



MODELING OF UPLIFTING MECHANISM IN UNANCHORED LIQUID STORAGE TANKS SUBJECTED TO SEISMIC LOADING

Maria VATHI¹ and Spyros KARAMANOS²

ABSTRACT

The present paper focuses on the seismic response of unanchored (self-anchored) liquid storage tanks. Ground supported liquid-storage cylindrical tanks that are unanchored at their base, when subjected to strong seismic loading may exhibit uplifting of their bottom plate, which has significant effects on their dynamic behavior and strength. Those effects mainly concern: (a) the increase of axial (meridional) compression at the tank base, resulting in premature buckling in the form of elephant's foot and (b) the significant plastic deformation at the vicinity of the welded connection between the tank shell and the bottom plate that may cause failure of the welded connection. Base uplifting mechanics and the corresponding effects are examined numerically through a detailed finite element shell model of the tank for nonlinear static analysis, capable of describing the state of stress and deformation at different levels of loading. Numerical results for typical steel liquid storage tanks are compared with the above design provisions.

INTRODUCTION

Liquid storage tanks are used in water distribution systems in refineries and in industrial plants for storing oil or other petrochemical liquids. In numerous practical applications, relatively broad aboveground liquid storage tanks are constructed unanchored, in the sense that their bottom plate is in simple contact with the ground, without anchor bolts. In such a case, under strong seismic loading, the tank may exhibit uplifting of its bottom plate, when the magnitude of the overturning moments exceeds a threshold value. Although uplift does not necessarily result in the collapse of the tank, its consequences may lead to serious damage to any piping connections that are incapable of accommodating severe vertical displacements, possible damage to the uplifted bottom plate, damage of the connection between the tank shell and the bottom plate and an increase in the axial stress acting on the tank wall which remains in contact with the ground, leading to a precipitation of "elephant's foot" buckling.

Valuable insight into the uplifting resistance of a plate may be gained from the solution of the somewhat simpler problem of a prismatic beam uplifted at one end. With this objective, several solutions of the beam problem have been attempted in the past. In the simpler of these analyses, the effect of membrane forces is ignored (Wozniak and Mitchell, 1978; Leon and Kausel, 1986), with the result that the maximum load capacity for the beam is reached as soon as two plastic hinges develop: one at the uplifted end, and the other at the section of maximum moment within the beam. An approximate solution that incorporates the effects of the membrane forces was proposed by Cambra (1982) using, on the basis of experimental data, certain simplifying assumptions regarding the

¹ Ph.D. Student, University of Thessaly, Volos, mvathi@mie.uth.gr

² Associate Professor, University of Thessaly, Volos, skara@mie.uth.gr

magnitude of the axial and shearing forces in the beam. A second-order beam theory that provides for the effects of membrane forces more accurately was used by Auli et al. (1985). Their analysis, however, did not provide for the effects of flexible end constraints and of plastic yielding within the beam. Ishida and Kobayashi (1988) made use of the finite element method to solve the beam problem in their analysis; they made a series of simplifying assumptions regarding the distribution of forces in the base plate and the foundation in order to analyze the rocking response of tanks. Malhotra and Veletsos (1994a) studied extensively the uplifting behavior of the bottom plate of an unanchored tank by idealizing the base plate as a uniformly loaded semi-infinite, prismatic beam that rests on a rigid foundation. The effect of elastic end constraints, the influence of the axial forces associated with large deflections and the effect of plastic yielding in the beam were considered. In a recent publication, Ahari et al. (2009) used a tapered beam to model base uplift of unanchored tanks.

The previous beam models did not take into account the effect of hoop deformation stresses, which develop in the base plate close to the junction with the shell wall. As an alternative to the beam-type approach, a fundamental step in the response analysis of liquid storage tanks is the investigation of the behavior of the partially uplifted base plate. Some more recent studies on plate uplifting were carried out by a combination of finite difference and energy technique (Peek and Jennings, 1988; Peek, 1988), and by the Ritz energy method (Haroun et al., 1987; Haroun and Badawi, 1988; Lau and Clough, 1989). Both axisymmetric and asymmetric solutions have been presented in these studies and the results compared with those obtained from experiments. A significant contribution of these studies has been the demonstration of the importance of the membrane forces on the load-carrying mechanism. The reported methods are generally rather complex and non-efficient for dynamic response analyses of uplifting tanks, in which the solution of the plate problem must be obtained at numerous integration steps. The complex nature of these analyses has also precluded an in-depth parametric study of the numerous factors involved and a simple interpretation of the results. Additionally, those studies did not consider the effects of load reversals and of the associated energy dissipation due to yielding; these effects may be important in dynamic response analyses. Malhotra and Veletsos (1994b) attempted to improve this approach by computing the vertical and rocking resistances of a uniformly loaded circular plate to forces that are distributed along its boundary. They described an exact method for axisymmetric vertical uplifting and an approximate method for asymmetric rocking uplifting. The latter was modeled by a series of semi-infinite prismatic beams. The two analyses were compared afterwards for checking the accuracy of the approximate method.

Studies on the dynamic response of unanchored tanks supported on rigid foundations have shown that base uplifting influences significantly the dynamic behavior of tanks, leading to axial compressive stresses in their walls that are substantially higher than those in similarly excited fixed-base systems, as noted by Peek (1986). Babcock and Shih (1981) performed scale model tests in an effort to gain a better understanding of the response and the failure criterion of the tank. The results of these experiments showed that the response of the unanchored tank is different than that predicted by current seismic codes and is highly influenced by the uplift mechanism. Natsiavas and Babcock (1987; 1988a; 1988b; 1990) presented analytical simplified models for determining the dynamic response and the hydrodynamic loads developed on unanchored liquid-filled tanks under horizontal base excitation. Notable contributions on the seismic response of anchored and unanchored liquid storage tanks have been presented by Fischer (1979), Rammerstorfer et al. (1988), Fischer et al. (1991), with particular emphasis on design implications. Malhotra and Veletsos (1994c, 1995), based on their previous works (1994a; 1994b), used the results they derived and thoroughly investigated the effects of uplifting of the bottom plate of the tank on the entire tank-liquid system, for a rigid foundation. In subsequent publications, Malhotra (1995; 1997) revisited the uplifting problem for a flexible foundation.

In the present study, uplifting of the tank bottom is being examined using numerical simulation tools. For this purpose, two typical liquid storage tanks are modeled using finite elements, and their behavior is discussed with respect to the seismic provisions of API 650 in Appendix E and the relevant provisions of EN 1998-4 in Annex A. A non-linear static analysis is conducted, applying the distribution of hydrodynamic pressure due to impulsive and convective motion through an incremental pushover analysis. Finally, in order to examine the influence of the roof to the uplifting mechanism, a parametric study of 6 liquid storage tanks with different aspect ratios has been conducted.

DESCRIPTION OF TANKS

Two typical fixed-roof liquid storage tanks are considered and simulated numerically. They represent real cases, in areas of seismic activity. The tanks have been designed using the relevant API 650 rules, without considering the new version of Appendix E, launched in 2007.

The first tank is a moderately-broad tank, referred to as Tank I. It is a 27.8-meter-diameter tank with a total height of 16.5 meters. The tank is unanchored. The filling height of the tank H is equal to 15.7, which corresponds to an aspect ratio of the tank, $\gamma(= H / R)$, equal to 1.131. The tank thickness varies from 6.4 mm at its top course to 17.7 mm at its bottom course and the bottom plate is 6 mm thick with an 8-mm-thick annular plate.

The second tank considered, referred to as Tank II, is a broad 46.9-meter-diameter tank with a total height of 19.95 meters. The filling height of the tank H is equal to 18.37 resulting to an aspect ratio, $\gamma(= H / R)$, equal to 0.783. The tank thickness varies from 8 mm at its top course to 22.23 mm at its bottom course.

In both tanks, the contained liquid is water ($\rho = 1000 \text{ kg/m}^3$) and the material of the tank shell, the bottom plate and the roof is structural steel S235 (equivalent to A36 steel) with yield stress $\sigma_y = 235 \text{ MPa}$. For the purposes of the present study, the tanks are simulated as both anchored and unanchored. The tank geometries are depicted in Fig.1.

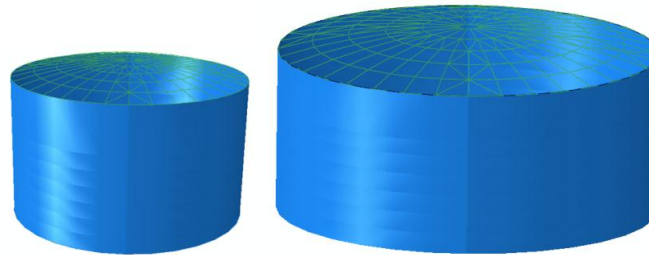


Figure 1. Tanks used for the parametric study

NONLINEAR STATIC ANALYSIS

Model Description

General-purpose finite element program ABAQUS is used for the simulation of the structural behavior of the tank under static loading, assuming anchored and unanchored conditions.

For the case of unanchored conditions, the tank shell and the bottom plate are modeled with shell finite elements, with special attention on the annular base plate of the tank. More specifically, the tank and the bottom plate are modeled with four-node reduced-integration shell elements (S4R). In such a case, the effects of uplifting of the unanchored tank are simulated with the use of appropriate contact conditions between the bottom plate of the tank and the ground. The ground is simulated using solid eight-node reduced-integration elements (C3D8R). The friction coefficient between the tank base plate and the ground is assumed to be 0.3. The tank roof is modeled with the use of two-node (linear) beam-type finite elements (B31).

The material of the tanks is described through a von Mises (J_2) plasticity model, with isotropic hardening. The yield stress of the S235 material is equal to 235 MPa, the Young's modulus is $E = 210,000 \text{ MPa}$, and the hardening modulus is assumed equal to 210 MPa which corresponds to 1/1000 of Young's modulus. For the ground, an elastic material has been used with Young's modulus $E = 110,000 \text{ MPa}$, Poisson's ratio $\nu = 0.3$ and mass density $\rho_g = 7500 \text{ kg/m}^3$.

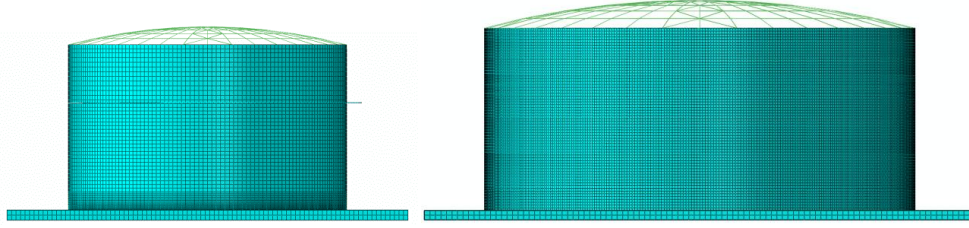


Figure 2. Finite element modeling of the two tanks used for the parametric study

An incremental non-linear static analysis is conducted in three steps. In the first step, gravity of the tank is applied, whereas in the second step, hydrostatic pressure of the liquid is activated. Finally, in the third step, horizontal hydrodynamic loading is applied, through an incremental elastic-plastic “pushover” analysis with large displacements. The distribution of hydrodynamic pressures on the tank wall follows the corresponding solution of the hydrodynamic problem (Wozniak and Mitchell 1978; Ibrahim 2005), for both the impulsive and convective component of liquid motion. More specifically, the distribution of hydrodynamic pressure is assumed for the impulsive motion:

$$P_I(\xi, \zeta, \theta, t) = C_I(\xi, \zeta) \rho H \cos \varphi S_I \quad (1)$$

where C_I is a spatial function of the non-dimensional coordinates of the tank which depends on the tank aspect ratio, ρ is the liquid density, H is the liquid height, R is the tank radius, S_I is the impulsive acceleration, $\xi = r/R$ and $\zeta = z/H$ are the non-dimensional coordinates assuming a cylindrical coordinate system, r, φ, z , with origin at the center of the tank bottom and the vertical axis. For the convective motion, only the first mode is considered.

$$P_C = 1.146R\rho \frac{\cosh(1.841\gamma\zeta)}{\cosh(1.841\gamma)} J_1(1.841\xi) \cos \varphi S_C \quad (2)$$

where J_1 is the Bessel function of first order and S_C is the convective acceleration. In the above expressions, r is taken equal to R ($\xi = 1$) for determining the pressure on the lateral surface, whereas z is taken equal to zero ($\zeta = 0$) for the pressure on the bottom plate.

The hydrodynamic pressure distribution is applied on the inner surface of the tank shell and the bottom plate through a special-purpose load-user subroutine in the ABAQUS model.

Numerical Results for Uplifting Tanks

Representative results of the behavior of the tank under consideration are displayed in the present section. Numerical results for the unanchored tank are examined and subsequently they are compared with results from the anchored tank. The general configuration of the deformed tank is shown schematically in Fig.3.

The average top horizontal displacement of the tank ($u = (u_1 + u_2) / 2$) is plotted in Fig.4 in terms of the overturning moment M due to hydrodynamic loading. The value of M is normalized by WH , where W is the total weight of the contained liquid and H is the maximum design product level. The graph shows that the increase of the overturning moment results in an increase of the top horizontal displacement. Furthermore, at the level of normalized overturning moment equal to 0.03 the slope of the curve significantly changes, indicating the beginning of the uplifting of the tank and the occurrence of inelastic deformation.

The global behavior of the tanks is also represented in Fig.5, which shows the maximum vertical uplift of the simulated unanchored tanks in terms of the normalized overturning moment at the

tank base. The present results indicate a smaller sensitivity of the uplifting on the value of tank aspect ratio, than the one suggested by the provisions of EN 1998-4 (2006).

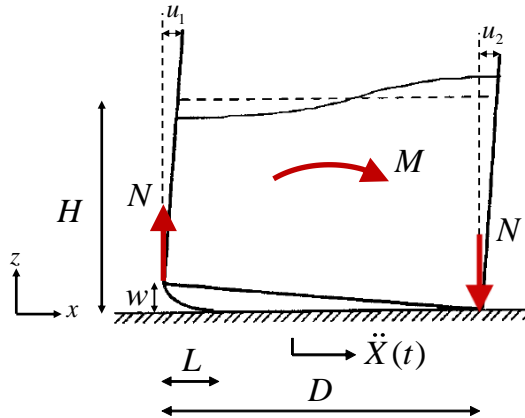


Figure 3. Uplifting parameters of an unanchored tank, also used in the following diagrams

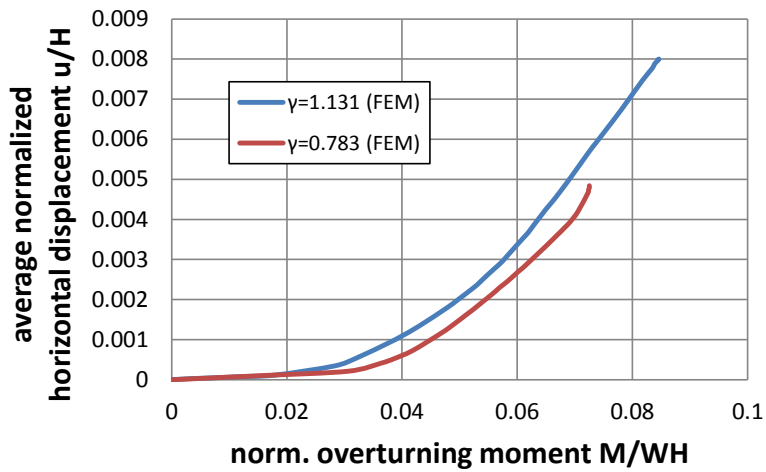


Figure 4. Top horizontal displacement u versus overturning moment M , subjected to finite element pushover analysis

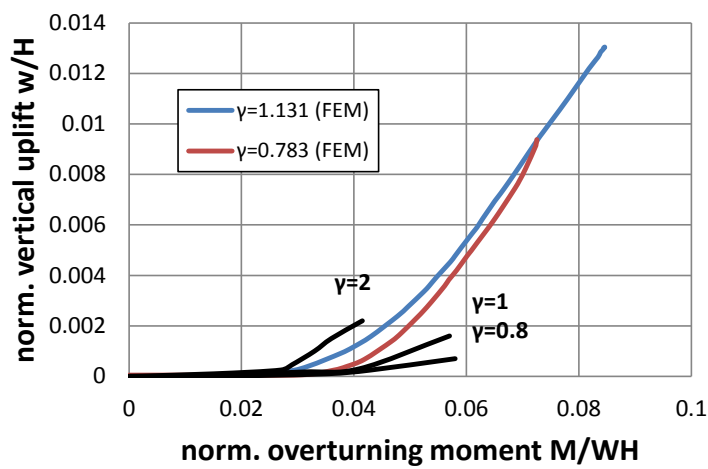


Figure 5. Maximum vertical uplift of the tank w versus overturning moment M ; the present finite element results are compared with the curves for $\gamma = H/R = 0.8, 1.0, 2.0$, adopted by EN 1998-4 (2006)

One should note that the curves for $\gamma = H/R = 0.8, 1.0, 2.0$ are the results from the work of Scharf (1990), adopted by EN 1998-4 (2006). The comparison between the finite element results and those in EN 1998-4 also shows that the EN 1998-4 curves appear to be non-conservative, predicting a smaller maximum uplift of the tank than the uplift obtained in the present finite element analysis. For estimating the radial membrane stresses in the plate, the relation of length L and the uplifting size w of the uplifted part of the tank bottom is necessary. The relationship between the normalized uplifted length (divided by R , the radius of the tank) and the normalized overturning moment is shown in Fig.6. Increase of the value of the overturning moment causes an increase of the value of the length of the uplifted base plate, as expected.

Combining the results from Fig.5 and Fig.6 one can develop the graph of Fig.7. It is interesting to note that once uplift occurs, the dependence of L on the vertical uplift w is almost linear. Comparing the present numerical results with those in paragraph A.9 of EN 1998-4, it is shown that the EN 1998-4 curves are quite accurate, yet somewhat conservative. With the normalization of parameters in Fig.7 one may easily produce Fig.8, which gives the relationship between the vertical uplift and uplifted length of the tank in a more universal manner. Fig.9 shows the increase of the compressive membrane force in the case of the unanchored tanks compared to the one of the anchored tanks.

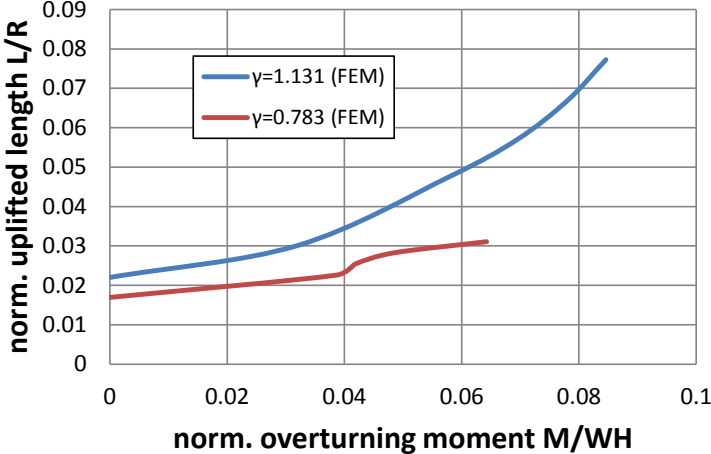


Figure 6. Normalized length of uplifted part of the base plate L versus overturning moment.

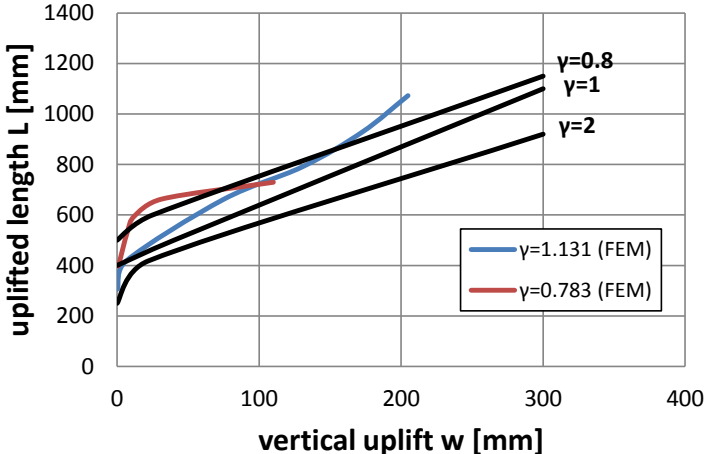


Figure 7. Length of uplifted part of the base as a function of the vertical uplift at the edge; present finite element results and curves for $\gamma = H/R = 0.8, 1.0, 2.0$, adopted by EN 1998-4

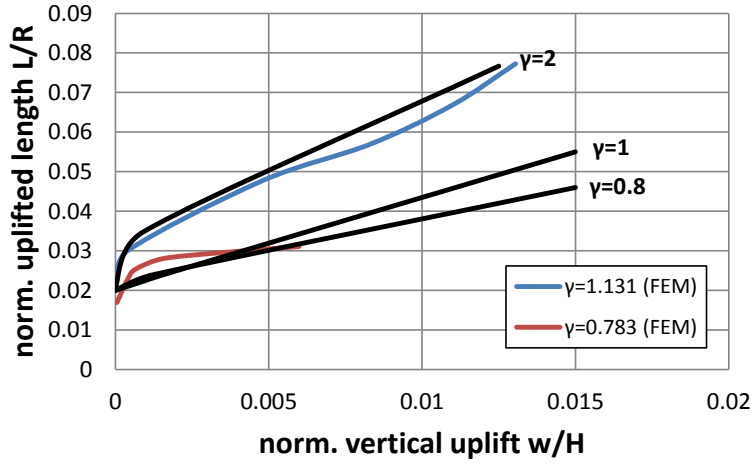


Figure 8. Normalized length of uplifted part of the base as a function of the normalized vertical uplift at the edge; present finite element results and curves for $\gamma = H/R = 0.8, 1.0, 2.0$, adopted by EN 1998-4

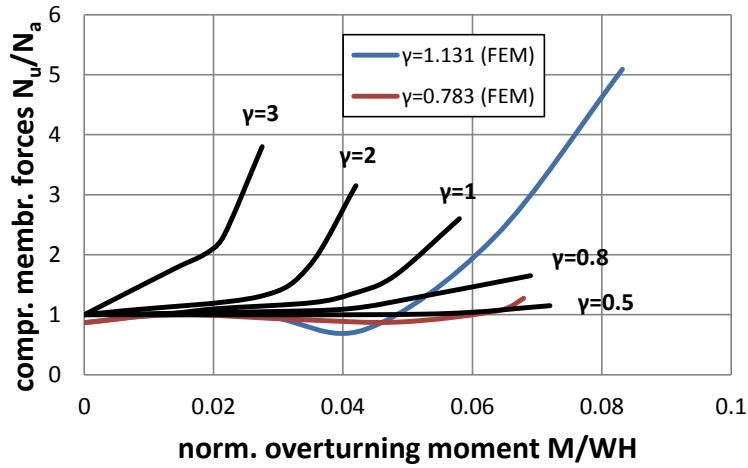


Figure 9. Ratio of maximum compressive axial membrane forces for the unanchored over the anchored tank versus the overturning moment; present finite element results and curves for $\gamma = H/R = 0.8, 1.0, 2.0$, adopted by EN 1998-4

The relationship between the membrane forces, at the uplifting and compression side of the anchored and unanchored tanks, is depicted in Fig.10 and Fig.11. These forces are also compared with the analytical formula of Eq.(3)

$$N = \pm \frac{M}{\pi R^2} - w_t \tag{3}$$

where M is the ringwall moment (i.e. the overturning moment due to hydrodynamic loading just above the base plate), R is the radius of the tank and w_t is the load per unit circumferential length because of shell and roof weight acting at the base of shell. The positive sign, “+”, is used for the calculation of the uplifting membrane force and the negative sign, “-“, for the compressive membrane force. Here, for the sake of uniformity, the compressive membrane forces are given in absolute values. The value of w_t is computed as follows:

$$w_t = \frac{W_s}{\pi D} + w_{rs} \tag{4}$$

where W_s is the total weight of the tank shell and appurtenances and w_{rs} is the roof loading acting on the shell including 10% of the roof design snow load. Particularly, Fig.10 shows that compressive membrane forces are the same for the anchored and unanchored tank for small values of overturning moment. At the level of normalized overturning moment equal to 0.03 the slope of N_u significantly increases due to the beginning of the uplifting of the tank. Moreover, the simple analytical formula gives good results for the case of the anchored tank.

The comparison of the uplifting membrane forces and the simple analytical formula (Fig.11) shows that there is good agreement between the three lines when there is no uplift of the tank. In both figures, when uplifting occurs, the analytical formula can't describe the uplifting behavior of the tanks.

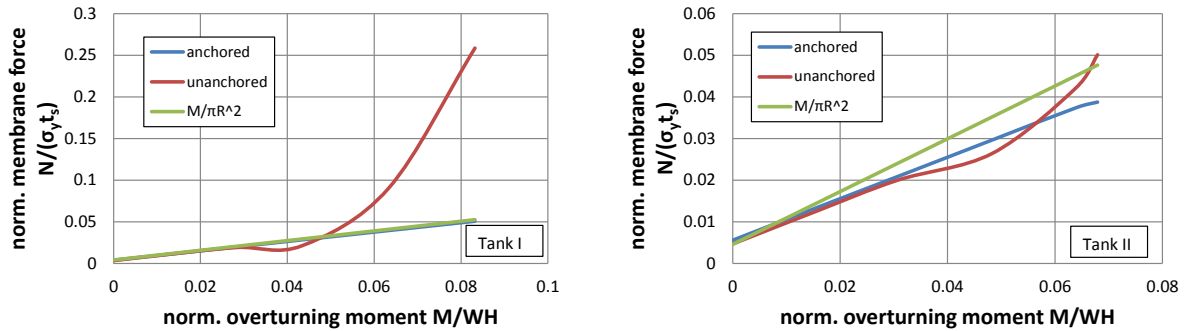


Figure 10. Absolute values of the membrane compressive axial forces in the case of the anchored and unanchored simulated tanks and the analytical formula in terms of the overturning moment for Tank I and Tank II.

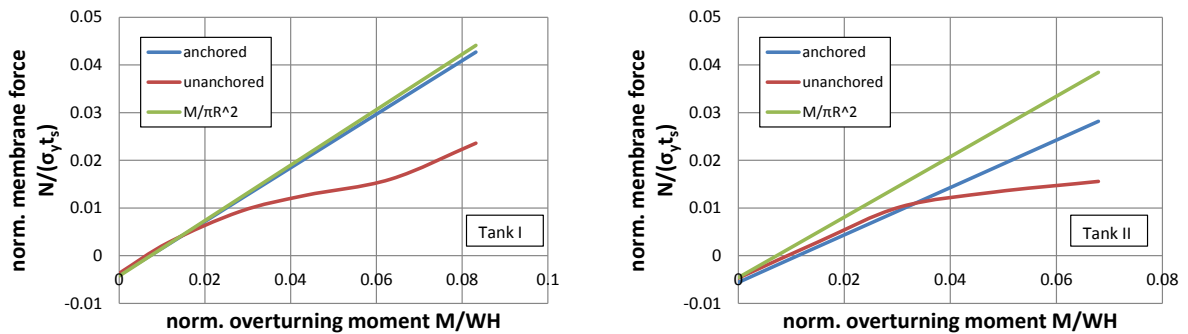


Figure 11. Membrane uplifting forces in the case of the anchored and unanchored simulated tanks and the analytical formula in terms of the overturning moment for Tank I and Tank II.

The equivalent plastic strain of the tank base plate is shown in Fig.12 and Fig.14, for Tank I and II respectively. The figures show that the equivalent plastic strain becomes maximum at two points, (a) at the junction between the tank wall and the base plate and (b) at a point at a small distance from the weld. At these two locations significant plastic yielding begins to develop. These points are also depicted in Fig.13 and Fig.15, for Tank I and II respectively, where the uplifting mechanism of the tanks is shown.

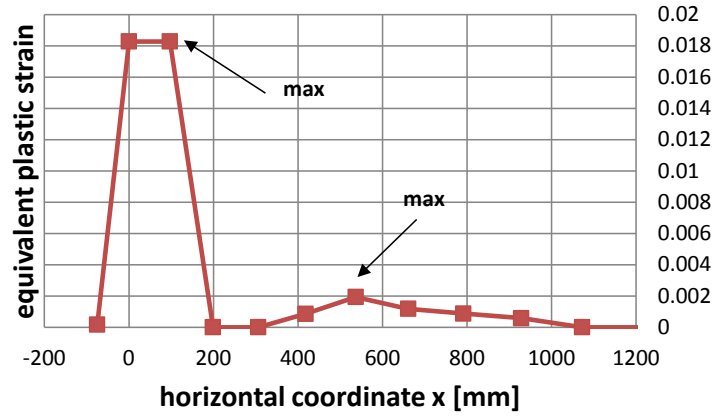


Figure 12. Equivalent plastic strain at the tank base plate in terms of the horizontal coordinate x for Tank I, on the inner surface of the base plate

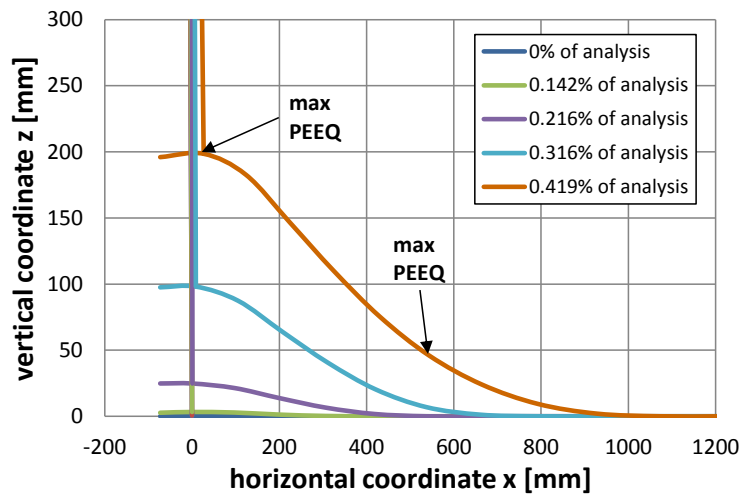


Figure 13. Deformed bottom plate of Tank I. In the figure are also shown the two points where the equivalent plastic strain becomes maximum

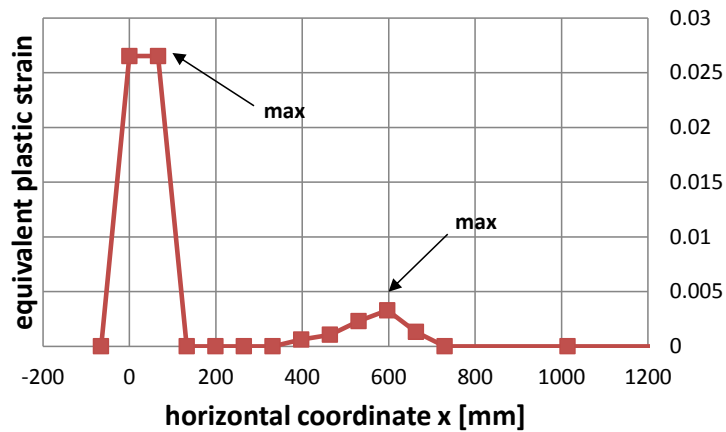


Figure 14. Equivalent plastic strain at the tank base plate in terms of the horizontal coordinate x for Tank II, on the inner surface of the base plate

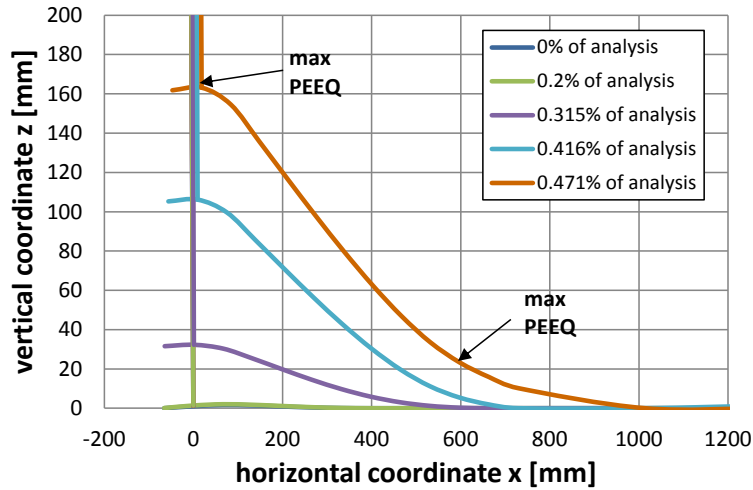


Figure 15. Uplifting mechanism for Tank II. In the figure are also shown the two points where the equivalent plastic strain becomes maximum

EFFECT OF ROOF ON UPLIFTING

An important issue that needs to be examined is the effect of the roof on the uplifting mechanism. For this purpose, a parametric study of 6 tanks has been made using finite elements. These tanks are unanchored and have been modeled with and without a roof. All have a total height of 12 m and a filling height of 10 m and aspect ratios ranging from 0.625 to 2. Their shell thickness has been designed according to the provisions of API 650 (2007). The results of this parametric study are shown in Fig.16.

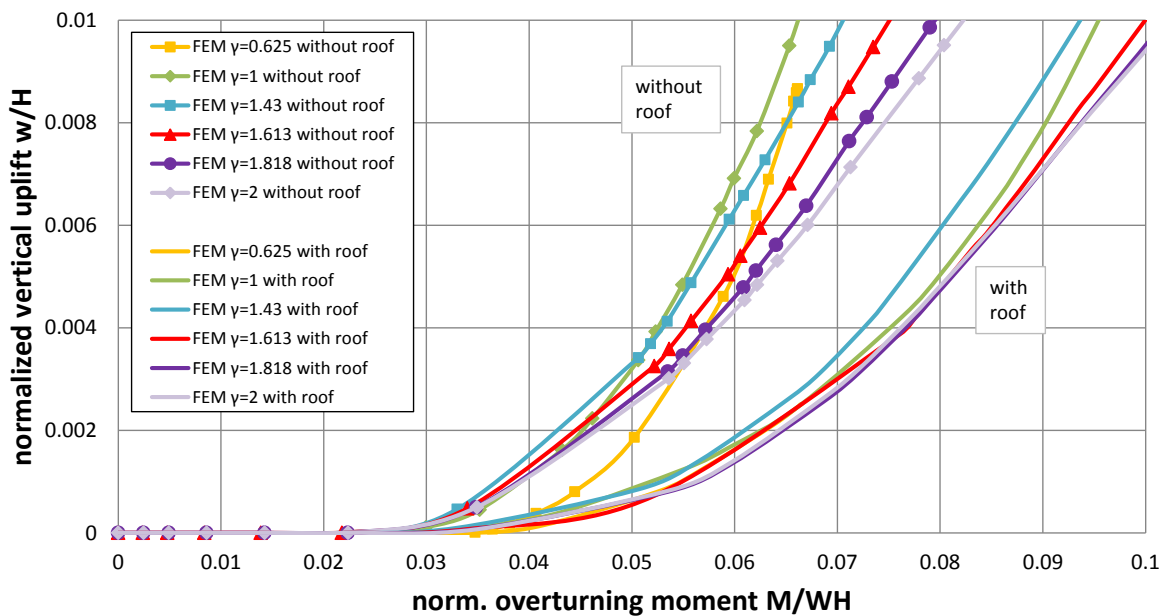


Figure 16. Parametric study of 6 unanchored tanks with different aspect ratios, modeled with and without a roof

It is shown that the existence of the roof affects the uplifting of the tanks and particularly it leads to a decrease of the vertical uplift of the tanks. This behavior can also be seen in Fig.17 for the two case studies.

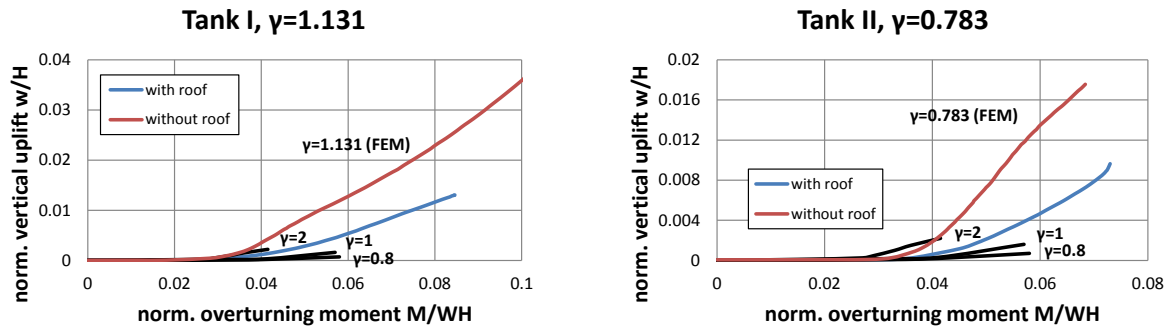


Figure 17. Maximum vertical uplift of the tank w versus overturning moment M ; present finite element results for Tank I and Tank II, with and without roof

CONCLUSIONS

Effects of base uplifting on the seismic response of unanchored liquid storage tanks are examined. Two typical liquid storage tanks were modeled both as anchored and unanchored using non-linear finite elements. Considering the distribution of hydrodynamic pressure on the tank wall and bottom plate, a non-linear static pushover analysis is conducted. For the tanks considered in the present analysis, uplifting of the tank occurs at values of M / WH exceeding 0.03. Furthermore, this uplift is associated with inelastic deformation at the uplifted area of the base plate. It is also observed that the increase of the overturning moment leads to an increase of the top horizontal displacement, the uplift size and the length of the uplifted part of the tank. Moreover, it leads to an increase of the axial compressive stresses acting on the tank wall. The comparison of the membrane forces in the case of the unanchored and anchored tank with Eq.(3) shows that the simple analytical formula gives good results when there is no uplift of the tank. The equivalent plastic strain becomes maximum at the junction between the tank wall and the base plate and at the point where there is a change of the slope of the base plate curve. These are the two locations where plastic yielding also begins to develop. Finally, using a parametric study of 6 tanks with different aspect ratios, it is found that the existence of the roof has a significant influence on the uplifting mechanism, apart from safeguarding the tank contents.

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