MEASURING RESIDUAL RISK IN EARTHQUAKES

Friedemann WENZEL¹, Ellen FREY², James DANIELL³

ABSTRACT

Even if buildings are built according to codes that assure non-collapse at a certain ground motion level (for instance 475 year return period) local ground motion can easily exceed this level even in medium-size earthquakes. This has been dramatically demonstrated in the Christchurch earthquake of 22.02.2011 (EERI, 2011) and is reflected in discussions regarding the value of hazard assessment (Wyss et al., 2010). The ground motion above the code level gives rise to the residual risk in an earthquake-prone area. Logically one would like to define the code level from a residual risk perspective: Is a certain residual risk acceptable? If not the safety margins need to be raised. This, however, requires the availability of measures of residual risk; two of which we discuss here. We quantify residual risk by studying the probability of building collapse by using standard hazard models and building types (HAZUS typology) by looking at the collapse probabilities beyond the design ground motion level. Our measure of residual risk for a certain building type is the collapse probability dependent on return periods of ground motion or the unconditional annual collapse probability for all PGA-values of the hazard curve. Another measure - or better proxy - of residual risk in a seismotectonic zone would be the annual probability with which the code ground motion is exceeded anywhere in the zone. Although this is not a loss quantity and thus not a direct measure of risk it can be easily translated in risk with information on average population density, capital stock, etc. in the zone. We explain the probabilistic evaluation of the residual risk proxy and discuss it further with data from the moderate seismicity Upper Rhinegraben in Western Europe.

INTRODUCTION

The UNISDR (2009) defines residual risk as, the risk that remains in unmanaged form, even when effective disaster risk reduction measures are in place, and for which emergency response and recovery capacities must be maintained. In insurance residual risk often refers risks that are not insurable for a given market, that underwriters do not want (Pfeffer, 1974). With specific view on earthquakes the Earthquake Engineering Handbook (Scawthorne, 2002) states: 'The basis for this (emergency) plan should be the residual risk, that is, that the portion of the baseline risk that the implemented mitigation alternatives cannot feasibly reduce. Understanding, preparing for, and clearly communicating this residual risk, that is, the loss that has been deemed acceptable, is necessary to avoid surprises and unnecessary recriminations following the earthquake'.

For residential and commercial buildings and facilities subject to code provisions, the ground motion above the code level gives rise to the residual risk in an earthquake-prone area. As Scawthorne (2002) stresses knowledge of the residual risk is critical in order to know and communicate the accepted risk and to prepare for the case of exceedance of mitigated risk level. Logically one would like to define the code level from a residual risk perspective: Is a certain residual risk acceptable? If not the safety margins need to be raised.
In insurance the standard way to assess (earthquake) risks to a portfolio is to estimate Probable Maximum Loss (PML) curves that represent the loss to the portfolio in one earthquake dependent on return periods. This is usually done by stochastically generating hazard-consistent earthquakes, evaluate the loss of each of them to the portfolio and counting the exceedance rates of losses. This is a common procedure applied in insurance to develop the pricing of risk but also to determine the necessary solvency of the insurer. In this paper we develop two ways to measure residual risk. The first refers to specific buildings, which are characterized by capacity and fragility curves. The fragility expressed in ground motion parameters can be directly related to hazard (=annual ground motion exceedance) at the site of the building. The second approach is not a direct measure but rather a proxy. It quantifies the area within which the design ground motion will be exceeded in a given return period.

We quantify residual risk by studying the probability of building collapse for standard hazard models and building types (HAZUS typology) by looking at the collapse probabilities beyond the design ground motion level. HAZUS (2003) provides the conditional probability of complete structural damage, given a certain PGA level. These curves are parameterized by a cumulative log-normal distribution, which requires only two parameters – the median and the variance. Our measure of residual risk for a certain building type is the collapse probability depended on return periods of ground motion or the unconditional annual collapse probability for all PGA-values of the hazard curve. We show results for C2M buildings exposed to very high and moderate hazard conditions. Apart from the quantitative description of the residual risk one can also address the question of whether and to what extent buildings designed for the same return period ground motion (for instance 475 years) have the same residual risk. In addition one can estimate the unconditional probability of collapse.

Further we define a new quantity, the areal ground motion exceedance as residual risk proxy. We assume a fairly uniform hazard in an area with earthquake activity described by a Gutenberg-Richter frequency magnitude relation $N(m)$. With a ground motion attenuation relation – in the simplest case it depends on magnitude and distance only – the average area within which a specified ground motion value, for instance the design ground motion $z_d$ is exceeded in an earthquake of magnitude $m$ can be evaluated. This area of ground motion exceedance is experienced with a return period given by the magnitude frequency relation. By eliminating the magnitude one can construct a relation between the return period and the associated area within which $z_d$ is exceeded. Although this area value is not a loss quantity and thus not a direct measure of risk it can be easily translated in risk with information on average population density, capital stock, etc. in the zone. We explain the probabilistic evaluation of the residual risk proxy and discuss it further with data from the moderate seismicity Upper Rhinegraben in Western Europe.

**Residual risk measures for a particular building type**

HAZUS provides model fragility functions depended on PGA in the functional form of a cumulative log-normal distribution, so that the probability of complete damage (= collapse) is available as function of PGA. Thus, the fragility function depends only on the median value of ground motion and the variance parameter that determines the range of PGA values within which the probability soars up from low to high values around the median. Interestingly HAZUS does not vary the variance parameter with building height or code level. It always has a value of 0.64. The building we consider in this paper is a concrete sheer wall building. The vertical components of the lateral – force–resistive system in these buildings are concrete sheer walls that are usually bearing walls. We look at midrise buildings with 4 to 7 but typically 5 stories. This building is characterized in HAZUS as C2M.

HAZUS classifies damage into four categories: slight, moderate, extensive, and complete structural damage. Complete structural damage – we also use the word collapse for this damage state means that the structure has collapsed or is in imminent danger of collapse due to failure of most of the shear walls and failure of some critical beams or columns. Typically 10% of the total area of C2M buildings is expected to be collapsed. For residual risk evaluations we only look at the damage class of complete structural damage in high hazard areas as compared to low hazard areas. As typical example of a high hazard area we take Los Angeles (Palos Verde) and use a hazard curve that has been
developed by LLNL (Lawrence Livermore National Laboratory) for a nuclear power plant in the area on hard rock (solid class B) in J. Savy and B. Foxall (2003). We use hazard assessment for nuclear power plants, as they are usually carefully done for long return periods. As typically low hazard we use the hazard curve of the Swiss nuclear power plant Beznau; the assessment for which has been done in the context of the PEGASOS (2004) project. We simplify the hazard curves to power laws characterized by the reference ground motion at a hundred year return period and different exponents. With ground motion \( z \) in PGA, annual exceedance rate \( H \) we have

\[
H(z) = H_0 \cdot \left( \frac{z_0}{z} \right)^k
\]  

(1)

or specifically for Beznau

\[
H_b(z) = \frac{1}{100 \text{ yr}} \cdot \left( \frac{0.31 \text{ m/s}^2}{z} \right)^{3.17}
\]

and Los Angeles

\[
H_h(z) = \frac{1}{100 \text{ yr}} \cdot \left( \frac{1.79 \text{ m/s}^2}{z} \right)^{4.50}
\]

(2)

(3)

The return period associated with a given ground motion level is

\[
T(z) = \frac{1}{H(z)}
\]

It is obvious, that the high hazard is characterized by an almost six times higher ground motion level at 100 year return period, but also by a much deeper slope, as expressed in the higher power law exponent of 4,50 (Los Angeles) as compared to 3,17 (Beznau). The respective ground motion levels at 475 year return period are 0,51 m/s² (Beznau) and 2,53 m/s² (Los Angeles) so that at this higher hazard level the relation is one to five rather than one to six, caused by the higher power law exponent for Los Angeles. HAZUS provides model fragilities dependent on PGA for its building types on different code levels. They are parameterized as cumulative log-normal distribution

\[
P(C|z) = \Phi \left( \frac{1}{\beta} \ln \left( \frac{z}{z_0} \right) \right)
\]

(4)

with the cumulative normal distribution \( \Phi(x) \). \( P(C|z) \) is the probability of complete damage at ground motion \( z \). It can be expressed in terms of return period

\[
P(C|T) = \Phi \left( \frac{1}{\beta} \ln \left( \frac{z_0 \cdot (H_0 \cdot T)^{0.5}}{\bar{z}} \right) \right)
\]

(5)

So that complete damage probabilities can be estimated for return periods beyond the design return period ground motion (here assumed as 475 years). Table 1 shows the median value of the fragility function for complete damage in units of gravity (g) and the variance parameter for the 4 code levels used. We assume initially that a C2M building has to be built according to high code standards in Los Angeles and according to low code standards in Beznau. If this is the case and using the values of table 1 we find a probability for complete damage of \( P(C|T_h) = 0.8\% \) for Los Angeles and \( P(C|T_h) = 0.05\% \) for Beznau, which represents a sixteen time smaller collapse probability at the same 475 year return period hazard level of the in both places.
Table 1. Median and variance values for HAZUS C2M complete damage fragility functions

<table>
<thead>
<tr>
<th>Code level</th>
<th>( \bar{\tau}(g) )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>1.95</td>
<td>0.64</td>
</tr>
<tr>
<td>Moderate</td>
<td>1.02</td>
<td>0.64</td>
</tr>
<tr>
<td>Low</td>
<td>0.63</td>
<td>0.64</td>
</tr>
<tr>
<td>Pre</td>
<td>0.5</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Using the complete damage probabilities of 0.08 % (C2M high-code building in high hazard area) and 0.005% (C2M low-code building in low hazard area) we find their residual risk at a return period of 2.500 years of 5% and 1% and at 10.000 years of 2% and 1%. Thus the residual risk of the C2M high-code building in the high hazard area remains higher than the residual risk of the C2M low-code building in the low hazard area even for rather large return periods, although the difference becomes smaller with growing return period. It falls off from a factor of 16 time higher at 475 years to 5 times higher at 2.500 years to only 2 times higher at 10.000 years.

In order to develop some understanding of the meaning of a 0.1 % collapse probability we use the HAZUS description of this damage as 10 % collapsed floor area. Assuming that due to occupancy and/or time of the day 5 % of inhabitants are killed, and estimating the total number of residents in such a building between 25 and 50 we expect typically 2 persons killed in one of these buildings at the collapse state. Using the probability of 0.1 % and a return period of 500 years this represents a probability of 4 times 10^{-6} per person per year being killed. If we compare this value with the chance of dying in a traffic accident in Germany we find that 3.500 fatal accidents per year in a population of 80 million people results in about a chance of 40 times 10^{-6} per year per person of being killed. This is 10 times higher as the earthquake death probability. Therefore one can take the 0.1 % collapse probability as a reasonable value of safety at the designed ground motion level. In the following we do not use low and high code levels anymore but use the somehow artificial assumption that in both – low and high hazard – areas buildings of C2M type have been constructed with equal probability of collapse (\( P(C|T_H) = 0.1\% \)) at the design ground motion level at a return period of 475 years. These ‘artificial’ buildings are used in order to compare the residual risks on a fair basis. We are very well aware of the fact that buildings are usually built much better than the code level for which its safety against collapse has to be demonstrated in the code application procedure. However, with this approach we can better demonstrate the systematic deviation of safety from a residual risk prospective.

As for low and high hazard areas we do not predominantly compare the ground motion levels but the return periods of those ground motion levels we express the probability of complete damage as a function of the return period. Using the simple functional forms for hazard and fragility we come up with the following formula:

\[
P(C|T) = \Phi\left(\frac{1}{\beta} \ln \left(\frac{\tau_g \cdot (H_0 \cdot T)^{\alpha}}{\bar{\tau}}\right)\right) = \Phi\left(q_H + \frac{1}{\beta \cdot k} \ln \left(\frac{T}{T_H}\right)\right)
\]

The probability depends on the design return period, the variance parameter of the fragility function and the hazard power law exponent. In addition it is controlled by the probability level of collapse at the design return period. Figure 1 shows the case of the ‘artificial’ buildings in both hazard areas which are characterized by the identical collapse probability at 475 years return period. It is obvious that at the design return period both buildings show the same collapse probability of 0.1 %. For higher return periods the building in the low hazard area shows a higher collapse probability as compared to the building in the high hazard area. This allows the statement that buildings at the same safety level at the designed turn period may have significantly different collapse probabilities for higher return periods. At a return period of 2.500 years the probability of complete damage for a C2M building in the low hazard area is 1%, and only 0.55% in the high hazard area. At a return period of 10.000 years the probability of complete damage for a C2M building in the low hazard area is 6%, and

---

Using the complete damage probabilities of 0.08 % (C2M high-code building in high hazard area) and 0.005% (C2M low-code building in low hazard area) we find their residual risk at a return period of 2.500 years of 5% and 1% and at 10.000 years of 2% and 1%. Thus the residual risk of the C2M high-code building in the high hazard area remains higher than the residual risk of the C2M low-code building in the low hazard area even for rather large return periods, although the difference becomes smaller with growing return period. It falls off from a factor of 16 time higher at 475 years to 5 times higher at 2.500 years to only 2 times higher at 10.000 years.

In order to develop some understanding of the meaning of a 0.1 % collapse probability we use the HAZUS description of this damage as 10 % collapsed floor area. Assuming that due to occupancy and/or time of the day 5 % of inhabitants are killed, and estimating the total number of residents in such a building between 25 and 50 we expect typically 2 persons killed in one of these buildings at the collapse state. Using the probability of 0.1 % and a return period of 500 years this represents a probability of 4 times 10^{-6} per person per year being killed. If we compare this value with the chance of dying in a traffic accident in Germany we find that 3.500 fatal accidents per year in a population of 80 million people results in about a chance of 40 times 10^{-6} per year per person of being killed. This is 10 times higher as the earthquake death probability. Therefore one can take the 0.1 % collapse probability as a reasonable value of safety at the designed ground motion level. In the following we do not use low and high code levels anymore but use the somehow artificial assumption that in both – low and high hazard – areas buildings of C2M type have been constructed with equal probability of collapse (\( P(C|T_H) = 0.1\% \)) at the design ground motion level at a return period of 475 years. These ‘artificial’ buildings are used in order to compare the residual risks on a fair basis. We are very well aware of the fact that buildings are usually built much better than the code level for which its safety against collapse has to be demonstrated in the code application procedure. However, with this approach we can better demonstrate the systematic deviation of safety from a residual risk prospective.

As for low and high hazard areas we do not predominantly compare the ground motion levels but the return periods of those ground motion levels we express the probability of complete damage as a function of the return period. Using the simple functional forms for hazard and fragility we come up with the following formula:

\[
P(C|T) = \Phi\left(\frac{1}{\beta} \ln \left(\frac{\tau_g \cdot (H_0 \cdot T)^{\alpha}}{\bar{\tau}}\right)\right) = \Phi\left(q_H + \frac{1}{\beta \cdot k} \ln \left(\frac{T}{T_H}\right)\right)
\]

The probability depends on the design return period, the variance parameter of the fragility function and the hazard power law exponent. In addition it is controlled by the probability level of collapse at the design return period. Figure 1 shows the case of the ‘artificial’ buildings in both hazard areas which are characterized by the identical collapse probability at 475 years return period. It is obvious that at the design return period both buildings show the same collapse probability of 0.1 %. For higher return periods the building in the low hazard area shows a higher collapse probability as compared to the building in the high hazard area. This allows the statement that buildings at the same safety level at the designed turn period may have significantly different collapse probabilities for higher return periods. At a return period of 2.500 years the probability of complete damage for a C2M building in the low hazard area is 1%, and only 0.55% in the high hazard area. At a return period of 10.000 years the probability of complete damage for a C2M building in the low hazard area is 6%, and
only 2% in the high hazard area. Thus the spread in residual risk grow with return period from a factor of 2 to a factor of 3.

The simple functional forms we use for hazard and fragility allows understanding this affect in a systematic way. The collapse probabilities as function of return period in low and high hazard areas

\[ P_l(C|T) = \Phi \left( q_h + \frac{1}{\beta_l \cdot k_l} \ln \left[ \frac{T}{T_h} \right] \right) \]  

\[ P_h(C|T) = \Phi \left( q_h + \frac{1}{\beta_h \cdot k_h} \ln \left[ \frac{T}{T_h} \right] \right) \]

(7a)  

(7b)

can intersect only at one point (the design return period). The exceptional case occurs if \( \beta_l \cdot k_l = \beta_h \cdot k_h \), then both curves being equal. Only in this case equal safety exists for all return periods. Otherwise the derivatives of the probabilities at the design return period \( T_h \) controls which building has higher residual risk as compared to the other. It is easy to show that the derivative is

\[ \frac{dP_l(C|T)}{dT} \bigg|_{T_h} = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{k_l \cdot \beta_l \cdot T_h} \cdot \exp \left( -\frac{1}{2} \frac{q_h^2}{\beta_l \cdot T_h} \right) \]  

\[ \frac{dP_h(C|T)}{dT} \bigg|_{T_h} = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{k_h \cdot \beta_h \cdot T_h} \cdot \exp \left( -\frac{1}{2} \frac{q_h^2}{\beta_h \cdot T_h} \right) \]

(8a)  

(8b)
If $\beta_i \cdot k_i < \beta_h \cdot k_h$ the residual risk for the low hazard area building is higher than the one for the high hazard area.

If $\beta_i \cdot k_i > \beta_h \cdot k_h$ the residual risk for the low hazard area building is lower than the one for the high hazard area.

If $\beta_i \cdot k_i = \beta_h \cdot k_h$ the residual risk for the low hazard area building is equal to the one for the high hazard area.

The annual unconditional probability of complete damage can be evaluated analytically if the simple relations for hazard (power law) and fragility (cumulative log-normal) apply and if we believe that these relations – although established for a limited range of ground motion only – are valid from arbitrarily small to arbitrarily large ground motion. The annual probability of complete damage can be written as

$$P_a(C) = \int P(C|z) \cdot \left| \frac{dH(z)}{dz} \right| dz$$

with the analytic solution (derived by McGuire et al., 2000)

$$P_a(C) = \frac{1}{T_H} \cdot \exp(k \cdot \beta \cdot q_H) \cdot \exp\left\{ \frac{1}{2} (k \cdot \beta)^2 \right\}$$

A 0.1 % safety level of $P(C|T_H) = 0.1\%$, $q_H = -3.090$ provides a return periods for complete damage of 55.000 years and 32.000 years in the high and low hazard region. Thus the risk for the C2M buildings with identical safety level at the ground motion associated with a 475 return period differs by almost a factor of 2 regarding the overall residual risk. This does not change significantly if the safety level is increased, but still based on return periods. If we take 975 years instead of 475 years the respective return periods are 110.000 yrs. and 65.000 years for the high and low hazard areas. Although the residual risk decreases in both areas, the difference between the residual risks remains at a factor of 2.

Frequency of areal ground motion exceedance

With a ground motion prediction equation – in the simplest case it depends on magnitude and distance only – the average area within which a specified ground motion value, for instance the design ground motion $z_d$, is exceeded in an earthquake of magnitude $m$ is given by

$$A(m, Z > z_d) = \int_0^\infty 2\pi \cdot r \cdot P(Z > z_d|m, r)dr = \int_0^\infty 2\pi \cdot r \cdot \Phi\left( \frac{1}{\sigma_z} \ln\left( \frac{z_d}{g(m, r)} \right) \right)dr$$

with GMPE $z = g(m, r) \pm \sigma_z$ and cumulative normal distribution $\Phi(x)$. An earthquake of magnitude in excess of $m$ occurs with a return period

$$T(m, z_d) = \frac{1}{N(m)}$$

where the frequency is given by a Gutenberg-Richter law.
By eliminating the magnitude m one can construct a relation between the return period and the associated area within which \( z_d \) is exceeded. In order to demonstrate the features of this risk proxy we initially assume a ground motion prediction equation for instance in PGA as \( A(T, Z > z_d) \).

In order to demonstrate the implications of the estimated exceedance area we use the case of earthquakes in the Upper Rhinegraben in Germany. This area is characterized as zone 1 in the German code (DIN 4149/2005, see for instance Schwarz and Grünthal, 2005) where the hard rock PGA design value is 0.4 m/s². As the code zones were developed on the base of the probabilistic DACH hazard map (Grünthal, 1996, 1998) which uses macro-seismic intensities, the code establishes a relationship between intensities and PGA. In case of zone 1, intensities of 6.5 to 7 are the 475 year return period hazard values. This applies to buildings of importance class II (residential) and would be multiplied by a factor of 1.2 and 1.4 for classes III and IV. The shift in intensities is for class III 7.8 to 8.4 and for class IV 9.1 to 9.8.

The seismicity of the Upper Rhinegraben follows a Gutenberg/Richter relation. We take the values from Grünthal et al. (2010): \( \dot{N}(m_0 = 3.8) = 0.05427 \text{ yr}^{-1} \quad b = 0.9233 \). In this publication the higher magnitudes are reduced by a truncated Pareto distribution to a maximum magnitude value. Here – for simplicity - we retain the distribution without truncation.

For ground motion modelling the GMPE of Stromeyer and Grünthal (2009) is employed

\[
I = I_0 - 2.95 \log_{10} \left( \frac{r^2}{h^2 + 1} \right) - 0.00252 \left( \sqrt{r^2 + h^2} - h \right) \pm \sigma_i
\]

(14)

with the epicentral intensity \( I_0 \), epicentral distance \( r \) in km, depth \( h \) in km, and variance of \( \sigma_i = 0.643 \). The epicentral intensity is related to the moment magnitude by

\[
I_0 = 1.5 \cdot (m - 0.3 \log_{10}(h) + 0.1)
\]

(15)

so that, for instance, a \( m = 6 \) earthquake results in an \( I_0 = 8.7 \) epicentral intensity. Both formulae together provide the GMPE \( I = g(m, r) \). The areal exceedance function in intensities is

\[
A(m, I > I_d) = \int_0^\infty 2\pi \cdot r \cdot P(I > I_d|m, r) dr = \int_0^\infty 2\pi \cdot r \cdot \Phi(I_d - g(m, r)) dr
\]

(16)

Figure 2 shows the \( A(m, I > I_d) \) as a function of \( I_d \). Note, that the epicentral intensity is 8.7. Without ground motion scatter this value would not be exceeded except at one point (the epicenter). The variance of ground motion is responsible that this value is exceeded in about 200 km². The spatial distribution of these 200 km² depends on the ground motion correlation properties. Figure 3 shows the exceedance area of the design ground motion intensity \( I_d = 7 \) dependent on the return period of earthquakes in the region. For instance, an earthquake with return period 1.000 years (\( m = 5.7 \)) anywhere in the Upper Rhinegraben will result in 900 km² where the design ground intensity \( I_d = 7 \) is exceeded. An earthquake with return period 500 years (\( m = 5.4 \)) anywhere in the Upper Rhinegraben will result in 550 km² where the design ground intensity \( I_d = 7 \) is exceeded. Although at each site of the region the ground motion of \( I_d = 7 \) will be exceeded only every 500 years (more
precisely 475 years) somewhere in the entire region this ground motion will be exceeded in an area of 550 km². This characterized the potential of residual risk, dependent on population density and values exposed. Figure 4 shows the exceedance area for the higher design ground motion $I_d = 9.8$ associated with importance class IV. In this case the exceedance area for the 500 year return period is almost 0. However, on a 2,500 year level ($m = 6.1$), even this high level will be exceeded in 20 km² somewhere in the region.
Figure 4. Exceedance area for the higher design ground motion $I_d = 9.8$ associated with importance class IV. In this case the exceedance area for the 500 year return period is almost 0. However, on a 2,500 year level ($m = 6.1$), even this high level will be exceeded in 20 km$^2$ somewhere in the region.

CONCLUSIONS
We discuss two ways to look at residual risk for earthquakes, understood as the risk beyond the design level caused by infrequent but possible high ground motion. At the level of individual buildings we have to study fragilities for complete damage dependent on ground motion parameters. Here we only used the HAZUS model functions, which may deviate from the ‘real’ fragilities modelled from capacity curves and response spectra. The key issue we address is whether the paradigm of equal safety holds if buildings are designed for ground motion specified by a return period. We show that—from a residual risk perspective—might not be the case.

The second measure, or better proxy, for residual risk is established with the new quantity of areal ground motion exceedance. We ask the question: Given a ground motion design level in a seismo-tectonic area, how frequently will this level be exceeded in an area of which size. Buildings in this area will be affected by ground motion above the design level and thus be exposed to residual risk even if built code-compliant. We show for the moderately active Upper Rhinegraben that the 475 year design ground motion is exceeded in about 500 km$^2$ with the same return period.

REFERENCES


