0.661\times\sqrt{\lambda^*\beta^*}$	1.5	3.0



(a) H sections



(b) Box sections

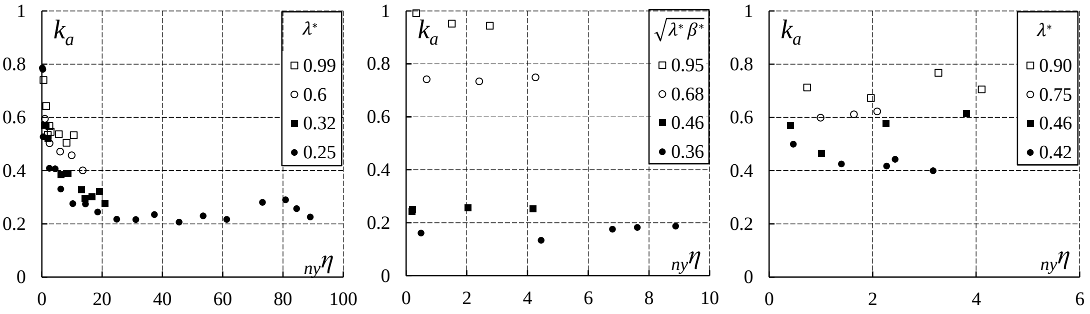


(c) Circular sections

Figure 13. Relation of parameters of target strength curve vs. slenderness ratio, width-thickness ratio

Table 3. Regression equation of target strength curve

	c_1	c_2	c_3
H	$0.522 - 0.490 \times \sqrt{\lambda^* \beta^*}$ $\left(\sqrt{\lambda^* \beta^*} \leq 1.065\right)$	$0.478 + 0.885 / (\lambda^* - 0.159)$ $\left(0.159 < \lambda^*\right)$	$0.027 + 0.354 / (\lambda^* - 0.173)$ $\left(0.173 < \lambda^*\right)$
Box	$0.735 - 0.447 \times \sqrt{\lambda^* \beta^*}$ $\left(\sqrt{\lambda^* \beta^*} \leq 1.644\right)$	$0.027 + 1.504 / (\lambda^* - 0.035)$ $\left(0.035 < \lambda^*\right)$	$0.115 + 0.265 / (\lambda^* - 0.060)$ $\left(0.060 < \lambda^*\right)$
Circular	$0.730 - 1.677 \times \sqrt{\lambda^* \beta^*}$ $\left(\sqrt{\lambda^* \beta^*} \leq 0.435\right)$	$1.327 + 0.088 / (\lambda^* - 0.381)$ $\left(0.381 < \lambda^*\right)$	$0.009 + 0.088 / (\lambda^* - 0.345)$ $\left(0.345 < \lambda^*\right)$



(a) H sections (b) Box sections (c) Circular sections

Figure 14. Relation of cumulative tensile plasticity rate vs. secant stiffness

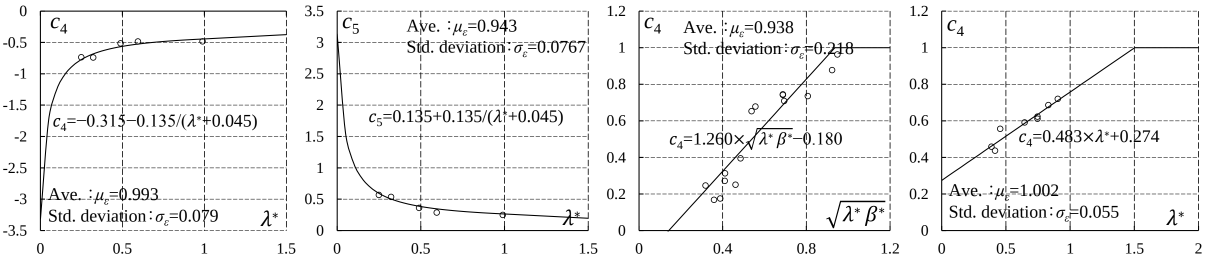


Figure 15. Relation of parameters of secant stiffness vs. slenderness ratio, width-thickness ratio

Table 4. Regression eq. of secant stiffness

	c_4	c_5
H	$-0.315-0.135/(\lambda^*+0.045)$	$0.135+0.135/(\lambda^*+0.045)$
Box	$1.260\times\sqrt{\lambda^*\beta^*}-0.180$ $\left(0.143\le\sqrt{\lambda^*\beta^*}\right)$	—
Circular	$0.483\times\lambda^*+0.274$	—

Comparison of the proposed hysteresis model and test result

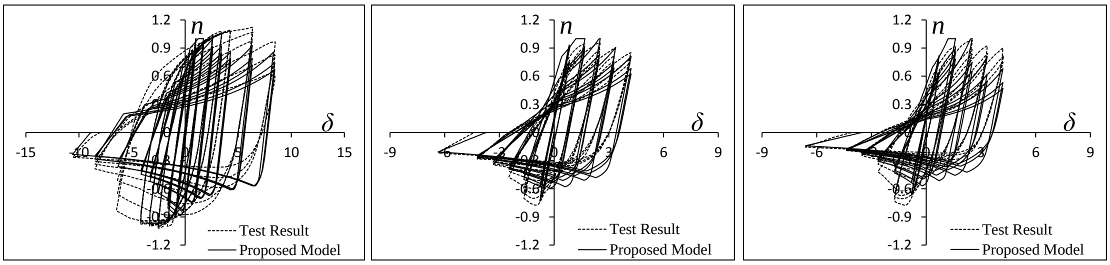
Comparisons of the analytical results by the restoring force characteristic model using the hysteresis rules formulized by the above analysis and experimental results are shown in Fig.16. From the comparisons, the proposed model can purchase the deterioration behavior of strength and rigidity about test result very well.

CONCLUSIONS

In this paper, the past experimental data about the steel compression member was analyzed and the restoring force characteristic model in considering deterioration behabior was formulized.

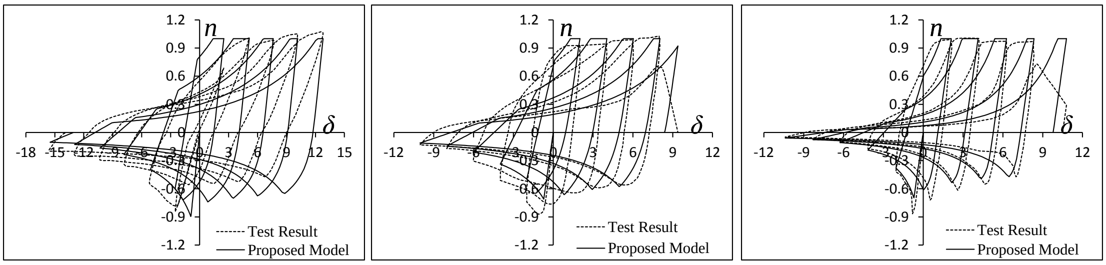
1) Inelastic behavior of steel compression member is explained based on each deformation and plasticity of steel compression member with reference about hysteresis model and mechanical property.
2) Database about past test data on reference is structured. From the comparison with past test results and analytical results of past hysteresis model proposed in Japan, the cause of error and accuracy of the model are analyzed. Furthermore it is analyzed that the relation between inelastic behavior and nondimennsional slenderness ratio λ^*, nondimensional width-thickness ratio β^*, cumulative tensile plasticity rate ${}_{ny}\eta$. According to the past references, the hysteresis behavior are expressed with 1) mechanism line, 2) its translation rule and hysteresis behavior during unloading. And also, they are formulated by regression analysis.
3) The mechanism lines are formulated with parameters as λ^* and β^*.

4) It is observed that the translation rule of mechanism line of Wakabayashi model can pursue the test results accurately, and it underestimates the past test results. From these observations, translation rule of Wakabayashi model is applied for proposal model in this paper.
5) It is observed that the stiffness of unloading with Wakabayashi model sometimes exceeds the elastic stiffness. So the limitation is imposed. Furthermore, the target strength curve of compressive side is formulated with parameters as λ^* and β^*. And also, if the unloading strength of tensile side exceeds yield strength, it is observed that the secant stiffness k_a connecting the unloading point and target strength of compressive side is gradually decreased as ${}_{ny}\eta$ is increased. From this observation, k_a is formulated with parameters as ${}_{ny}\eta$, λ^* and β^*.
6) Finally, the proposed hysteresis characteristics model compares with past test results, it shows the good agreements with each other.
7) Although the hysteresis model proposed in this paper is based on the Wakabayashi model, it is a model considering the effects of β^* and plastic amplitude history other than λ^*. Furthermore, although the applicable limitation of sectional type and slenderness ratio are provided at the past hysteresis model, the model proposed in this paper extends an applicable range.



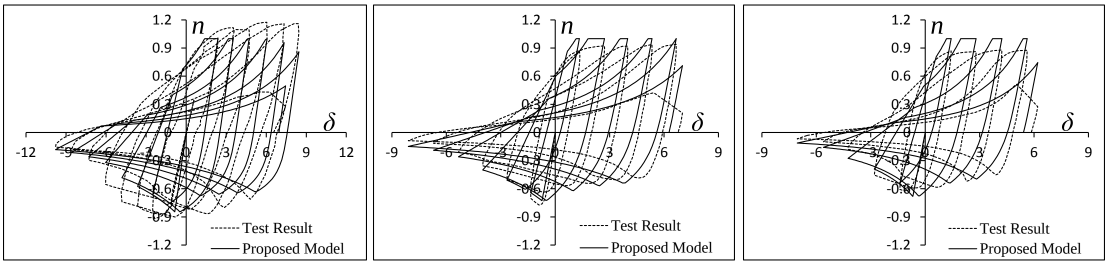
(a-1) λ^*=0.31, β^*=0.31 (a-2) λ^*=0.62, β^*=0.31 (a-3) λ^*=0.62, β^*=0.51

(a) H sections



(b-1) λ^*=0.16, β^*=1.36 (b-2) λ^*=0.43, β^*=0.71 (b-3) λ^*=0.43, β^*=1.33

(b) Box sections



(c-1) λ^*=0.39, β^*=0.04 (c-2) λ^*=0.55, β^*=0.04 (c-3) λ^*=0.54, β^*=0.07

(c) Circular sections

Figure 16. Comparison of hysteresis curve of past test results vs. analytical results of proposed model

REFERENCES

Wakabayashi M., Shibata M. (1982). Mathmatical expression of hysteretic behavior of braces, part 1. Transactions of the Architectural Institute of Japan. Volume 316, pp.18-24.