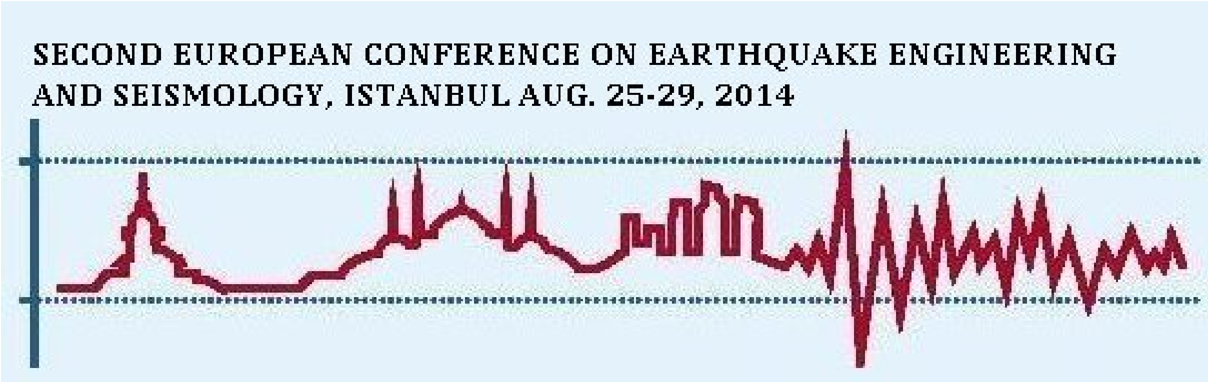

COMPLETE WAVEFIELD MODELLING OF AMBIENT VIBRATIONS FROM A DISTRIBUTION OF CORRELATED ALEATORY SURFACE SOURCES: COMPUTATION OF HVSR

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Two alternative models have been so far proposed to describe the average features of the ambient vibration wavefield (*e.g.*, García-Jerez *et al.*, 2012a,b). In both of them the subsoil structure is approximated by a 1D geometry, *i.e.*, as a stack of horizontal homogeneous and isotropic strata, overlaying an half-space. In the first model (*e.g.*, Sánchez-Sesma *et al.*, 2011), named Diffuse Field Approach (DFA), the ambient vibrations are assumed to be a diffuse wavefield. In the alternative model (*e.g.*, Field and Jacob, 1993; Lachet and Bard, 1994; Arai and Tokimatsu, 2004; Lunedei and Albarello, 2010), ambient vibrations are the effect of an aleatory distribution of harmonic uncorrelated point-like forces lying on the Earth's surface or close to it (Distributed Surface Sources or DSS model).

Both the models allow the computation of theoretical HVSR (Horizontal-to-Vertical Spectral Ratio) or H/V curves, which can be used to interpret empirical observations. In general, computations of theoretical H/V curves in the DFA model are much faster than in the case DSS model, except when the surface-wave component of the ambient vibrations is considered only (Albarello and Lunedei, 2011); this fact surely prevents any possibility to apply the full-wavefield DSS model in inversion procedures. Another limitation of the DSS model is the unrealistic assumption that surface sources are mutually uncorrelated, irrespective to the relative distance and to the wavelength of concern.

To overcome these last drawbacks of the DSS model, a new theoretical scheme is here proposed, where both the ambient-vibration displacement field and its generating force field are described as stochastic processes stationary in time as well as in space, *i.e.*, on the Earth's surface (which is the horizontal plane in the adopted 1D geometry). Since small-size oscillations are considered only, in the wavenumber-frequency domain power spectra of the ground-motion and of its generating forces are expected to be linked by a linear relationship, *via* the square modulus of the Green's function. In particular, in the hypothesis that the spectral powers of the three force-field Cartesian-components are isotropic in the horizontal plane, the overall spectral power of the displacement along the *i*-th Cartesian direction $p_{U,i}(\omega)$ at the generic angular frequency ω is the sum of the powers on all the wavenumbers:

$$p_{U,i}(\omega) \propto \int_0^{+\infty} \sum_{j=x,y,z} |\hat{G}_{ij}(k,\omega)|^2 \cdot h_{F,j}(k,\omega) \cdot k\, dk\,, \qquad (1)$$

where $\hat{G}_{ij}(k,\omega)$ is a element of the Green's matrix as a function of the horizontal-wavenumber modulus and of the angular frequency, while $h_{F,j}(k,\omega)$ is the wavenumber-frequency power-spectrum of the force-field component along the *j*-th Cartesian direction. They are the Hankel-transform of the Green's function in the space domain and of the spatial covariance of the force field, respectively. Relationship given by Equation (1) links the power properties of the force stationary

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stochastic process with the ones of the ground-motion stationary stochastic process. The dependency of the Green's matrix on the horizontal-wavenumber direction produces constant factors in Equation (1), as a consequence of the uniform and isotropic distribution of sources on the horizontal plane, as well as of the 1D geometry assumed for the Earth. The fact that $h_{F,j}(k,\omega)$ just depends on the wavenumber modulus k derives from the assumed isotropy of each force-field Cartesian component and reflects, in the wavenumber domain, the covariance among sources as a function of their reciprocal distance. The situation of mutually uncorrelated sources (that means a force field that is a purely random process) is allowed only in the case that each $\left(|\hat{G}_{ij}(k,\omega)|^2 \cdot k\right)$ is summable as a function of k on [0,+∞]. However, this is not the case when both ground-motion and sources are located at the surface (as reasonably expected), as the above term behaves as $1/k$ for $k \rightarrow +\infty$. In this situation, each $h_{F,j}(k,\omega)$ is required to decrease with k fast enough to warrant the convergence of the integral in Equation (1) and this means that the existence of some degree of spatial correlation in the force field is necessary. This convergence results as faster as larger the spatial correlation radius is. *Vice versa*, when this radius decreases (*i.e.*, the force field approximates a white noise with respect to space) the convergence rate also decreases. In any case, convergence of the integral giving the ground-motion power spectrum (Equation 1) is ensured by a suitable choice of the force power spectrum with respect to the space. In Figure 1, the relationship between the covariance function (in the space

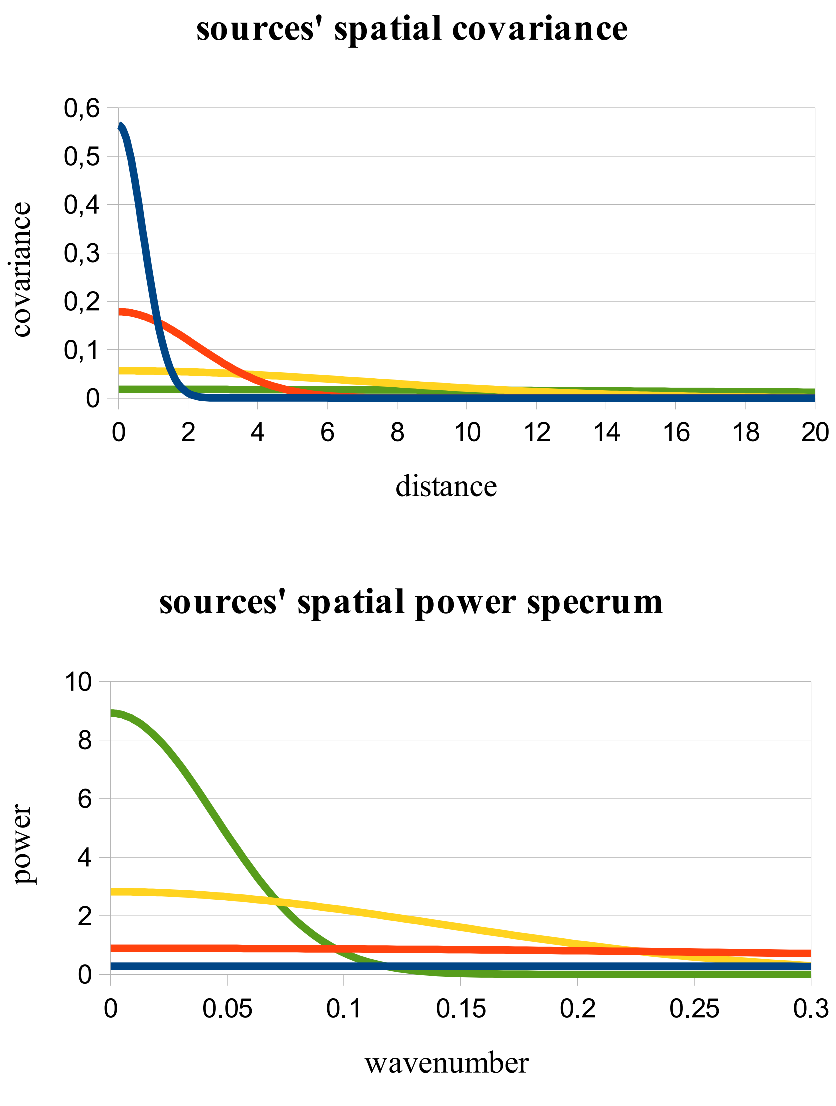

Figure 1. Example of the dualism between the spatial covariance and the correspondent spatial power spectrum of the force process for a Gaussian-like covariance function; colours refer to different values of its parameter

domain) and the power-spectrum (in the wavenumber domain) is shown in the case that a Gaussian-like covariance function is considered for the force field.

By supposing that the force-field component along the three Cartesian directions are uncorrelated one to each other, the H/V curve can be computed by the formula:

$$HV(\omega)=\sqrt{\frac{p_{U,x}(\omega)+p_{U,y}(\omega)}{p_{U,z}(\omega)}}\,, \tag{2}$$

where the two horizontal $p_{U,x}(\omega)$ and $p_{U,y}(\omega)$ and the vertical $p_{U,z}(\omega)$ power spectra are computed by Equation (1). It is clear the *HV* is a (very complicated) function of the spatial correlation among ambient vibration wavefield sources, their strength and soil 1D configuration.

From a computational point of view, one can see that, differently from previous formulations (Lunedei and Albarello, 2010), an integration in the wavenumber domain is only required for obtain the horizontal and the vertical ground-motion displacement power-spectra and this considerably accelerates computing the H/V curve by the DSS model, for any fixed subsoil configuration.

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