AN IMPROVED STOCHASTIC POINT SOURCE METHOD FOR SIMULATION OF STRONG GROUND MOTION

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ABSTRACT

Stochastic method is one of the most widely used approaches for simulation of strong ground motion, especially for high frequency ground motion. The stochastic point source method using effective distance (R_{EFF}) and finite fault method with dynamic corner frequency are two powerful tools for simulation. In the present study, general characteristics of above two methods are conducted first by comparing the Fourier Acceleration Spectra (FAS) and averaged Peak Ground Acceleration (PGA) for sites with different closest distance (R_{CD}). Then a modification to point source method using equivalent distance (R_{EQL}), called as equivalent point source method for short, as well as corresponding Matlab code are developed following the idea of R_{EFF}.

It is found that normal point source method with R_{EFF} is consistent with finite fault method and is independent of sub-fault size only if the fault slip distribution is uniform, which might be unrealistic in most cases usually. While our proposed equivalent point source method with R_{EQL} agrees well with finite fault method under any slip distribution of fault and has higher computational efficiency. As an example to illustrate the effectiveness, the PGA fields generated by three popular simulating methods demonstrate that our improved method in the present study is of better adaptability and practicability.

INTRODUCTION

Seismic ground motions of high-frequency (usually greater than 1Hz) are of significant importance for both seismologist and civil engineers as they provide not only data basic for the understanding of source processes but also the motions that structures must be designed to withstand (Boore, 1983). These needs has been increasing due to recent development of performance-based seismic design for civil engineering structures and software tools as well as modeling techniques. Although recorded ground motions under conditions similar to the design earthquake are often used, it may be impossible to collect an adequate suite of such data in terms of tectonic structure, earthquake source mechanism and local geology, especially for regions has little earthquake recordings and incomplete seismic station network. Simulation of strong ground motion method is a good solution.

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Earthquake is processes of wave radiation from a fault rupture, propagation through the curst and modifications by local site conditions. Stochastic simulation for seismic activity is one of the most popular approaches that accommodate the above processes, which is first proposed by Boore (1983) based on far-field model of Brune (1970) and further developed later (Boore, 2003). The idea of this method is to generate a time series of filtered and windowed Gaussian white noise whose amplitude spectrum approximates the acceleration spectrum given by physical considerations. The model used here considers fault as a single point and no geometry information of the fault is involved, which is the so-called stochastic point-source method. However, this is an inappropriate assumption when sites are near the epicenters of large earthquakes. Rupture directivity and hanging wall effect should be reflected near the fault. To this end, Beresnev and Atkinson (1997) followed Hartzell's (1978) idea to divided a large fault into N sub-faults and each sub-fault is considered as a small point source, which is the so-called stochastic finite-fault source method. The ground motions of sub-faults, each of which is calculated by the stochastic point-source method, are summed with a proper time delay in time domain. However, it is found that total radiated energy from fault is not conserved and is heavily dependent on the number of sub-faults. In order to remove this constrain, a new finite-fault source method using dynamic corner frequency was developed (Motazedian and Atkinson, 2005). Then, researchers found that the results of point-source method and finite-fault source method are not the same even for a small earthquake at a substantial distance (Campbell, 2008). Boore (2009) discussed this inconformity of the two methods and made several modifications, one of which is replacing closest distance (R_{CD}) with a so-called effective distance measure (R_{EFF}, discussed later) into point-source method. Although point-source method using R_{EFF} and finite-fault source method are equivalent, even for large (e.g., $M_w \geq 7$) earthquakes, the necessary condition is the slip distribution must be uniform, which seems unrealistic since most large earthquakes have non-uniform slip distribution (e.g. Northridge earthquakes in 1994, Wenchuan earthquake in 2008).

In order to make point source method more practical, we propose an improved stochastic point-source method using equivalent distance (R_{EQL}) that is able to consider non-uniform slip distribution of fault rupture and at the same time agrees well with finite-fault source method. The software for the present method is developed in Matlab (http://www.mathworks.com) language. Comparisons and examples shown below demonstrate that the present method is effective and practical with high computational efficiency.

1 IMPROVEMENT TO POINT-SOURCE METHOD

1.1. Point-source Method with R_{EFF}

The total spectrum ($Y(M_0, R, f)$) of the motion used to normalize the spectrum amplitude of filtered and windowed Gaussian white noise, can be separated into several contributions from earthquake source (E), path (P), site (G) and type of motion (I), thus:

$$Y(M_0, R, f) = E(M_0, f)P(R, f)G(f)I(f)L(f) \tag{1}$$

where M_0 is the seismic moment, R is distance between site and epicenter and f is frequency.

Similarly, for finite-fault source method, the total spectrum for each sub-fault is

$$Y_{ij}(M_{0ij}, R_{ij}, f) = E_{ij}(M_{0ij}, f)P_{ij}(R_{ij}, f)G_{ij}(f)I_{ij}(f)L_{ij}(f) \tag{2}$$

The meaning of each item in equation (2) is the same with that in equation (1) except that the subscript i, j represents the ij^{th} sub-fault. To be more specific,

$$E_{ij}(M_{0ij}, f) = CM_{0ij} S_{ij}(f) \tag{3}$$

$$P_{ij}(R_{ij}, f) = Z(R_{ij}) \exp(-\pi f R_{ij} / Q(f) C_Q) \tag{4}$$

where C is a constant, $S(f)$ is the displacement source spectrum, $Z(R)$ is geometrical spreading function, $Q(f)$ is seismic attenuation Q function and C_Q is the seismic velocity used in the determination of Q. For a certain site, when the fault slip distribution is uniform, differences between sub-faults come from equation (4) so that it is possible to modify the single distance used in the point-source simulation to capture the effect of the range of distances (Boore, 2009). The following equation can be used in the place of R_{CD},

$$P(R_{EFF}, f_Q) = [\frac{1}{N}\sum_{i=1}^{N} P(R_i, f_Q)^2]^{1/2} \tag{5}$$

where R_i is the distance from the site to i^{th} sub-fault and f_Q is set as 10 Hz. The right side is calculated and then R_{EFF} is found iteratively such that the left side of the equation equals the right side of the equation.

1.2. Point-source Method with R_{EQL}

The point-source simulation with R_{EFF} tries to adopt a comprehensive average value of distances between sub-fault and site for calculation. It is obvious that it is only related to equation (4) -- that is only related to distance. However, as mentioned above, fault slip distribution is usually non-uniform, which means seismic moment for each sub-fault will be different (sub- seismic moment is distributed by slip distribution). Then, equation (5) will become invalid as it has no item to consider seismic moment change.

Here we propose an improved point-source method that involves with any fault slip distribution. First consider a relative (slip matrix divided by sum of elements) non-uniform slip distribution $S_R^*(i, j)$, it will change values of E_{ij} for each sub-fault by equation (3), which will be equal with each other under relative uniform slip distribution $S_R^1(i, j)$. Since point-source method with R_{EFF} will be consistent with finite-fault method under uniform slip distribution, it is possible to convert the change of seismic moment to the change of distance between sub-fault and site with the following equations

$$Sratio(i, j) = S_R^*(i, j) / S_R^1(i, j) \tag{6}$$

$$P(R^*_{ij}, f_Q) = Sratio(i, j) \times P(R^1_{ij}, f_Q) \tag{7}$$

where $Sratio(i, j)$ is the ratio of non-uniform to uniform slip distribution. R_{ij}^* and R_{ij}^1 are distance from site to ij^{th} sub-fault under non-uniform and uniform slip distribution respectively. Then, with equation (7), we can get a new set of distances and the slip distribution become uniform under these distances.

The newly obtained distances are not sufficient to get a final distance for point-source simulation. It is found that results of simulation under non-uniform slip distribution is $\boldsymbol{\beta}$ times larger than that under uniform slip distribution. The ratio $\boldsymbol{\beta}$ is calculated with equation (8), (9) and (10),

$$P(R^1_{EFF}, f_Q) = [\frac{1}{N}\sum_{i=1}^{N} P(R^1_i, f_Q)^2]^{1/2} \tag{8}$$

$$P(R_{EQL}, f_Q) = [\frac{1}{N}\sum_{i=1}^{N} P(R^*_i, f_Q)^2]^{1/2} \tag{9}$$

$$\beta = P(R^*_{EFF}, f_Q) / P(R^1_{EFF}, f_Q) \tag{10}$$

where R_{EQL} and R_{EFF}^1 are effective distance under non-uniform and uniform slip distribution respectively. At last, total spectrum Y in equation (1) is calculated with R_{EFF}^1 but then multiplied by $\boldsymbol{\beta}$.

2 VERIFICATION FOR EQUIVALENT POINT-SOURCE METHOD

2.1. Program for Calculation

Two available stochastic simulation Fortran programs are SMSIM (Boore, 2000) and EXSIM (Motazedian and Atkinson, 2005) corresponding to point-source and finite-fault method respectively. However, structure of SMSIM seems complicated as it is a set of tools not only for simulation of ground motion but also for other usages. In order to embed the modification mentioned in this study and facilitate output of various results and parameters, a program is developed in Matlab language and named as EQSIM (equivalent distance is involved). The program has been validated by carefully comparing results computed by SMSIM and EQSIM using several input files in the package of SMSIM, and it is testified that EQSIM generates the same outcome with SMSIM with various parameters. Thus, the software used for verification of the equivalent point-source method in this study are EQSIM and EXSIM.

2.2. Input Parameters

Table 1 gives the essential input parameters for the simulations, most of which follow the parameters used by Boore (2009). Since the purpose of the present study is to check whether the proposed point-source method is equivalent with finite-fault method under non-uniform slip distribution, only one magnitude (M_w=7) and vertical fault plane are adopted as these parameters have no impact on the verification of the proposed method. Like most previous studies, Fourier acceleration spectrum and peak ground acceleration are selected as comparison measures.

Figure 1 is the relative geometry of fault and sites used in the present study. The line is the upper edge of fault plane and A, B are endpoint of the fault. Point 1 (R_{CD}=1/3L), 2 (R_{CD}=1/3L) and 3 (R_{CD}=5L) are on the perpendicular bisector, while point 4, 5 and 6 have the same R_{CD} but on the line perpendicular to B. These points represent near-filed and far-field sites and at same time simulated sites with different azimuth angle. Figure 2 shows different slip distributions of fault for simulation. Distribution 1 is actually the uniform slip distribution and distribution 2 to 4 describe the moving of seismic moment from A to B gradually.

Table 1. Parameters used in the present simulation

Parameter	Value	Parameter	Value
Shear-wave velocity (V_S)	3.7 km/sec	*Geometric spreading: b=	-1.3（0-70km）
Density (ρ)	2.8 g/cm^3		+0.2（70-140km）
High frequency (κ_0)	0.005 s		-0.5（>140km）
Quality factor (Q)	max(1000,893$f^{0.32}$)	*Path duration, d=	0 （0-10km）
Stress drop ($\Delta\sigma$)	140 bars		+0.16 （10-70km）
Rupture propagation speed	0.8V_S		-0.03 （70-130km）
Fault dip	90°		+0.04 （>130km）
Fault length and width	**M** 7: 29.4 km × 9 km	Type of window	Exponential
Type of sub-fault	Dynamic corner frequency	Slip distribution	Non-uniform distribution

* see user manual of SMSIM for the meaning of these parameters（http://daveboore.com/software_online.html）

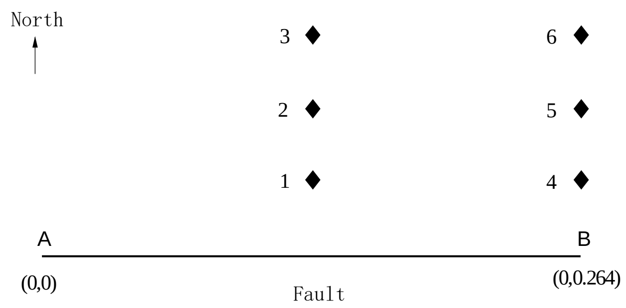

Figure 1. Sketch of relative geometry of fault and sites (M=7)

1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1

Distribution 1

2	3	5	4	3	2	1	1	1	1
2	3	5	4	3	2	1	1	1	1
2	3	5	4	3	2	1	1	1	1

Distribution 2

1	2	3	4	5	5	4	3	2	1
1	2	3	4	5	5	4	3	2	1
1	2	3	4	5	5	4	3	2	1

Distribution 3

1	1	1	1	2	3	4	5	3	2
1	1	1	1	2	3	4	5	3	2
1	1	1	1	2	3	4	5	3	2

Distribution 4

Figure 2. 4 slip distribution models for simulation

3 RESULTS AND DISSCUSSION

FAS of the 6 sites with different slip distribution model are calculated and plot for a given earthquake of magnitude 7, as is shown in Figure 3. In general, the proposed method, equivalent point-source method, can reflect slip information of fault plane effectively and has the same results with finite-fault simulation. The point-source method with R_{EFF}, which has been proved to be equal with finite-fault method under uniform slip distribution, will only match the black lines in the Figure. Also, geometry spreading and sub-fault seismic moment are two critical parameters for the results of different cases, that is different fault information with different sites (related to equation (3) and (4)). For example, for site 1 to 3, when sub-faults seismic moment moves closer to site (from distribution 1 to 3), ground motion level of site will increase. As R_{CD} increases, the impact of sub-faults seismic moment get weak gradually. This dual function of distance and seismic moment can be presented more clearly by FAS of site 4 to 6. For site 4, when sub-fault seismic moment moves to sub-faults that have long distances (distribution 2), value of FAS drops comparing to uniform distribution, and vice versa (distribution 4). But for site 5, value of FAS under distribution 2 exceeds uniform distribution, which means when geometry differences between sub-faults become small, seismic moment become the dominant factor. Thus, when the site is getting further(site 6), any non-uniform distribution will get bigger value than uniform distribution and at the same time results between all distributions get closer.

In order to exhibit the effect of equivalent point-source method and characteristics of non-uniform slip distribution, the PGA fields are calculated and delineated by EXSIM, EQSIM and SMSIM respectively, as is shown in Figure 4. The first two graphs demonstrate that equivalent point-source method agrees well with finite-fault method under non-uniform slip distribution even near the fault. Results of SMSIM using R_{EFF} could only reflect information about distance between sub-fault and site, whose contour lines are symmetry. Moreover, PGA values attenuate slower near the sub-faults with higher seismic moment and near-fault values calculated under non-uniform distribution are a little higher than uniform distribution on the whole.

At last, Table 2 shows a small numerical experiment for detecting the computation efficiency of EQSIM and EXSIM with 10 simulations. Generally speaking, EQSIM uses less time than EXSIM under different conditions. The predominance of EQSIM will appear when the number of sub-faults increases. Moreover, EQSIM has stable computation efficiency as the elapsed time for 10 simulations seems independent of number of sub-faults. For both method, large distance between site and fault will slow down the computation as it increase the length of various time history.

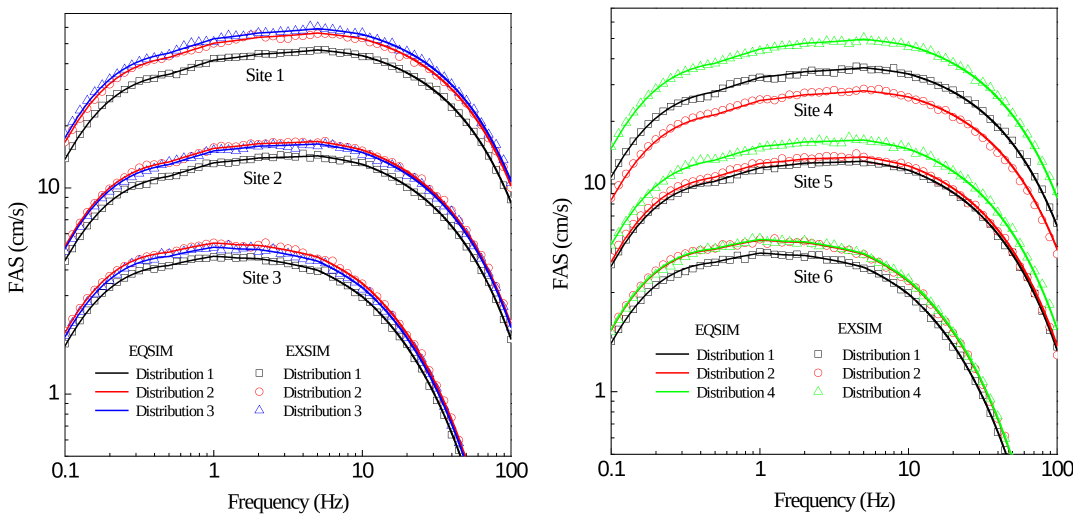

Figure 3. FAS from EQSIM and EXSIM simulations for the 6 sites.

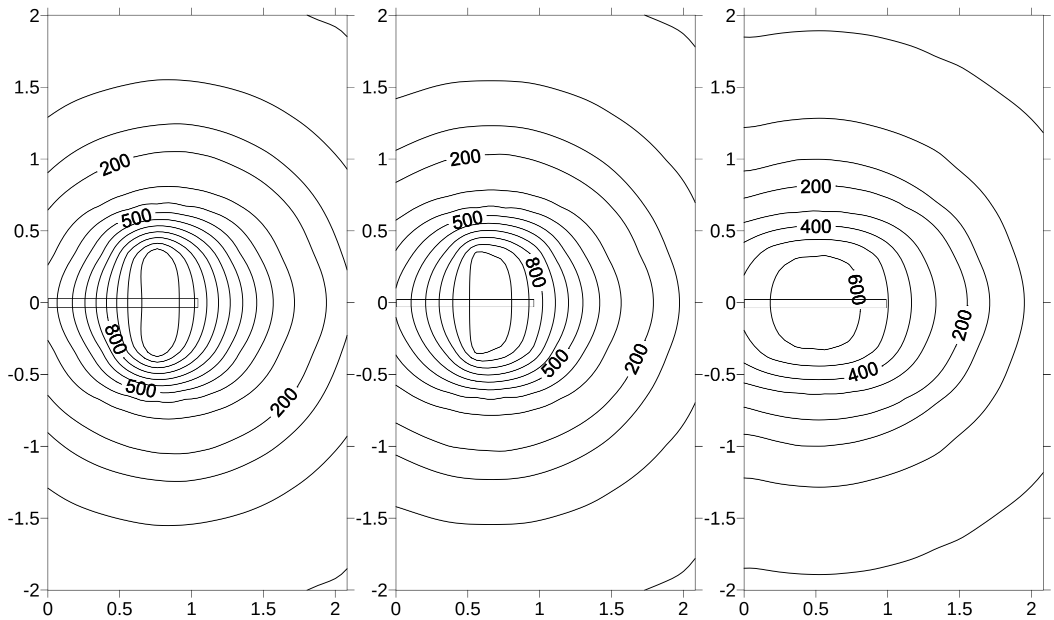

Figure 4. PGA field from EXSIM (left), EQSIM (middle) and SMSIM (right). X axis is ratio of horizontal distance between sites and endpoint A (0, 0)of fault to fault length. Y axis is similar.

Table 2. Elapsed time of EQSIM and EXSIM for 10 simulations under different conditions

Number of sub-faults and R_{CD}		EQSIM	EXSIM
N=4	R_{CD}=29.4km	1.5s	1.1s
	R_{CD}=294km	2.1s	1.2s
N=30	R_{CD}=29.4km	1.1s	2.6s
	R_{CD}=294km	2.1s	3.0s
N=126	R_{CD}=29.4km	1.3s	8.1s
	R_{CD}=294km	2.3s	9.5s

4 CONCLUSIONS

In the present study, we propose the equivalent stochastic point-source method for simulation of strong ground motion. When fault slip distribution is non-uniform, ground motion for sites will change comparing to uniform distribution. This method, unlike the previous point-source method, is demonstrated to be able to reflect fault slip information and agrees well with finite-fault method but with higher computation efficiency. It is found that the finite-fault method has internal relation with point-source method. Geometry spreading $Z(R)$ and seismic moment in source $E(M_0, f)$ are two most important parameters that characterize the stochastic method.

Indeed, we only consider FAS as comparison measure that contain frequency information, but the pseudovelocity response spectra (PSV) and pseudoabsolute acceleration response spectra (PSA) are ignored here. It is argued by Boore (2009) that these measures calculated by point-source method will not be consistent with finite-fault method if some input parameters are not handled carefully, for example the sub-fault rise time. Moreover, PGA generated by point-source and finite-fault method have a large value range so that the stability of time history should also be compared. It is worthy being studied further, because point-source method has relatively simple algorithm with high computation efficiency and the more important is that it will stay consistent with finite-fault method.

REFERENCES

Boore DM (1983) Stochastic simulation of high-frequency ground motions based on seismological models of the radiated spectra, *Bulletin of the Seismological Society of America*, 73(6A), 1865-1894

Boore DM (2003) Simulation of ground motion using the stochastic method, *Pure and applied geophysics*, 160(3-4), 635-676

Boore DM (2009) Comparing stochastic point-source and finite-source ground-motion simulations: SMSIM and EXSIM. *Bulletin of the Seismological Society of America*, 99(6), 3202-3216.

Boore DM (2000) SMSIM–Fortran programs for simulating ground motions from earthquakes: version 2.0—A revision of OFR 96-80-A US Geol Surv, Open-File Rept OF 00-509, 1-59

Brune JN (1970) Tectonic stress and the spectra of seismic shear waves from earthquakes, *Journal of geophysical research*, 75(26), 4997-5009

Beresnev IA, Atkinson GM (1997) Modeling finite-fault radiation from the ω_n spectrum, *Bulletin of the Seismological Society of America*, 87(1), 67-84

Hartzell SH (1978) Earthquake aftershocks as Green's functions, *Geophysical Research Letters*, 5(1), 1-4

Atkinson GM, Assatourians, K, Boore, DM, Campbell, K, Motazedian, D (2009) A guide to differences between stochastic point-source and stochastic finite-fault simulations, *Bulletin of the Seismological Society of America*, 99(6), 3192-3201

Motazedian D, Atkinson GM (2005) Stochastic finite-fault modeling based on a dynamic corner frequency, *Bulletin of the Seismological Society of America*, 95(3), 995-1010

Campbell KW (2008, October) Hybrid empirical ground motion model for PGA and 5% damped linear elastic response spectra from shallow crustal earthquakes in stable continental regions: Example for eastern North America, *In Proc. 14th World Conf. Earthq. Eng*, 12-17