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PROBABILISTIC STRENGTH DOMAINS OF MASONRY WALLS REINFORCED WITH EXTERNALLY BONDED COMPOSITES

Fulvio PARISI[1] and Carmine GALASSO[2]

ABSTRACT

Unreinforced masonry (URM) has been used to build up a large number of structures and infrastructures since ancient times, and is still employed in modern construction. In recent earthquakes, URM buildings have sustained a high degree of damage due to in-plane loading, demonstrating a pressing need for retrofitting. In the last years, externally bonded fibre-reinforced polymers (FRPs) and fabric-reinforced cementitious matrices (FRCMs) have been proposed as effective solutions for seismic retrofit of URM walls and their use is rapidly increasing worldwide.

In this paper, the in-plane lateral strength of tuff stone masonry walls in both as-built and strengthened conditions is investigated. Diagonal FRP strips and FRCM composite, applied on both sides of walls and through single plies, are considered as strengthening systems. Based on capacity models as well as statistics and probability distributions for material properties, geometry and models, a probabilistic analysis was carried out by using plain Monte Carlo simulation. The output of that analysis consists of shear force versus axial force strength domains corresponding to the 16th, 50th and 84th percentile levels. Simplified equations, in a dimensionless format, are finally provided by using robust regression. Such an output represents a practice-oriented tool for both design and assessment of externally-strengthened masonry walls.

INTRODUCTION

Recent earthquakes have produced extensive damage in a large number of existing unreinforced masonry (URM) buildings or components, showing an increasing need for their retrofit especially when they are not in compliance with recent building code provisions. As a consequence, a number of international guidelines have been issued in recent years to predict the in-plane lateral strength of masonry walls strengthened with externally bonded composites such as fibre-reinforced polymers (FRPs) (e.g., INRC, 2004; ACI, 2010) and fabric-reinforced cementitious matrices (FRCMs) (e.g., ACI, 2013). The use of externally bonded FRP composite materials for seismic strengthening of masonry structures is significantly increased in the last two decades (e.g., Stratford et al., 2004; Prota et al., 2008; Marcari et al., 2011) and is now well established worldwide. Besides, new attractive applications involve the use of strengthening systems based on FRCMs, where mortar-based matrices allow to overcome some limitations of resin-based composites, such as sensitivity to high temperatures, moisture impermeability, flammability, and poor bond to the existing masonry substrate. Several experimental programmes have been carried out to investigate both in-plane and out-of-plane behaviour of FRCM-strengthened masonry walls (e.g., Papanicolaou et al., 2007; Prota et al., 2006; Parisi et al., 2013). Although some analytical capacity models have been proposed and included in guidelines (e.g., Parisi et al., 2011; Babaeidarabad et al., 2013), a reliability-based assessment of the

[1] Assistant Professor, University of Naples Federico II, Naples, fulvio.parisi@unina.it
[2] Lecturer, Newcastle University, Newcastle upon Tyne, carmine.galasso@ncl.ac.uk

safety level for such strengthened masonry walls is still lacking. In fact, little work has been done on statistical descriptors for material properties and member strengths in the case of masonry structures, eventually strengthened with FRP/FRCM systems. There are several reasons why guidelines dealing with innovative strengthening systems should follow a probabilistic format, including compatibility with design codes for other materials and the fact that reliability-based codes offer a relatively uniform level of risk. Experience over the past three decades in developing probability-based limit state design criteria for conventional construction materials points the way forward for making similar advances in guidelines for design and evaluation of structural components and systems that employ composite materials (e.g., Wang et al., 2010).

This paper aims at assessing the influence of different uncertainty sources on strength domains for masonry walls strengthened with FRP laminates or FCRM composites. In particular, a Monte Carlo simulation technique is used to compute statistics describing the lateral capacity of case-study walls, allowing the incorporation of known (or assumed) distribution for material, geometry and model uncertainty with relative ease. The following masonry wall type and strengthening schemes are considered in the preliminary investigation discussed in this paper: tuff stone masonry wall; double-side strengthening; single-ply FRP composite with diagonal layout; single-ply FRCM composite. Tuff stone masonry is investigated because it is one of the most common masonry types worldwide and is frequently used in earthquake-prone regions. Double-side strengthening means that the composite system is applied to both sides of the masonry wall. This strengthening scheme typically provides a more ductile behaviour and ultimate capacity increase than its single-side counterpart (e.g., Parisi et al., 2013). Especially in the case of FRCM systems, this study represents a fundamental first step toward the development of probabilistic design guidelines and codes. The availability of such standards and guidelines for design and assessment of FRCM-strengthened structural elements will advance the acceptance of FRCM systems and their use in construction practice.

CAPACITY MODELS FOR AS-BUILT URM WALLS

As-built URM walls may suffer one of the following failure modes under in-plane lateral loading: toe crushing; diagonal tension cracking; stair-stepped diagonal sliding; and bed-joint sliding. The first failure mode is flexure-controlled, while the others are shear-controlled. In this study, the lateral strengths associated with such failure modes are respectively denoted as V_{tc}, V_{dt}, V_{sd}, and V_{bj}. Lateral strength was computed through stress-based formulations discussed, amongst others, by Parisi and Augenti (2013). These formulations are in compliance with current building codes, such as Eurocode 8 (or EC8; CEN, 2004) and the Italian Building Code (or IBC08; IMIT, 2008).

The URM wall is assumed to have length l, height h and thickness t, resulting in a sectional area $A_w = l\ t$. Masonry was supposed to be an equivalent homogeneous material with zero tensile strength, uniaxial compressive strength f_m, diagonal tension shear strength at zero confining stress τ_{dt}, diagonal sliding shear strength at zero confining stress τ_{sd}, and friction coefficient μ_a. In the case of existing masonry structures, IBC08 (IMIT, 2008) defines the design strength as mean strength (herein assumed as nominal strength) divided by a partial safety factor γ_m times a confidence factor FC (herein assumed equal to 1, neglecting the influence of the knowledge level). In seismic conditions, IBC08 sets $\gamma_m = 2$. Assuming an elastic-perfectly plastic (EPP) stress–strain model for masonry, the actual compressive strength was reduced to $f_{md} = 0.85 f_m$, in order to account for nonlinear distributions of axial strains over cross section. The ultimate axial load of the wall was then set to $N_u = f_{md}\ l\ t$. It is convenient to normalise internal forces to the nominal axial load capacity, that is, $\bar{N} = N/N_u$ and $\bar{V} = V/N_u$.

The normalised lateral strength associated with toe crushing in flexure was predicted through Eq. (1) based on the stress block criterion:

$$\bar{V}_{tc} = \frac{l}{2h_0}\left(\bar{N} - \bar{N}^2\right) \tag{1}$$

where h_0 is the distance between the section where flexural capacity is attained and the contraflexure point. That distance changes with boundary conditions, h_0 ranging between $0.5h$ and h

from fixed-fixed (shear-type wall) to fixed-free (cantilever wall) conditions.

The normalised lateral strength associated with diagonal tension cracking of the wall was computed as follows:

$$\bar{V}_{dt} = \beta\sqrt{1+\frac{\bar{N}}{p\,\beta}} \tag{2}$$

In Eq. (2), $\beta = \tau_{dt}/f_{md}$ and p is a shear stress distribution factor related to the wall aspect ratio h/l, as follows: $p = 1$ if $h/l \leq 1$ (squat wall), $p = 1.5$ if $h/l \leq 1$ (slender wall), and $h/l\ p = h/l$ if $1 < h/l < 1.5$.

The normalised lateral strength associated with stair-stepped diagonal sliding was evaluated via the Mohr-Coulomb friction law as follows:

$$\bar{V}_{sd} = \frac{1}{p}\left(\gamma + \mu_a \bar{N}\right) \tag{3}$$

where: $\gamma = \tau_{sd}/f_{md}$; μ_a is a fictitious friction coefficient of masonry ranging which is assumed equal to 0.4 by Eurocode 6 (or EC6; CEN, 2005a) and IBC08 (IMIT, 2008).

Finally, the normalised lateral strength associated with bed-joint sliding was predicted through Eq. (3) by assuming $\mu_a = 0.17/\bar{N}^{2/3}$, and hence that friction reduces as the confining pressure increases. In other words, this normalised lateral strength was computed as follows:

$$\bar{V}_{bj} = \frac{1}{p}\left(\gamma + 0.17\sqrt[3]{\bar{N}}\right) \tag{4}$$

It is frequently assumed that shear strengths (at zero confining stress) associated with diagonal tension cracking and diagonal sliding are the same, resulting in $\beta = \gamma$ (e.g., IBC08). Nevertheless, different estimates were derived for τ_{dt} and τ_{sd} in this research.

The normalised lateral strength of the URM wall subjected to in-plane loading was thus defined as follows:

$$\bar{V}_m = \min\left(\bar{V}_{tc};\bar{V}_{dt};\bar{V}_{sd};\bar{V}_{bj}\right) \tag{5}$$

CAPACITY MODELS FOR FRP-STRENGTHENED MASONRY WALLS

The capacity model for masonry walls externally strengthened through FRP laminates with diagonal layout is not provided by current standards and guidelines, such as ACI440.7R-10 (ACI, 2010) and CNR-DT200 (INRC, 2004). The FRP-strengthened wall may fail in bending or shear, so its normalised in-plane lateral strength was predicted as follows:

$$\bar{V}_n = \min\left(\bar{V}_{ns};\bar{V}_{nf}\right) \tag{6}$$

where $\bar{V}_{ns}$ and $\bar{V}_{nf}$ are the normalised lateral strengths corresponding to shear and flexural failure, respectively. The normalised lateral strength associated with shear failure was computed through the following equation (e.g., Marcari et al., 2007; Prota et al., 2008, Marcari et al., 2011):

$$\bar{V}_{ns} = \bar{V}_m + \bar{V}_f \tag{7}$$

where $\bar{V}_m$ and $\bar{V}_f$ are the shear strength contributions from masonry and FRP reinforcement, respectively. In the case of external FRP strengthening with diagonal configuration, a truss resisting

mechanism develops under in-plane lateral loading. This increases the lateral strength associated with shear failure without significant enhancement in bending capacity (INRC, 2004). As a result, if the as-built URM wall is expected to fail in flexure, the FRP diagonal reinforcement is not effective to increase the in-plane lateral strength. In that case, the lateral strength corresponding to flexural failure is that evaluated for the as-built wall, that is:

$$\bar{V}_{nf} = \bar{V}_{tc} \tag{8}$$

In Eq. (7), the shear contribution from FRP reinforcement with diagonal layout was estimated according to Prota et al. (2008), which used a strut-and-tie model where a diagonal masonry strut withstands compression stresses and an FRP strip carries tensile stresses. The other FRP strip in compression is neglected. Based on these assumptions, the shear contribution from diagonal FRP reinforcement layout was predicted as the horizontal component of the maximum tensile force transmitted by the FRP strips in tension (here denoted by F_{frp}), as follows:

$$\bar{V}_f = \frac{F_{frp} \cos\alpha}{N_u} \tag{9}$$

where α is the angle of diagonal FRP strips in tension to the horizontal direction. The tensile strength of the FRP strips is given by:

$$F_{frp} = \varepsilon_{fd} E_f A_{fd} \tag{10}$$

where: ε_{fd} is the design strain of FRP reinforcement; E_f is the Young's modulus of FRP reinforcement; A_{fd} is the area of diagonal FRP strips in tension. In both ACI and INRC guidelines, the design strain may be determined as a function of an environmental reduction factor (depending on the fibre type and exposure conditions) and the *manufacturer-guaranteed* (i.e., the nominal) ultimate rupture strain of the FRP reinforcement, defined as the mean ultimate rupture strain minus three standard deviations (ACI guidelines) or as characteristic tensile strain (i.e., having a 95% probability of exceedance according to material specifications) in the CNR-DT200 (INRC, 2004)[3]. This truss model was also applied by Stratford et al. (2004), Krevaikas and Triantafillou (2005), and Marcari et al. (2011). The area of diagonal FRP strips is $A_{fd} = N_{plies}\ w_{frp}\ t_{frp}$, where: N_{plies} is the number of FRP plies ($N_{plies} = 2$ in the case of single FRP plies on both wall sides); w_f is the width of FRP ply; t_f is the thickness of FRP ply. According to CNR-DT200[4], ε_{fd} is the minimum between the ultimate tensile strain ε_{fdu} and the debonding strain of FRP ε_{fdd}. Prota et al. (2008) recommended to amplify the debonding strain provided by CNR-DT200, which is associated with end debonding of FRP strips. Actually, such a failure mode does not take place if diagonal strips are connected by horizontal collar strips both at the base and top of the walls (Marcari et al., 2007). The FRP-strengthened wall may then suffer intermediate debonding instead of end debonding. If the end debonding strain is defined as:

$$\varepsilon_{fdd,end} = \frac{1}{\gamma_{f,d}\sqrt{\gamma_m}} \sqrt{\frac{2\Gamma_{fm}}{E_f t_f}} \tag{11}$$

the intermediate debonding strain is given by $\varepsilon_{fdd} = k_{cr}\ \varepsilon_{fdd,end}$, where k_{cr} is set to 3. In Eq. (11), Γ_{fm} is the mean specific fracture energy of the FRP-strengthened masonry. This energy depends on the mean compressive strength of masonry f_m and mean tensile strength of masonry f_{tm}, as follows:

$$\Gamma_{fm} = c_1 \sqrt{f_m f_{tm}} \tag{12}$$

[3] According to CNR-DT200, a material *partial safety factor* $\gamma_f = 1.10$ to the characteristic tensile strain and a *partial factor for resistance models*, γ_{Rd} (equal to 1.00 for bending and 1.20 for shear) are also applied.
[4] ACI guidelines offer similar provisions.

where c_1 is an experimentally determined coefficient. Based on analytical-experimental comparisons for FRP reinforcement, Prota et al. (2008) recommended to assume c_1 equal to 0.045 in the case of natural stone masonry walls and 0.3 in the case of clay brick masonry walls. The guidelines CNR-DT200 (INRC, 2004) suggest $c_1 = 0.015$ and $f_{tm} = 0.1 f_m$ unless experimental data are available, and to assume $\gamma_{f,d} = 1.20$ as material partial safety factor for debonding into Eq. (11).

CAPACITY MODELS FOR FRCM-STRENGTHENED MASONRY WALLS

The in-plane capacity model for URM walls externally strengthened with FRCM composites is provided by guidelines ACI549-13 (ACI, 2013). The normalised lateral strength was defined through Eqs. (6), (7) and (8), thus neglecting the flexural strength increase provided by the FRCM reinforcement. More rigorously, a flexural capacity model should be used considering the mechanical behaviour of the FRCM constituents separately (e.g., Parisi et al., 2011), or that of the whole FRCM composite (ACI, 2013). Nonetheless, neglecting the flexural contribution from FRCM reinforcement may be in favour of structural safety. The in-plane shear capacity of the FRCM-strengthened wall was computed according to Babaeidarabad et al. (2013), consistently with ACI (2013), where the FRCM contribution is considered only after the occurrence of masonry cracking as follows:

$$\overline{V}_f = \frac{2nA_f l\, f_{fv}}{N_u} \tag{13}$$

where: n is the number of FRCM layers; A_f is the area of the fabric reinforcement by unit length of masonry wall; l is the wall length; f_{fv} is the tensile strength of the FRCM composite. The latter is defined by $f_{fv} = E'_f\, \varepsilon_{fv}$ where: E'_f is the tensile Young's modulus of the cracked FRCM composite and it is assumed equal to the average value from experimental tests; ε_{fv} is the tensile strain in the FRCM shear reinforcement. Such a strain is assumed to be $\varepsilon_{fv} = \varepsilon_{fu} \leq 0.4\%$, being ε_{fu} the ultimate tensile strain of the FRCM composite, assumed equal to the average minus one standard deviation derived from tensile tests conducted as per AC434 (ICC Evaluation Service, 2013). ACI549-13 establishes design provisions by limiting the increase in shear capacity provided by the FRCM reinforcement to 50% of the as-built wall capacity. This provides an upper bound to the total force per unit width transferred to the masonry substrate. Additionally, a strength reduction factor for shear, ϕ_v, is set to 0.75.

Figures 1a and 1b show the nominal (or design) limit strength domains of a case-study masonry wall strengthened with FRP diagonal strips and FRCM composite, respectively. The case-study wall was assumed to have $l = h = 2$ m and $t = 0.5$ m. Nominal values of material properties were assumed in accordance to IBC08 (IMIT, 2008), CNR-DT200 (INRC, 2004) and ACI549-13 (ACI, 2013), as also discussed above. The as-built masonry wall was assumed to be a doubly-fixed wall, so h_0 was set to $0.5h$. The walls was then supposed to be strengthened on both sides with: (a) single-ply carbon fibre reinforced polymer (CFRP) with diagonal layout, nominal thickness of 1.10 mm and reinforcement ratio equal to 0.02%; (b) single-ply carbon FRCM composite with fibre area by unit width equal to 0.05 mm^2/mm.

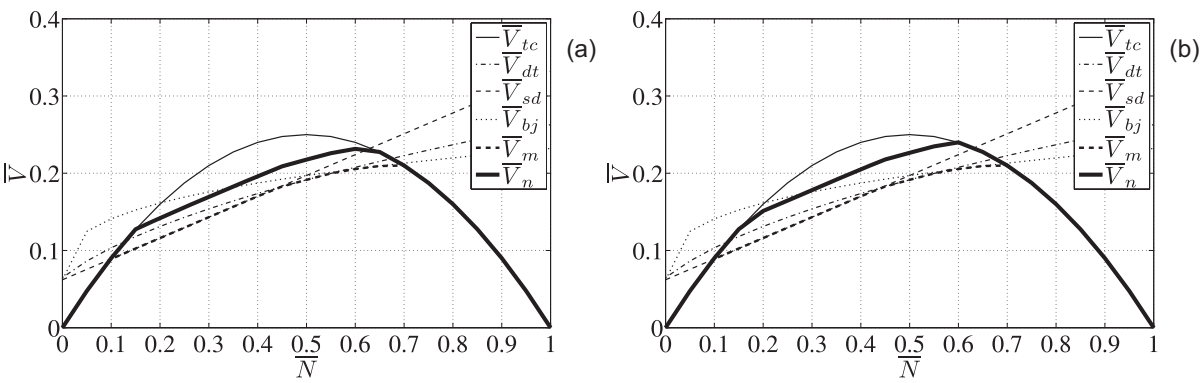

Figure 1. Nominal limit strength domains: (a) FRP-strengthened wall; (b) FRCM-strengthened wall

Although the amount of reinforcement in the case of FRP was relatively high, the increase in lateral strength provided by this strengthening system was limited by the fact that code prescriptions are rather on the safe side as a result of the environmental conversion factor, partial safety factor and partial safety factor for resistance model (INRC, 2004). Both FRP and FRCM systems increased the lateral strength corresponding to shear failure of the URM wall without contributing to the flexural strength. This occurred at intermediate axial load levels. If a normalised axial load between 0.15 and 0.7 is considered, the average increase in lateral strength turns out to be 15% in the case of FRP-strengthened wall and 19% in the case of FRCM-strengthened wall.

UNCERTAINTY CHARACTERIZATION

As FRP and FRCM systems are relatively new means of strengthening existing structures, much of the early work has been conducted in a deterministic fashion, especially when dealing with rehabilitation of masonry structures. In fact, research efforts have generally been concerned with gaining an understanding of the mechanical behaviour of these systems and modelling the interaction between FRP/FRCM and the existing structure. In practice, due to inherent uncertainties in material properties, geometrical dimensions, and the equations used to compute member strengths, the actual lateral capacity of an URM wall (be it as-built or strengthened with externally bonded composites) differs from its design (i.e., nominal) lateral capacity which is calculated based on nominal values. For that reason, the basis for assessing the safety levels for different guidelines at the state-of-the-art with respect to design of strengthening systems based on externally bonded composites is to accurately develop a statistical model for the lateral strength of the wall subjected to in-plane loading. The estimation of such lateral strengths (and the consequent calibration of safety factors to be used in design of retrofit systems) must necessarily be expressed in probabilistic terms because most, if not all, the factors possibly affecting the lateral capacity of URM and reinforced masonry walls are uncertain despite the values assumed in design. Currently, calibration of the safety factors used in codes and guidelines, particularly for innovative materials and strengthening systems, seems to be based mainly on engineering judgment rather than scientifically sound assessments.

To assess the probability distribution of the capacity model for FRP- and FRCM- strengthened URM walls, 280 case-study walls were derived by varying:

1) Wall geometry: this study considers tuff stone masonry walls obtained by varying the height h between 2 m and 5 m (with a step of 0.50 m) and the wall aspect ratio h/l between 1 and 5 (with a step of 1). Transverse slenderness ratio h/t (i.e., the wall height to the thickness) is also varied between 5 and 20, with a step of 5, while h_0 is assumed equal to $0.5h$. Such assumptions were based on a comprehensive review of past practice rules, historical documents, handbooks, and both past and current codes.
2) Reinforcement type: a double-side strengthening scheme is considered consisting of (a) single-ply CFRP with diagonal layout or (b) single-ply carbon FRCM composite.

A Monte Carlo sampling procedure was applied to accomplish the capacity assessment, using an approach similar to Iervolino and Galasso (2012) and Galasso et al. (2014). Representative statistics and appropriate probability distributions for all the basic resistance variables were selected from previous related studies. In particular, a literature review was carried out to select the statistical characterization for each random variable (RV) referring to materials (i.e., masonry, FRP and FRCM characteristics), geometry (i.e., cross-section dimensions and reinforcement area), and models. The resulting assumptions, corresponding to average-quality construction, are summarised in Table 1 and described in the following sub-sections. The parameters given in Table 1 are the mean and coefficients of variation (or CoV; i.e., the ratio of standard deviation to mean). All RVs considered were treated as stochastically independent; possible correlation between masonry properties is currently under investigation by the authors.

Materials

The uncertainty involving masonry properties is modelled by assuming a Lognormal distribution for

the uniaxial compressive strength f_m, the diagonal tension shear strength at zero confining stress τ_{dt} and the diagonal sliding shear strength at zero confining stress τ_{sd}; a Uniform distribution is assumed for the friction coefficient μ_a. For masonry properties, the results of a comprehensive statistical analysis were used as input in the Monte Carlo simulation. In particular, the online MAsonry DAtabase (MADA; Augenti et al., 2012), including experimental results on mechanical properties of masonry and its constituents (i.e., masonry units and mortar), is used here. Masonry tensile strength was not considered in the computation of flexural strength, as its effect is negligible.

Similarly, FRP is a material that is subjected to variability in its own properties, particularly when it is manufactured on site using manual processes such as wet layup. Thus, data in terms of strength and modulus obtained from CFRP composite panels that are fabricated during a case-study rehabilitation process itself (then using the same equipment and personnel and under the same conditions), are used here (Atadero et al., 2005). In that study, it was found that all of the tested theoretical distributions (i.e., Weibull, Gamma, Normal and Lognormal) can be considered acceptable fits to the experimental data. However, a Normal distribution is assumed here for simplicity. This assumption is well established in literature and in codes (e.g., in ACI guidelines).

In the case of FRCM, the mechanical properties of carbon FRCM described in Babaeidarabad et al. (2013) and based on experimental data are used here. FRCM was composed of a sequence of one or four layers of cement-based matrix reinforced with dry-fibre fabrics. The fabric consisted of a balanced network of carbon-fibre toes disposed along two orthogonal directions at a nominal spacing of 10 mm; the equivalent nominal fibre thickness was 0.048 mm. A Normal distribution is assumed for both modulus of elasticity and ultimate tensile strain of the FRCM samples.

Table 1. Statistics and distributions of random variables

Category	*System*	*Variable*	*Mean*	*CoV [%]*	*Distribution*
Material	Masonry	f_m	4.17 MPa	29	Lognormal
		τ_{dt}	0.23 MPa	31	Lognormal
		τ_{sd}	0.22 MPa	47	Lognormal
		μ_a	0.54	27	Uniform
	FRP	E_f	70.4 GPa	13.4	Normal
		ε_{fd}	1.48%	18	Normal
	FRCM	E'_f	69 GPa	19	Normal
		ε_{fu}	0.52%	21	Normal
Geometry	Masonry	l	Nominal value	5	Normal
		h	Nominal value	5	Normal
		t	Nominal value	5	Normal
	FRP	ρ_f	0.02%	–	Deterministic
		t_f	1.10 mm	5	Normal
	FRCM	A_f	0.05 mm^2/mm	–	Deterministic
Model	As-built masonry wall	V_{exp}/V_{theor}	1.02	5	Normal
	FRP-strengthened masonry wall	V_{exp}/V_{theor}	1.03	4	Normal
	FRCM-strengthened masonry wall	V_{exp}/V_{theor}	0.90	10	Normal

Sectional geometry

Uncertainties in geometry (or *fabrication*, Nowak and Szerszen, 2003) account for the heterogeneity in the dimensions of the considered structural element due to construction quality. The considered statistical parameters for URM walls are based on Xiao et al. (2012). In particular, the bias factor (i.e., the ratio of the sample mean to the reported nominal value) is assumed to be equal to 1.00 for length, height and thickness with a CoV = 5% for URM walls. A Normal model was assumed for all case-study sections. Each FRP strip was assumed to have a width corresponding to a reinforcement ratio ρ_f = 0.02% (deterministic value) and a thickness lognormally distributed (Atadero et al., 2005). The area

of FRCM (fibre area by unit width) was treated as a determinist value, equal to 0.05 mm^2/m (Babaeidarabad et al., 2013).

Mechanical models

Model (or *professional*, Nowak and Szerszen, 2003) uncertainties characterise the heterogeneity in sectional capacity estimation which is caused by design equations. In fact, such uncertainties are generally measured by comparing the lateral capacity obtained in experimental tests with the corresponding values obtained via analytical formulations. Model statistical properties are comprehensively documented in Prota et al. (2008) for both URM walls and masonry walls strengthened with FRP laminates and in Babaeidarabad et al. (2013) for masonry walls strengthened with FCRM composites. The Normal distribution is typically used to represent these modelling factors, whose mean and CoV depend on the type of reinforcement. Table 1 outlines the ratio V_{exp}/V_{theor} between experimental and theoretical lateral strength of the wall.

METHODOLOGY

The probability distributions and statistics for the normalised lateral strength, $\bar{V}$, are determined using a Monte Carlo sampling procedure, employing the uncertainty characterization discussed above. To achieve this aim, the authors developed an ad hoc MATHWORKS-MATLAB® script. In particular, each case-study wall was defined by a set of nominal material strengths, nominal dimensions and strengthening system (FRP or FRCM). For each wall and $\bar{N}$-level (assumed to be a deterministic demand parameter ranging between 0 and 1 with step 0.05), the following steps were carried out:

1) Given the wall nominal characteristics, a set of material properties and dimensions was randomly generated from the statistical distributions of each variable that affects $\bar{V}$ and discussed above. That set of material and geometrical properties, plus a randomly-generated value of the model error, was used to estimate the wall theoretical capacity $\bar{V}$. The latter was computed based on Eqs. (1)–(13).
2) This procedure was repeated 1,000 times, enabling the probability distribution of $\bar{V}$ to be determined numerically. Then, the median and both 16% and 84% fractiles from this distribution were evaluated. A Shapiro-Wilk parametric hypothesis test (Mood et al., 1974) was performed to assess the assumption of Lognormal distribution for lateral capacity, conditional to a given level of axial load. That hypothesis was not rejected with 5% significance level.
3) Robust regression was employed to develop simplified equations for the median lateral strength of each case-study wall as a function of the axial load. It is emphasised that robust regression is a form of regression analysis designed to circumvent some limitations of traditional parametric and non-parametric methods. For instance, ordinary least squares are based on certain assumptions, such as a normal distribution of errors in the observed responses and homoscedasticity, i.e., the variance of the error term is constant for all values of the explanatory variables. If the distribution of errors is asymmetric - as in the present study - model assumptions are invalidated, and parameter estimates, confidence intervals, and other computed statistics become unreliable.

Figure 2a gives an example of probabilistic limit strength domains for the case-study as-built wall discussed in previous section. Conversely, the lateral strength of that wall was disaggregated in terms of failure mode for each axial load level, as shown in Figure 2b. When a normalised axial load between 0.15 and 0.8 was considered, there were some cases of bed-joint sliding failure for the as-built wall. The contribution of that failure mode to the lateral strength exceeded 30% for a normalised axial load between 0.4 and 0.7. Probabilistic limit strength domains of FRP-strengthened walls are shown in Figure 3a, whereas Figure 3b shows the fitted Lognormal probability density function (PDF) of lateral strength under a given axial load level. A similar output was obtained for FRCM-strengthened walls and is not reported here for the sake of brevity.

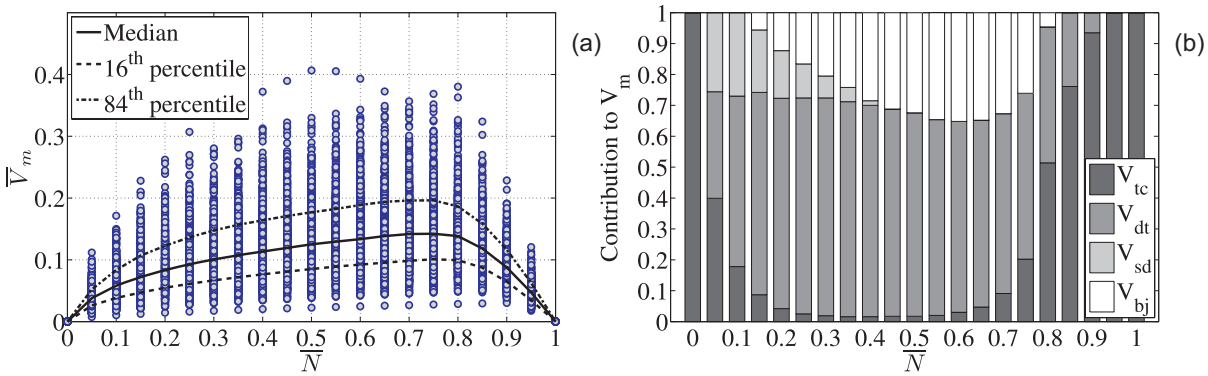

Figure 2. (a) Probabilistic limit strength domains of as-built walls; (b) disaggregation of lateral strength

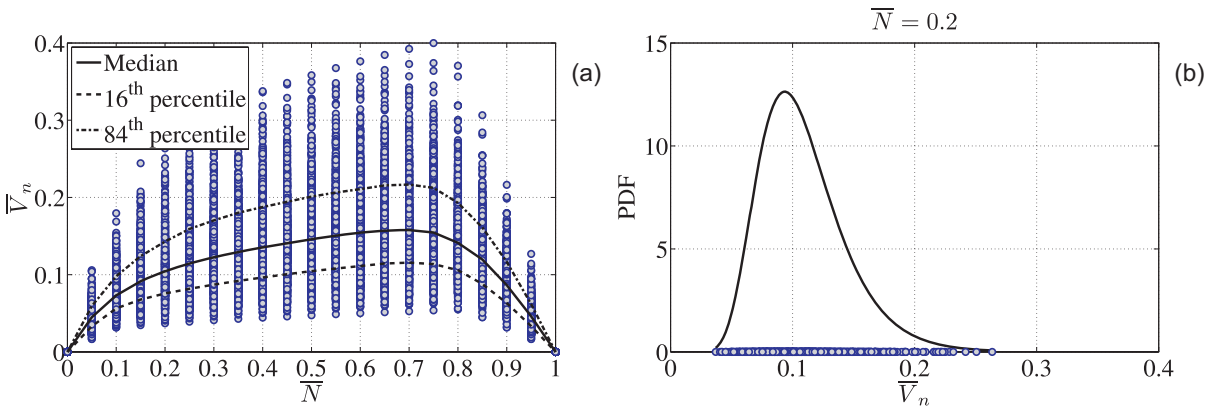

Figure 3. FRP-strengthened wall: (a) probabilistic limit strength domains; (b) PDF of lateral strength

PROBABILITY-BASED DESIGN EQUATIONS FOR LATERAL STRENGTH

Median limit strength domains were derived for all the case-study walls considered. Figure 4a shows those median domains associated with the same aspect ratio of wall are very close each other in the case of FRP-strengthened walls. A higher dispersion was found in the case of FRCM-strengthened walls, as shown in Figure 5a especially at aspect ratios equal to 1. The dispersion of lateral strength as a function of the normalised axial load was also investigated and was between 25%–35% in the case of FRP-strengthened walls (Fig. 4b), and 26%–39% in the case of FRCM-strengthened walls (Fig. 5b).

Nonlinear regression analysis was carried out for each case-study wall, assuming a quadratic polynomial function for the normalised lateral strength, as follows:

$$\overline{V} = c_1 \overline{N} + c_2 \overline{N}^2 \tag{14}$$

The coefficient of determination was found to be $R^2 > 0.9$ in the case of FRP-strengthened walls and $R^2 > 0.87$ in the case of FRCM-strengthened walls. As a quadratic function is used to predict the lateral strength corresponding to flexural failure of walls (see Eq. (1)), most of dispersion was caused by the occurrence of shear failure modes at intermediate axial load levels. The regression coefficients in Eq. (14) were estimated by robust regression as linear functions of the aspect ratio of walls, that is:

$$c_1 = a + b\frac{h}{l} \qquad c_2 = c + d\frac{h}{l} \tag{15}$$

Robust regression lines for these coefficients are shown in Figures 4c and 4d for FRP-strengthened walls, and Figures 5c and 5d for FRCM-strengthened walls where dispersion of median

domains at aspect ratios equal to 1 (see Fig. 5a) is confirmed. Expected values for coefficients in Eq. (15) are reported in Table 2 for both as-built and strengthened walls.

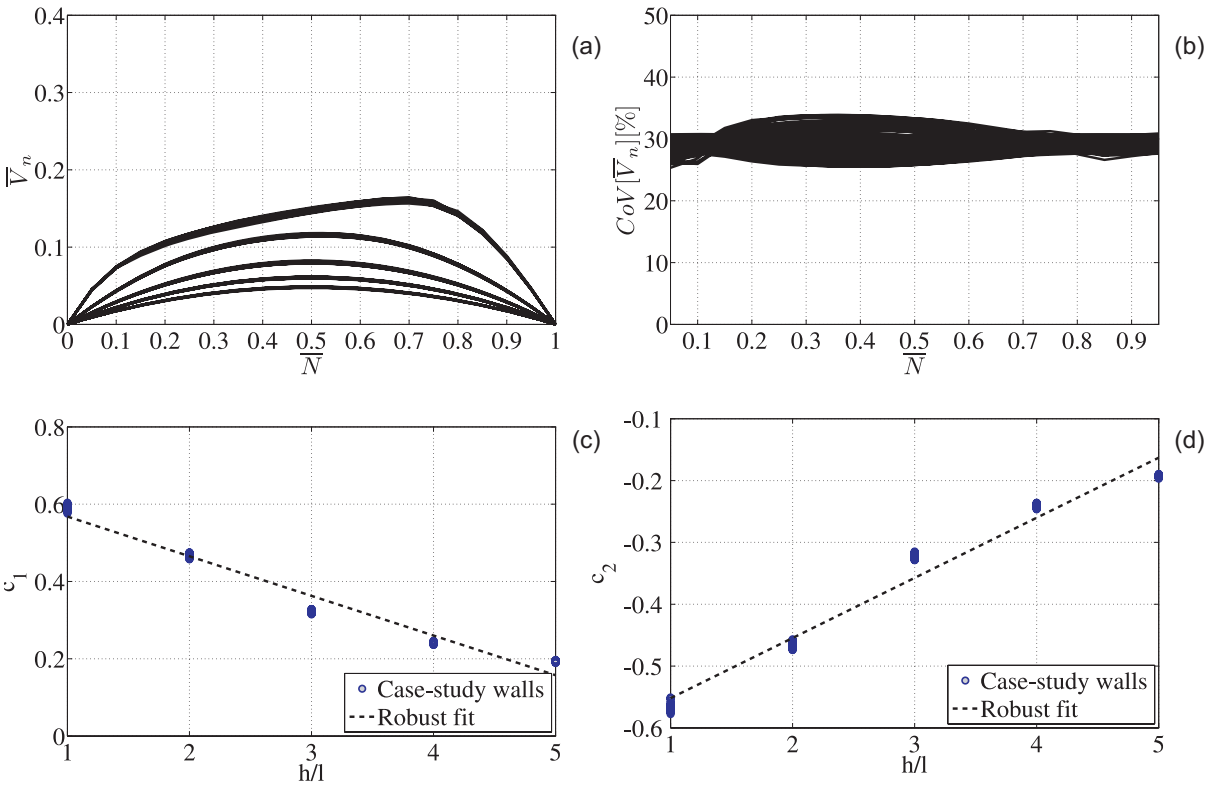

Figure 4. FRP-strengthened wall: (a) median limit strength domains; (b) lateral strength dispersion versus normalised axial load; (c) robust regression on c_1; (d) robust regression on c_2

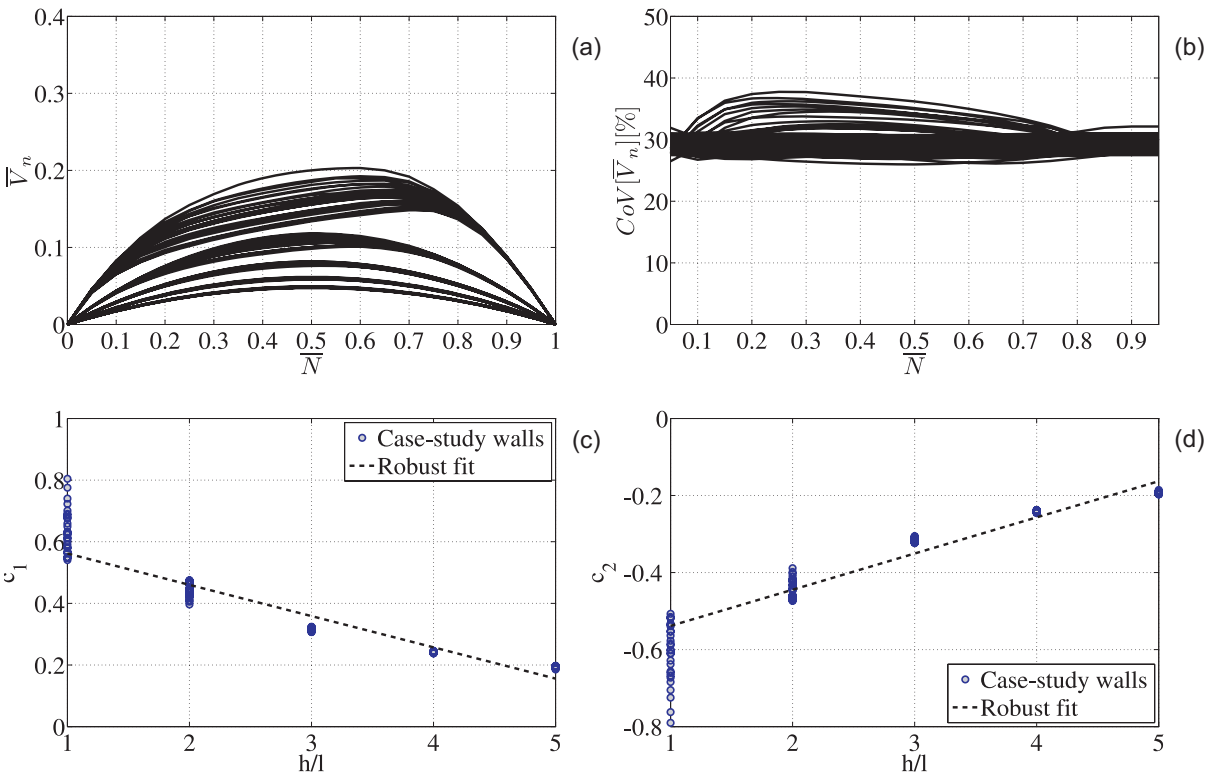

Figure 5. FRCM-strengthened wall: (a) median limit strength domains; (b) lateral strength dispersion versus normalised axial load; (c) robust regression on c_1; (d) robust regression on c_2

The dimensionless equations presented above allow designers to predict the in-plane lateral strength of masonry walls as normalised lateral strength times the nominal value of ultimate axial load. Any percentile of axial load demand provided by structural analysis may be used to this aim.

Based on results in Table 2, c_1 and c_2 have respectively positive and negative values for any aspect ratio, according to Eq. (1). It is underlined that IBC08 (IMIT, 2008) establishes that URM walls with $h/l > 3$ should not be included in the capacity model of a masonry building. A more conservative prescription is given in EC8 (CEN, 2004) for seismic design of URM buildings. Indeed, EC8 suggests to design walls with $h/l \leq 2$. Based on Eqs. (14)–(15) and estimates in Table 2, the reduction in lateral strength under varying aspect ratio was investigated and was computed with respect to that associated with $h/l = 1$. Considering all as-built and strengthened walls, the CoV of lateral strength reduction is 8%–11% for h/l equal to 2 and 3, and not greater than 1% for larger aspect ratios. The average reduction in lateral strength of any wall is 17% and 34% for h/l equal to 2 and 3, respectively, reaching 84% and 97% for h/l equal to 4 and 5. These results substantiate the recommendation in IBC08, while highlighting a high degree of conservativeness for the recommendation in EC8. Nevertheless, the authors will confirm such strength reductions after that effects of large uncertainty in robust regression at $h/l = 1$ will be further investigated for FRCM-strengthened walls.

Table 2. Robust regression coefficients of design equations for lateral strength

Wall type	*a*	*b*	*c*	*d*
As-built	0.56	–0.08	–0.52	0.07
FRP-strengthened	0.66	–0.10	–0.63	0.10
FRCM-strengthened	0.67	–0.10	–0.65	0.10

Median capacity increase factors (i.e., the ratios of median lateral strengths of strengthened and as-built walls) for URM walls externally strengthened with the FRP and FRCM systems under study were estimated through Eqs. (14) and (15). If $\bar{N} \in [0.1,0.7]$ (typical compression level in masonry buildings) and $h/l \in [1,3]$, the average increase in lateral capacity is between 16% and 18% regardless of the strengthening system (CoV equal to 5%–25% in the case of FRP-strengthened wall and 4%–14% in the case of FRCM-strengthened wall). In the same range of $\bar{N}$, the average strength increase reaches 27% (CoV = 16%) and 32%–40% (CoV = 5%–6%) for FRP- and FRCM-strengthened walls, respectively, showing a higher effectiveness of FRCM strengthening. The opposite effect is observed if $\bar{N} \in [0.8,1]$ and $h/l \in [1,4]$ as the capacity increase by diagonal FRP strengthening is approximately 10% greater than that provided by FRCM strengthening. Finally, if $\bar{N} \in [0.8,1]$ and $h/l = 5$, both strengthening systems provides an average strength increase of 53% (CoV equal to 28% and 14% in the case of FRP- and FRCM-strengthened wall, respectively).

CONCLUSIONS

Probabilistic limit strength domains of as-built, FRP-strengthened, and FRCM-strengthened tuff stone masonry walls have been presented. Capacity models available in literature, guidelines and codes were used. Based on statistics and probability distributions for material properties, geometry and models, a Monte Carlo simulation was run to estimate strength domains at several percentile levels. Lateral strength of walls was found to be lognormally distributed with dispersion varying with the axial load level. Finally, robust regression allowed to derive probability-based simplified equations for prediction of lateral strength as a function of the aspect ratio and normalised axial load. Based on such equations, it has been found that: (1) the median lateral strength of case-study walls with aspect ratios larger than 3 is notably lower than that related to walls with unit aspect ratios; (2) both FRP and FRCM systems provide an average increase in lateral capacity between 16% and 18% for typical compression levels.

Robust regression will be performed in a future work to provide the dispersion of lateral strength as a function of axial load, allowing to estimate any fractile of lateral strength for design of masonry walls externally strengthened with FRP and FRCM systems. The methodology presented in this study is also being applied to brick masonry walls and glass FRP/FRCM systems. Effects of reinforcement ratios, strengthening configurations (e.g., grid FRP layout, multiple plies) and more

refined capacity models different from those considered in this study will be investigated.

REFERENCES

American Concrete Institute (ACI) (2010) Guide for the design and construction of externally bonded fiber-reinforced polymer systems for strengthening unreinforced masonry structures, ACI440.7R-10, Farmington Hills, USA

American Concrete Institute (ACI) (2013) Design and construction guide of externally bonded FRCM systems for concrete and masonry repair and strengthening, ACI549-13, Farmington Hills, USA

Atadero R, Lee L and Karbhari VM (2005) "Consideration of material variability in reliability analysis of FRP strengthened bridge decks," *Composite Structures*, 7:430-443

Augenti N, Parisi F and Acconcia E (2012) "MADA: Online experimental database for mechanical modelling of existing masonry assemblages", *Proceedings of the 15th World Conference on Earthquake Engineering*, Lisbon, Portugal, 24-28 September 2012, Paper No. 2897 [database accessible online at http://www.reluis.it/index.php?option=com_mada&Itemid=160&lang=en]

Babaeidarabad S, De Caso F and Nanni A (2013) "URM walls strengthened with fabric-reinforced cementitious matrix composite subjected to diagonal compression," *Journal of Composites for Construction*, DOI: 10.1061/(ASCE)CC.1943-5614.0000441

Comité Européen de Normalisation (CEN) (2004) Eurocode 8 - Design of structures for earthquake resistance - Part 1: General rules, seismic actions and rules for buildings, EN 1998-1, Brussels, Belgium

Comité Européen de Normalisation (CEN) (2005a) Eurocode 6: Design of masonry structures - Part 1-1: General rules for reinforced and unreinforced masonry structures, EN 1996-1-1, Brussels

Galasso C, Maddaloni G and Cosenza E (2014) "Uncertainty analysis of flexural overstrength for new designed RC beams," *Journal of Structural Engineering*, DOI: 10.1061/(ASCE)ST.1943-541X.0001024

ICC Evaluation Service (2013) Proposed acceptance criteria for masonry and concrete strengthening using fiber-reinforced cementitious matrix (FRCM) composite systems, AC434, Whittier, USA

Iervolino I and Galasso C (2012) "Comparative assessment of load–resistance factor design of FRP-reinforced cross sections," *Construction and Building Materials*, 34:151-161

Italian Ministry for Infrastructures and Transportation (IMIT) (2008) DM 14.01.2008: Norme tecniche per le costruzioni, Rome, Italy [in Italian]

Italian National Research Council (INRC) (2004) Guide for the design and construction of externally bonded FRP systems for strengthening existing structures, CNR-DT200, Rome, Italy

Krevaikas TD and Triantafillou TC (2005) "Computer-aided strengthening of masonry walls using fibre-reinforced polymer strips," *Materials and Structures*, 38:93-98

Marcari G, Manfredi G, Prota A and Pecce M (2007) "In-plane shear performance of masonry panels strengthened with FRP," *Composites Part B*, 38 (7-8):887-901

Marcari G, Oliveira DV, Fabbrocino G and Lourenço PB (2011) "Shear capacity assessment of tuff panels strengthened with FRP diagonal layout," *Composites Part B*, 42(7):1956-1965

Mood MA, Graybill FA and Boes DC (1974) Introduction to the theory of statistics, McGraw-Hill, New York, USA

Nowak AS and Szerszen MM (2003) "Calibration of design code for buildings (ACI 318) - Part 1: Statistical models for resistance," *ACI Structural Journal*, 100(3):377-382

Papanicolaou CG, Triantafillou TC, Karlos K and Papathanasiou M (2007) "Textile reinforced mortar (TRM) versus FRP as strengthening material of URM walls: in-plane cyclic loading," *Materials and Structures*, 40(10):1081-1097

Parisi F and Augenti N (2013) "Seismic capacity of irregular unreinforced masonry walls with openings," *Earthquake Engineering and Structural Dynamics*, 42:101-121

Parisi F, Iovinella I, Balsamo A, Augenti N and Prota A (2013) "In-plane behaviour of tuff masonry strengthened with inorganic matrix–grid composites," *Composites Part B*, 45:1657-1666

Parisi F, Lignola GP, Augenti N, Prota A and Manfredi G (2011) "Nonlinear behavior of a masonry subassemblage before and after strengthening with inorganic matrix-grid composites," *Journal of Composites for Construction*, 15(5):821-832

Prota A, Manfredi G and Nardone F (2008) "Assessment of design formulas for in-plane FRP strengthening of masonry walls," *Journal of Composites for Construction*, 12(6):643-649

Prota A, Marcari G, Fabbrocino G, Manfredi G and Aldea C (2006) "Experimental in-plane behavior of tuff masonry strengthened with cementitious matrix-grid composites," *Journal of Composites for Construction*, 10(3):223-233

Stratford T, Pascale G, Manfroni O and Bonfiglioli B (2004) "Shear strengthening masonry panels with sheet glass-fiber-reinforced polymer," *Journal of Composites for Construction*, 8(5):434-443

Wang N, Ellingwood BR and Zureick A-H (2010) "Reliability-based evaluation of flexural members strengthened with externally bonded fiber-reinforced polymer composites," *Journal of Structural Engineering*, 136(9):1151-1160

Xiao LL, Wang XT, Li Y and Bulleit WM (2012) "Reliability analysis on shear capacity of reinforced masonry walls due to earthquakes," *Journal of Applied Mechanics and Materials*, 105-107:360-365