



## GROUND MOTION PREDICTION EQUATION FOR SOUTH ICELAND

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### ABSTRACT

A ground motion prediction equation is presented based on data from South-Iceland, one of the test areas in the European joint project UPStrat-MAFA. The model is based on a point source model that is specially adapted to Icelandic earthquakes. The model parameters are derived from the strong motion acceleration records obtained in South-Iceland in the period 1986-2008. The earthquakes were grouped into magnitude bins:  $M 6.5 \pm 0.2$ ,  $M 6 \pm 0.2$ ,  $M 5 \pm 0.2$ ,  $M 4 \pm 0.2$  and  $M 3 \pm 0.2$ . A set of parameters is estimated for each magnitude bin, obtained by fitting the model to the PGA values from the records by using constrained optimization. In this work a special attention is given to the duration parameter that is modelled by a functional form and the functions parameters were estimated for each magnitude range based on selecting intervals of the records containing 85% of the total energy in the record.

### INTRODUCTION

As a part of Task C in the UPStrat-MAFA (Calibration of the input parameters in pilot test area and completion of dataset) Project a ground motion model is presented based on strong motion data from one of the projects test areas, South-Iceland, the other being Mt Vesuvius, Campi Flegrei and Mt Etna in Italy and the Azores Islands. The dataset is composed of data in the magnitude range from approximately  $M 3$  to  $M 6.5$ , recorded by the Icelandic Strong Motion Network in the period 1986-2008.

The FINSIM (Beresnev and Atkinson, 1998) has been applied in the UPStrat-MAFA Project and input parameters have been determined for the larger events in the Icelandic dataset (Galluzzo et al., 2012). A similar stochastic model based on Brune's point source model has been developed for Icelandic strong motion data. The model is a theoretical model based on the stochastic approach (Boore, 1983) and was originally presented in Ólafsson and Sigbjörnsson (1999) and Ólafsson (1999). Many of the input parameters are the same for both aforementioned models. Strong motion duration and source dimensions and geometric attenuation function are for example used in both models.

The Icelandic stochastic model was originally developed based on data from a  $M 6$  earthquake in Vatnafjöll (Ólafsson et al., 1998) but has since been adapted to  $M 6.5$  earthquakes. In this paper the model will also be adapted to lower magnitude earthquakes that is  $M = 5.0$ ,  $4.0$  and  $3.0$ . The parameters for the ground motion model have been estimated based on the dataset presented in UPStrat-MAFA Project and are presented in the report (Ólafsson and Sigbjörnsson, 2014).

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## THE ICELANDIC STRONG MOTION DATA

The strong motion data included in this study is a dataset that was prepared for the UPStrat-MAFA project. In a part of the project called Task C: Calibration of the input parameters in pilot test area and completion of dataset. South-Iceland was one of the selected test areas, the other being. Mt Vesuvius, Campi Flegrei and Mt Etna in Italy and the Azores Islands. The main focus is on earthquakes in Volcanic areas. In the dataset for the Icelandic test area there are 53 earthquakes and a total of 684 recorded in South-Iceland. The dataset is a subset of records obtained by the The Icelandic Strong-Motion Network (IceSMN) that run by the Earthquake Engineering Research Centre of the University of Iceland. Strong-motion records obtained in earthquakes in Iceland by the IceSMN have been included in the Internet Site for European Strong-Motion, <http://www.isesd.hi.is> (Ambraseys et al., 2004) and can be freely downloaded. The magnitude of the earthquakes in the dataset range from about  $M 3$  to  $M 6.5$  and most of them have epicentral distance less than 100 km.

The earthquakes in the dataset are recorded in South-Iceland, both in the South Iceland Seismic Zone (SISZ) and the Western Volcanic Zone (WVZ) which is closer to Reykjavík. The SISZ is one of two transform zones, the other being off the shore of North Iceland, where the largest earthquakes in Iceland occur. The largest earthquakes can reach up to around  $M 7$  and have strike-slip mechanism. The earthquakes in the WVZ can exceed  $M 6$ . The earthquakes there may have a more complex mechanism. These seismic zones are within 100 km distance from Reykjavík and surrounding urban areas and have been extensively studied. The SISZ is a rather well defined, 70 km long and 20 km wide, and reflects the basic kinematic features referred to as bookshelf tectonics with shallow right lateral strike-slip earthquakes on near vertical faults (Einarsson, 1991).

## GROUND MOTION MODEL

The theoretical GMPE model is based on Brune's far-field source model (Brune, 1970) and is derived by using the Parsevals theorem, which equates rms-acceleration with an integral of the acceleration Fourier spectra squared, i.e. the periodogram. Similarly Brunes's model in the near-field is used for determining a closed form equation for the rms-acceleration in the near-field (Ólafsson and Sigbjörnsson, 2012). The attenuation model or GMPE is derived based on the far-field model and the near-field model can be used to constrain the theoretical attenuation model close to the source, especially for those magnitudes where near-field data is not available. The Fourier amplitude spectrum in the far-field is given as follows, where Brune's model is extended with an exponential term to account for the spectral decay at higher frequencies:

$$|A(\omega)| = \frac{2C_p R_{\theta\phi} M_0}{4\pi\beta^3\rho R} \frac{\omega^2}{(1+(\omega/\omega_c)^2)} \exp(-\frac{1}{2}\kappa\omega) \quad (1)$$

where  $M_0$  is the seismic moment;  $R_{\theta\phi}$  is the radiation pattern;  $C_p$  is a reduction factor accounting for the partitioning of the energy into two horizontal components;  $R$  is the distance to the fault;  $\beta$  is the shear-wave velocity;  $\omega_c$  ( $= 2\pi f_c$ ) is the corner frequency, and  $\rho$  is the material density of the crust. Furthermore, it is assumed, as an engineering approximation, that the spectral decay parameter,  $\kappa = R/Q\beta$  ( $Q$  is a path-averaged quality factor), increases very slowly with distance and is near constant within a certain radius of the earthquake source. The geometrical spreading function  $G(R)$  is here assumed to represent  $(2C_p R_{\theta\phi} M_0 / 4\pi\beta^3\rho R)$ . The distance to the fault,  $R$ , can, for example, be taken to represent the hypocentral distance or the closest distance to the fault. More detailed models would take into account a faster rate of attenuation close to the fault than  $1/R$ .

$$\log_{10}(a_{rms}) = \log_{10}\left(\frac{(2\sqrt{7})^{2/3} C_p R_{\theta\phi} \Delta\sigma^{2/3}}{2\sqrt{\pi} \beta\rho\sqrt{\kappa}}\right) + \frac{1}{2}\log_{10}\left(\frac{\Psi}{T_d}\right) + \frac{1}{3}\log_{10}(M_o) - \log_{10}(R) \quad (2)$$

here  $T_d$  represents the strong-motion duration,  $M_0$  represents the seismic moment,  $\beta$  is shear wave velocity,  $R_{\theta\phi}$  is the radiation pattern,  $C_p$  is a partitioning factor  $(2)^{-1/2}$ ,  $\rho$  is the density of the crust,  $\Delta\sigma$  is the seismic stress drop and  $\Psi$  represents a dispersion function of the variable  $\lambda = \kappa\omega_c$ , and can be evaluated by a closed form expression. The peak ground acceleration can be evaluated as  $a_{\text{peak}} = pa_{\text{rms}}$  by using a peak factor  $p$  obtained by applying the theory of locally stationary Gaussian processes (Vanmarke and Lai, 1980). The dispersion function  $\Psi$  can be represented in closed form as:

$$\Psi = 1 - \frac{1}{2}\lambda \text{ci}(\lambda)(\lambda \cos(\lambda) + 3 \sin(\lambda)) - \frac{1}{2}\lambda \text{si}(\lambda)(\lambda \sin(\lambda) - 3 \cos(\lambda)) \quad (3)$$

Here,  $\text{ci}(\bullet)$  and  $\text{si}(\bullet)$  represent cosine and sine integrals with  $\lambda = \kappa\omega_c$  where  $\omega_c$  is the corner frequency of the Brune spectrum.

## GEOMETRIC SPREADING FUNCTION

An important term in Eq.(2) is the geometric spreading function that describes the varying rate of attenuation with distance from source. The following expression is suggested for the geometrical spreading function (Ólafsson, 1999):

$$R = \begin{cases} D_2^{1-n} D^n & D_1 < D \leq D_2 \\ D & D_2 < D \leq D_3 \end{cases} \quad (4)$$

where  $1 < n \leq 2$  and  $R$  is a distance defined as:

$$D = \sqrt{d^2 + h^2} \quad (5)$$

Here,  $d$  is the epicentral distance and  $h$  is a depth parameter. The parameters  $D_1$ ,  $D_2$  and  $D_3$  are used to set the limits for the different zones of the spreading function. The first zone can be thought of as a crude approximation for the intermediate field. Hence, the quantity  $D_1$  can be approximated by  $h$ ;  $D_2$  quantifies the size of the zone representing the intermediate field, which is related to the magnitude of the earthquake (as represented by the seismic moment) and the thickness of the seismogenic zone; while  $D_3$  can be thought of as the distance where cylindrical waves begin to dominate the wave field.

## DURATION

A required quantity of the GMPE of Eq.2 is the strong motion duration,  $T_d$ . For the near-field model the duration is the time it takes for the fault to break, that is the source duration termed  $T_0$ . Further away from the fault there is an increase in the duration with distance due to the dispersion of the seismic waves (Trifunac and Brady, 1975). The method selected here is to select the most important part of the strong motion record based on the energy contained in the measured ground motion (acceleration in this case).

The following simplified relationship describes this increase in the duration with respect to epicentral distance,  $d$ :

$$T_d = c_1 \frac{r}{\beta} + c_2 d^{c_3} + \sigma_T \quad (5)$$

Here  $r$  and  $c_1$ ,  $c_2$ ,  $c_3$  are regression coefficients,  $\sigma_T$  is the standard deviation,  $r$  is the radius of the dislocation and  $\beta$  the shear wave velocity.

In this article a similar process as is in Ólafsson and Sigbjörnsson (2012) is applied to determine the coefficients  $c_1$ ,  $c_2$ ,  $c_3$ . The duration is computed as an interval containing a certain fraction of the total energy in the signal. In this article the duration is chosen based on discarding the part of the signal containing the first 5% and the last 10% of the data points. It should also be mentioned that in Snaebjörnsson and Sigbjörnsson (2012) a duration model for Icelandic earthquakes is presented

## APPLICATION TO DATA

The theoretical GMPE presented in this article (see Eq.(2)) has been applied to a set of earthquakes from South-Iceland that were included in the UPStrat-MAFA EU Project. In the dataset there are a total of 684 records from 53 earthquake recorded in the period from August 1986 to May 2008, with a range of magnitudes from  $M$  3 to  $M$  6.5. The largest events are the two  $M$ 6.5 earthquakes which occurred on June 17<sup>th</sup> and June 21<sup>st</sup> in the year 2000. On May 29<sup>th</sup> 2008 a  $M$ 6.3 earthquake occurred near the town Hveragerdi in South-Iceland and the strong motion records from that event are the last ones to be included in the dataset.

Most of the events have a strike slip mechanism with a near vertical fault plane. A few of the earthquakes are, however, from the Western Volcanic Zone and some of them have normal source mechanism. Do to how few records there are the majority of them are strike slip earthquakes, no attempt is made to sort the earthquakes by mechanism.

For the processing the earthquakes are sorted into 5 bins according to magnitude that are close to the following central magnitude values:  $M$  6.5,  $M$  6,  $M$  5,  $M$  4 and  $M$  3. Earthquakes with magnitudes that fall within the range of  $\pm 0.2$  of these central magnitude values are selected into the bins. For optimum model fit the parameters of the theoretical model need to be obtained by estimating the source parameters for Eq.(2) and duration function of Eq.(5). It was decided to use calculate the duration of the strong motion based on selecting the interval that contained 85% (start limit 5% and end limit 90% of the cumulative energy). The parameters  $c_1$ ,  $c_2$  and  $c_3$  for each bin were obtained by fitting the function of Eq.(5) to the duration values as a function of distance from source. The resulting curves for the time functions can be seen in Fig.1. The curves represent duration functions for magnitudes  $M$  6.5 (red),  $M$  6 (blue),  $M$  5 (green),  $M$  4 (magenta) and  $M$  3 (black).

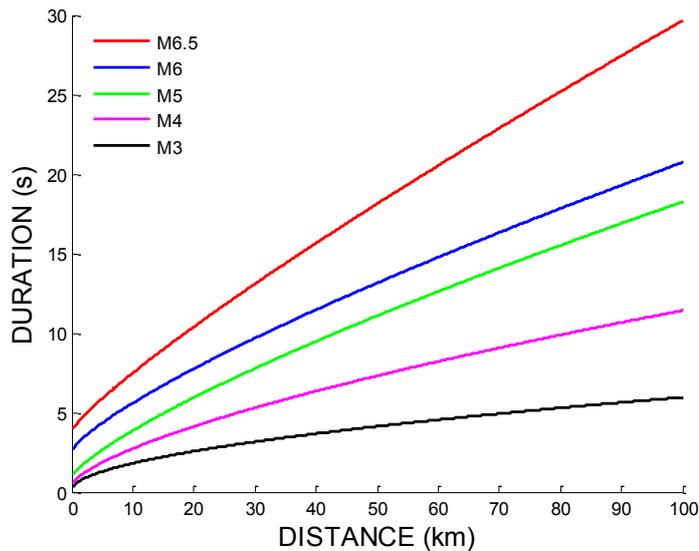


Figure 1. Duration function of Eq.(5) fit with constrained optimization to duration based on 90% cumulative energy. The curves represent duration functions for magnitudes  $M$  6.5 (red),  $M$  6 (blue),  $M$  5 (green),  $M$  4 (magenta) and  $M$  3 (black).

Using the estimated parameters for the durationa functional form of Eq.(5) the theoretical GMPE model parameters in Eq.(1) ( $\kappa$ ,  $r$ ,  $\Delta\sigma$ ,  $h$ ,  $D_2$ ) were estimated using constrained optimization. Apriori information about the model parameters was used to constrain the parameters. In Fig.2 the models are shown using the estimated optimum parameter and the PGA obtained from the recorded ground motion for a)  $M$  3, b)  $M$  4, c)  $M$  5, and d)  $M$  6. The solid black curve represents the mean value given by the GMPE and the dotted red curves represent the mean value  $\pm$  one standard deviation ( $\pm 1\sigma$ ). The parameters are not shown here but can be found in report by Ólafsson and Sigbjörnsson (2014).

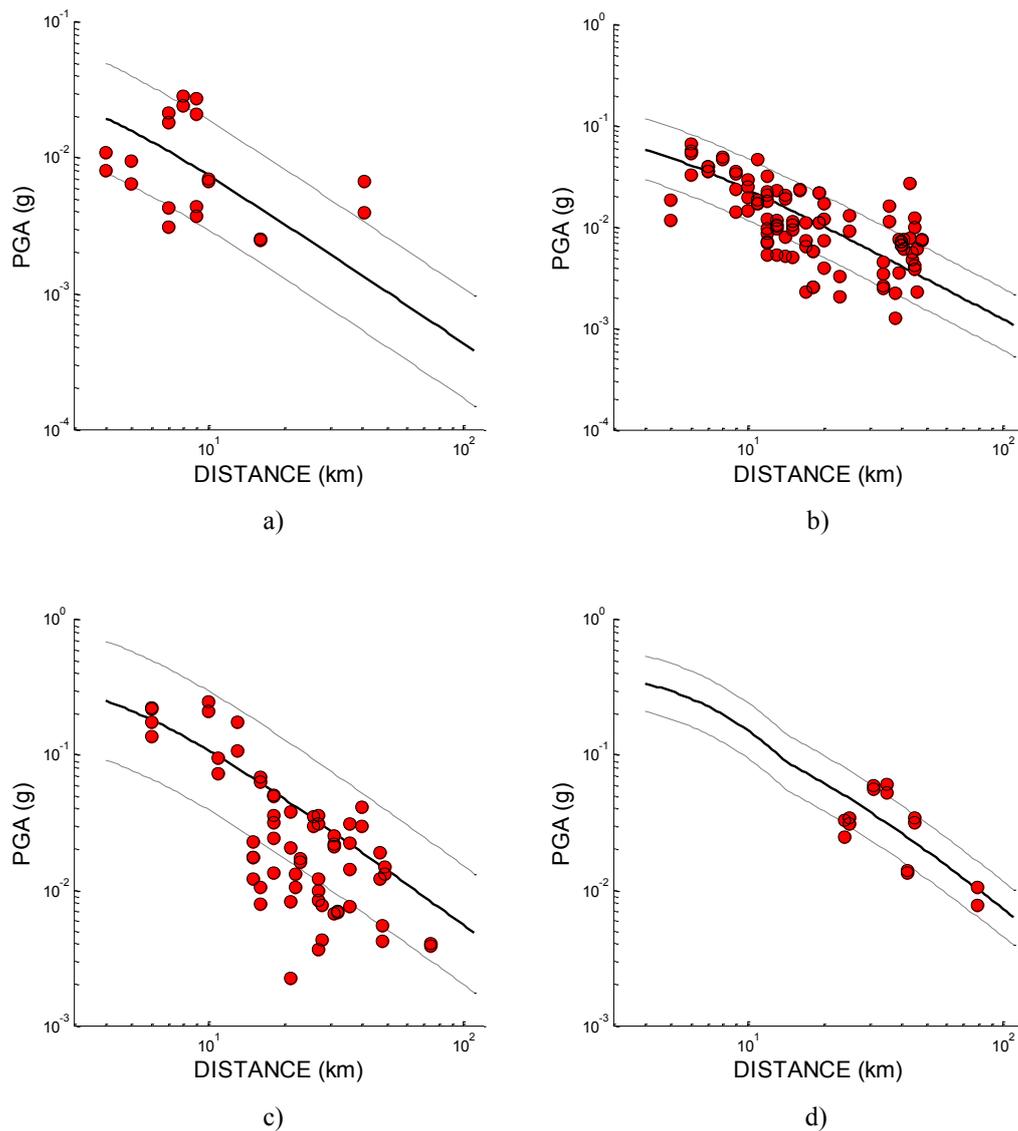


Figure 2. a) Theoretical GMPE applied to data for different magnitudes. The solid black curve represent the mean value and the dotted curve dotted red curves represent mean value  $\pm$  one standard deviation ( $\pm 1\sigma$ ). a)  $M$  3 b)  $M$  4 c)  $M$  5 d)  $M$  6

Fig.3 shows the result of applying the theoretical GMPE to the strong motion records contained in the  $M$  6.5 magnitude bin. Records from the two earthquakes in June 2000 from June 17<sup>th</sup> and June 21<sup>st</sup> are contained in this magnitude bin. The solid black curve represent the mean value and the dotted black curve represent mean value  $\pm$  one standard deviation ( $\pm 1\sigma$ ). The red solid circles represent PGA from strong motion data recorded in South-Iceland on June 17<sup>th</sup>, 2000. The blue triangles represent PGA obtained from strong motion recorded four days later, June 21<sup>st</sup>, 2000.

In Fig.4 the mean value curves for the 5 earthquake bins are plotted on the same graph where in a) the scale is log-log but in b) the scale is linear on both axis (lin-lin). The curves represent from top to bottom  $M$  6.5 (red),  $M$  6 (blue),  $M$  5 (green),  $M$  4 (magenta) and  $M$  3 (black).

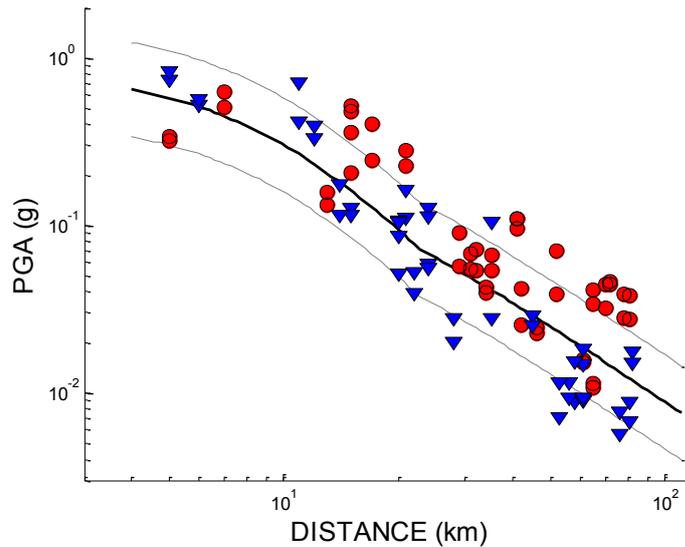


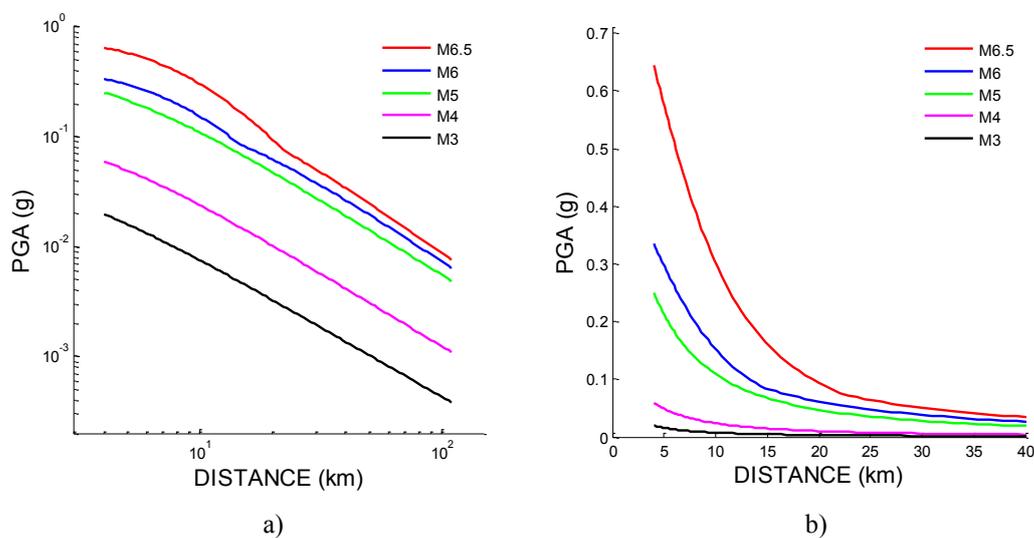
Figure 3. Theoretical GMPE applied to PGA data from two  $M$ 6.5 earthquakes. The solid black curve represent the mean value and the dotted black curve are represent mean value  $\pm$  one standard deviation ( $\pm 1\sigma$ ). The red solid circles represent PGA from strong motion data recorded in South-Iceland on June 17<sup>th</sup>, 2000. The blue triangles represent PGA obtained from strong motion recorded four days later, June 21<sup>st</sup>, 2000.

It is interesting to note that the break in the curve for the intermediate zone, (i.e.  $D_1 < D < D_2$  in Eq.(4) that represents the geometric attenuation function) can be clearly seen in Figs.4 a) and b), but only for the larger magnitudes (i.e.  $M$  6.5 and  $M$  6). This is due to the fact that  $D_2$  is a function of size of the earthquake, being approximately in the interval  $[3r, 4r]$  where  $r$  is the radius of the dislocation (fault plane). So for the smaller magnitudes the zone is so close to the epicentre that it cannot be noticed on the graphs. Also no near field measurements are available so close to the fault. This also applies to  $M$  6. But for  $M$  6.5 (see Fig.3) the effect of the geometric attenuation function is clearly visible and without the change in the rate of geometric attenuation given by Eq.(4) the model would not provide a good fit the the data closer to the fault. This would lead to an underestimation of the acceleration in the near-field which in term of potential damage is the most relavent zone.

In Fig.2 for b)  $M$  4 and c)  $M$  5 it can be observed that there are several points that extend outside of the one standard deviation dotted line and can be viewed as outlier. For b)  $M$  4 there are several points that line up outside of the dotted line between at a 40 to 50 km distance from the epicentre. It is possible that this is due to Moho bounce. For Fig.2 c), however, there are several points that lie outside of the dotted line. There is the possibility that their local magnitude which is used for classifying the records into bins, does not give a correct estimate of the earthquake size. A better method of determining the magnitude would be to calculate the seismic moment and from those results calculate the moment magnitude. It is also possible that these records are from events from

lower stress drop than the other earthquakes. A steeper rate of attenuation for these events could also play a role. This is also matter that can be investigated further by doing an estimat all the relevants source model parameters for these records.

Several other strong motion records exist from Iceland that could be used to validate the models for the different magnitudes (shown in Figs.2 and 3). The model for  $M$  6.5 was validated af few years back when a  $M$ 6.3 earthquake occurred close to the town of Hvergerði on May 29<sup>th</sup> 2008. The estimation of the source parameters from all available strong-motion records could also lead to an improved model. An investigation of whether there is a difference in source parameter between the various zones within Iceland needs to be carried out. An estimate of source parameters for all the available strong motionn data was performed by Ólafsson, 1999, but needs to be repeated due to the increase in the number of strong motion records obtained by the Icelandic Strong Motion Network (since 1999). The question of whether is different characteristic for the earthquakes in with regard to their loacation in North and South Iceland, as well as examining whether there is difference between earthquake characteristic.



**Figure 4.** The curves represent the mean value of the theoretical GMPE applied to Icelandic strong motion data recorded in earthquakes of different magnitudes and are shown in Fig.2 along with the data. The two graphs show the curves in: a) Log-Log scale b) Linear-Linear scale. The curves represent GMPEs based on data from magnitudes:  $M$  6.5 (red curve),  $M$  6 (blue curve),  $M$  5 (green curve),  $M$  4 (magenta),  $M$  3 (black curve).

## CONCLUSIONS

A theoretical ground prediction model was applied to the subset of strong ground motion recordings from South-Iceland that was included in the EU project UPStrat-MAFA. Up to this point the ground motion model had mainly been applied to the larger earthquakes ( $M$  6 and  $M$  6.5) but in this study the model was extended to include the smaller magnitude events. The earthquakes were grouped into magnitude bins:  $M$   $6.5 \pm 0.2$ ,  $M$   $6 \pm 0.2$ ,  $M$   $5 \pm 0.2$ ,  $M$   $4 \pm 0.2$  and  $M$   $3 \pm 0.2$ . A duration model was then estimated for each magnitude range based on selecting intervals of the records containing 90% of the total energy in the record. The other parameters of the ground motion model were then obtained by using constrained optimization.

The main reasons for applying a theoretical GMPE to the Icelandic earthquakes, instead of the more traditional empirical regression type models, were originally the lack of data and also the fact that GMPEs from other areas did not fit the Icelandic data very well. The advantage of using a theoretical model is that most of the parameters have a direct physical meaning and the same model that can estimate be used to estimate PGA at each site can be used to simulate ground motion records

using the stochastic method. Furthermore, the parameters for the model can be estimated directly from the strong ground motion records or a priori information about earthquake source properties in a certain area.

The ground motion modelling study presented here is a step towards of complete model for Iceland. However, still further work needs to be done. A complete recalculation of the source parameters for all strong motion records available in Iceland is required. With an estimate of the seismic moment a new estimate of the magnitude (moment magnitude) will be obtained, which possibly improves the classification of the earthquakes into magnitude bins. Also to see whether or not source parameters vary between areas, such as the seismic zones and the volcanic zones or North and South Iceland.

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