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A COMPARISON OF SITE RESPONSE ANALYSIS METHOD AND ITS IMPACT ON EARTHQUAKE ENGINEERING PRACTICE

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ABSTRACT

Site effects are quantified via site response analysis, which involves the propagation of earthquake motions from the base rock, through the overlying soil layers, to the ground surface. Frequency domain (FD) equivalent linear (EQL) and time domain (TD) nonlinear (NL) analyses are the most common approaches used for performing 1D seismic site response analysis. The dynamic responses computed via these methods can vary considerably because of the inherent differences in the numerical approaches (FD vs. TD) and the differences in how nonlinear soil response is modeled (EQL vs. fully NL). To evaluate these two approaches systematically, this study performs a series of site response analyses that consider different input motions, intensities of input motions, depths of soil columns, and nonlinear properties. The effect of different approaches on engineering practices is discussed based on the analysis results.

INTRODUCTION

Observations from earthquakes over the past 40 years have shown that local soil conditions can significantly influence the characteristics of ground shaking during earthquakes. These site effects should be considered when specifying ground shaking levels for seismic designs to prevent earthquake damage. Site effects are quantified via site response analysis, which involves the propagation of earthquake motions from the base rock, through the overlying soil layers, to the ground surface. Site response analysis provides surface acceleration time series, surface acceleration response spectra, and spectral amplification factors based on the dynamic response of local soil conditions.

In most cases, 1D site response analysis is performed to assess the effect of soil conditions on ground shaking because vertically propagating and horizontally polarized shear waves dominate the earthquake ground motion wave field. Frequency domain (FD) equivalent linear (EQL) (e.g. Schnabel et al., 1972) and time domain (TD) nonlinear (NL) analyses (e.g. Hashash and Park, 2001) are the most common approaches used to perform 1D seismic site response analysis. The dynamic responses computed via these methods can vary considerably because of the inherent differences in the numerical approaches (FD vs. TD solutions) and differences in how nonlinear soil response is modeled (EQL vs. fully NL).

TD-NL site response analysis does not only propagate input ground motion through the soil deposit in TD, but also varies soil properties with time. This approach allows more realistic modeling of NL soil response than EQL, which only approximates transient nonlinear behavior as a strain compatible parameter. Therefore, the nonlinear method is generally assumed to provide more accurate site response results, especially for high-intensity input motions. However, for capturing energy dissapation TD analysis requires to use of frequency-dependent Rayleigh damping, which is against

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the generally known frequency-independent behavior of soil damping. Such damping formulation can potentially impact the nonlinear analysis result, particularly for low-intensity input motions. Park and Hashash (2004) proposed extended Rayleigh damping for minimizing the effect of frequency-dependent damping in TD analysis. Phillips and Hashash (2009) further formulated "frequency-dependent Rayleigh damping" for NL analysis. The later proposed formula is debatable and requires further investigation because Rayleigh damping used in TD analysis is inherently frequency dependent.

In brief, FD-EQL and TD-NL approaches are commonly understood to be different. FD-EQL is mostly used due to its simpility while TD-NL analysis might be more suitable for analyzing strong motions. However, studies that quantifies the differences between the two approaches are few. Moreover, guidelines for selecting a better approach between the two are absent while different scenarios are encountered during site response analysis. Most inportantly, discussion on how these differences can affect engineering practices is not yet available. In this study, a comprehensive comparison between the two analytical approaches for site response analysis was conducted. Evaluation was performed from hypothetical events with various depths of soil columns, input motions, and intensities. Such evaluation could help explain inherent differences in the analytical approaches. The possible effects of using different approaches on engineering practices were also discussed.

ANALYSIS APPROACH

The code DEEPSOIL (Hashash et al., 2012), which is capable of performing TD-NL and FD-EQL analyses, is adopted for performing 1D site response analysis. The Mississippi Embayment (ME), located in central United States, is selected as the study site because of its thick soil deposit up to 1000m. As mentioned earlier, TD analysis with the sue of Rayleigh damping potentially impact the nonlinear analysis result. Tsai and Chen (2013) revisited the issue of selecting the target value for viscous damping frequencies to minimize the effect of frequency-dependent damping in TD analysis. They concluded that aside from the site frequency (SF), as suggested by previous studies (e.g. Kwok et al., 2007), the predominant frequency (PF) of ground motions should also be considered. Therefore, the site frequency and predominate frequency of the input motion is selected as target frequencies for Rayleigh damping in all TD analysis performed in this study.

Soil Profile and Nonlinear Properties

Site response analysis requires information on dynamic soil properties that include (1) the shear-wave velocity profile as well as (2) the modulus reduction and damping curves. The shear-wave velocity profiles of Romero and Rix (2001), which represent the ME upland regions, are considered in the site response analyses (Figure 1). Moreover, average shear wave velocity (Vs) of upper 30m soil of the uplands is 314 m/s, which is within the National Earthquake Hazards Reduction Program (NEHRP) site class D. The soil profile is truncated at 30, 100, 300, 500, and 1000 m for studying the effect of soil thickness on the two analysis results. Paleozoic rock with a shear-wave velocity of 3000 m/s (Romero and Rix, 2001) is assumed at the base of all profiles.

With regard to the modulus reduction and damping curves, two sets of dynamic soil properties, namely, ME (Park and Hashash, 2005) and EPRI (EPRI, 1993), are available for describing depth (pressure)-dependent nonlinear soil behavior for thick deposits in ME. ME properties are specifically obtained from back analysis of earthquak recordings in the ME while EPRI properties are developed by laboratory test results as well as literature review of available dynamic curves for a gneneral case. The generic EPRI curves have been adopted by other studies (e.g. Silva, 2005; Toro and Silva, 2001) to analyze nonlinear site effect with deep soil deposit. As shown in Figure 2, EPRI properties use lower small-strain damping than ME properties and exhibit higher nonlinearity, particularly for deeper soil layers.

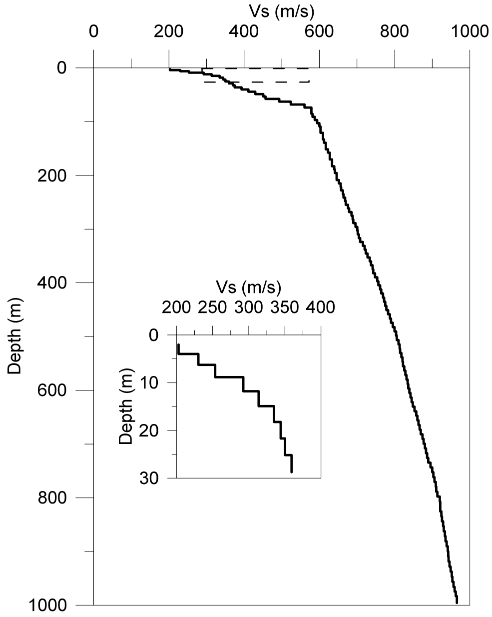

Figure 1 Vs profile of ME

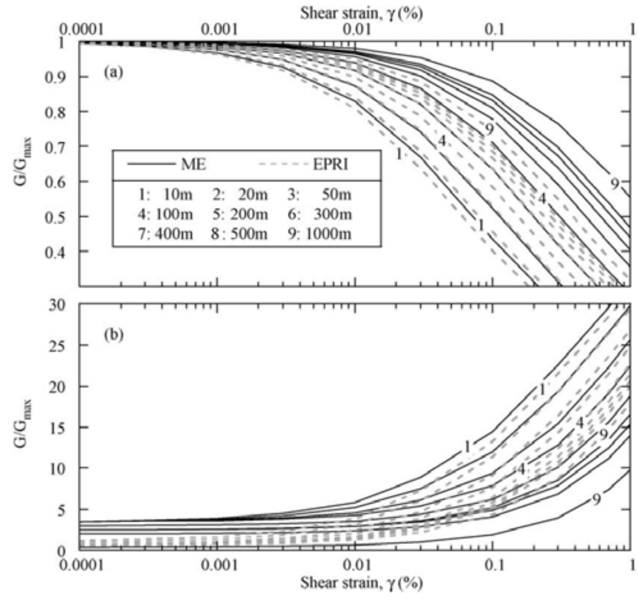

Figure 2 Two sets of dynamic soil properties: ME (Park and Hashash, 2005) and Electric Power Research Institute (EPRI, 1993)

Input Motions

Three recorded and two synthetic ground motions on hard rock (NEHRP site class A/B), as listed in Table 1, are used as input motions. The recorded strong motions are selected from the PEER database and used as outcrop strong motions on hard rock. Synthetic motion is commonly used when strong ground motion record is limited (for example, ME) and, hense, are considered as one type of input motions. The two approaches are adapted for developing synthetic motions.

- The first synthetic motion is a numerically simulated time series generated via seismological simulations using a theoretical Fourier amplitude spectrum and stochastic simulation methods by Stochastic Model SIMulation (SMSIM) (Boore, 2002). The time history [peak ground acceleration (PGA) = 0.62 *g*], representing a scenario of Mw = 8.0 and R = 25 km, is simulated. Such motion has more high-frequency components than recorded motions, as shown in Figure 3.
- The second synthetic motion is generated by spectrally matching the selected (seed) motion to fit a target response spectrum (RS) according to Abrahamson (1992) and Hancock et al. (2005). The no. 2 recorded motion (LOM) is selected as the seed motion for matching the target spectrum of Abrahamson and Silva (2008) for a given scenario (R = 10 km, Mw = 8.0 Vs = 1100 m/s). As shown in the Figure 3, the seed motion has a loosely matched target spectrum, except for the spike in nature, and exhibits lower amplitudes at long periods. The characteristic of seed motions (such as frequency content) is modified after spectral matching, and the RS exhibits a smooth curve.

All motions are scaled to 0.001, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, and 1.0 g to evaluate nonlinear soil effect.

Table 1. Input Base Motions

No.	Name	Type	Event	Mw	R (km)	PGA (g)	Vs_{30} (m/s)
1	TAB	Recorded	Tabas, Iran	7.4	55	0.81	767
2	LOM	Recorded	Loma Prieta	6.9	28	0.44	1428
3	KOC	Recorded	Izmit Kocaeli,	7.5	3.6	0.22	881
4	SMSIM	Synthetic	-	8.0	25	0.6	
5	LOM_M	Synthetic	-	8.0	10	0.52	1100

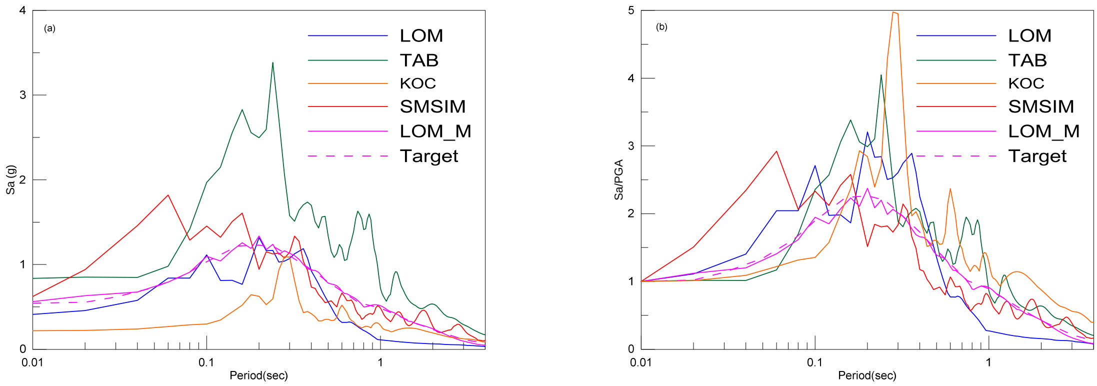

Figure 3 The RS of input motion (a) Unscaled RS. (b) Normalized RS by PGA

ANALYSIS RESULT

Relative Difference

The difference between NL and EQL analyses is initially compared using the surface response spectrum (RS). Figure 4 and Figure 5 show examples of responses using the TAB input motion scaled to PGA = 0.001 *g* and PGA = 1.0 *g*, respectively. For the case of 0.001 *g*, the analyses show almost identical results regardless of soil column depth. Soil response is linear under such small events; thus, the minor difference is attributed to the inherent difference between TD and FD analyses and Rayleigh damping used in TD as discussed previously. However, once the input motion becomes extremely strong (that is, PGA = 1.0 *g*), the two methods exhibit significant differences. EQL and NL approaches are the major reasons for the difference because soil response is highly nonlinear. Similar results can be observed in other input motions. The relative difference (RD) in the RS can be quantified as

$$RD = \frac{RS_{TD} - RS_{FD}}{RS_{FD}} \times 100\% \quad (1)$$

Figure 6 shows the RD for various intensities of input motions. The thin lines represent the RD for each input motion and the bold lines represent the average results for four input motions. The differences between the two approaches become more obvious as PGA of input motions increases. NL is lower than EQL between 0.01 s and 0.1 s for 30 m and 100 m soil columns at 0.001 *g*. The lowest result occurs at 0.05 s to 0.06 s. As depth increases, NL becomes higher than EQL in most period ranges except for 0.1 s, in which a relative low point occurs. Such difference is attributed to the inherent difference between TD and FD approaches and the use of frequency-dependent Rayleigh damping in TD because soil is under a linear condition. The trend in RD variation changes with the increase in PGA. The RD scenario is totally modified at 1 *g*. PGA (0.01 s SA), calculated by TD, is significantly lower. The SA of TD is mostly lower within period range up to approximately 1.0 s and becomes similar with the SA of FD at longer periods. A relatively high point of RD between 0.1 s and 0.2 s is observed, but TD remains lower. The overall maximum RD is up to 40%. A similar trend can be found in different soil columns but the transition periods may be slightly different. Figure 6 shows that the period range during which TD is lower becomes wider with increasing soil column. For example, TD is lower than FD for a period of less than 1.0 s in a 30 m soil column. The result increases to 3.0 s in a 1000 m soil column. Meanwhile, the RD becomes smaller with increasing soil column. It is widely believed that the site response of deep soil columns requires TD analysis. However, our analysis result indicates that a more significant difference occurs when the analyzed soil column is shallow because soil nonlinearity mostly occurs at shallow depths and imposes the largest discrepancies between NL and EQL approaches. Although soil column becomes deeper, average soil nonlinearity becomes less significant when considering less nonlinear responses at greater depths. Thus, the RD is reduced.

The effect of soil properties on the performance of NL and EQL analyses are also evaluated in the present study. In addition to ME properties, this study considers ERPI properties in site response analysis. Figure 7 shows the RD in the RS when using ME and EPRI properties with the same input motion (TAB). The most significant change when EPRI properties are used is that the NL results are higher than the EQL results between the periods of 0.1 s and 0.3 s, which is not observed when ME properties are used. The RD in this period range becomes more obvious as soil column depth increases by up to 70% because the module reduction curves of ME and EPRI begin to deviate as depth increases, as shown in Figure 7. EPRI curves exhibit more nonlinearity, whereas ME curves exhibit less. Therefore, if a site has soil with higher nonlinearity (such as sand), the EQL analysis result may potentially underestimate ground response in a short period range.

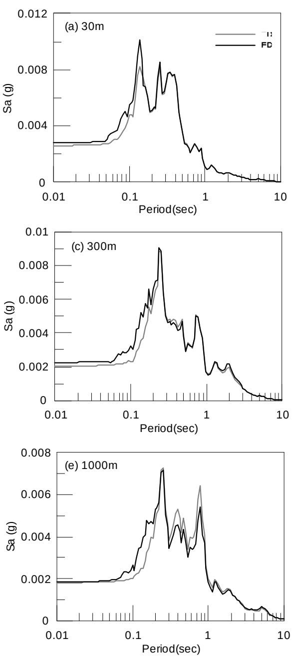

Figure 4 The surface RS by NL and EQL analyses with Tabas, Iran input motion PGA = 0.001 g

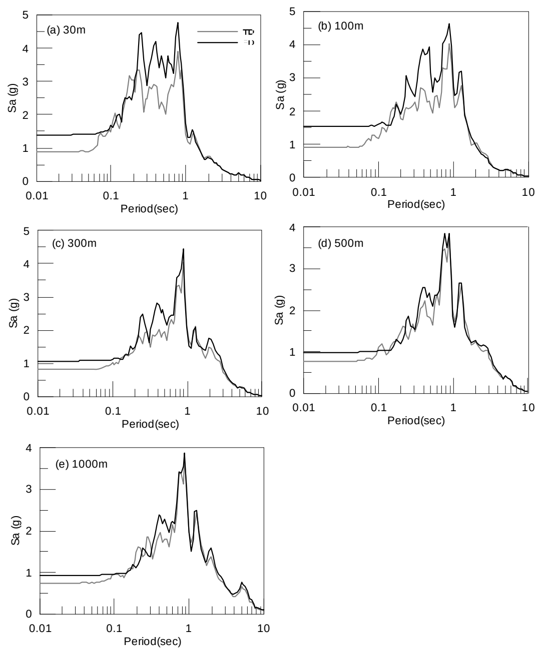

Figure 5 The surface RS by NL and EQL analyses with Tabas, Iran input motion PGA = 1.0 g

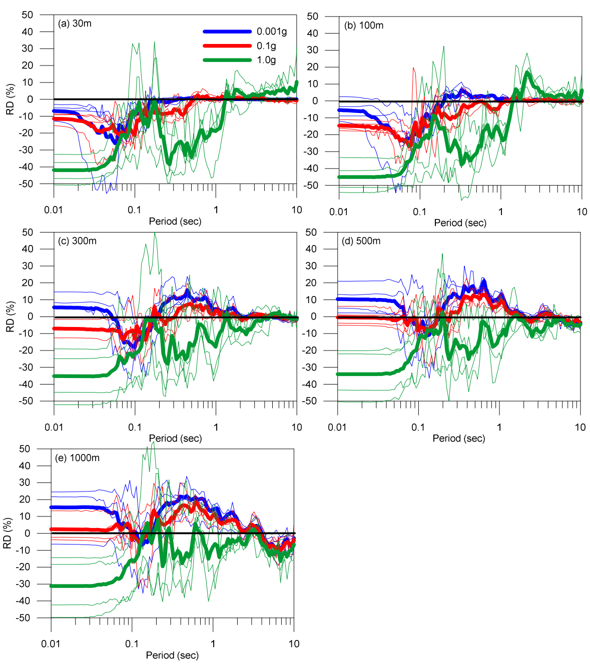

Figure 6 The RD in the RS (average of five motions)

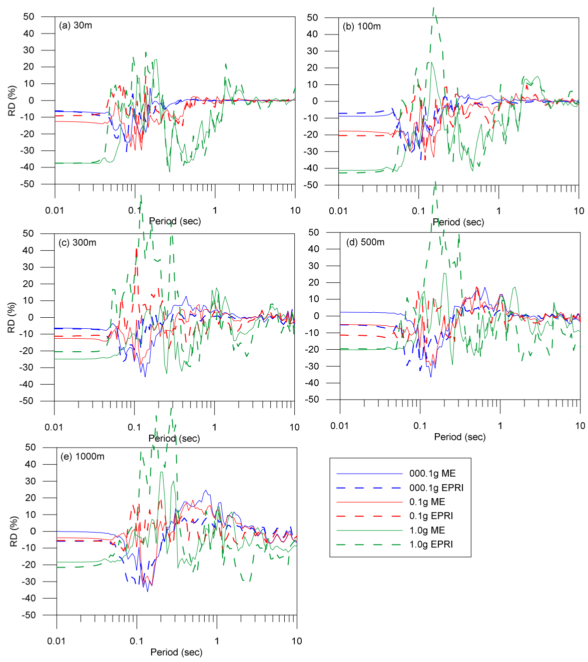

Figure 7 The RD in the RS using ME and EPRI properties

Threshold Intensity

The difference in site response of the two approaches is further identified by PGA, 0.2 s SA and 1.0 s SA. PGA is a frequently used ground motion intensity parameter, whereas 0.2 s SA and 1.0 s SA are also significant because they dominate the shape of a design spectrum. To compare the overall difference at short and long periods calculated by TD and FD analyses, 0.2 s SA and 1.0 s SA are considered as the average of the surface spectrum between 0.1 s and 0.5 s and between 0.4 s and 2.0 s, respectively. This process is similar to the method for deriving site factor in the NEHRP provision (Borcherdt, 1994). Figure 8 shows the comparison between surface PGA, 0.2 s SA and 1.0 s SA, calculated by TD and FD methods.

Three main features can be observed.

1. The two approaches exhibit more differences for shallow soil columns and less for deep soil columns. This condition is attributed to the nonlinearity that occurs at shallow depths which imposes the largest difference in the two approaches. While the depth of the soil column increases, the nonlinearity becomes less significant through the average response of the deep soil column. Consequently, the differences become insignificant.
2. The PGA plots reveal more significant difference. Intermediate difference is exhibited in 0.2 s SA, whereas less difference is exhibited in 1.0 s SA. This result implies that if the critical design parameter is long period SA, such as in bridges and high-rise buildings, then no significant difference is observed when the two approaches are used. However, if the short period is significant for the design (such as PGA for liquefaction evaluation), then the two approaches can produce a significant difference. In general, NL analysis is more suitable for analyzing strong motions. Therefore, the NL approach may be better than the EQL approach for estimating short period parameters. However, performing EQL analysis to account for site factor at long periods is still acceptable.
3. The threshold PGA that causes the NL result to deviate from that of the EQL is difficult to identify. Figure 9 shows the RD of SA. The RD increases as the intensity of input motion increases (that is, FD becomes higher). The difference increases gradually with the PGA of input motion and no break point clearly shows the threshold PGA. The maximum RD is approximately 20% to 30% for most cases, except for the PGA case, which has an RD of up to 60%. For the PGA case, the threshold PGA of input motion is probably 0.2 *g* for shallow soil columns and 0.4 *g* for deep soil columns given the acceptable 20% of the RD. For 0.2 s SA, 0.2 *g* is probably the threshold for shallow cases. For 1.0 s SA, the RD is within 20%.

In general, EQL analysis predicts higher spectrum acceleration at the surface, which is more conservative. If NL analysis is believed to be more accurate, then the analysis results imply that typical EQL analysis can potentially overestimate the seismic hazard.

Amplification Factor

The difference in the two approaches is further evaluated by site amplification factors that are used for adjusting ground motion intensity from a reference motion. Site amplification factor *SF* is the ratio between the 5% damped spectrum acceleration of ground shaking *SA* and the reference ground motion for the site *SAr*, as follows:

$$SF(T) = SA\,(T)/SA_r(T) \quad (2)$$

where T is the period of the spectrum. Site factors *Fa* and *Fv* represent the amplifications for a short period (0.2 s) and a long period (1.0 s), respectively. These factors are vital in the seismic design procedure. *Fa* is based on the average of *SF* between 0.1 s and 0.5 s, whereas *Fv* is based on the average of *SF* between 0.4 s and 2.0 s according to the method for deriving site factor in the NEHRP provision (Borcherdt, 1994).

Site factor is determined by two components: (1) linear site amplification that represents the amount of amplification under a small ground motion and (2) the nonlinear degree associated with the change that occurs when ground motion intensity increases. The site factor derived by NL and EQL analyses can show the significant difference. Figure 10 shows the comparison between the site factor determined by both analysis approaches and those recommended in the NEHRP. SF of TD (solid line) and FD (dashed line) are the average of five imput motioms using ME properties. SF of NEHRP provisions was developed empirically for relatively small ground motion levels (PGA near 0.1 *g*) and have nonlinearity levels derived from simulation with EQL analysis, in which suits of profiles from sites in California and Mexico City were analyzed by Seed et al. (1994) and Dobry et al. (1994).

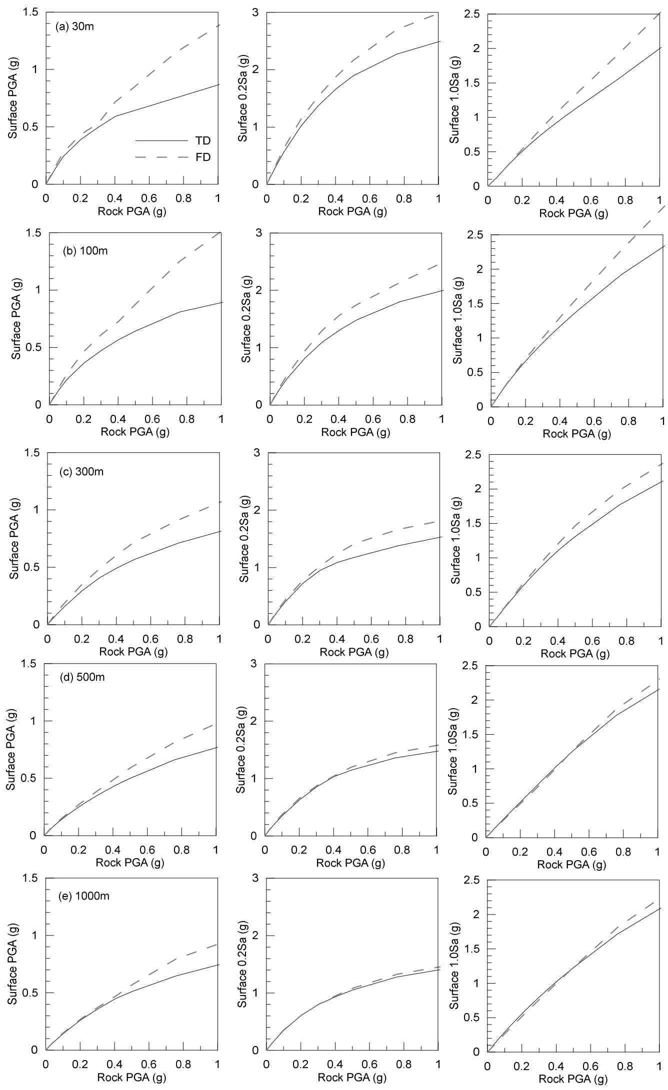

Figure 8 The comparison of surface PGA, 0.2 s SA and 1.0 s SA, calculated using TD and FD methods (average of all motions)

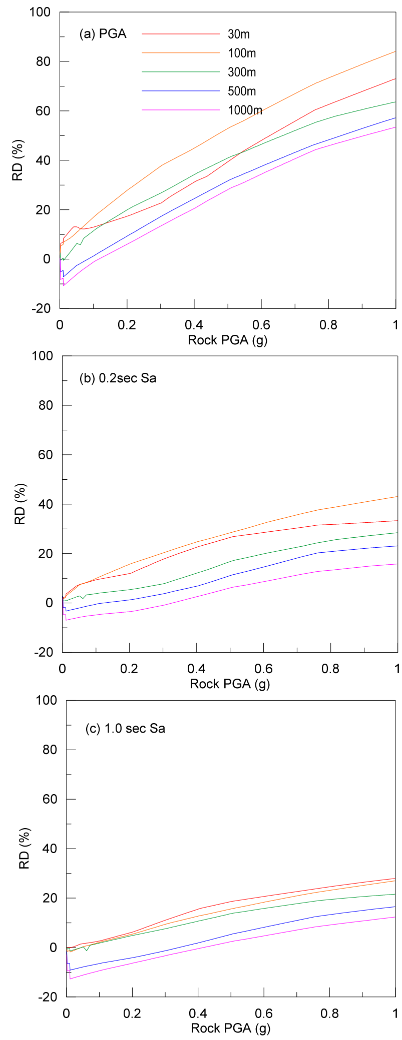

Figure 9 The RD of PGA, 0.2 s SA and 1.0 s SA, vs. PGA of input motion

It can be seen that substantial scattering of SF among NL, EQL, and NEHRP in Figure 10. The SF derived from the analysis of the 300 m soil column is closer to that of NEHRP under low intensity motion. Site amplification under low intensity is dominated by linear properties (such as Vs profile) and unrelated to NL or EQL approaches; thus, the difference in SF is primarily attributed to the analyzed profiles different from the underground condition of recorded motions used by NEHRP. SF in NEHRP generally includes more variety of uncertainties such as different ground motions, depths of soil columns, and Vs profiles. Therefore, this SF should be considered as more representative than the values derived in the present study.

By contrast, SF is also dependent on the intensity of reference motions because of the nonlinear effect of soil. The comparison of such nonlinearity effects obtained from the present study with those in NEHRP is more rational because intensity-dependent relationship in NEHRP is derived from the simulation. Therefore, all site factors are normalized with those at a small ground motion level to examine the nonlinear effect. Figure 11 presents the normalized SF using ME properties. It shows that SF by NL exhibits more nonlinearities than that by EQL, particularly for *Fv* because EQL adopts an equivalent (averaged) module, whereas NL actually models nonlinear behavior. In addition, the figure also implies that the depth of the analyzed column has a limited effect on nonlinearity because all curves for different depths fall within a narrow band. For *Fa*, the nonlinearity derived from both NL and EQL are relatively consistent with NEHRP, but SF of EQL is closer to that of NEHRP. These results are expected because nonlinearity in NEHRP is also based on EQL simulation. For *Fv*, however, a significant difference among NL, EQL, and NEHRP is observed. NEHRP exhibits the largest nonlinearity, whereas EQL presents the least nonlinearity. Choi and Stewart (2005) (CS) performed further comparison with SF, which was fully developed empirically for weak to strong ground motion levels. NL agrees well with the aforementioned study at *Fv*, thus implying that NL could be a better method for modeling soil nonlinear behavior in seismic site response analysis.

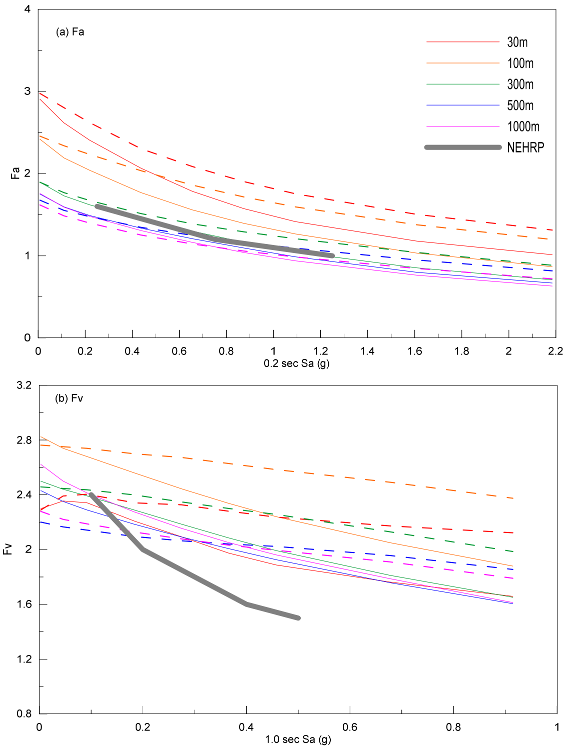

Figure 10 Comparisons of site factors Fa and Fv calculated from TD (solid line) and FD (dashed line) analyses with NEHRP

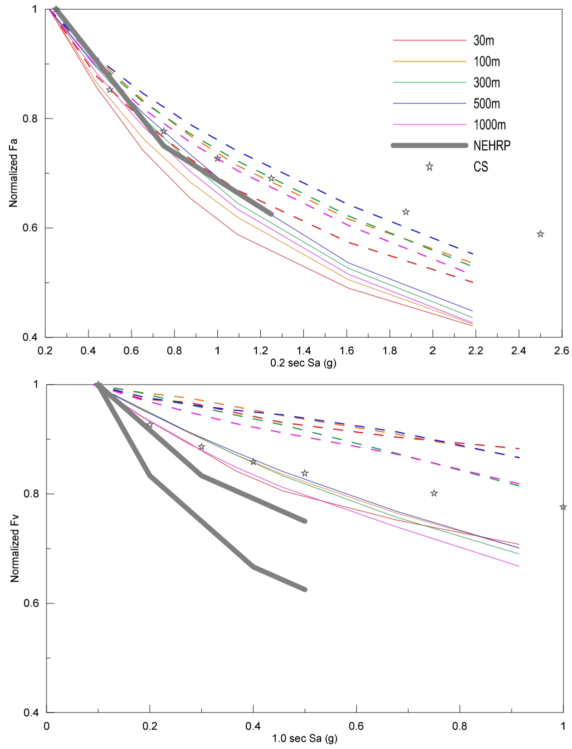

Figure 11 Comparisons of normalized site factors Fa and Fv calculated from TD (solid line) and FD (dashed line) analyses with NEHRP.

CONCLUSIONS

FD-EQL analysis is widely used in engineering practices for estimating seismic site response although its accuracy is less than that of TD-NL. To evaluate the two approaches systematically, the present study performs a series of site response analyses that consider different input motions, intensities of input motion, depths of soil columns, and nonlinear properties. The main conclusions are as follows.

1. The two approaches exhibit more differences for shallow soil columns and less for deep soil columns, which contradicts the general concept that TD-NL analysis is required for deep soil columns.
2. The PGA plots reveal more significant differences. SA at 0.2 s exhibits an intermediate difference, whereas SA at 1.0 s exhibits less difference. Engineers should be cautious when selecting an analysis approach that depends on the range of interest of the period.

3. No clear threshold PGA that causes the NL result to deviate from that of EQL is found. The RD increases as the intensity of input motion increases (that is, FD becomes higher). For the PGA case, the threshold PGA of input motion is 0.2 *g* for shallow soil columns and 0.4 *g* for deep soil columns, given that 20% of RD is acceptable. For the 0.2 s SA case, 0.2 *g* is probably the threshold for shallow cases. For the 1.0 s SA, the RD is within 20%.
4. The NL results are higher than the EQL results between the periods of 0.1 s and 0.3 s when more nonlinear soil properties (EPRI) are used. Therefore, if a site mostly consists of highly nonlinear soil (such as sand), then the result of the EQL analysis can underestimate ground response in a short period range.
5. SF by NL exhibits more nonlinearity than that by EQL, particularly for *Fv* because EQL adopts an equivalent (averaged) module, whereas NL actually models nonlinear behavior.

This study discusses the RD between the two approaches but no absolute accuracy has been evaluated. Through comparison with site factors developed empirically for weak to strong ground motion levels by Choi and Stewart (2005), NL is found to be better in capturing soil nonlinear behavior in site response analysis. Further comparison is under taken with recently developed empirical SF so that the accuracy of two approach can be identified.

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